# Reconstruction of the Higgs mass in H → ττ events

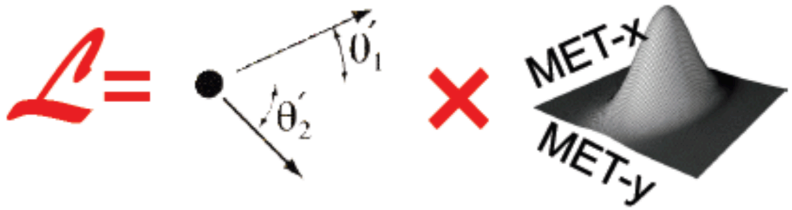
**Christian Veelken**

**NICPB Tallinn**

ττ Workshop DESY, November 16th 2015

# Reminder: What is SVfit ?

Algorithm for reconstruction of Higgs mass in Higgs → ττ events, based on **likelihood method**, using as input:




- Measured e, μ, $\tau_h$-jet momenta
- Reconstructed $E_T^{miss}$ and event-by-event estimate of $E_T^{miss}$ resolution ($E_T^{miss}$ reconstructed by a multivariate regression technique)

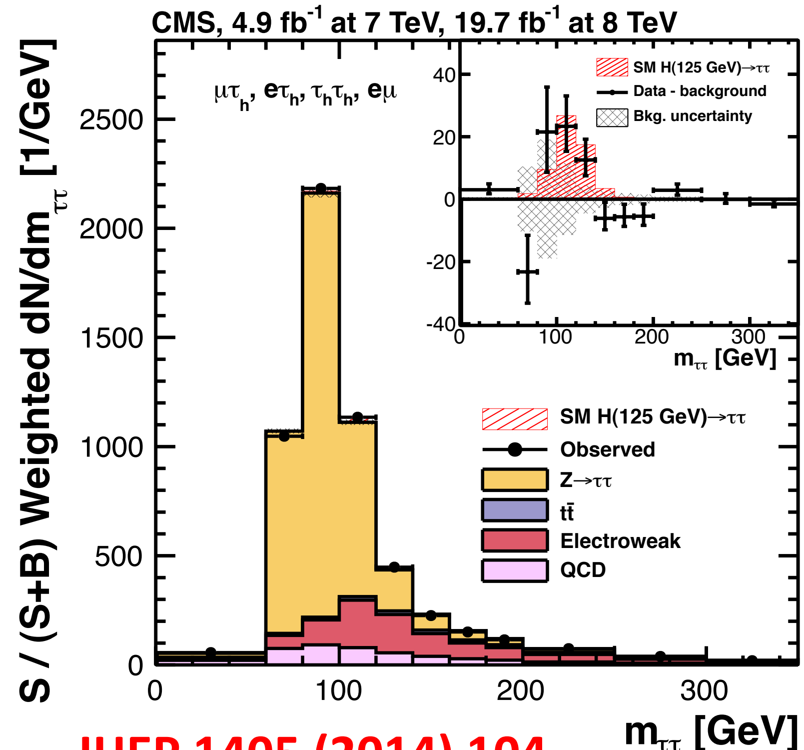
**Focus of today's presentation:**

- New version of SVfit rigorously based on matrix element (ME) method
- Comparison with Missing Mass Calculator (MMC)

# Motivation for ME version of SVfit

$H \rightarrow \tau\tau$ signal shows-up as small bump on tail of large irreducible $Z/\gamma^* \rightarrow \tau\tau$ background

➔ Sensitivity of the $H \rightarrow \tau\tau$ analysis crucially depends on good mass resolution to separate the signal from the background

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Was "old" version of SVfit using an assumption that degrades the resolution ?

ME method provides clear description how to account for energy/$p_T$ resolution of $\tau_h$-jet, which is non-negligible in CMS

Develop formalism to handle τ decays in ME method, which we can use for future applications in other analyses

# Parametrization of $\tau$ Decays

## Leptonic $\tau$ decays:

$\tau \rightarrow \ell\nu\nu$ decays parametrized by 3 variables

- $\theta^*$
- $\phi^*$
- $m_{\nu\nu}$

decay angles in $\tau$ restframe

## Hadronic $\tau$ decays:

Treated as two-body decays into a hadronic system $\tau_h$ and a $\nu_\tau$

$\tau \rightarrow \tau_h\nu_\tau$ decays parametrized by 2 variables

- $\theta^*$
- $\phi^*$

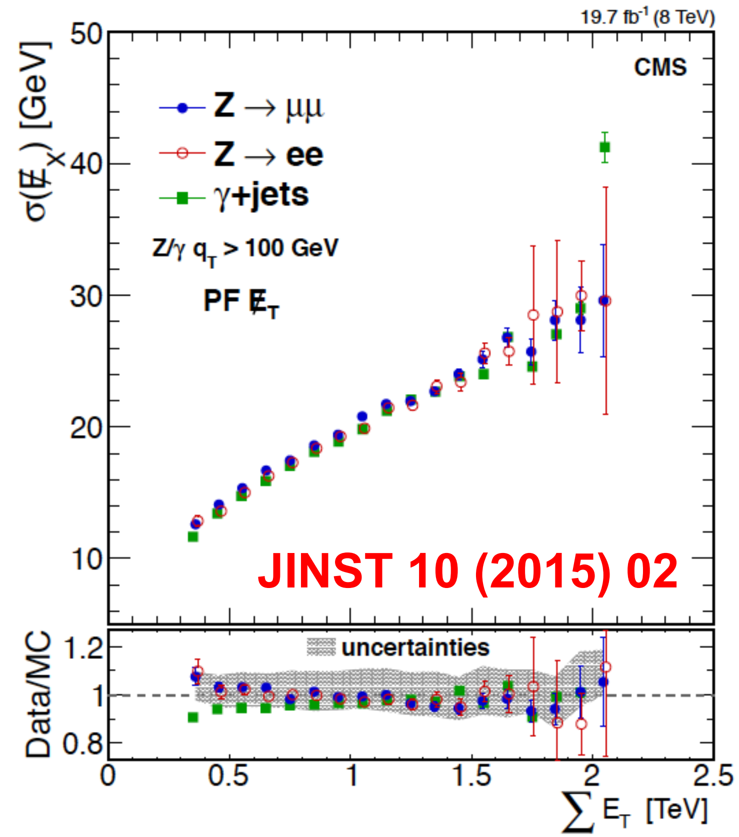
➔ 4 unknown variables in $\tau\tau \rightarrow \tau_h\tau_h$, 5 in $\tau\tau \rightarrow \ell\tau_h$, 6 in $\tau\tau \rightarrow \ell\ell$ decays

# The Problem

Unknown variables constrained by 2 observables only:

- $\Sigma\, p_x^{\nu} = E_x^{miss}$
- $\Sigma\, p_y^{\nu} = E_y^{miss}$

These are not even "hard" constraints, as $E_T^{miss}$ is reconstructed with a resolution of 10-20 GeV




➔ Problem of reconstructing $m_{\tau\tau}$ is underconstrained

# The Solution

Compute $m_{\tau\tau}$ using a likelihood approach:

$$\mathcal{L}(m_{\tau\tau}^{(i)}) = \int_{\Omega} d\vec{x}\, f(\vec{x},\vec{y})\, \delta\left(m_{\tau\tau}^{(i)} - m(\vec{x},\vec{y})\right)$$

x: unknown variables $\theta^*$, $\phi^*$, $m_{\nu\nu}$

y: measured observables $E_x^{miss}$, $E_y^{miss}$,
momenta of "visible" $\tau$ decay products (electrons, muons and $\tau_h$)

The likelihood function $\mathcal{L}$ is computed for a series of test mass hypotheses $m_{\tau\tau}^{(i)}$
The value of $m_{\tau\tau}^{(i)}$ that yields the maximal is taken as the best estimate $m_{\tau\tau}$ for the Higgs boson mass in a given event

The integral over dx has the following interpretation:
The value of $m_{\tau\tau}$ is computed by taking an "average" over all possible values of the unknown variables x that are compatible with the measured values y

The integral is computed numerically, using the VEGAS algorithm

J.Comput.Phys. 27 (1978) 192

# The Matrix Element Method

Theory motivates a specific choice for the function f:

PDF x flux factor

Energy and momentum conservation

$$P(\boldsymbol{y}|a) = \frac{1}{\sigma(a)} \int \frac{f(x_a) f(x_b)}{2\, x_a\, x_b\, s} (2\pi)^4\, \delta^4(x_a\, \boldsymbol{p}_a + x_b\, \boldsymbol{p}_b - \sum_i^n \boldsymbol{p}^{(i)}) \, |\mathcal{M}(\boldsymbol{p}, a)|^2 \, W(\boldsymbol{y}|\boldsymbol{p}) \, dx_a\, dx_b\, d\boldsymbol{p}$$

Matrix element

Transfer function

**y**: Observables measured in the detector ($E_x^{miss}$, $E_y^{miss}$, momenta of e, μ, $\tau_h$)

**p**: "true" momenta of all particles in the final state

$x_a$, $x_b$: Bjorken scaling variables

a: unknown model parameter (here: "true" value of $m_{\tau\tau}$)

**N.B.**: Normalization factor 1/σ ensures that P(**y**|a) is a probability density, i.e.

$$\int P(\mathbf{y}|a)\, d\mathbf{y} = 1\,.$$

# Matrix Elements

Gluon fusion ME from literature/Madgraph

ME for τ decays

$$|\mathcal{M}(\boldsymbol{p}, a)|^2 = |\mathcal{M}_{\mathrm{gg}\to\mathrm{H}\to\tau\tau}(\boldsymbol{p}, a)|^2 \cdot |\mathcal{M}_{\tau}^{(1)}(\boldsymbol{p})|^2 \cdot |\mathcal{M}_{\tau}^{(2)}(\boldsymbol{p})|^2$$

Use narrow-width approximation for τ decays:

$$\rightarrow \quad |\mathcal{M}_{\tau}^{(i)}|^2 = |\mathrm{BW}_{\tau}|^2 \cdot |\mathcal{M}_{\mathrm{decay}}^{(i)}|^2$$

with $|\mathrm{BW}_{\tau}|^2 = \frac{\pi}{m_{\tau}\Gamma_{\tau}} \delta(q_{\tau}^2 - m_{\tau}^2)$

For $|\mathcal{M}_{\mathrm{decay}}^{(i)}|^2$, we use ME from literature for leptonic τ decays:

$$|\mathcal{M}_{\tau\to\ell\overline{\nu}_{\ell}\nu_{\tau}}|^2 = 128\, G_F^2 \, (\boldsymbol{p}_{\tau} \cdot \boldsymbol{p}_{\overline{\nu}_{\ell}}) \, (\boldsymbol{p}_{\ell} \cdot \boldsymbol{p}_{\nu_{\tau}})$$

# Matrix Elements (cont'd)

For hadronic τ decays, we use the simplified model, assuming a two-body decay into $\tau_h$ and $\nu_\tau$ and further assume the ME to be a constant:

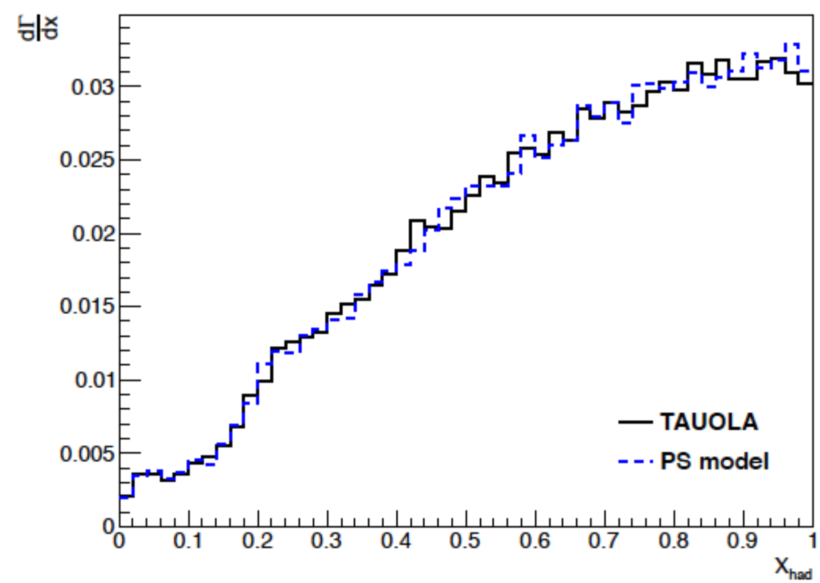
$$|\mathcal{M}^{\text{eff}}_{\tau \to \tau_{\text{h}} \nu_\tau}|^2 = \boxed{\frac{16\pi \, m_\tau^3}{m_\tau^2 - m_{vis}^2} \cdot \frac{\hbar}{\Delta t} \cdot \mathcal{B}(\tau \to \tau_{\text{h}} \, \nu_\tau)}$$

Factor chosen such that $B(\tau \rightarrow \tau_h \nu) = 64.8\%$ is reproduced

$\Delta t$: τ lepton lifetime = $290 \bullet 10^{-15}$ s

➔ Simplified model compares well with sum of all hadronic τ decay modes simulated by τ decay library TAUOLA

**Comput.Phys.Commun. 76 (1993) 361**



# Transfer Functions (TF)

The transfer functions $W(\boldsymbol{y}|\boldsymbol{p})$ represent the probability density for measuring observables **y** in the detector, given that the “true” values of the momenta of all particles in the final state is **p**

# TF for visible τ Decay Products

For now, we assume that $p_T$, η and ϕ of electrons, muons and $\tau_h$ are measured with infinite precision

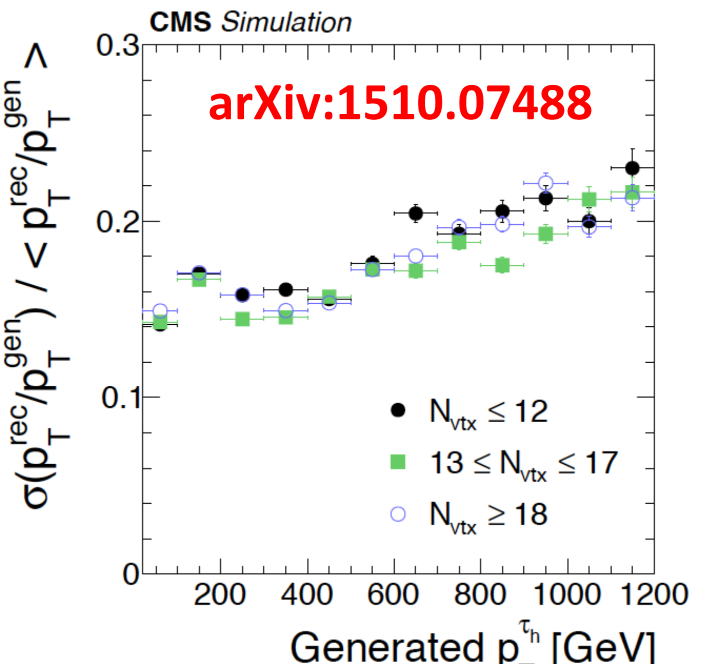
➔ The TF factorizes into a product of δ-functions:

$$W_\ell(p_T^{rec}, \eta^{rec}, \phi^{rec} | p_T^{true}, \eta^{true}, \phi^{true})$$
$$= \frac{\sin^2 \theta^{rec}}{p_T^{rec}} \delta(p_T^{rec} - p_T^{true}) \, \delta(\theta^{rec} - \theta^{true}) \, \delta(\phi^{rec} - \phi^{true})$$

Experimental resolution on $p_T$ of $\tau_h$ actually not negligible

➔ Work ongoing to model resolution for $\tau_h$ via transfer functions

Expected to improve $m_{\tau\tau}$ resolution by O(10%)



# TF for $E_T^{miss}$

$$W_\nu(E_x^{miss}, E_y^{miss} | \hat{E}_x^{miss}, \hat{E}_y^{miss}) =$$

$$\frac{1}{2\pi\sqrt{|V|}} \exp\left( -\frac{1}{2} \left( \begin{array}{c} \Delta E_x^{miss} \\ \Delta E_y^{miss} \end{array} \right)^T \cdot V^{-1} \cdot \left( \begin{array}{c} \Delta E_x^{miss} \\ \Delta E_y^{miss} \end{array} \right) \right)$$

with:
$$\begin{array}{rcl} \Delta E_x^{miss} & = & E_x^{miss} - \hat{E}_x^{miss} \\ \Delta E_y^{miss} & = & E_y^{miss} - \hat{E}_y^{miss} \end{array}$$

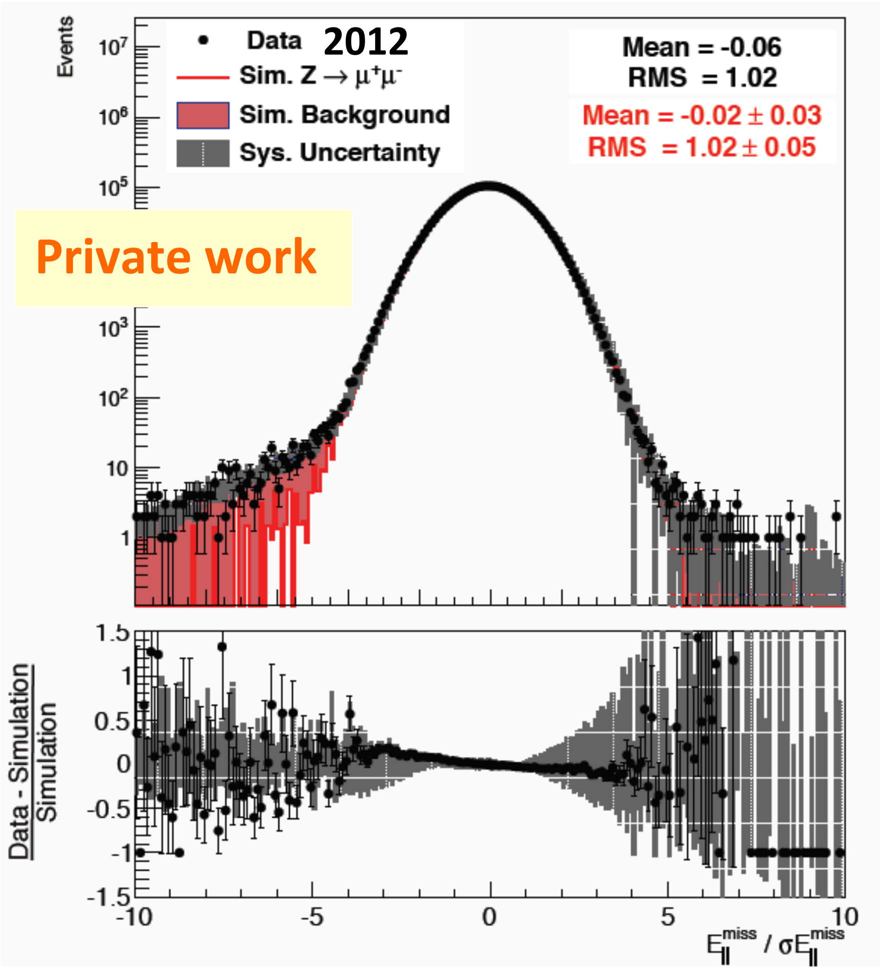
$E_x^{miss}$, $E_y^{miss}$: x and y components of reconstructed $E_T^{miss}$

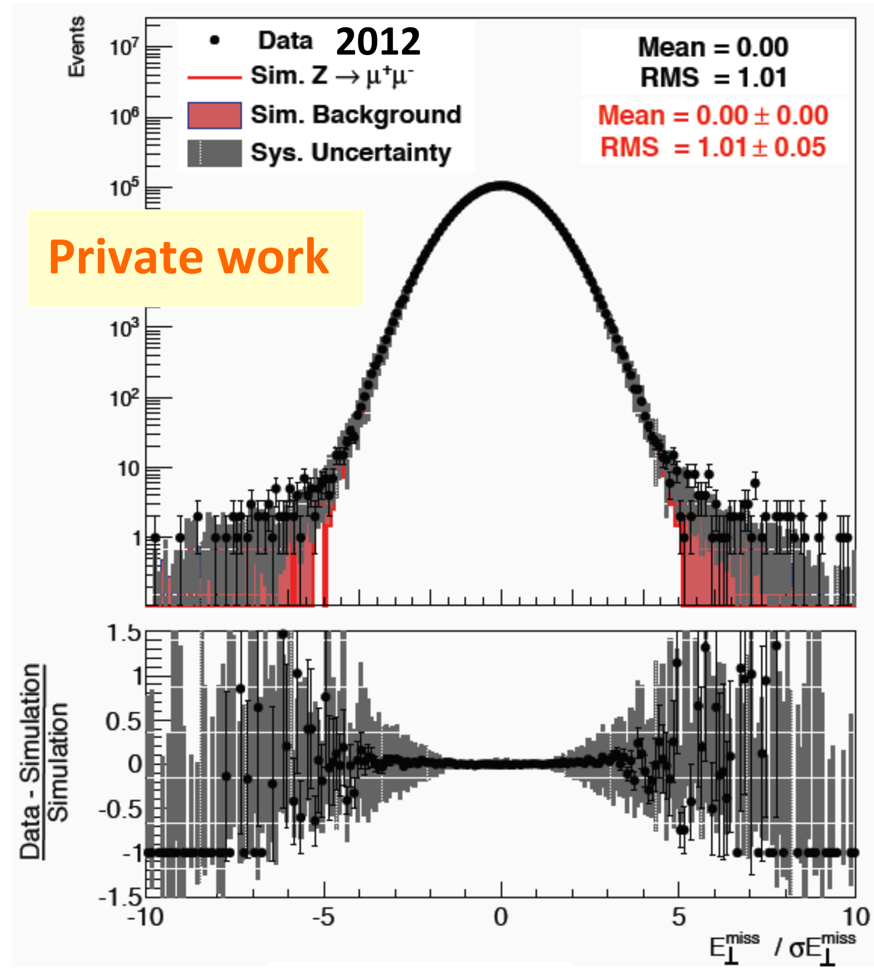
$\hat{E}_x^{miss}, \hat{E}_y^{miss}$ : x and y components of the sum of momenta of all neutrinos produced in the τ decays

$E_T^{miss}$ resolution is given by covariance matrix $V = \left( \begin{array}{cc} \sigma_x^2 & \rho\,\sigma_x\,\sigma_y \\ \rho\,\sigma_x\,\sigma_y & \sigma_y^2 \end{array} \right)$

computed on event-by-event basis using resolution functions obtained from the MC simulation **JME-10-009**

# Validation of TF for $E_T^{miss}$




Distribution of pulls approximately Gaussian

Data and MC simulation agree within systematic uncertainties

# Computation of Phase-space Integral

$$d\boldsymbol{p} = \begin{cases} d\Phi^{(1)}_{\tau_{\rm h}\,\nu_\tau}\; d\Phi^{(2)}_{\tau_{\rm h}\,\nu_\tau} & \text{if } \tau\tau \to \tau_{\rm h}\,\nu_\tau\,\tau_{\rm h}\,\nu_\tau\,, \\ d\Phi^{(1)}_{\ell\,\overline{\nu}_\ell\,\nu_\tau}\; d\Phi^{(2)}_{\tau_{\rm h}\,\nu_\tau} & \text{if } \tau\tau \to \tau_{\rm h}\,\nu_\tau\,\ell\,\overline{\nu}_\ell\,\nu_\tau\,, \\ d\Phi^{(1)}_{\ell\,\overline{\nu}_\ell\,\nu_\tau}\; d\Phi^{(2)}_{\ell\,\overline{\nu}_\ell\,\nu_\tau} & \text{if } \tau\tau \to \ell\,\overline{\nu}_\ell\,\nu_\tau\,\ell\,\overline{\nu}_\ell\,\nu_\tau\,, \end{cases}$$

(12-dim)

(15-dim)

(18-dim)

where

$$d\Phi^{(i)}_{\tau_{\rm h}\,\nu_\tau} = d^3\boldsymbol{p}^{(i)}_{vis}\, d^3\boldsymbol{p}^{(i)}_{\nu}$$

$$d\Phi^{(i)}_{\ell\,\overline{\nu}_\ell\,\nu_\tau} = d^3\boldsymbol{p}^{(i)}_{vis}\, d^3\boldsymbol{p}^{(i)}_{\nu}\, d^3\boldsymbol{p}^{(i)}_{\overline{\nu}}\,.$$

Product of phase-space element and τ decay ME simplified analytically before numeric integration:

$$|\mathcal{M}^{(i)}_\tau|^2\, d\Phi^{(i)}_{\tau_{\rm h}\,\nu_\tau} = \frac{\pi}{m_\tau \Gamma_\tau}\, f_{\rm h}\left(\boldsymbol{p}^{\rm vis(\boldsymbol{i})}, m^{{\rm vis}(i)}, \boldsymbol{p}^{\rm inv(\boldsymbol{i})}\right)\, \frac{d^3\boldsymbol{p}^{\rm vis}}{2E_{\rm vis}}\, dz\, d\phi_{\rm inv}$$

$$|\mathcal{M}^{(i)}_\tau|^2\, d\Phi^{(i)}_{\ell\,\overline{\nu}_\ell\,\nu_\tau} = \frac{\pi}{m_\tau \Gamma_\tau}\, f_\ell\left(\boldsymbol{p}^{\rm vis(\boldsymbol{i})}, m^{{\rm vis}(i)}, \boldsymbol{p}^{\rm inv(\boldsymbol{i})}\right)\, \frac{d^3\boldsymbol{p}^{\rm vis}}{2E_{\rm vis}}\, dz\, dm^2_{\rm inv}\, d\phi_{\rm inv}$$

# Phase-space Integration (cont'd)

$$f_h\left(\mathbf{p^{vis}}, m_{vis}, \mathbf{p^{inv}}\right) = \frac{\left|\mathcal{M}^{eff}_{\tau \to \tau_h \nu_\tau}\right|^2}{512\pi^6 \left|\mathbf{p^{vis}}\right| z^2}$$

$$f_\ell\left(\mathbf{p^{vis}}, m_{vis}, \mathbf{p^{inv}}\right) = \frac{\left|\mathcal{M}^{eff}_{\tau \to \ell \bar{\nu}_\ell \nu_\tau}\right|^2}{512\pi^6} \cdot \frac{I_{inv}}{\left|\mathbf{p^{vis}}\right| z^2}$$

with

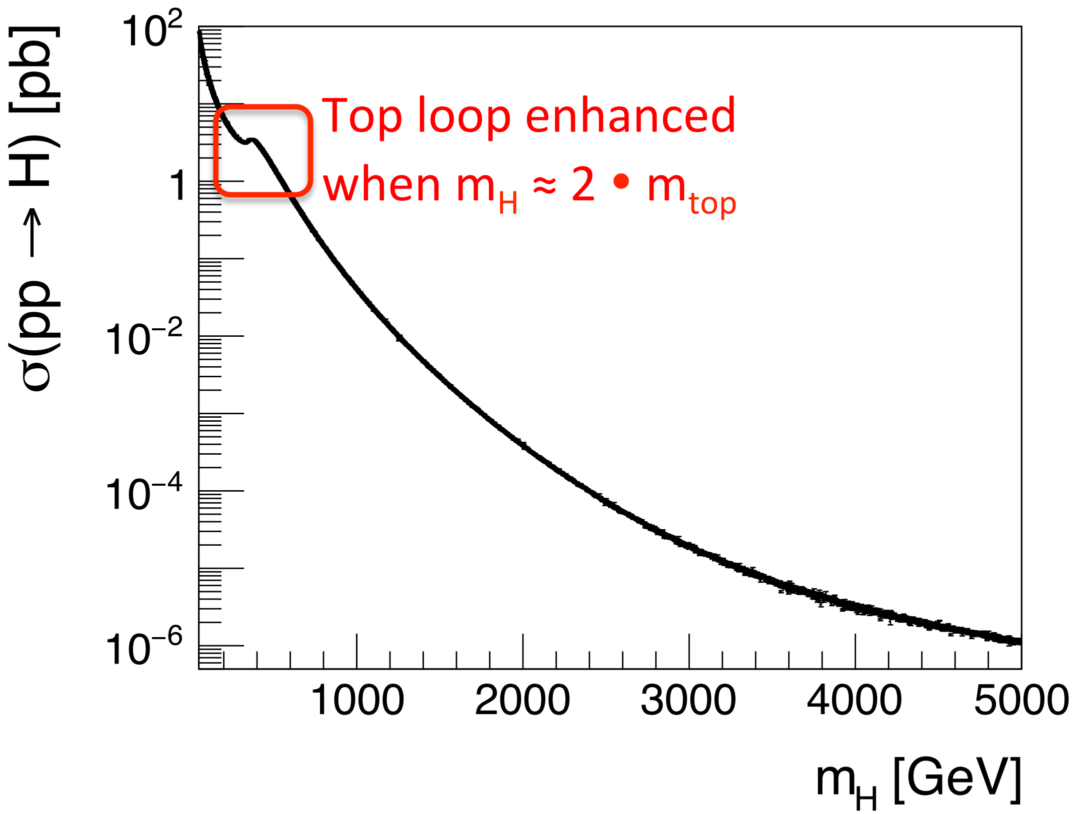
$$I_{inv} = \frac{G_F^2}{\pi^2} m_{inv}^2 \left(2\, E_\tau\, E_\ell - \frac{2}{3}\sqrt{E_\tau^2 - m_\tau^2}\sqrt{E_\ell^2 - m_\ell^2}\right)$$

The τ lepton momenta can be computed as function of $\mathbf{p^{vis}}$ and the integration variables z and $\phi_{inv}$ respectively z, $\phi_{inv}$ and $m_{inv}^2$

This then allows to evaluate the squared modulus of the ME $\left|\mathcal{M}_{\mathrm{pp}\to\mathrm{H}\to\tau\tau}(\boldsymbol{p}, m_{\mathrm{H}})\right|^2$ and the PDF in the same way as if the τ leptons were stable particles

# Normalization

Cross-section σ = 1/Normalization factor is steeply falling as function of $m_{\tau\tau}^{test} = m_H$




➔ Normalization factor 1/σ increases probability to reconstruct events in high mass tail compared to "standard" SVfit

➔ Add artificial "penalty" term of form $k \cdot Log(m_{\tau\tau})$ to reduce high mass tail

# Artificial Regularization Term

Rather than obtaining $m_{\tau\tau}$ via maximization of $P(\mathbf{y}|m_{\tau\tau}^{(i)})$,
we may choose to maximize

$$\mathrm{Log}(\ P(\mathbf{y}|m_{\tau\tau}^{(i)})\ ) - k \bullet \mathrm{Log}(m_{\tau\tau}^{(i)})$$

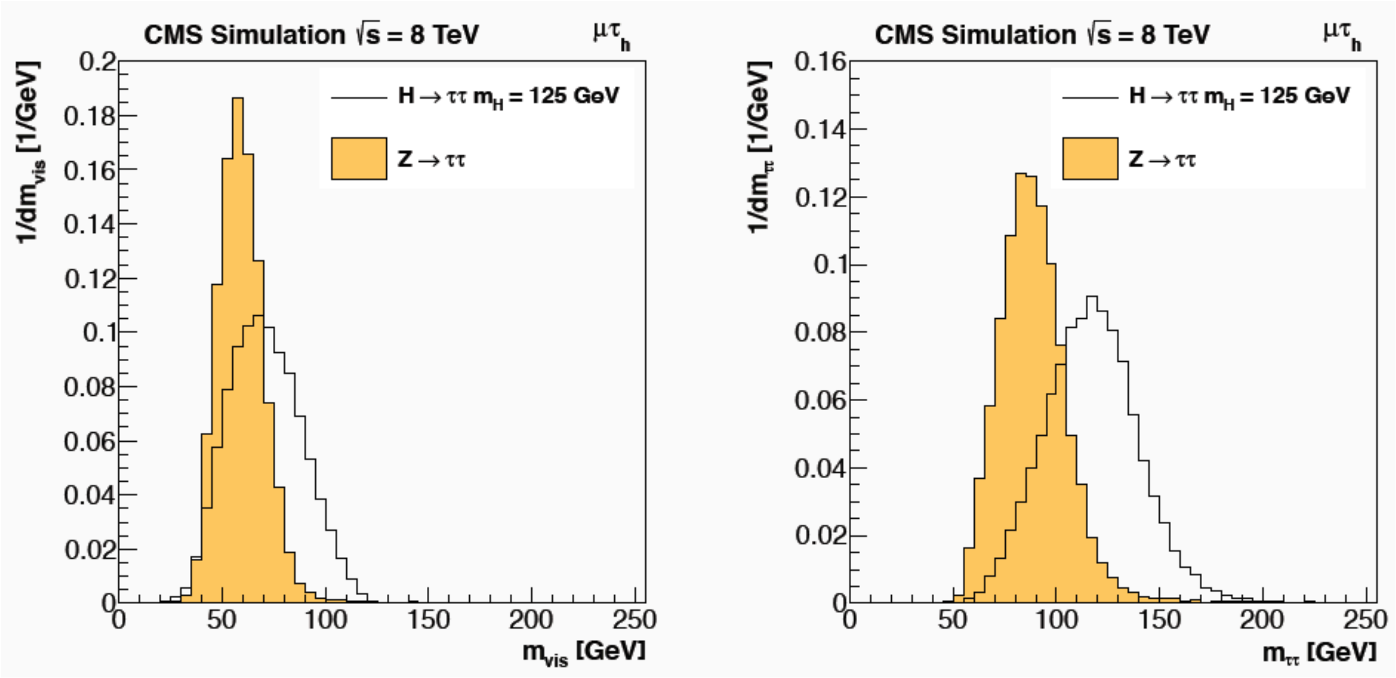
instead.

The parameter k is found by optimizing the $m_{\tau\tau}$ resolution for a sample of events:

We find that a value k = 5 significantly reduces the high $m_{\tau\tau}$ tail for the irreducible $Z/\gamma^* \rightarrow$ background, while causing a small bias on the $m_{\tau\tau}$ distribution reconstructed in Higgs → ττ signal events

**N.B.**: Adding the artificial term $k \bullet \mathrm{Log}(m_{\tau\tau})$ is known as penalized likelihood method in the literature

# Mass Resolution




**J.Phys.Conf.Ser. 513 (2014) 022035**

SVfit algorithm achieves resolution of typically 15-20% relative to true value of $m_{\tau\tau}$

This resolution significantly improves separation between Higgs → ττ signal and irreducible Z/γ* → ττ background compared to alternative mass observables

Reducible backgrounds (W+jets, QCD, tt) approximately flat in $m_{\tau\tau}$

➔ Effective background contribution decreases if Higgs signal is concentrated in narrow mass window

# Performance in CMS Higgs → ττ Analysis

Events analyzed in different event categories:

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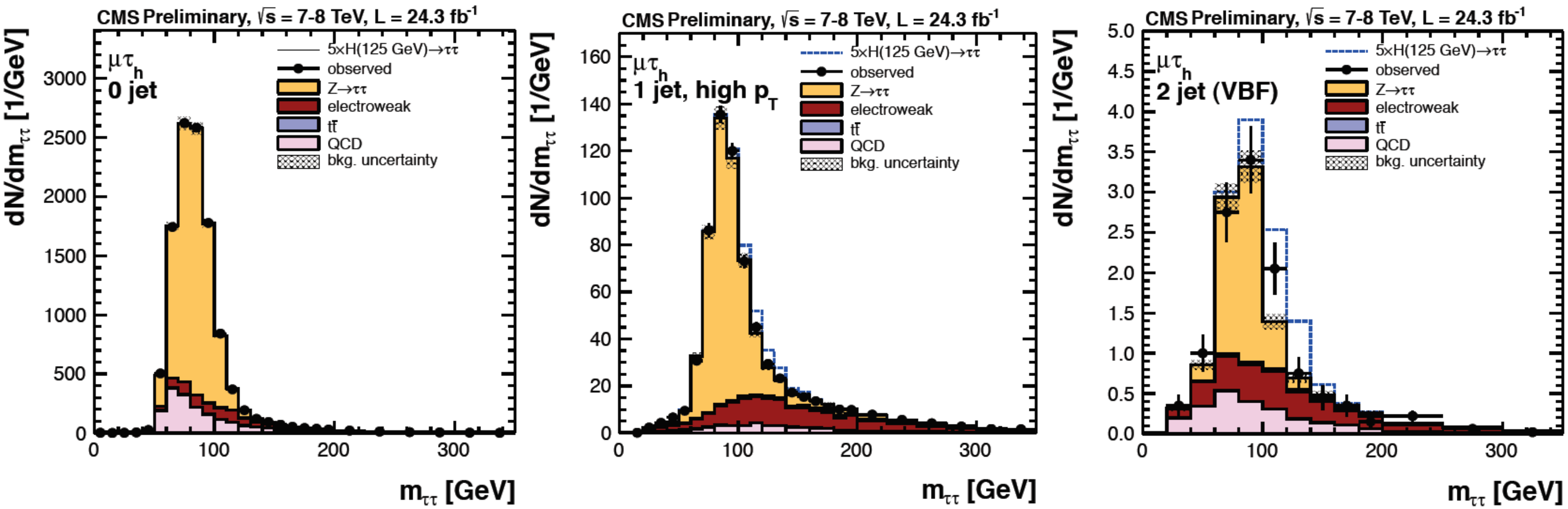



Most sensitive categories are 1 jet and VBF, in which Higgs is typically boosted:

→ SVfit resolution improves compared to 0 jet category in which visible τ decay products are typically "back-to-back" in transverse plane

Overall sensitivity of CMS SM Higgs → ττ analysis improves by ≈40% when using $m_{\tau\tau}$ reconstructed by SVfit algorithm compared to $m_{vis}$

# ATLAS Higgs boson ML challenge

ATLAS setup a kaggle contest for improving the sensitivity of their SM Higgs → ττ analysis https://www.kaggle.com/c/higgs-boson

ATLAS provided Ntuples with MMC* mass, the momenta and energies of the visible tau decay products plus $E_T^{miss}$

➔ Information sufficient to compute SVfit mass for events in ATLAS Ntuples

➔ Compare SVfit vs. MMC mass resolution for SM Higgs → ττ events using the event categories of our SM Higgs → analysis **HIG-13-004**

**Note:**

- ATLAS does not (yet) estimate the $E_T^{miss}$ resolution on an event-by-event basis
  ➔ I estimated ATLAS' $E_T^{miss}$ resolution based on public performance numbers
- The ATLAS $\tau_h$ reconstruction assumes the $\tau_h$ to be massless
  ➔ I extended SVfit to take an average over the distribution of true $\tau_h$ mass

*Missing Mass Calculator **arXiv:1012.4686**

Completed • $13,000 • 1,785 teams

# Higgs Boson Machine Learning Challenge

Mon 12 May 2014 – Mon 15 Sep 2014 (4 months ago)

Dashboard

## Private Leaderboard - Higgs Boson Machine Learning Challenge

This competition has completed. This leaderboard reflects the final standings.

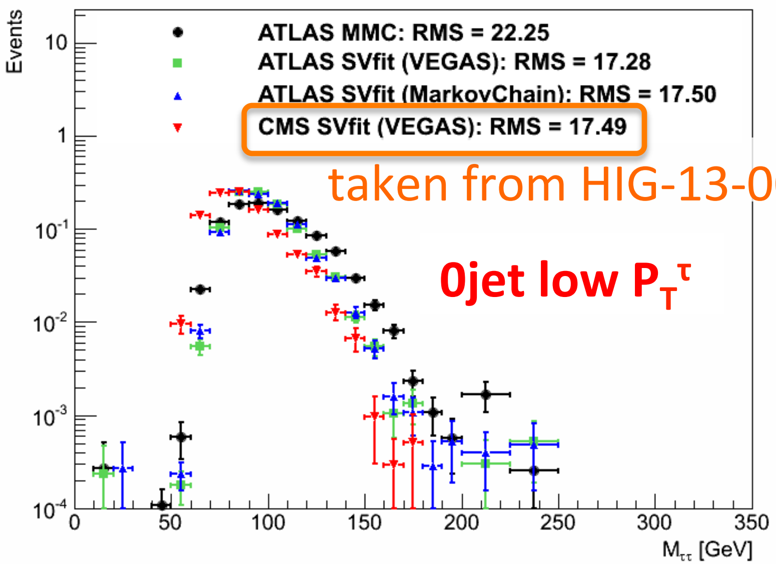
See someone using multiple accounts?
Let us know.

| # | Δrank | Team Name ‡ model uploaded * in the money | Score | Entries | Last Submission UTC (Best – Last Submission) |
|---|---|---|---|---|---|
| 1 | ↑1 | Gábor Melis ‡ * | 3.80581 | 110 | Sun, 14 Sep 2014 09:10:04 (-0h) |
| 2 | ↑1 | Tim Salimans ‡ * | 3.78913 | 57 | Mon, 15 Sep 2014 23:49:02 (-40.6d) |
| 3 | ↑1 | nhlx5haze ‡ * | 3.78682 | 254 | Mon, 15 Sep 2014 16:50:01 (-76.3d) |
| 4 | ↑38 | ChoKo Team | 3.77526 | 216 | Mon, 15 Sep 2014 15:21:36 (-42.1h) |
| 5 | ↑35 | cheng chen | 3.77384 | 21 | Mon, 15 Sep 2014 23:29:29 (-0h) |
| 6 | ↑16 | quantify | 3.77086 | 8 | Mon, 15 Sep 2014 16:12:48 (-7.3h) |
| 7 | ↑1 | Stanislav Semenov & Co (HSE Yandex) | 3.76211 | 68 | Mon, 15 Sep 2014 20:19:03 |
| 8 | ↓7 | Luboš Motl's team<br>• Luboš Motl<br>• Christian Veelken | 3.76050 | 589 | Mon, 15 Sep 2014 08:38:49 (-1.6h) |
| 9 | ↑8 | Roberto-UCIIIM | 3.75864 | 292 | Mon, 15 Sep 2014 23:44:42 (-44d) |
| 10 | ↑2 | Davut & Josef | 3.75838 | 161 | Mon, 15 Sep 2014 23:24:32 (-4.5d) |

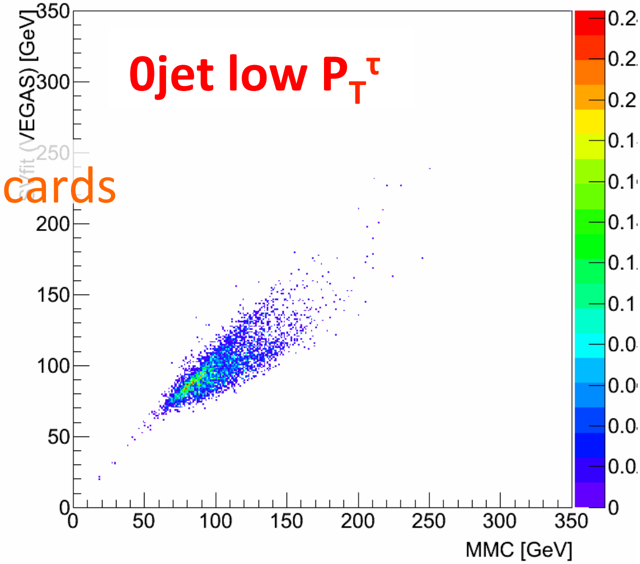
➔ SVfit finished 8$^{th}$ place out of 1785 teams!

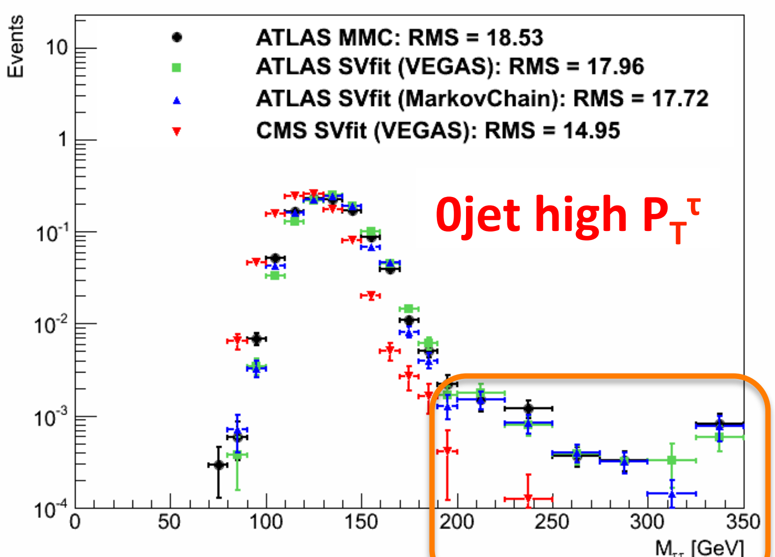
# Comparison SVfit vs. MMC

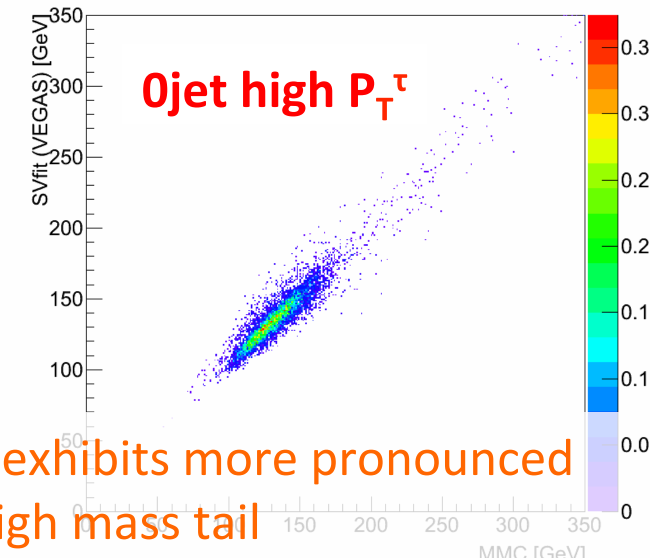
(on gg → Higgs + VBF Higgs signal events)




taken from HIG-13-004 datacards




MMC exhibits more pronounced high mass tail

# Conclusions: SVfit vs. MMC

- Difference between MMC and SVfit is small in boosted events, in which $m_{\tau\tau}$ is well constrained by $E_T^{miss}$
- MMC likelihood model seems not optimal for 0jet events, in which both taus are typically "back-to-back"

# Extensions of SVfit Algorithm

Combination of likelihood approach and matrix element method is very universal, can be used for estimating any quantity of interest

**Recent developments:**

- Estimation of Higgs boson $p_T$, $\eta$ and $\phi$ in addition to $m_{\tau\tau}$
- Reconstruction of Higgs boson mass in search for lepton flavor violating (LFV) Higgs → $\mu\tau$ → $\mu\tau_h$ and Higgs → $e\tau$ → $e\tau_h$ decays
- Reconstruction of di-Higgs mass $m_{hh}$ in search for X → hh → bbττ decays of high mass resonances X
- Reconstruction of "transverse" Higgs mass

  $$m_{T\tau\tau} = \sqrt{(E_T^{\tau_1} + E_T^{\tau_2})^2 - ((p_x^{\tau_1} + p_x^{\tau_2})^2 + (p_y^{\tau_1} + p_y^{\tau_2})^2)}$$

  in CMS MSSM Higgs → ττ analysis

➔ In most cases SVfit algorithm yields a significant improvement in analysis sensitivity

# Summary

- SVfit algorithm allows to reconstruct Higgs boson mass in Higgs → ττ events with a resolution of typically 15-20% relative to the true value of $m_{\tau\tau}$
- This resolution significantly improves the separation of the Higgs → ττ signal from the irreducible Z/γ* → ττ background, improving the sensitivity of CMS SM Higgs → ττ analysis by ≈40%
- The approach is very universal and has been customized for applications to LFV Higgs decays, the search for X → hh decays and for the CMS MSSM Higgs → ττ analysis
- I expect the formalism to handle τ lepton decays in the ME method, developed for the SVfit algorithm, to be useful for future applications of the ME method to analyses with τ leptons in the final state

# Backup