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# Naturally light uncolored and heavy colored superparticles

Gautam Bhattacharyya

Saha Institute of Nuclear Physics, Kolkata

**G.B., B. Bhattacharjee, T.T. Yanagida and N. Yokozaki**

**PLB 725 (2013) 339 and PLB 730 (2014) 231**

**G.B., T.T. Yanagida and N. Yokozaki, PLB 749 (2015) 82**

# Motivation

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- Squarks and gluino are heavy
  - $m_h = 125 \text{ GeV}$   
 $\Rightarrow \text{stop} \sim \text{TeV}$  (large mixing) or more (small mixing)
  - FCNC  $\Rightarrow$  first two generation squarks heavy and degenerate
  - Non-observation of squarks and gluino in 7 and 8 TeV LHC
- Sleptons and weak gauginos may be light
  - Muon ( $g - 2$ ) has  $> 3 \sigma$  discrepancy  $\Rightarrow$  light smuons
  - Neutralino as DM expected in  $\mathcal{O}(100) \text{ GeV}$  range
  - Light staus may slightly alter Higgs diphoton rate
  - Collider bounds on them are not so strong

How to reconcile this splitting between colored and uncolored superparticles?

# GMSB – basic introduction

- Information on SUSY breaking is transmitted to observable sector by gauge interaction. FCNC is suppressed.
- 'Messenger sector' comprising of heavy chiral superfields which have gauge charges. SUSY is broken in messenger sector by interaction with 'spurion'. Consider a set of vector-like superfields  $M + \bar{M}$  (e.g.  $5 + \bar{5}$  and/or  $10 + \bar{10}$  of SU(5) GUT). **Complete multiplets do not spoil gauge coupling unification.**
- Minimal scenario:  $W = \lambda X M \bar{M}$ . The messenger fermions acquire a supersymmetric mass  $m = \lambda \langle X \rangle$  and messenger scalars are split:  $m_{\pm}^2 = m^2 \pm \lambda \langle F_X \rangle$ . SUSY breaking scale  $\Lambda \equiv \langle F_X \rangle / \langle X \rangle$ .
- Gaugino masses are generated at one-loop while sfermion masses are generated at two-loop. When  $\Lambda \ll M$  ( $\sim 100 \text{ TeV} < M < M_{Pl}$ )
$$m_{\tilde{\chi}_i} \simeq \frac{\alpha_i}{4\pi} \Lambda, \quad \tilde{m}^2 \simeq 2\Lambda^2 \frac{\sum_i c_i \alpha_i^2}{16\pi^2}$$
- Gravitino mass  $m_{\tilde{G}} \sim \frac{F}{M_{Pl}}$  is in general much lighter (than in supergravity). It can be  $\sim 100 \text{ eV}$ . In general gravitino is the LSP. Distinct signatures.
- $\mu$  and  $B_\mu$  problem! Essentially,  $B_\mu \sim \mu \Lambda$ .**

# Fusion of three issues

- Gauge coupling unification even with incomplete multiplets at string scale  $>$  GUT scale (Bachas, Fabre, Yanagida '96; Bastero-Gil, Brahmachari '97).
  - Adjoint octet ( $\Sigma_8$ ) of color SU(3), adjoint triplet ( $\Sigma_3$ ) of weak SU(2)
  - Origin of these states can be traced to the adjoint 24-plet of SU(5)
- Presence of intermediate states characterizing GMSB.

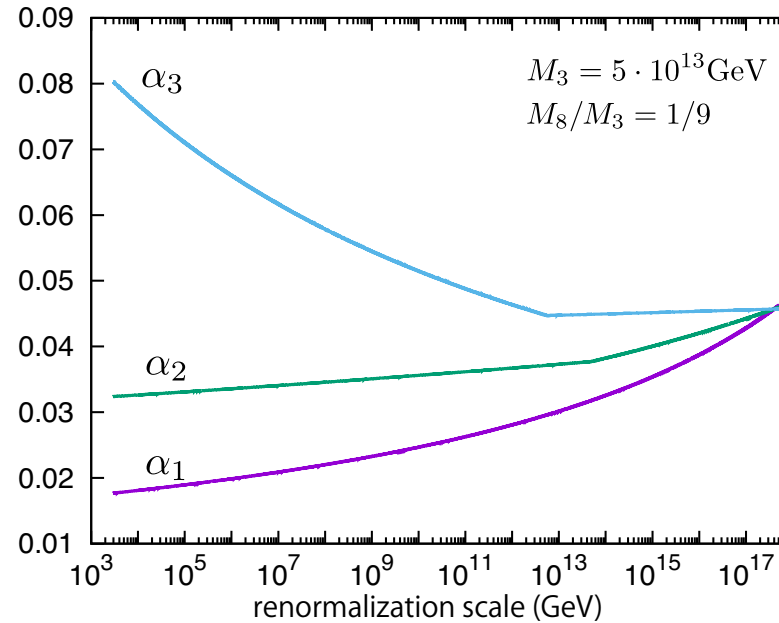
$$W_{\text{mess}} = (M_8 + \lambda_8 X) \text{Tr}(\Sigma_8^2) + (M_3 + \lambda_3 X) \text{Tr}(\Sigma_3^2)$$

$F$ -term vev of hidden sector field  $X$  transmits SUSY breaking to visible sector via messenger multiplets.

- Dynamically ensure  $\tilde{m}_{\text{color}} \gg \tilde{m}_{\text{uncolor}}$  by delinking the sources of mass generation for colored and uncolored super-particles (Han, Yanagida, Zhang '98).

Aim is to reproduce  $m_h$ ,  $(g - 2)_\mu$ , and other data

# Unification with $\Sigma_3$ and $\Sigma_8$



$$\alpha_1^{-1}(M_{\text{str}}) = \alpha_1^{-1}(m_{\text{SUSY}}) - \frac{(33/5)}{2\pi} \ln \frac{M_{\text{str}}}{m_{\text{SUSY}}}$$

$$\alpha_2^{-1}(M_{\text{str}}) = \alpha_2^{-1}(m_{\text{SUSY}}) - \frac{1}{2\pi} \ln \frac{M_{\text{str}}}{m_{\text{SUSY}}} - \frac{2}{2\pi} \ln \frac{M_{\text{str}}}{M_3}$$

$$\alpha_3^{-1}(M_{\text{str}}) = \alpha_3^{-1}(m_{\text{SUSY}}) - \frac{(-3)}{2\pi} \ln \frac{M_{\text{str}}}{m_{\text{SUSY}}} - \frac{3}{2\pi} \ln \frac{M_{\text{str}}}{M_8}$$

•  $m_{\text{SUSY}} \equiv (m_{Q_3} m_{\bar{U}_3})^{1/2}$  is the average stop mass.

•  $\alpha_{1,2,3}^{-1} \simeq (57, 31, 13)$  at  $m_{\text{SUSY}} = 3 \text{ TeV}$ .

# Unification – key issues

For unification,  $M_3 > M_8$  at one-loop.

For  $M_{\text{str}} = 10^{17}(10^{18})$  GeV,  $M_3/M_8 = 7(18)$ .

$$M_{\text{str}}^2 m_{\text{mess}} = M_{\text{GUT}}^3 \quad \text{where} \quad m_{\text{mess}} \equiv \sqrt{M_3 M_8}.$$

Late Unification avoids proton decay constraints:  $p \rightarrow K^+ \nu$  goes like  $1/m_{H_c}^2$  where  $m_{H_c} \sim M_{\text{str}} \sim 10^{17-18}$  GeV.

# Sparticle masses at mess scale

- Define  $\Lambda_8 \equiv \frac{\lambda_8 F_X}{M_8}$ ,  $\Lambda_3 \equiv \frac{\lambda_3 F_X}{M_3}$
- Recall  $M_3 > M_8$  (unification), tune  $\lambda_8$  and  $\lambda_3$  to ensure  $\Lambda_8 \gg \Lambda_3$
- Messenger scale spectrum

$$\begin{aligned} m_{\tilde{B}} &\simeq 0, \quad m_{\tilde{W}} \simeq \frac{g_2^2}{16\pi^2} (2\Lambda_3), \quad m_{\tilde{g}} \simeq \frac{g_3^2}{16\pi^2} (3\Lambda_8) \\ m_{\tilde{Q}}^2 &\simeq \frac{2}{(16\pi^2)^2} \left[ \frac{4}{3} g_3^4 (3\Lambda_8^2) + \frac{3}{4} g_2^4 (2\Lambda_3^2) \right], \quad m_{\tilde{D}}^2 = m_{\tilde{U}}^2 \simeq \frac{2}{(16\pi^2)^2} \frac{4}{3} g_3^4 (3\Lambda_8^2), \\ m_{\tilde{L}}^2 &\simeq \frac{2}{(16\pi^2)^2} \frac{3}{4} g_2^4 (2\Lambda_3^2), \quad m_{\tilde{E}}^2 \simeq 0 \end{aligned}$$

- No messenger is charged under U(1). Right-handed slepton and Bino masses are generated by Planck scale suppressed gravitational interaction and are of the order of the gravitino mass.

$$m_{\tilde{E}}(M_{\text{str}}) \sim M_{\tilde{B}}(M_{\text{str}}) \sim m_{3/2} \sim \frac{F_X}{M_P}$$

# Sample spectra with $\Sigma_{3,8}$

$\Lambda_3/\Lambda_8$	0.10
$\Lambda_8$	500 TeV
$M_{1/2}$	920 GeV
$M_{\text{mess}}$	$10^{13}$ GeV
$\tan \beta$	10
$\mu$	5.9 TeV
$m_{\text{stop}}$	8.2 TeV
$\delta a_\mu$	$1.24 \times 10^{-9}$
$m_{\text{gluino}}$	10 TeV
$m_{\text{squark}}$	9.4 TeV
$m_{\tilde{e}_L} (m_{\tilde{\mu}_L})$	601 GeV
$m_{\tilde{e}_R} (m_{\tilde{\mu}_R})$	258 GeV
$m_{\tilde{\tau}_1}$	98 GeV
$m_{\chi_1^0}$	315 GeV
$m_{\chi_1^\pm}$	851 GeV

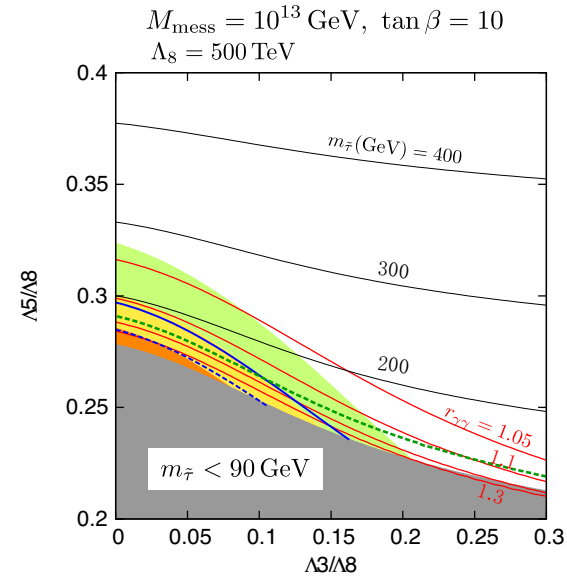
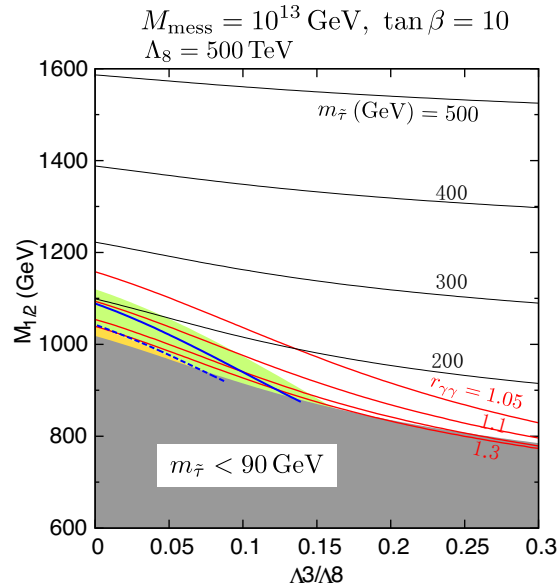


# Phenomenology with $\Sigma_{3,8}$

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- Due to large left-right stau mixing, one stau can be very light. Light stau can modify diphoton BR of Higgs by 10-20%.
- **Stau is the NLSP, with gravitino LSP.** However, stau is long-lived, as its decay to gravitino of the same order mass is suppressed. CMS limit:  $m_{\tilde{\tau}} > 340$  GeV. This is too heavy for sizable diphoton contribution.
- This implies smuon is too heavy to explain muon  $(g - 2)$  anomaly.
- **Way out:** Allow mild ( $\leq 10^{-7}$ ) RPV, so that stau can promptly decay to a lepton and neutrino. Then stau can be lighter than 340 GeV. Then muon g-2 can be explained at slightly better than  $2\sigma$  level. **Perhaps not any more!!**
- Can we avoid RPV?

# muon ( $g - 2$ )



**Introduction of  $(5 + \bar{5})$  messengers explains muon ( $g - 2$ ) better** (right panel).

**Key point:** Bino/stau and gravitino mass generation de-linked. Gravitino can be ultra-light, while bino/stau can weigh around 100 GeV (since 5-plets have non-zero  $Y$ ).

Bino/stau mass  $\propto \Lambda_5$ .

Unification is not affected by complete multiplets.

Region below blue solid line is excluded by vac stability limit arising from large LR slepton mixing, which sets an upper limit on  $\mu \tan \beta$ .

# Further improvements

- Including 3-loop corrections to  $m_h$ , stop mass in (3-5) TeV range even with minimal mixing can reproduce  $m_h = 125$  GeV (Feng et al '13).
- Since SUSY breaking scale comes down,  $\mu$  gets smaller.

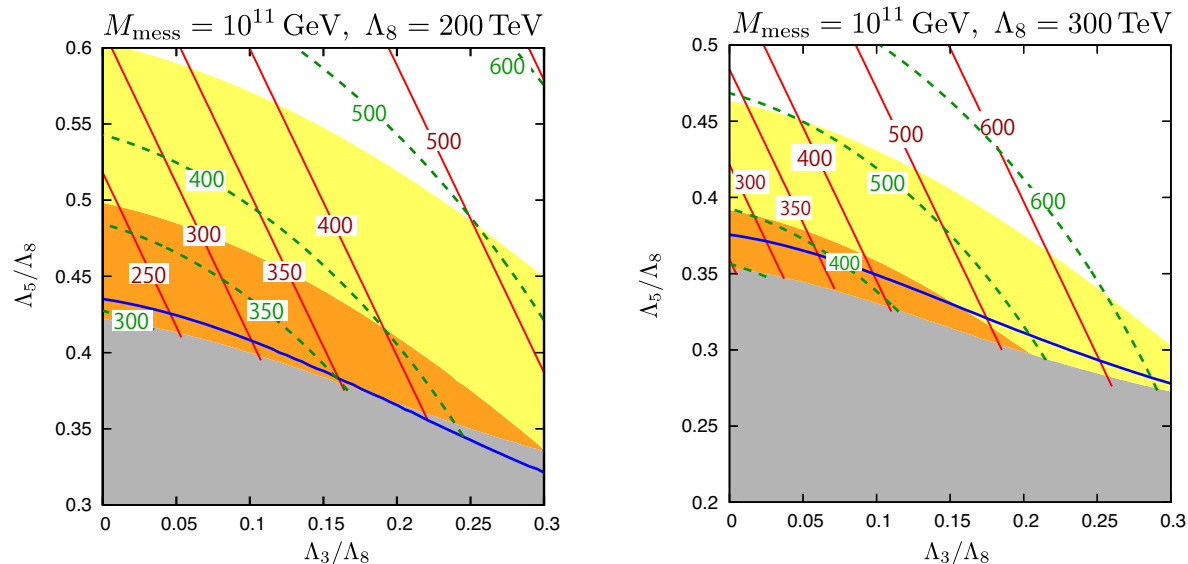
$$\mu^2 \sim (-m_{H_u}^2) \sim \frac{3}{4\pi^2} y_t^2 (m_{\text{stop}}^2) \ln \left( \frac{M_{\text{mess}}}{m_{\text{stop}}} \right)$$

- LR mixing also goes down. So  $\tilde{\tau}_1$  need not be that light.
- Bino/RH-slepton and gravitino masses de-correlated, thanks to 5-plets. Bino is NLSP.  $\text{At messenger scale } m_{\tilde{B}} = \frac{\alpha_1}{4\pi} \Lambda_5, \quad m_{\tilde{E}}^2 = \frac{1}{8\pi^2} \left[ \frac{3}{5} \alpha_1^2 \Lambda_5^2 \right]$ .
- Gravitino can be made light

$$m_{3/2} \simeq 0.01 \text{ GeV} \left( \frac{\Lambda_8}{200 \text{ TeV}} \right) \left( \frac{(\Lambda_3/\Lambda_8)}{0.2} \right) \left( \frac{M_8}{10^{11} \text{ GeV}} \right) \left( \frac{(M_3/M_8)}{10} \right)$$

- 100 GeV Neutralino decays into 10 MeV gravitino in a BBN safe way (Kawasaki et al '08). No need of RPV.

# Muon $(g - 2)$ (updated)



- $(g - 2)_\mu$  is dominated by bino-smuon loop. In the orange (yellow) region it is explained at 1 (2)- $\sigma$  level. In the gray region, stau is lighter than 90 GeV.

$$(\Delta a_\mu)_{\text{SUSY}} \simeq \frac{3}{5} \frac{g_1^2}{8\pi^2} \frac{m_\mu^2 \mu \tan \beta}{M_1^3} F_b \left( \frac{m_{\tilde{L}}^2}{M_1^2}, \frac{m_{\tilde{E}}^2}{M_1^2} \right)$$

- Viable regions are above the blue solid line where bino is NLSP. A stau NLSP is stable inside the detector (hence  $> 340$  GeV (CMS '13)), which makes smuons too heavy!

# Focus point

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A region where EWSB seems natural even if superparticles are very heavy. One or more fixed ratios of soft SUSY breaking parameters are introduced which reduce the fine-tuning of the potential.

In GMSB, F.P. was achieved with different number of weakly ( $N_2$ ) and strongly ( $N_3$ ) interacting messenger multiplets. But gauge couplings do not unify (Brummer, Buchmuller'12; Brummer, Ibe, Yanagida'13).

The EWSB conditions are

$$\frac{g_1^2 + g_2^2}{4} v^2 = \left[ -\mu^2 - \frac{(m_{H_u}^2 + \frac{1}{2v_u} \frac{\partial \Delta V}{\partial v_u}) \tan^2 \beta}{\tan^2 \beta - 1} + \frac{m_{H_d}^2 + \frac{1}{2v_d} \frac{\partial \Delta V}{\partial v_d}}{\tan^2 \beta - 1} \right]_{m_{\text{SUSY}}},$$
$$\frac{\tan^2 \beta + 1}{\tan \beta} = \left[ \frac{1}{B\mu} \left( m_{H_u}^2 + \frac{1}{2v_u} \frac{\partial \Delta V}{\partial v_u} + m_{H_d}^2 + \frac{1}{2v_d} \frac{\partial \Delta V}{\partial v_d} + 2\mu^2 \right) \right]_{m_{\text{SUSY}}}.$$

where  $\Delta V$  is the one-loop correction to the Higgs potential.

# RG running and cancellations

- $m_{H_u}^2$  (weak) receives negative contributions from colored super-partners.
- $m_{H_u}^2$  (weak) receives positive contribution from wino loop and tree level  $m_{H_u}^2$ .

$$\begin{aligned} m_{H_u}^2(3\text{TeV}) &= 0.704m_{H_u}^2 + 0.019m_{H_d}^2 \\ &- 0.336m_Q^2 - 0.167m_U^2 - 0.056m_E^2 \\ &+ 0.055m_L^2 - 0.054m_{\bar{D}}^2 \\ &+ 0.011M_{\tilde{B}}^2 + 0.192M_{\tilde{W}}^2 - 0.727M_{\tilde{g}}^2 \\ &- 0.003M_{\tilde{B}}M_{\tilde{W}} - 0.062M_{\tilde{W}}M_{\tilde{g}} - 0.010M_{\tilde{B}}M_{\tilde{g}} \end{aligned}$$

$$m_{H_u}^2(\text{weak}) \sim 0.9 m_{\text{uncolor}}^2 - 1.3 m_{\text{color}}^2$$

In minimal GMSB with 5 and  $\bar{5}$  messengers, the negative contributions substantially dominate over the positive contributions.

# RG invariant parameter

With only  $\Sigma_3$  and  $\Sigma_8$  messengers, introduce

$$r_3 \equiv \frac{\Lambda_3}{\Lambda_8} = \frac{\lambda_3 M_8}{\lambda_8 M_3}$$

This parameter is RG invariant

$$\lambda_{(3,8)}(t) = \lambda_{(3,8)}(t_0) \exp \left[ \int_{t_0}^t dt' (\gamma_X + 2\gamma_{\Sigma_{(3,8)}}) \right]$$

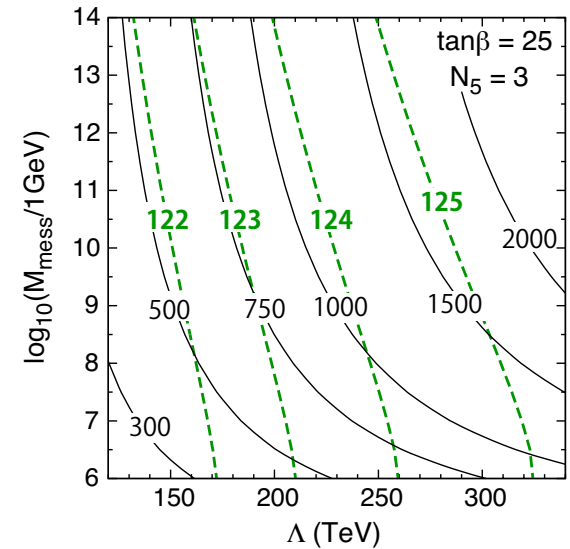
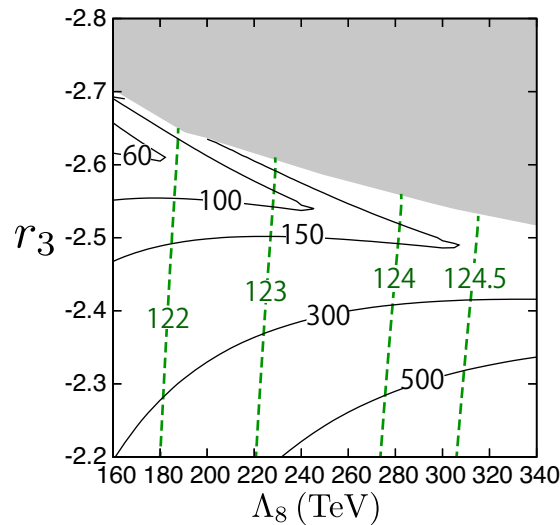
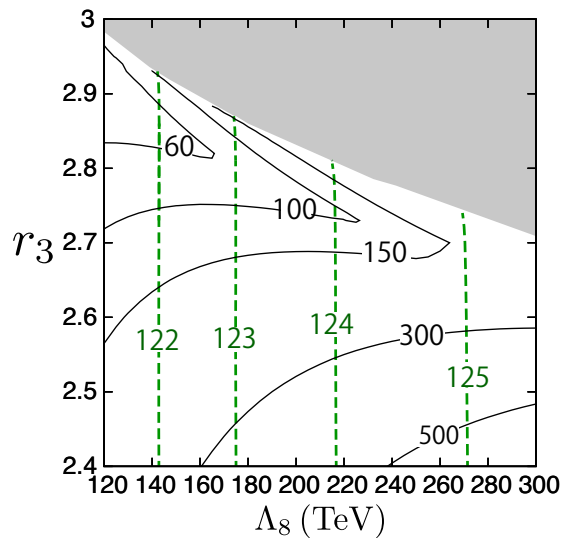
$$M_{(3,8)}(t) = M_{(3,8)}(t_0) \exp \left[ \int_{t_0}^t dt' (2\gamma_{\Sigma_{(3,8)}}) \right]$$

$$\frac{\lambda_3(t) M_8(t)}{\lambda_8(t) M_3(t)} = \frac{\lambda_3(t_0) M_8(t_0)}{\lambda_8(t_0) M_3(t_0)}$$

# Focus point in AM-GMSB

$$m_{H_u}^2(3\text{TeV}) \simeq [0.16 r_3^2 - 1.2] M_{\tilde{g}}^2$$

For  $r_3 \simeq 2.8, -2.6$  we achieve Focus Point region



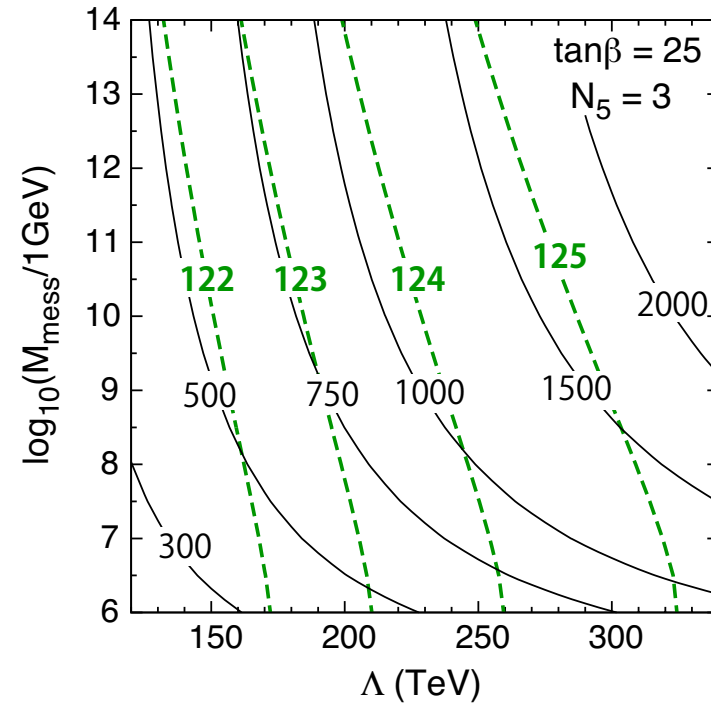
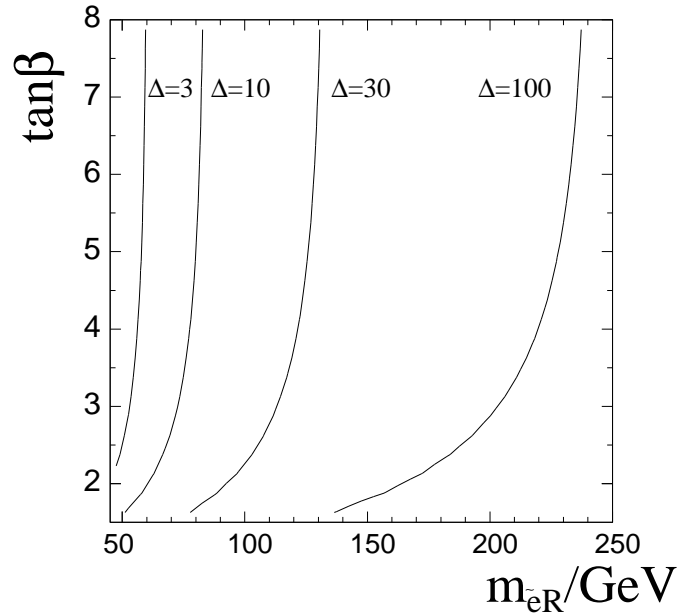
- In the gray region the EWSB does not occur.  $M_{\text{mess}} = 10^{13}$  GeV.
- $\Delta = 60 - 150$  for  $r_3 = 2.8$  to explain the observed  $m_h$ .
- For minimal GMSB,  $\Delta = 750 - 1000$  to explain  $m_h > 125$  GeV for  $M_{\text{mess}} > 10^9$  GeV.



# Sample spectra for Focus Point

P1		P2		P3	
$\Lambda_8$	180 TeV	$\Lambda_8$	280 TeV	$\Lambda_8$	230 TeV
$r_3$	2.8	$r_3$	8/3	$r_3$	-2.55
$\tan \beta$	15	$\tan \beta$	15	$\tan \beta$	15
$m_h$	123.1 GeV	$m_h$	125.1 GeV	$m_h$	123.0 GeV
$\Delta$	69	$\Delta$	156	$\Delta$	91
$\mu$	538 GeV	$\mu$	850 GeV	$\mu$	652 GeV
$m_{\text{gl}}$	3.6 TeV	$m_{\text{gl}}$	5.4 TeV	$m_{\text{gl}}$	4.5 TeV
$m_{\text{sq}}$	3.4 - 4.5 TeV	$m_{\text{sq}}$	5.1 - 6.7 TeV	$m_{\text{sq}}$	4.2 - 5.5 TeV
$m_{\text{st}}$	2.2, 4.1 TeV	$m_{\text{st}}$	3.4, 6.2 TeV	$m_{\text{st}}$	3.1, 5.1 TeV
$m_{\tilde{e}_L}$	3.1 TeV	$m_{\tilde{e}_L}$	4.5 TeV	$m_{\tilde{e}_L}$	3.6 TeV
$m_{\tilde{e}_R}$	473 GeV	$m_{\tilde{e}_R}$	727 GeV	$m_{\tilde{e}_R}$	618 GeV
$m_{\tilde{\tau}_1}$	221 GeV	$m_{\tilde{\tau}_1}$	399 GeV	$m_{\tilde{\tau}_1}$	394 GeV
$m_{\chi_1^0}$	128 GeV	$m_{\chi_1^0}$	124 GeV	$m_{\chi_1^0}$	131 GeV
$m_{\chi_1^\pm}$	550 GeV	$m_{\chi_1^\pm}$	870 GeV	$m_{\chi_1^\pm}$	670 GeV
$m_{\chi_2^\pm}$	2.6 TeV	$m_{\chi_2^\pm}$	3.8 TeV	$m_{\chi_2^\pm}$	3.1 TeV

# F.T. 'then' and 'now'



- Years ago,  $\Delta \sim 50$  for  $M \sim 10^5$  TeV and it was worse than mSUGRA then (G.B., Romanino 1997).
- In 20 years it has gone up by a factor of  $\sim 20$ .

# Conclusions

- Does naturalness demand that super-particles all have to be simultaneously heavy? OR, sleptons/weak gauginos can remain significantly lighter than squarks/gluino by internal dynamics?
- Key observation:** With unconventional choice of messenger multiplets, a color SU(3) octet and a weak SU(2) triplet, GMSB works:
  - unification at string scale (between GUT and Planck scale).
  - colored mass  $\gg$  uncolored mass of sparticles by intrinsic dynamics.
  - Introducing in addition the SU(5) 5-plets, it is possible to explain Muon  $(g - 2)$  within  $1\sigma$ . *Scenario fine-tuned with  $\mu \sim \text{few TeV}$ .*
  - If we give up  $(g - 2)$ , then with just  $\Sigma_3$  and  $\Sigma_8$ , Focus Point can be achieved introducing a RG-invariant parameter.
  - Lighter stau is in (100-400) GeV range which can be a target at ILC.

**GMSB with Adjoint Messenger multiplets ( $\Sigma_3$  and  $\Sigma_8$ ) is an attractive scenario**