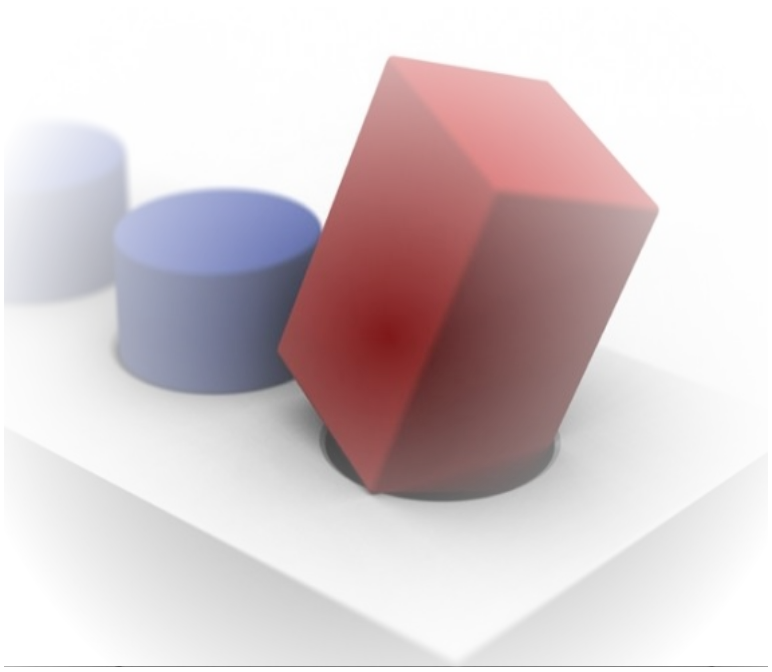
Determination of EW parameters using HERA data



A. Cooper-Sarkar, C. Gwenlan,
K. Klimek, V. Myronenko

ZEUS collaboration week
08.09.2015, Hamburg

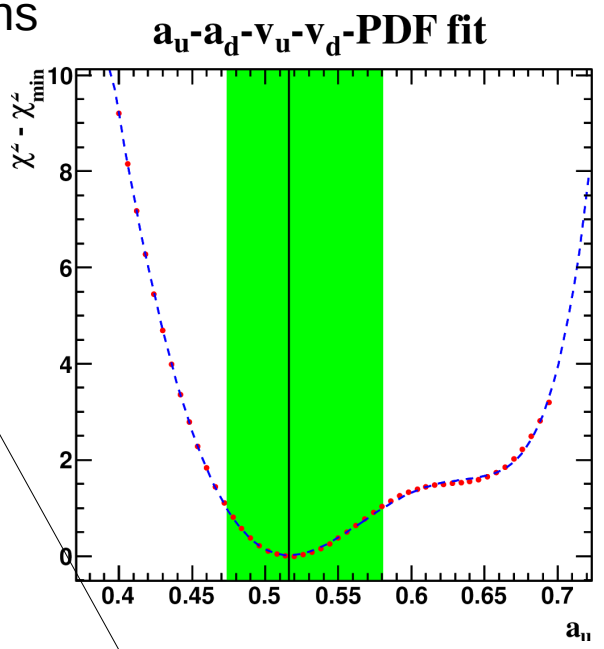
13p contour scanning (MINOS)

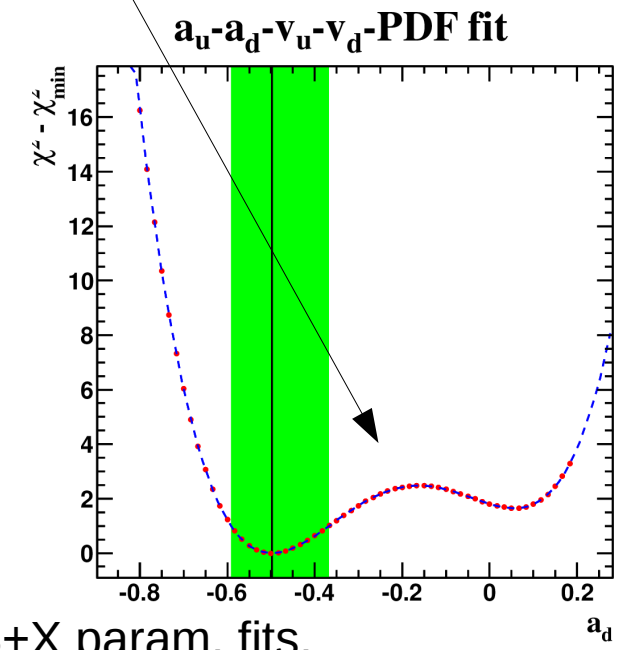
8 points out of 100.

Flips between solutions

```
ERR MATRIX NOT POS-DEF
MNCONT CANNOT FIND NEXT POINT ON CONTOUR.  ONLY   8 POINTS FOUND.
Y-AXIS: PARAMETER 107  vdEW
-0.6000000E-01 ..........................................................
                  .     X       .                                       .
-0.1000000      ...              .                                       .
                  .              .                                       .
-0.1400000      ...              .                                       .
                  .              .                                       .
-0.1800000      ...              .                                       .
                  .              .                                       .
-0.2200000      ...X             .                                       .
                  .              .                                       .
-0.2600000      ...              .                                       .
                  .             .X                                       .
-0.3000000      ...              .                                       .
                  .              .                                       .
-0.3400000      ...              .                                       .
                  .              .                                    X. .
-0.3800000      ...              .                                       .
                  .              .                                       .
-0.4200000      ...              .           X                           .
                  .              .                                       .
-0.4600000      ................ ........................................
                  .              .                                       .
-0.5000000      ...              .                                       .
                  .              .                                       .
-0.5400000      ...              .                                       .
                  .         X    .                                       .
-0.5800000      ...              .                                       .
                  .              .                                    X  .
-0.6200000      ...              .                                       .
                  .              .                                       .
-0.6600000      ...              .                                       .
                  .              .                                       .
-0.7000000      ...              .             X                         .
-0.7200000      ..........................................................
                /       /       /       /       /       /
           -.8000    -.6000    -.4000    -.2000     0.000    0.2000
                     ONE COLUMN=0.2000000E-01
                     X-AXIS: PARAMETER 105  adEW

 8 POINTS ON CONTOUR.   FMIN=   3588.9       ERRDEF=   2.30
      adEW        vdEW                        adEW        vdEW
  1 -0.78938     -0.21929                 5  0.27643     -0.34565
  2 -0.59540     -0.55821                 6 -0.26999     -0.41654
  3 -0.22450     -0.68039                 7 -0.50077     -0.26280
  4  0.82860E-01 -0.59817                 8 -0.69083     -0.60878E-01
                 PARAMETER 107, vdEW      RESTORED TO VARIABLE.
                 PARAMETER 105, adEW      RESTORED TO VARIABLE.
```



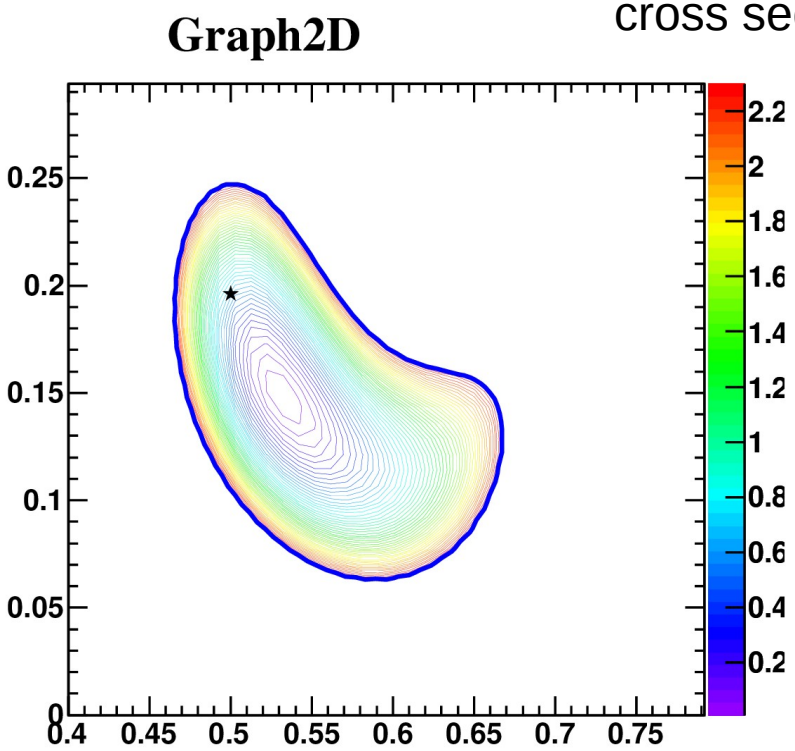


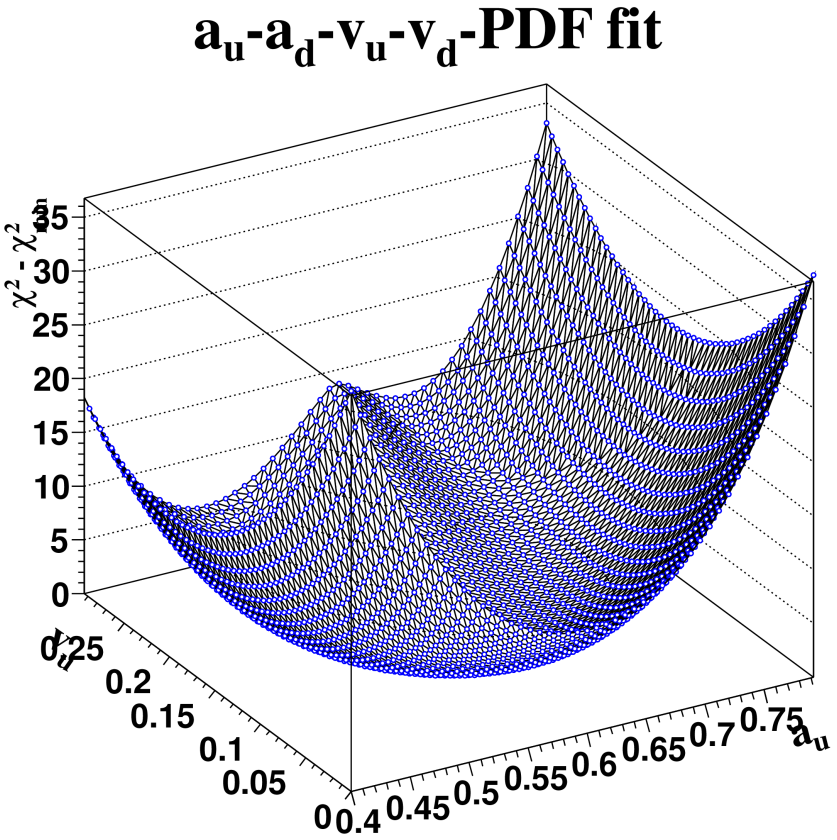
- Takes ~ a week to get the contours! Way too slow for 13+X param. fits.

Alternative to MINOS contour scan

- MINOS result
- Scan (grid of ~2500 MIGRAD/HESSE fits per given plane)

Using Delaunay triangles to define a surface and plotting cross section of the surface at certain chi2 - $chi2_{min}$





Takes 4 times less time for 10+4p; 3-2 times less time for 13+4p.

Can reveal possible multiple minima.

On condition of normal computer farm work :)

- Results agree with contours from MINOS at 10p+EW fit

Valence-like gluon at low Q^2

Very high Ag and Ag'
Positive Bg and Bg'
Different solutions from MIGRAD and MINOS

Regular and more precise high Ag and Ag'
Negative Bg and Bg'
Same solutions from MIGRAD and MINOS

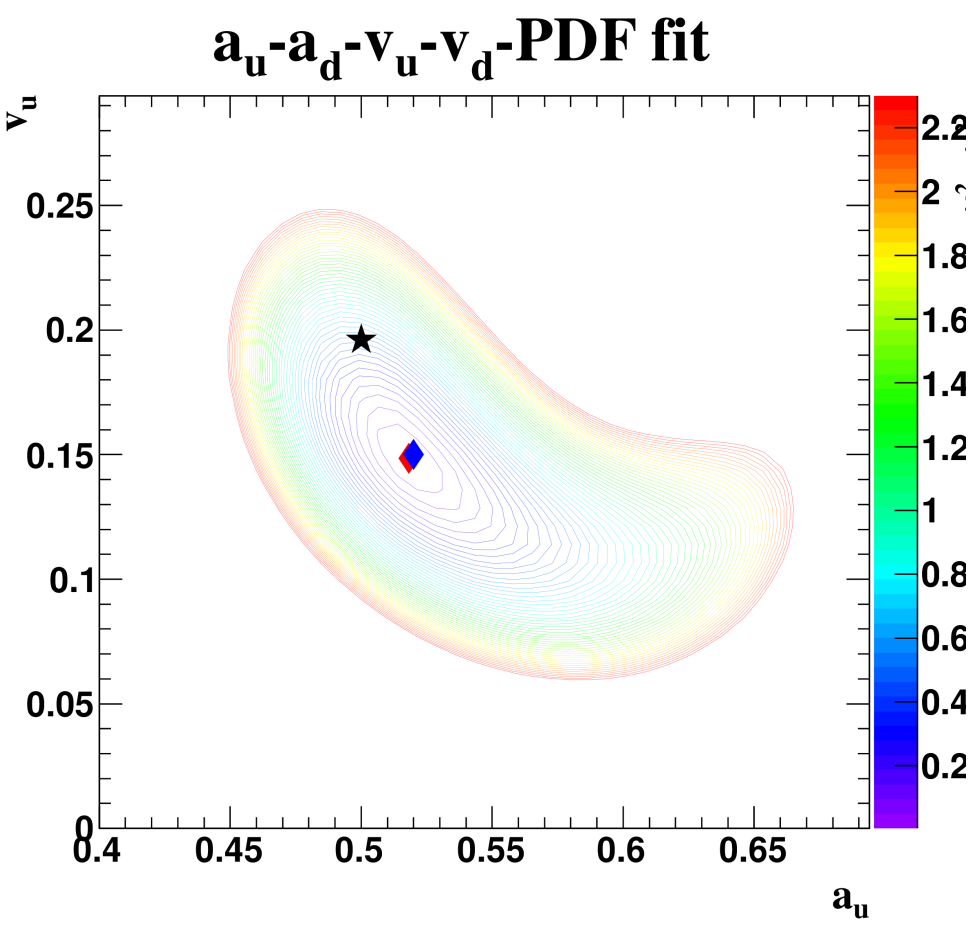
Parameter	Pos Bg MIGRAD	Pos Bg MINOS	Neg Bg MIGRAD	Neg Bg MINOS
'Bg'	0.189 ± 0.045	0.327 ± 0.064	−0.030 ± 0.079	−0.030 ± 0.075
'Cg'	10.0 ± 1.2	14.2 ± 2.1	8.63 ± 0.86	8.62 ± 0.82
'Aprig'	3.4 ± 3.2	33 ± 22	1.51 ± 0.70	1.50 ± 0.62
'Bprig'	0.30 ± 0.16	0.545 ± 0.086	−0.118 ± 0.067	−0.118 ± 0.067
'Cprig'	25.00	25.00	25.00	25.00
'Buv'	0.748 ± 0.026	0.761 ± 0.027	0.745 ± 0.026	0.745 ± 0.025
'Cuv'	4.712 ± 0.085	4.671 ± 0.091	4.708 ± 0.085	4.708 ± 0.084
'Euv'	9.3 ± 1.2	8.4 ± 1.2	9.4 ± 1.2	9.4 ± 1.2
'Bdv'	0.789 ± 0.081	0.817 ± 0.085	0.781 ± 0.079	0.781 ± 0.078
'Cdv'	4.40 ± 0.32	4.61 ± 0.34	4.37 ± 0.32	4.37 ± 0.32
'CUbar'	3.60 ± 0.48	3.52 ± 0.49	3.60 ± 0.47	3.59 ± 0.47
'ADbar'	0.1993 ± 0.0088	0.1978 ± 0.0091	0.1994 ± 0.0089	0.1994 ± 0.0088
'BDbar'	−0.1584 ± 0.0053	−0.1596 ± 0.0054	−0.1582 ± 0.0053	−0.1582 ± 0.0053
'CDbar'	4.4 ± 1.1	4.1 ± 1.1	4.4 ± 1.1	4.4 ± 1.1
'alphas'	0.1180	0.1180	0.1180	0.1180
'fs'	0.4000	0.4000	0.4000	0.4000
'auEW'	0.516 ± 0.062	0.516 ± 0.074	0.518 ± 0.061	0.518 ± 0.061
'adEW'	−0.52 ± 0.23	−0.51 ± 0.27	−0.52 ± 0.22	−0.52 ± 0.22
'vuEW'	0.148 ± 0.071	0.139 ± 0.078	0.148 ± 0.070	0.148 ± 0.070
'vdEW'	−0.44 ± 0.19	−0.47 ± 0.20	−0.44 ± 0.18	−0.44 ± 0.18
Fit status	converged	converged	converged	converged
Uncertainties	migrad-hesse	migrad-hesse	migrad-hesse	migrad-hesse
Total χ^2 / dof	3589 / 3231	3587 / 3231	3590 / 3231	3590 / 3231

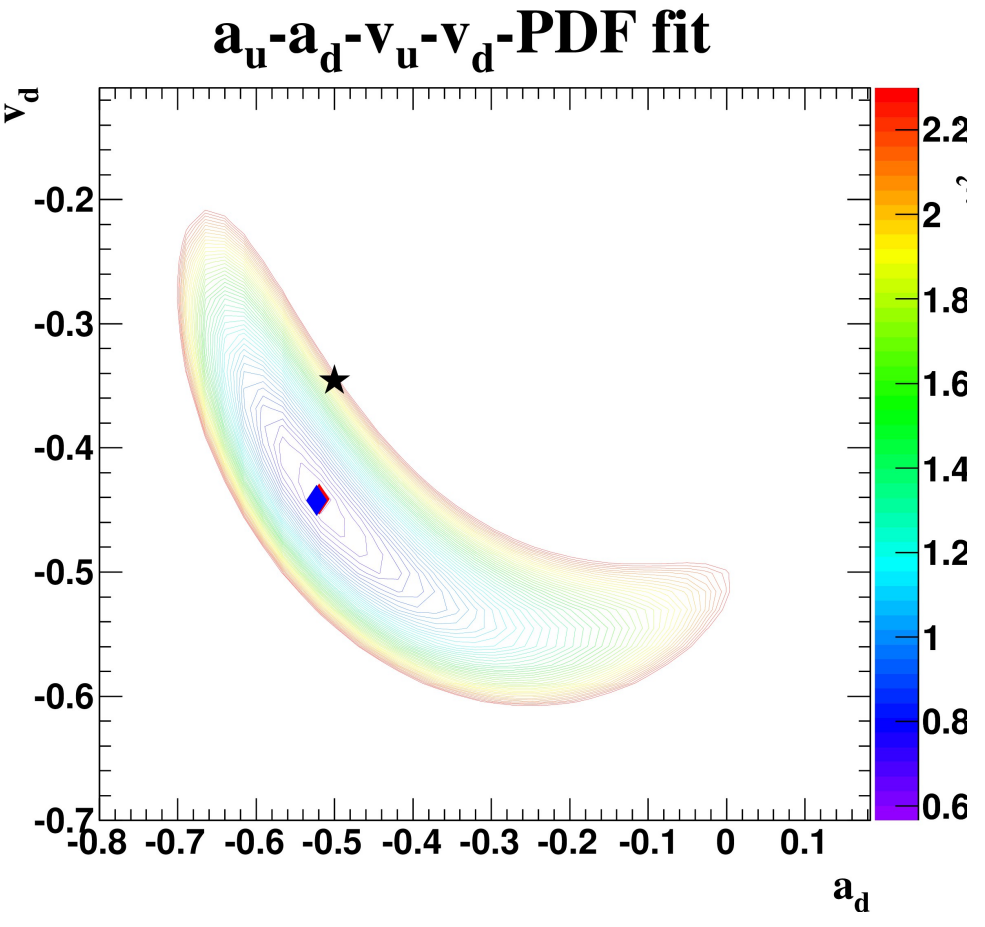
Chi2 does not change very significantly

Negative BG and Bg' MIGRAD and MINOS give the same minima
Boundaries on Bg and Bg' were set to [-100; 0.05]

13p-EW fit

- Contours now look way less banana-shaped.

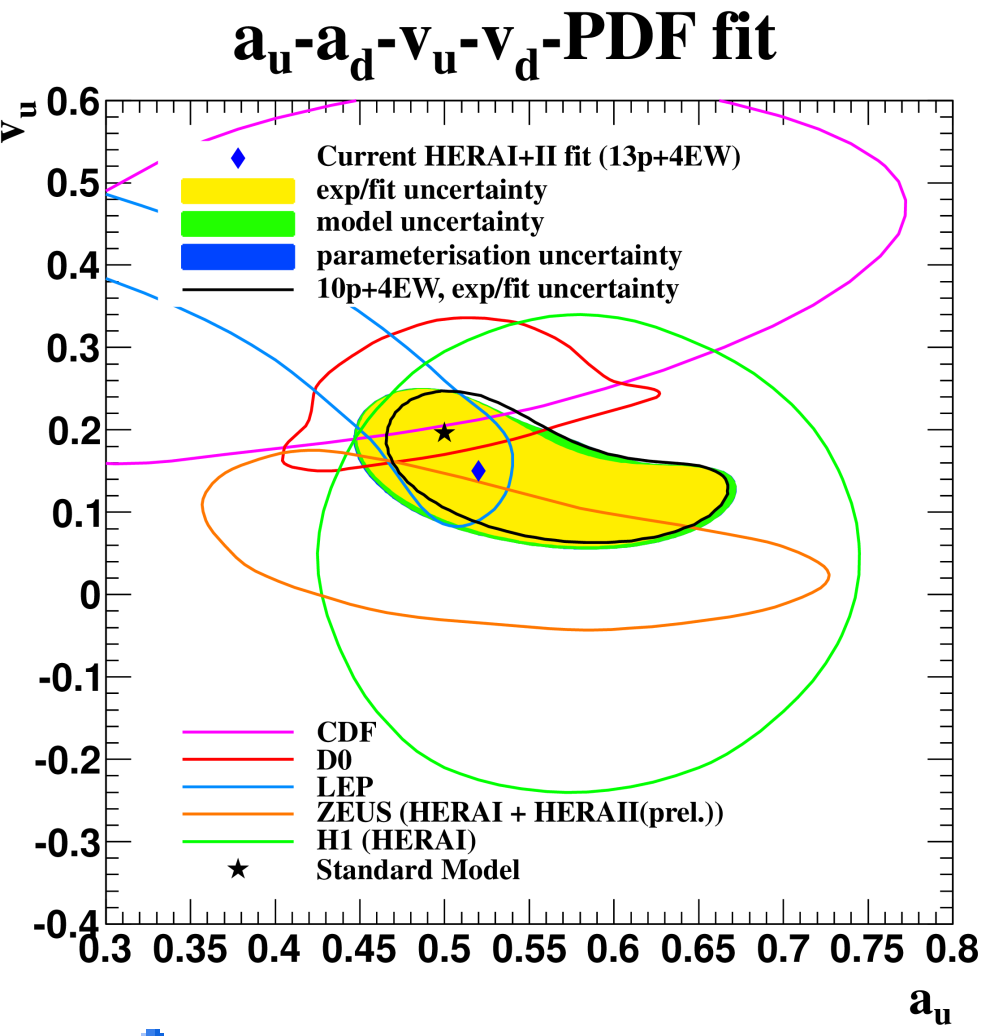


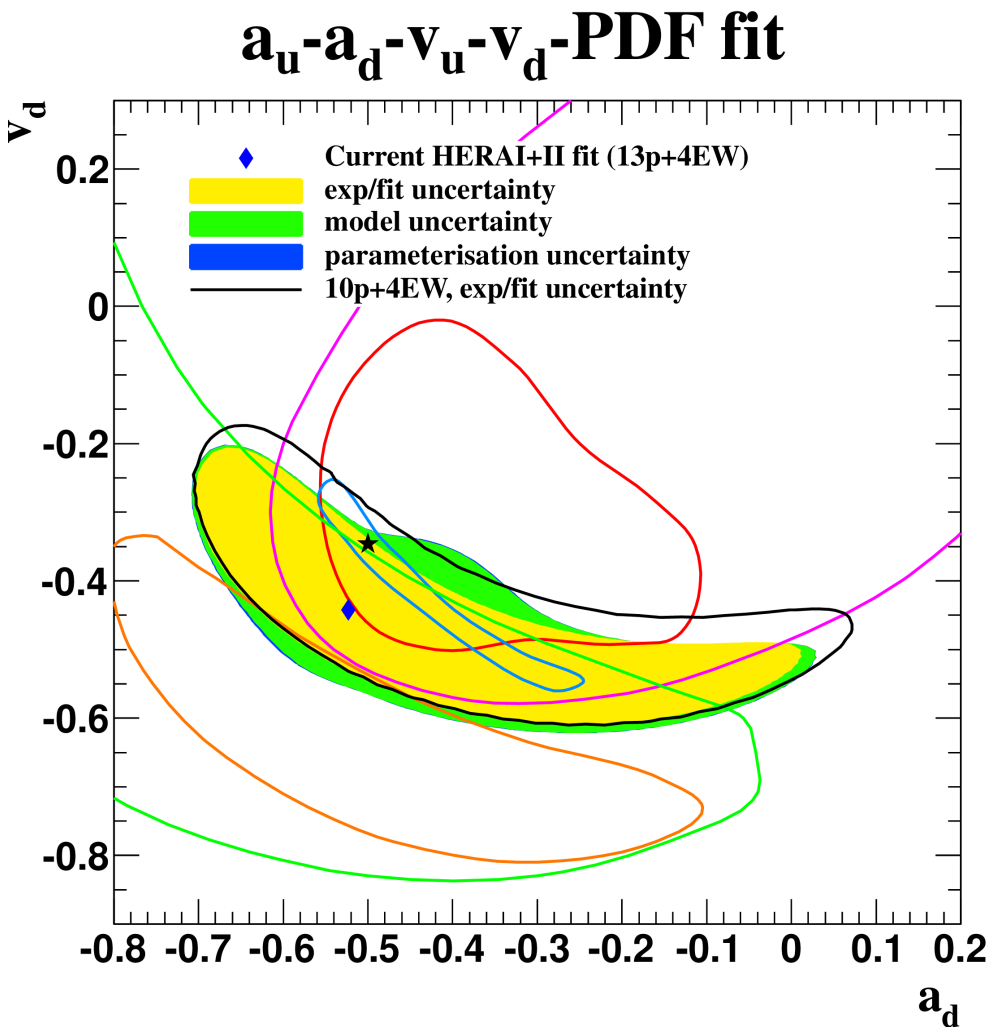


Contours are not accessible via MINOS at 13p.

- MIGRAD/HESSE and current scanned minima are the same for **13p+4EW fit**
- MIGRAD agree with the result from MINOS.

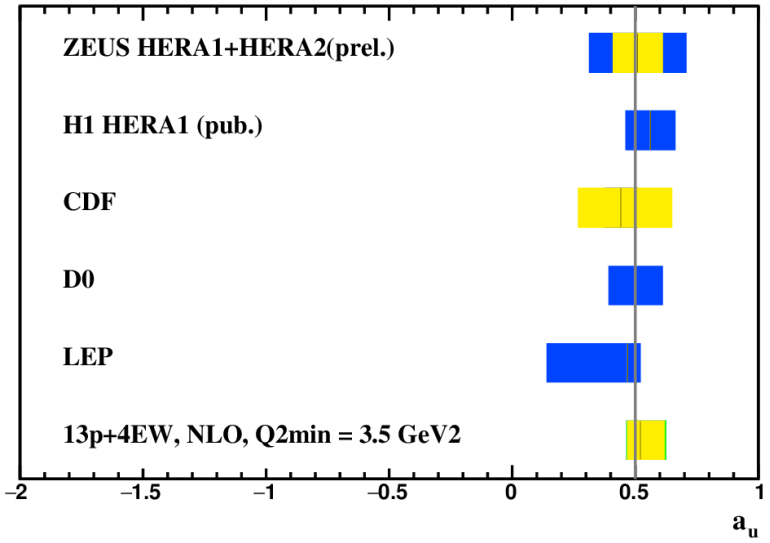
13p-EW fit

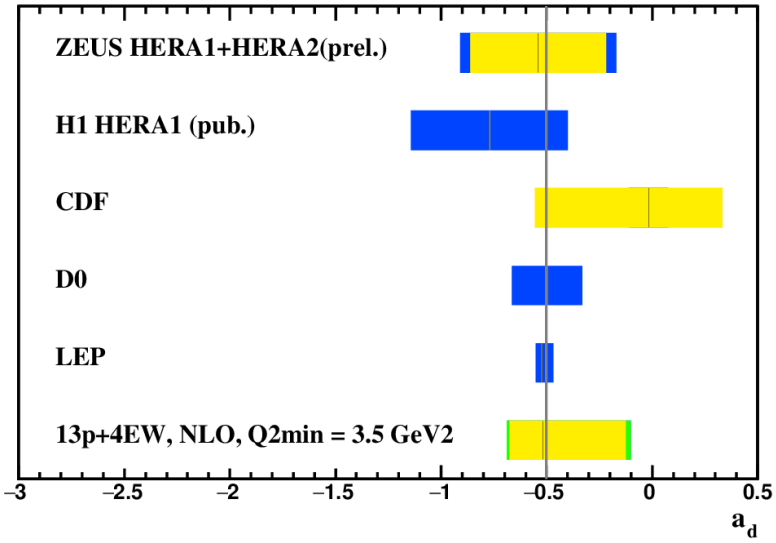


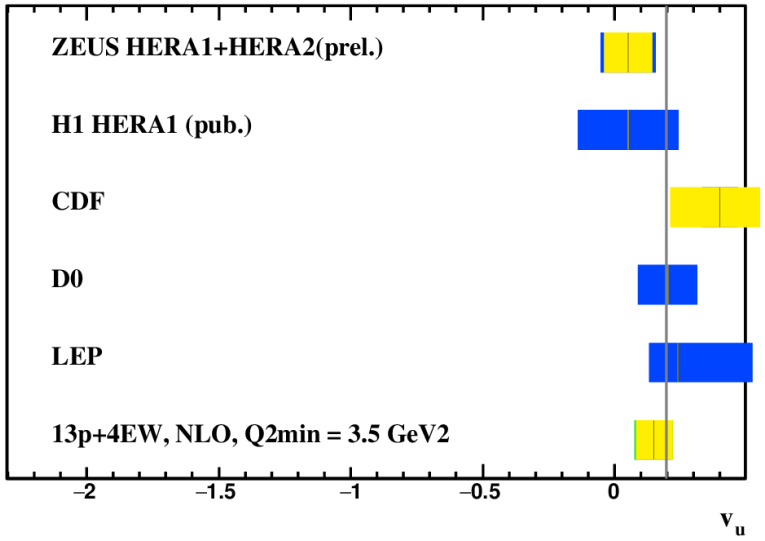


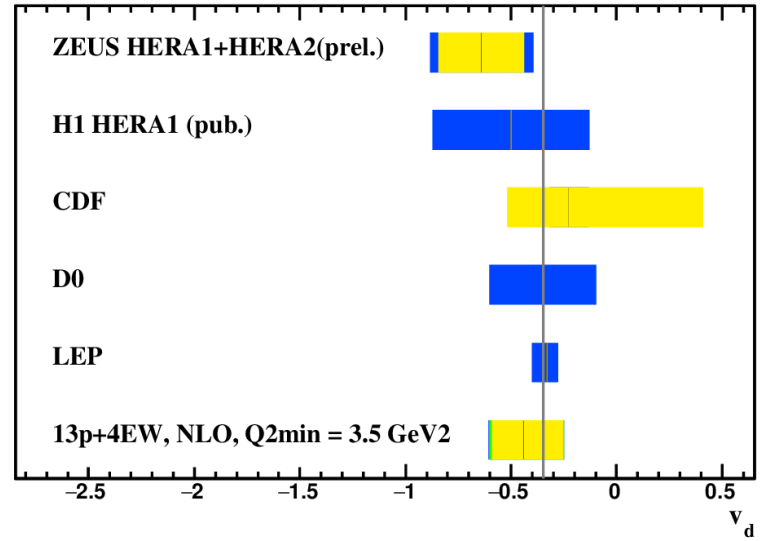
- Usual version version and unc. breakdown → in the backup.
- Overall, the results look compatible with ones published before.
- Use the scan technique for contours and MINOS for central values and 1D uncertainties.

World results (full uncertainties)









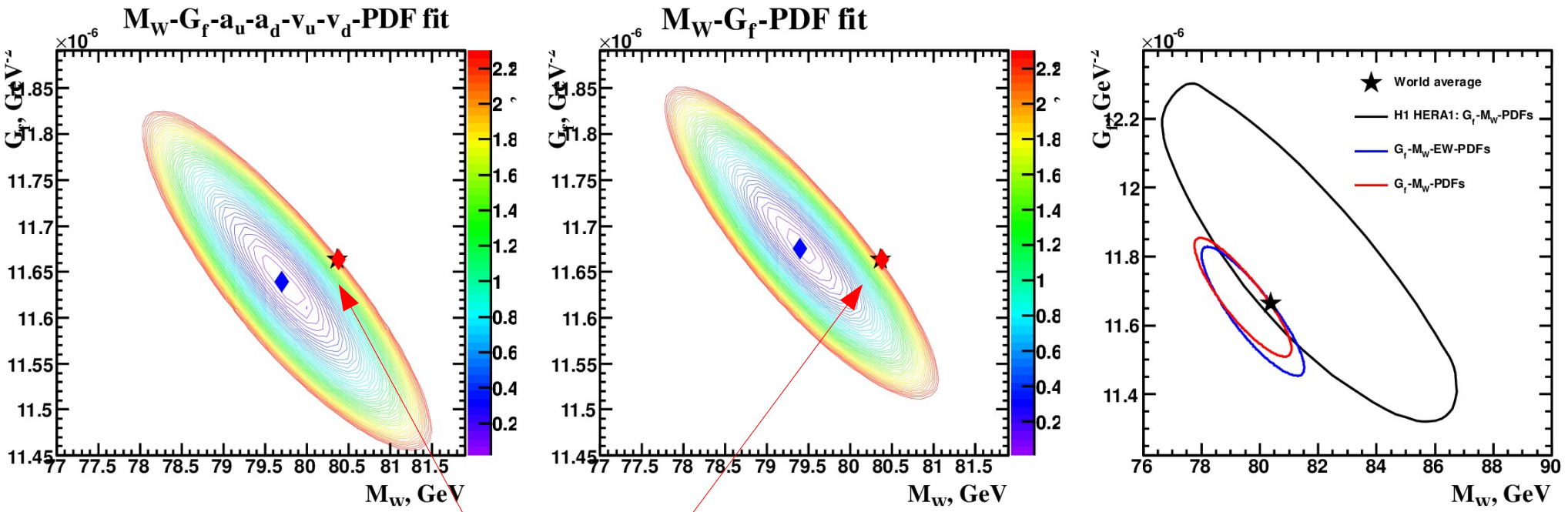
- Fit (experimental) uncertainty
- Paramatrisation uncertainty
- Model uncertainty
- Full (provided) uncertainty

World results (full uncertainties)

	a_u	a_d	v_u	v_d
LEP	$0.47^{+0.05}_{-0.33}$	$-0.52^{+0.05}_{-0.03}$	$0.24^{+0.28}_{-0.11}$	$-0.33^{+0.05}_{-0.07}$
D0	0.50±0.11	-0.50±0.17	0.20±0.11	0.35±0.25
CDF	$0.44^{+0.22}_{-0.19}$	$-0.02^{+0.36}_{-0.54}$	$0.40^{+0.17}_{-0.20}$	$-0.23^{+0.64}_{-0.30}$
H1: HERA1 (publ.)	0.56±0.10	-0.77±-0.37	0.05±0.19	-0.50±0.37
ZEUS: HERA1+2 (prel.)	0.51±0.20	-0.54±0.37	0.05±0.10	-0.64±0.24
H1+ZEUS: HERA1+2 13p+EW	$0.518^{+0.106}_{-0.054}$	$-0.52^{+0.42}_{-0.17}$	$0.148^{+0.070}_{-0.071}$	$-0.44^{+0.19}_{-0.16}$
SM	0.5	-0.5	0.196	-0.346

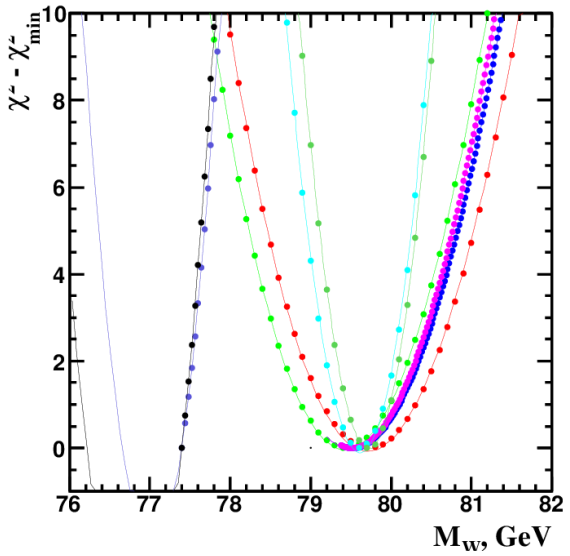
- Current <u>full</u> uncertainties are compatible with ones from other experiments.
  - Note precision on a_u and v_u.

10p-M_W-G_f(-EW) fit



- ZEUS CC data were translated back to non-reduced cross sections.
  - HERAPDF2.0 (PDG) settings are within uncertainty.
  - Uncertainties are reduced noticeably.
  - Correlation of M_W and G_f is confirmed (since H1 paper).
  - Fitting M_W and G_f with couplings demonstrates some cross-correlations.

Variants of M_W, $\sin^2\theta_W$, G_f fits

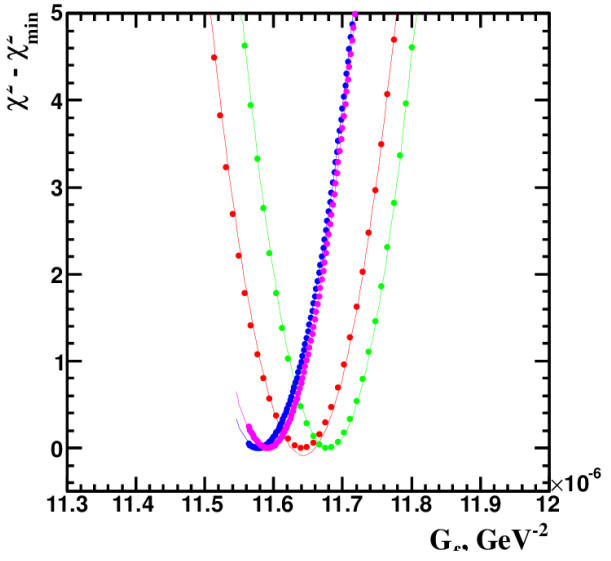


- M_W-G_f-EW-PDFs: M_W = 79.709 ± 0.877 GeV
- M_W-G_f-PDFs: M_W = 79.428 ± 0.831 GeV
- M_W-EW-PDFs: M_W = 79.504 ± 0.592 GeV
- M_W-PDFs: M_W = 79.495 ± 0.569 GeV

G_f->(M_W, $\sin^2\theta_W$)

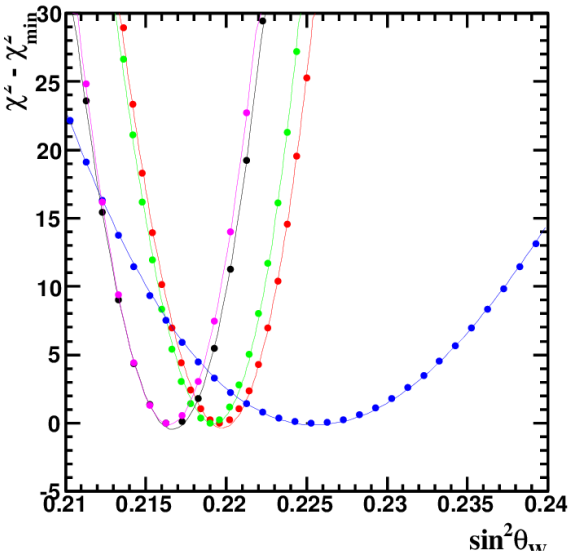
- M_W-$\sin^2\theta_W$-EW-PDFs: M_W = 79.617 ± 0.427 GeV
- M_W-$\sin^2\theta_W$-PDFs: M_W = 79.719 ± 0.401 GeV
- M_W-EW-PDFs: M_W = 77.031 ± 0.242 GeV
- M_W-PDFs: M_W = 76.819 ± 0.263 GeV

PDG: M_W = 80.363 ± 0.006 GeV



- M_W-G_f-EW-PDFs: G_f = 1.1643e-05 ± 9.07e-08 GeV^{-2}
- M_W-G_f-PDFs: G_f = 1.1680e-05 ± 8.56e-08 GeV^{-2}
- G_f-EW-PDFs: G_f = 1.1580e-05 ± 6.06e-08 GeV^{-2}
- G_f-PDFs: G_f = 1.1591e-05 ± 5.70e-08 GeV^{-2}

PDG: G_f = 1.1663787E-05 ± 6E-12 GeV^{-2}



- $\sin^2\theta_W$-PDFs: $\sin^2\theta_W$ = 0.22552 ± 0.003519

G_f->(M_W, $\sin^2\theta_W$)

- M_W-$\sin^2\theta_W$-EW-PDFs: $\sin^2\theta_W$ = 0.21974 ± 0.001624
- M_W-$\sin^2\theta_W$-PDFs: $\sin^2\theta_W$ = 0.21922 ± 0.001548
- $\sin^2\theta_W$-EW-PDFs: $\sin^2\theta_W$ = 0.21668 ± 0.001058
- $\sin^2\theta_W$-PDFs: $\sin^2\theta_W$ = 0.21647 ± 0.001019

$\sin^2\theta_w^{\overline{MS}}$ = 0.23126±0.00005

$\sin^2\theta_w^{On\text{-}shell}$ = 0.22333±0.00011

$\sin^2\theta_w^{eff}$ = 0.23155±0.000005

- Discussion of the results with H. Spiesberger is ongoing.

Summary

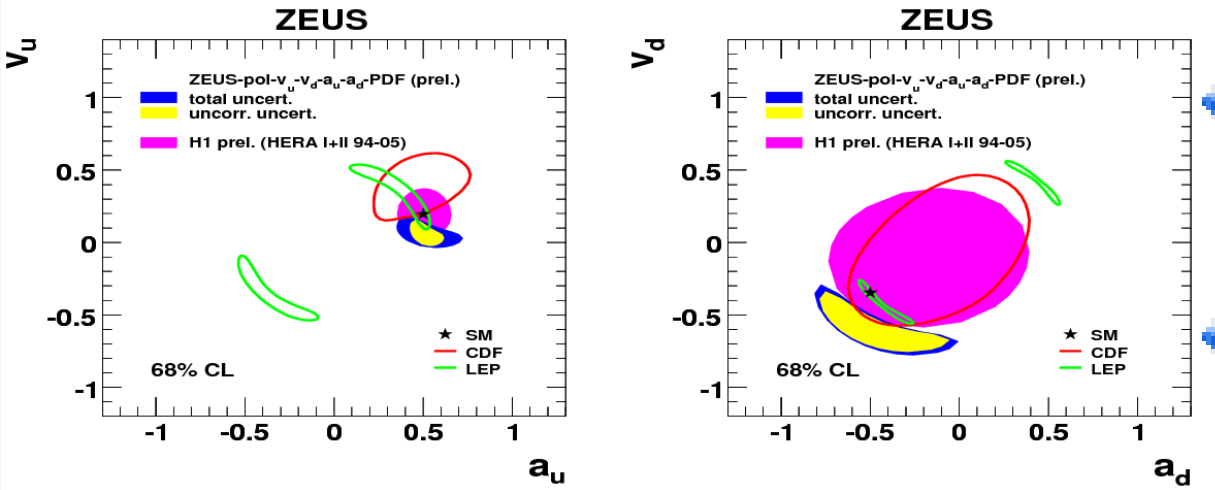
- The alternative to MINOS contours is introduced. (problem solved for 13p fits);
- Issue of positive Bg/Bg' is solved, MINOS and MIGRAD now agree;
- Model and parametrisation uncertainties variations are ready for 13p+4EW fit;
- Various fits of G_f, M_W and $\sin^2\theta_W$ were performed:
  - Uncertainties are reduced;
  - Fitted results agree with world averages;
  - Discussing the results with H. Spiesberger so far;

Plans

- Finalising the idea of what should go to the publication;
- Writing a paper.

Backup

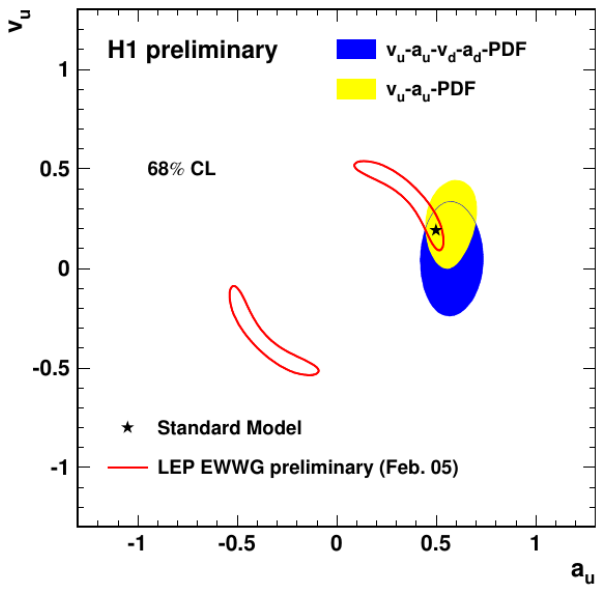
Motivation

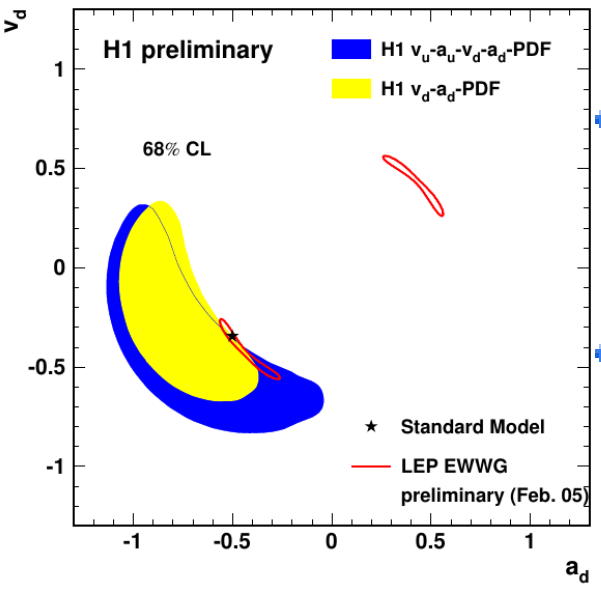


- Determination of EW par. by ZEUS

  ZEUS HERA I + HERA II e^-p (pol)

- ZEUS-prel-07-027





- Determination of EW par. by H1

  H1 HERA I (unpolarized)

- Phys. Lett. **B632**, 35, (2006)

- All H1 and ZEUS HERA I unpolarized and HERA II polarized data are now available

$$\tilde{F_2}^{\pm} = F_2 + k_Z(-v_e \mp Pa_e) \cdot F_2^{\gamma Z} + k_Z^2(v_e^2 + a_e^2 \pm 2Pv_e a_e) \cdot F_2^Z$$

$$x\tilde{F_3}^{\pm} = k_Z(\pm a_e + Pv_e) \cdot xF_3^{\gamma Z} + k_Z^2(\mp 2v_e a_e - P(v_e^2 + a_e^2)) \cdot xF_3^Z$$

$$(F_2, F_2^{\gamma Z}, F_2^Z) = x\sum(e_q^2, 2e_q v_q, v_q^2 + a_q^2)(q + \bar{q}) \qquad (xF_3^{\gamma Z}, xF_3^Z) = 2x\sum(e_q a_q, v_q a_q)(q - \bar{q})$$

$$\left.\frac{d^2\sigma^{\pm}}{dxdQ^2}\right|_{NC} = \frac{2\pi\alpha^2}{xQ^4}\left[H_0^{\pm} + \mathcal{P}H_{\mathcal{P}}^{\pm}\right], \qquad F_2^{0,\mathcal{P}} = \sum_i x(q_i + \bar{q}_i)A_i^{0,\mathcal{P}}$$

$$H_{0,\mathcal{P}}^{\pm} = Y_+ F_2^{0,\mathcal{P}} \mp Y_- x F_3^{0,\mathcal{P}} - \frac{y^2}{2} F_L^{0,\mathcal{P}} \qquad xF_3^{0,\mathcal{P}} = \sum_i x(q_i - \bar{q}_i)B_i^{0,\mathcal{P}}$$

$$A_i^0(Q^2) = e_i^2 - 2e_i v_i v_e P_Z + (v_e^2 + a_e^2)(v_i^2 + a_i^2)P_Z^2$$

$$B_i^0(Q^2) = -2e_i a_i a_e P_Z + 4a_i v_i v_e a_e P_Z^2$$

$$A_i^{\mathcal{P}} = 2e_i a_e v_i P_Z - 2a_e v_e (v_i^2 + a_i^2)P_Z^2$$
$$B_i^{\mathcal{P}} = 2e_i a_i v_e P_Z - 2a_i v_i (v_e^2 + a_e^2)P_Z^2.$$

$$P_Z = \frac{Q^2}{Q^2 + M_Z^2}\frac{1}{\sin^2 2\theta_W}$$

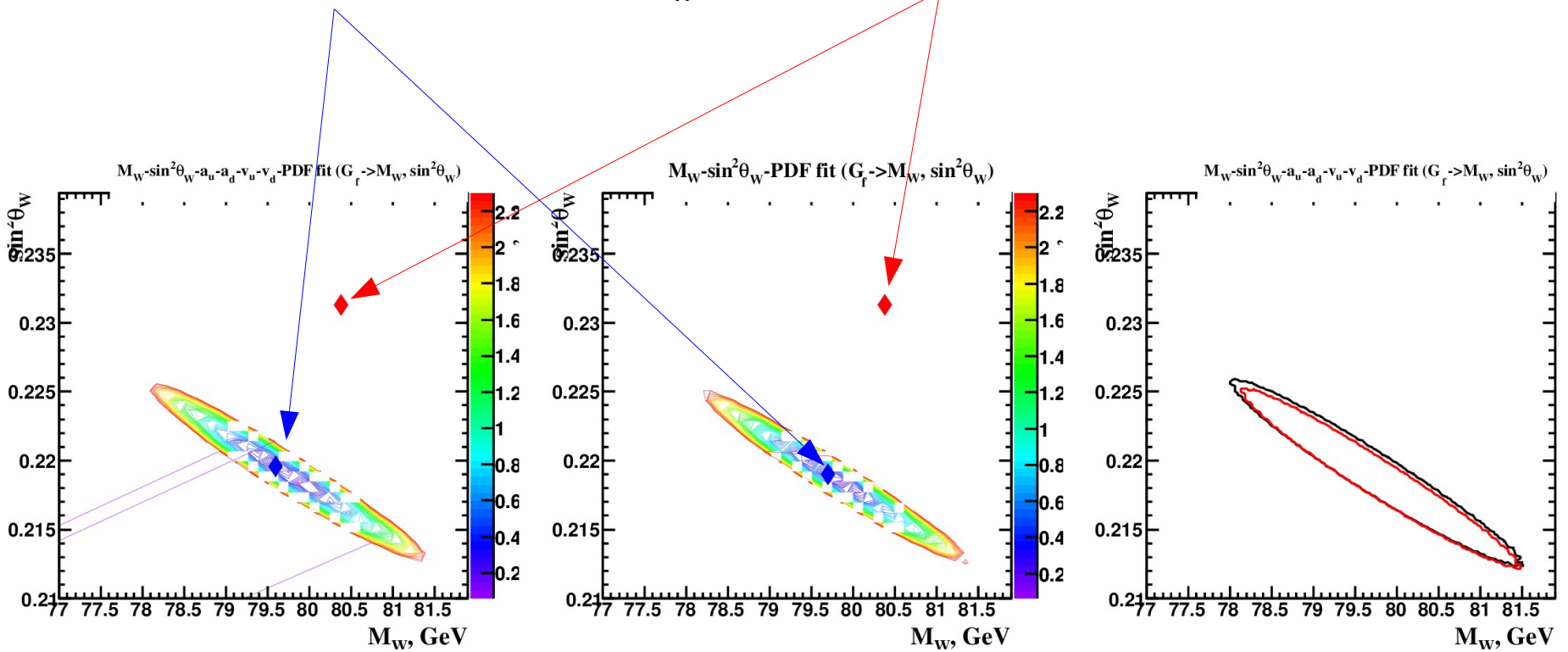
10p-M_W-$\sin^2\theta_W$(-EW) fit

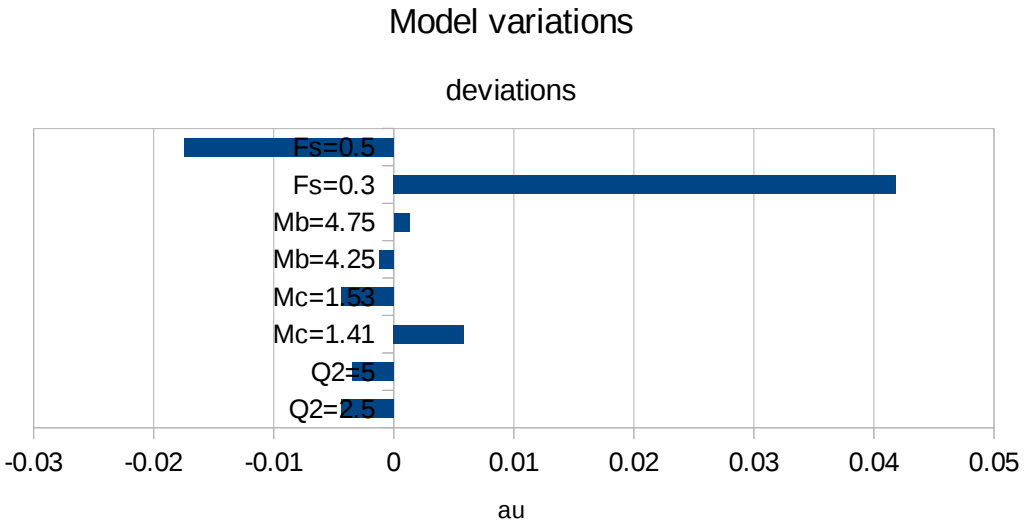
- One can redefine G_f in terms of M_W and $\sin^2\theta_W$. (on-shell scheme)

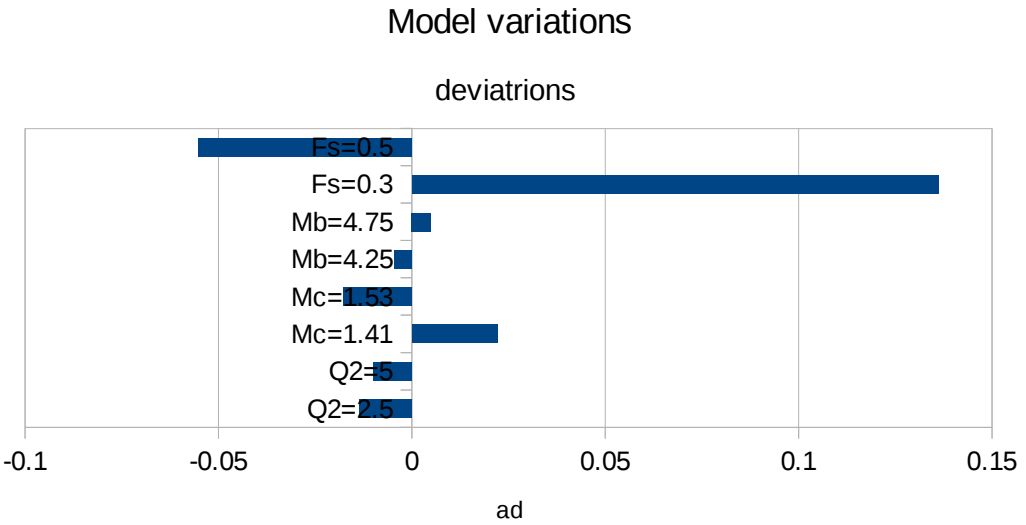
$$G_\mu = \frac{\pi\alpha}{\sqrt{2}s_w^2 M_W^2}\frac{1}{1-\Delta r}$$

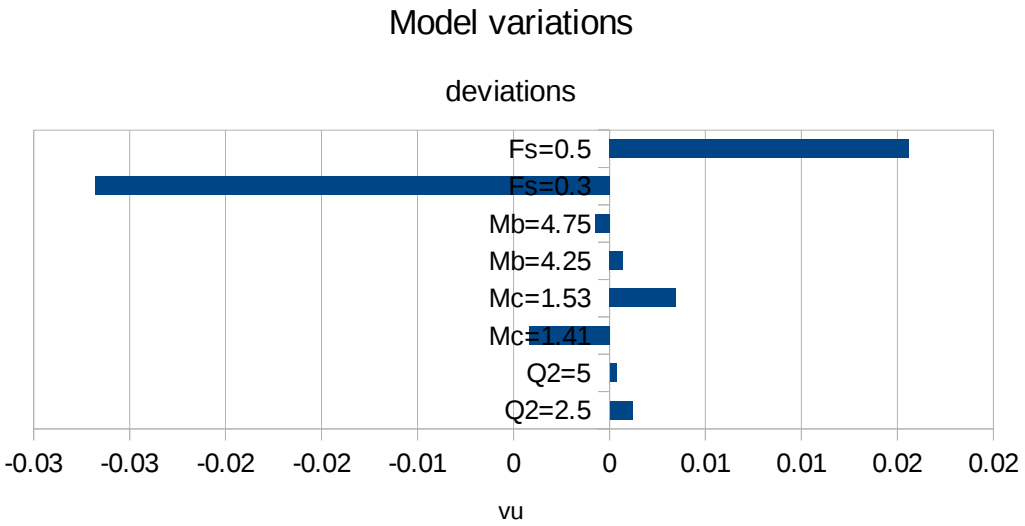
arXiv:hep-ph/9902277

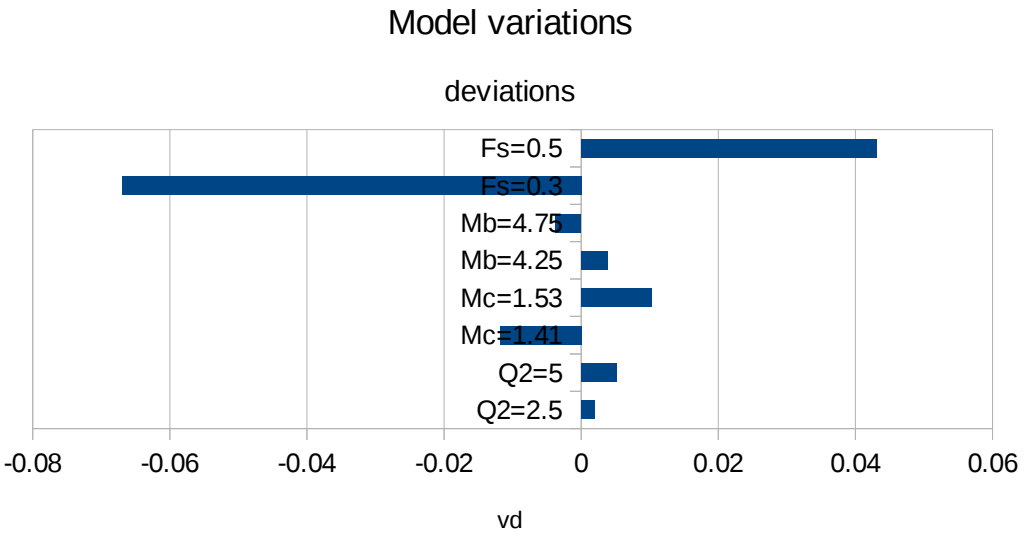
- Shows almost no difference between fits with released or fixed couplings.
- Fit points at considerably lover $\sin^2\theta_W$ than PDG (HERAPDF2.0).

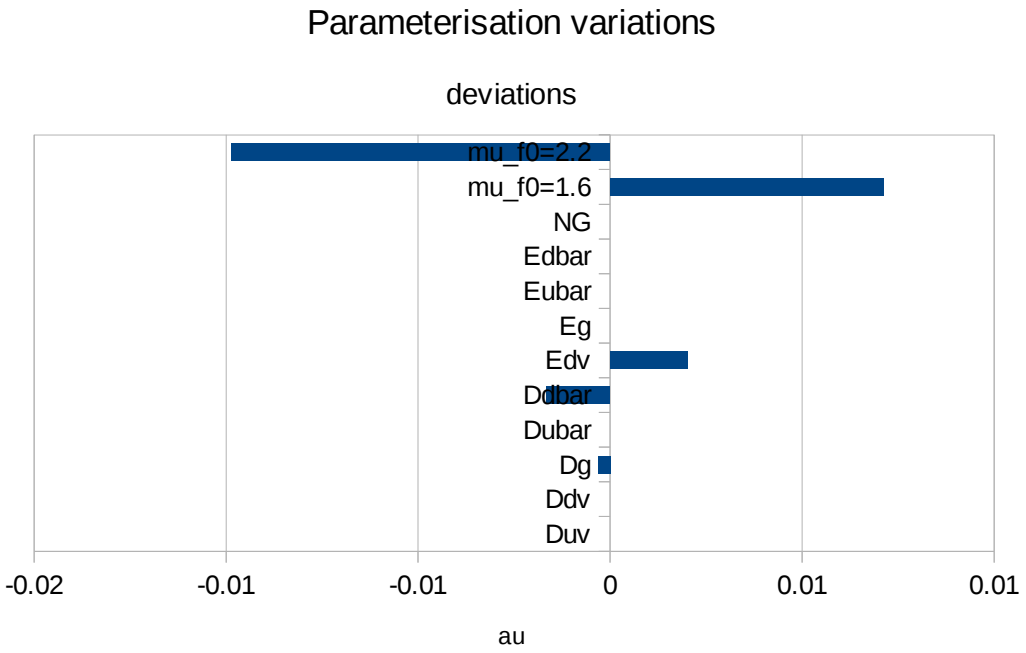


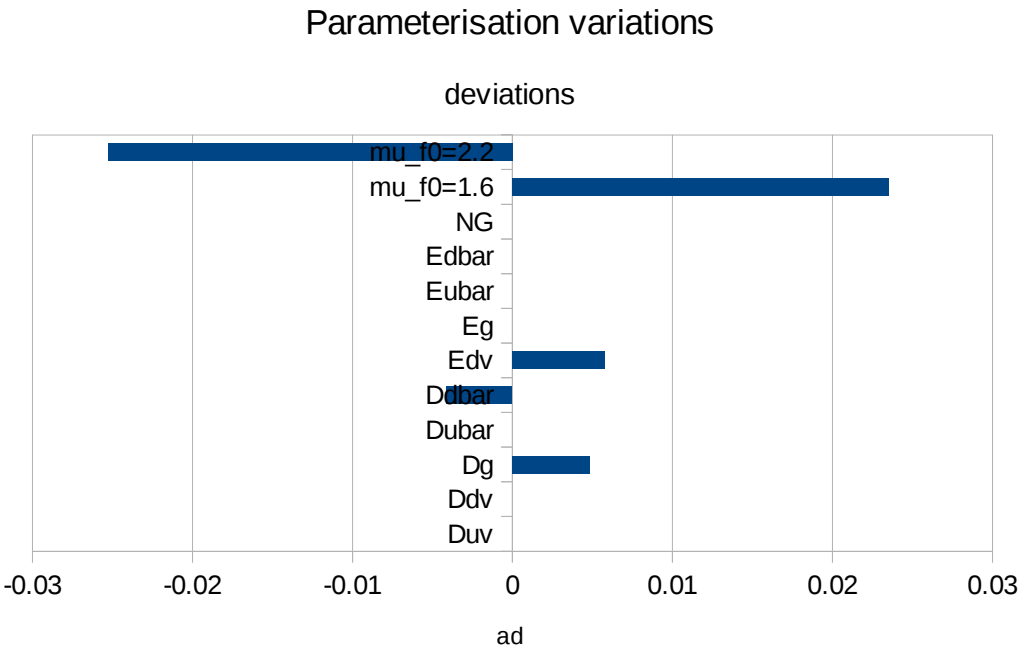

Model variations
deviations
Fs=0.5
Fs=0.3
Mb=4.75
Mb=4.25
Mc=1.53
Mc=1.41
Q2=5
Q2=2.5
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05
au

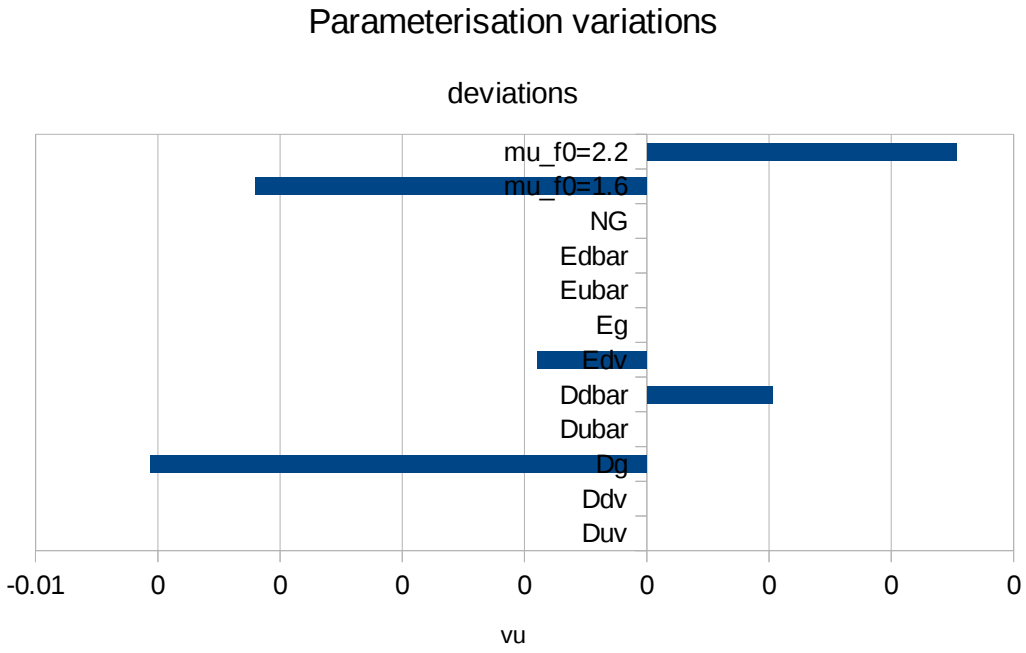

Model variations
deviatrions
Fs=0.5
Fs=0.3
Mb=4.75
Mb=4.25
Mc=1.53
Mc=1.41
Q2=5
Q2=2.5
-0.1
-0.05
0
0.05
0.1
0.15
ad

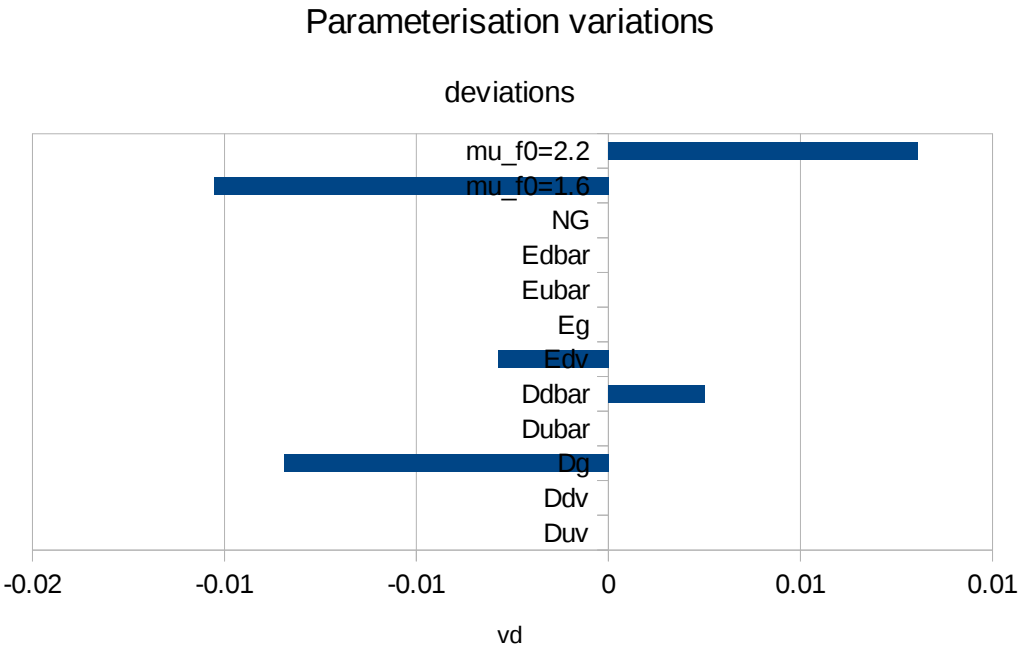

Model variations
deviations
Fs=0.5
Fs=0.3
Mb=4.75
Mb=4.25
Mc=1.53
Mc=1.41
Q2=5
Q2=2.5
-0.03
-0.03
-0.02
-0.02
-0.01
0
0
0.01
0.01
0.02
0.02
vu

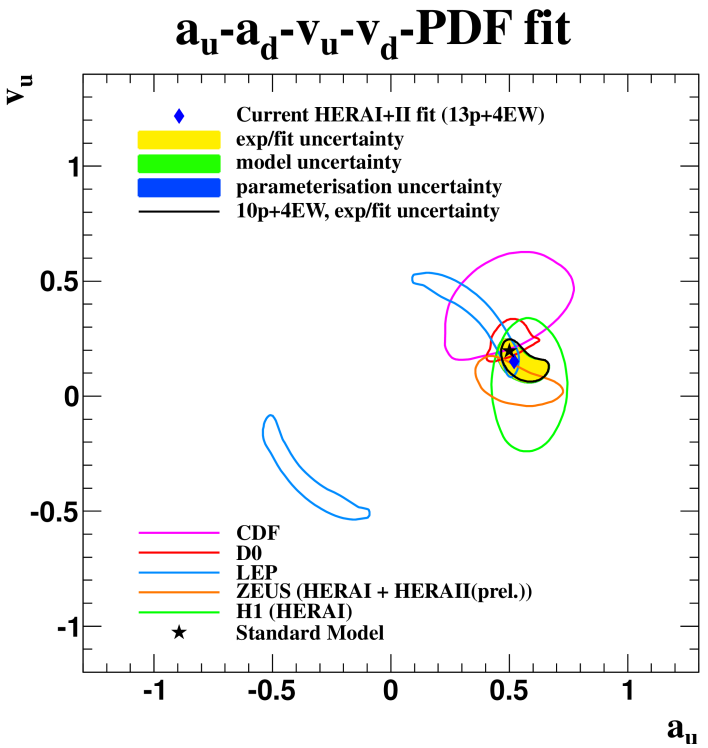

Model variations
deviations
Fs=0.5
Fs=0.3
Mb=4.75
Mb=4.25
Mc=1.53
Mc=1.41
Q2=5
Q2=2.5
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
vd

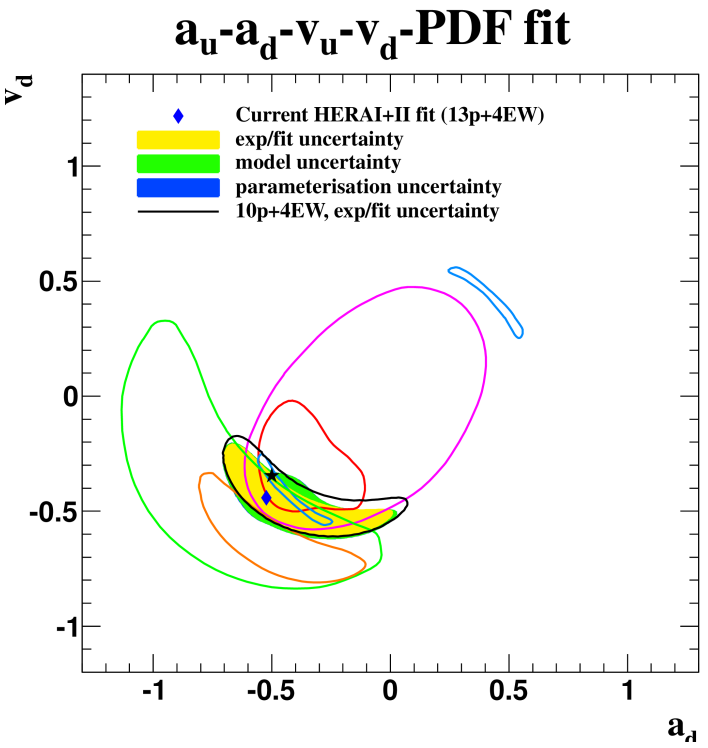

Parameterisation variations
deviations
mu_f0=2.2
mu_f0=1.6
NG
Edbar
Eubar
Eg
Edv
Ddbar
Dubar
Dg
Ddv
Duv
-0.02
-0.01
-0.01
0
0.01
0.01
au

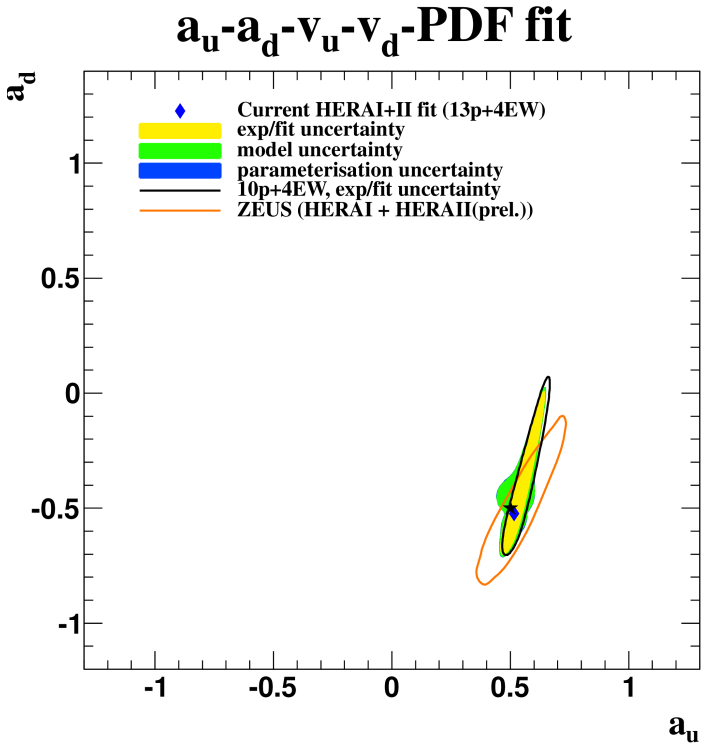

Parameterisation variations
deviations
mu_f0=2.2
mu_f0=1.6
NG
Edbar
Eubar
Eg
Edv
Ddbar
Dubar
Dg
Ddv
Duv
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
ad

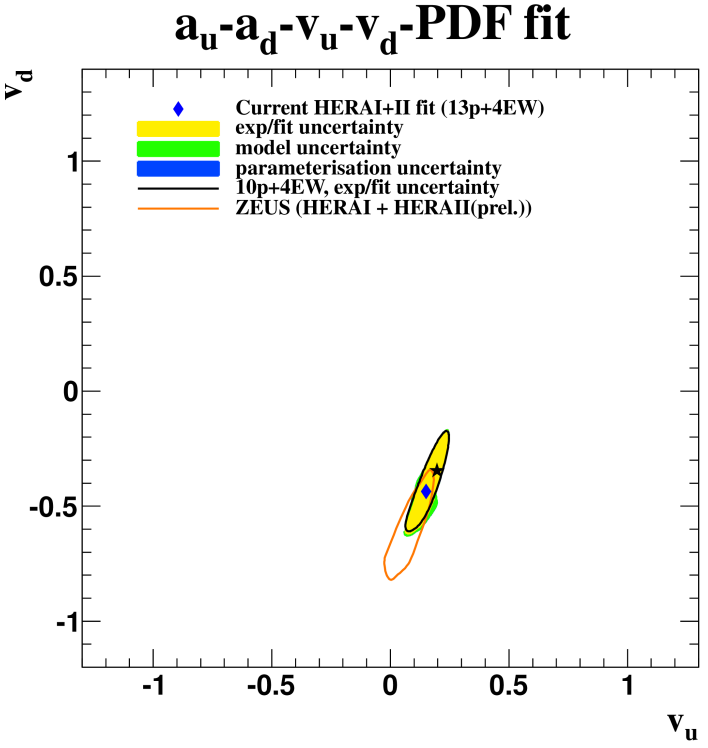

Parameterisation variations
deviations
mu_f0=2.2
mu_f0=1.6
NG
Edbar
Eubar
Eg
Edv
Ddbar
Dubar
Dg
Ddv
Duv
-0.01
0
0
0
0
0
0
0
0
vu


Parameterisation variations
deviations
mu_f0=2.2
mu_f0=1.6
NG
Edbar
Eubar
Eg
Edv
Ddbar
Dubar
Dg
Ddv
Duv
-0.02
-0.01
-0.01
0
0.01
0.01
vd


a_u-a_d-v_u-v_d-PDF fit
Current HERAI+II fit (13p+4EW)
exp/fit uncertainty
model uncertainty
parameterisation uncertainty
10p+4EW, exp/fit uncertainty
CDF
D0
LEP
ZEUS (HERAI + HERAII(prel.))
H1 (HERAI)
Standard Model
v_u
a_u


a_u-a_d-v_u-v_d-PDF fit
Current HERAI+II fit (13p+4EW)
exp/fit uncertainty
model uncertainty
parameterisation uncertainty
10p+4EW, exp/fit uncertainty
v_d
a_d


a_u-a_d-v_u-v_d-PDF fit
Current HERAI+II fit (13p+4EW)
exp/fit uncertainty
model uncertainty
parameterisation uncertainty
10p+4EW, exp/fit uncertainty
ZEUS (HERAI + HERAII(prel.))
a_d
a_u


a_u-a_d-v_u-v_d-PDF fit
Current HERAI+II fit (13p+4EW)
exp/fit uncertainty
model uncertainty
parameterisation uncertainty
10p+4EW, exp/fit uncertainty
ZEUS (HERAI + HERAII(prel.))
v_d
v_u

This is an old slide. This is to have it up just in case.

Parameter	10+4p I+II(unpol)	10+4p I+II(pol)	10+4p I+Zpol+H1unpol	10+4p I+Zunpol+H1pol	12p+4 I+II(unpol)
'Bg'	0.187 ± 0.026	0.189 ± 0.025	0.184 ± 0.024	0.181 ± 0.025	−0.17 ± 0.18
'Cg'	8.36 ± 0.54	8.55 ± 0.54	8.57 ± 0.53	8.25 ± 0.53	7.4 ± 1.1
'Aprig'	-	-	-	-	1.18 ± 0.64
'Bprig'	-	-	-	-	−0.26 ± 0.12
'Cprig'	-	-	-	-	25.00
'Buv'	0.722 ± 0.023	0.726 ± 0.023	0.729 ± 0.022	0.728 ± 0.022	0.729 ± 0.042
'Cuv'	4.777 ± 0.088	4.750 ± 0.078	4.738 ± 0.081	4.779 ± 0.080	4.75 ± 0.14
'Euv'	10.5 ± 1.3	10.2 ± 1.1	10.0 ± 1.1	10.4 ± 1.2	10.0 ± 2.2
'Cdv'	4.21 ± 0.23	4.20 ± 0.25	4.22 ± 0.24	4.11 ± 0.21	4.24 ± 0.35
'CUbar'	2.92 ± 0.41	3.42 ± 0.45	3.41 ± 0.44	3.36 ± 0.45	3.01 ± 0.66
'ADbar'	0.1922 ± 0.0080	0.1958 ± 0.0084	0.1961 ± 0.0082	0.1975 ± 0.0083	0.195 ± 0.013
'BDbar'	−0.1617 ± 0.0050	−0.1605 ± 0.0051	−0.1599 ± 0.0050	−0.1596 ± 0.0051	−0.1593 ± 0.0085
'CDbar'	4.91 ± 0.97	4.16 ± 0.89	4.21 ± 0.84	4.68 ± 0.90	4.9 ± 1.4
'auEW'	0.546 ± 0.098	0.532 ± 0.060	0.500 ± 0.040	0.585 ± 0.066	0.54 ± 0.14
'adEW'	−0.55 ± 0.43	−0.50 ± 0.23	−0.65 ± 0.13	−0.26 ± 0.27	−0.54 ± 0.63
'vuEW'	−0.21 ± 0.13	0.149 ± 0.067	0.152 ± 0.084	0.138 ± 0.054	−0.21 ± 0.20
'vdEW'	−0.49 ± 0.27	−0.44 ± 0.17	−0.29 ± 0.22	−0.524 ± 0.078	−0.50 ± 0.39
Total χ^2 / dof	1362 / 1131	3593 / 3234	3277 / 2928	3303 / 2985	1361 / 1129

Note the 2x uncertainty for HERAPDF2.0 combined data for EW couplings.

ZEUSpol+H1unpol: a_d is a bit more than 1 sigma away from SM.

a_u and a_d get somewhat lower uncertainty, BUT v_u and v_d become less precise!

Also, chi2 is a bit larger than from the all-polarised-data-fit.

PDF parametrisation choice

- 14 PDF parameters + 4 EW parameters → unstable fit + unphysical results.

10p	+Du_v	+Dd_v	+Dg	+$D\bar{U}$	+$D\bar{D}$	+Ed_v	+Eg	+$E\bar{U}$	+$E\bar{D}$	+NG
3592.723	3592.452	3592.570	3588.994	3573.382	3592.664	3592.554	3586.277 Not physical	3585.778 Not physical	3592.707	3590.645
+$D\bar{U}$	3571.435	3573.348	3573.338	-	3576.839	3573.771	3573.339	3571.827	3572.560	3570.587
+NG	Not converged	Not converged	Not converged	-	Not converged	Not converged	Not converged	Not converged	Not converged	-

HERAPDF1.0 (10p)

$$xg(x) = Ag x^{Bg}(1-x)^{Cg}$$

$Bu_v = Bd_v$

$$xu_v(x) = Au_v\, x^{Bu_v}(1-x)^{Cu_v}(1+Eu_v x^2)$$

$$xd_v(x) = Ad_v\, x^{Bd_v}(1-x)^{Cd_v}$$

$$x\bar{U}(x) = A\bar{U}\, x^{B\bar{U}}(1-x)^{C\bar{U}}$$

$$x\bar{D}(x) = A\bar{D}\, x^{B\bar{D}}(1-x)^{C\bar{D}}$$

11p used in prel. ZEUS EW fit

HERAPDF2.0 (14p)

$$xg(x) = Ag x^{Bg}(1-x)^{Cg} - A'g x^{B'g}(1-x)^{C'g}$$

$$xu_v(x) = Au_v\, x^{Bu_v}(1-x)^{Cu_v}(1+Eu_v x^2)$$

$$xd_v(x) = Ad_v\, x^{Bd_v}(1-x)^{Cd_v}$$

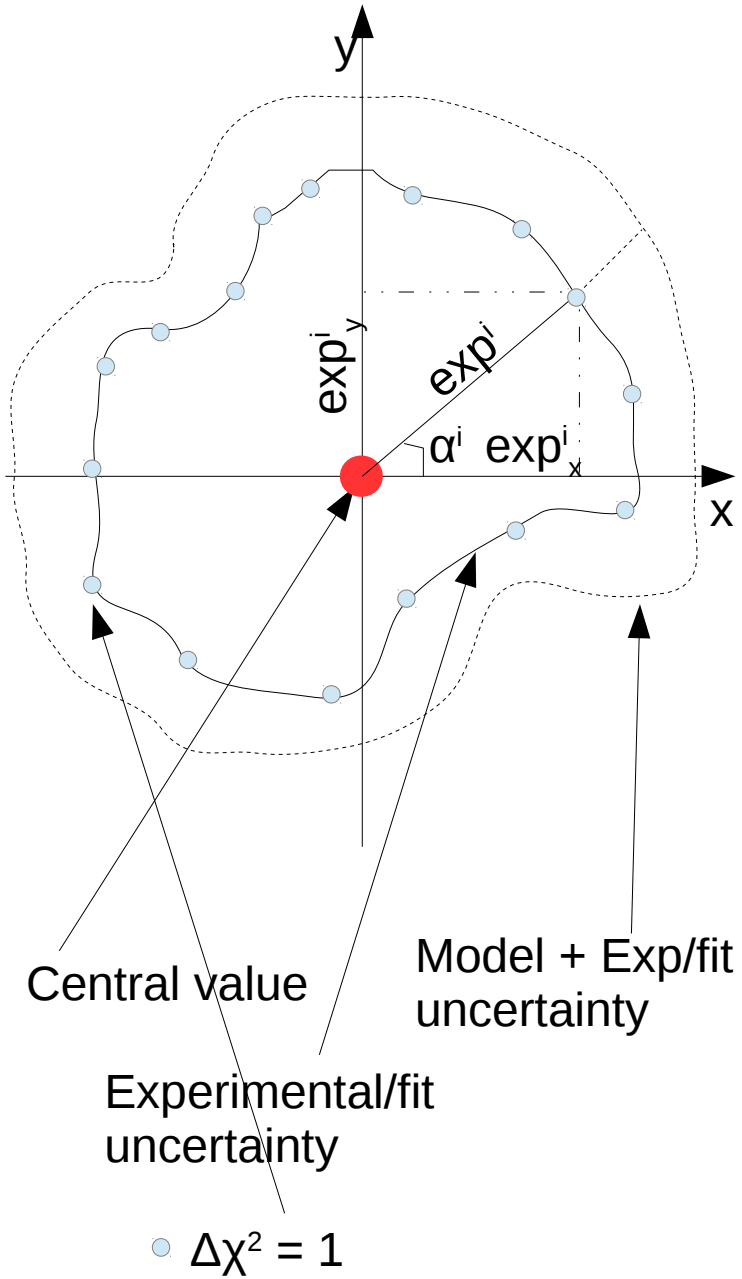
$$x\bar{U}(x) = A\bar{U}\, x^{B\bar{U}}(1-x)^{C\bar{U}}(1+D_{\bar{U}} x)$$

$$x\bar{D}(x) = A\bar{D}\, x^{B\bar{D}}(1-x)^{C\bar{D}}$$

HERAPDF2.0 param. scan

10p	+Duv	+Ddv	+Dg	+DUb	+DDb	+Euv	+Edv	+Eg	+EUb	+EDb	+NG
	1466	1457	1505	1460	1531	**1422**	1460	1494	1524	1482	1492
+Euv	1422	1422	1417	**1408**	1414		1422	11420	1415	1421	1420
+DUb	**1399**	1408	1408		1408		1408	1408	1408	1408	1403
+Duv		1399	1399		11399		13991	1398	1399	1399	**1392**
+NG											

Contours with exp + other uncertainty



$$\exp^i = \sqrt{\exp_x^{i\,2} + \exp_y^{i\,2}}$$

$$mod^i = \sqrt{(\cos\alpha\, mod_x)^2 + (\sin\alpha\, mod_y)^2}$$

$$\cos\alpha\,\frac{\exp_x^i}{\exp i} \qquad \sin\alpha\,\frac{\exp_y^i}{\exp i}$$

$$total^i = \sqrt{\exp^{i\,2} + mod^{i\,2}}$$

$$total_x^i = \cos\alpha\, total^i$$

$$total_y^i = \sin\alpha\, total^i$$