The QCD Axion and the Relaxion

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G. Grilli di Cortona, E.H, J. Pardo Vega, G. Villadoro:
arXiv:1510.?????

E.H.:
arXiv:1507.07525 , JHEP

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Motivation

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\theta_0 G\tilde{G}$$

cryoEDM

Experiment bounds $\theta = \theta_0 + \arg\,\det M_q \lesssim \mathcal{O}(10^{-10})$

Other phases in Yukawa matrices order 1

Non-decoupling contributions from new CP violating physics at arbitrarily high scales

Effects on large distance physics irrelevant for $\theta \lesssim 10^{-1} \div 10^{-2}$

Begs for a dynamical explanation!

Out of known solutions QCD axion probably simplest and most robust

Standard Model + extra pseudo-goldstone boson with coupling

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\left(\theta + \frac{a}{f_a}\right) G\tilde{G}$$

Strong coupling $\longrightarrow$ Axion potential & axion VEV that removes θ

$\longrightarrow$ Axion mass $\mathcal{O}(m_\pi f_\pi / f_a)$

In a large part of parameter space axion can nicely explain observed Dark Matter content of universe

"Generic" feature of string compactifications (modulo expected decay constants)

Small couplings mean discovery hard, but several ideas

Many including ADMX exploit resonance effects, and if axion is DM then may be possible to measure its mass with relative accuracy 10^{-6}

Depending on experiment other couplings as well

Could we exploit such a high precision experiment?

What could we learn?

Possible to infer the UV completion of the axion and its cosmology?

QCD Axion Outline

- Precision physics at zero temperature
- Physics at finite temperature
- Impact on cosmology and astrophysics

Lagrangian

UV Lagrangian can be written in the form

$$\mathcal{L}_a = \frac{1}{2}(\partial a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}\, a\, g^0_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} + j^{\mu}_{a,0}\frac{\partial_\mu a}{2f_a}$$

where $g^0_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\frac{E}{N}$ Anomalous EM coupling

$j^{\mu}_{a,0} = c^0_q \bar{q}\gamma^\mu\gamma_5 q$ Model dependent axial current

Convenient to perform a rotation

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 \frac{a}{2f_a} Q_a} \begin{pmatrix} u \\ d \end{pmatrix}, \qquad \mathrm{tr} Q_a = 1$$

Leads to

$$\mathcal{L}_a = \frac{1}{2}(\partial a)^2 + \frac{1}{4}\, a\, g_{a\gamma\gamma} F_{\mu\nu}\tilde{F}^{\mu\nu} + j_a^\mu \frac{\partial_\mu a}{2f_a} - \bar{q}_L M_a q_R + h.c.$$

Where

$$\boxed{M_a = e^{i\frac{a}{2f_a}Q_a} M_q\, e^{i\frac{a}{2f_a}Q_a}\,,} \quad M_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix},$$

All non-derivative couplings to QCD in quark mass matrix

Coupling to axial current only multiplicatively renormalised

Chiral Perturbation Theory

Axion can be treated as an external source, non-derivative couplings via dressed mass matrix and derivative couplings enters as an external axial vector current (Georgi, Kaplan, Randall)

Low energy correlators entirely captured by chiral perturbation theory, consistent perturbative expansion

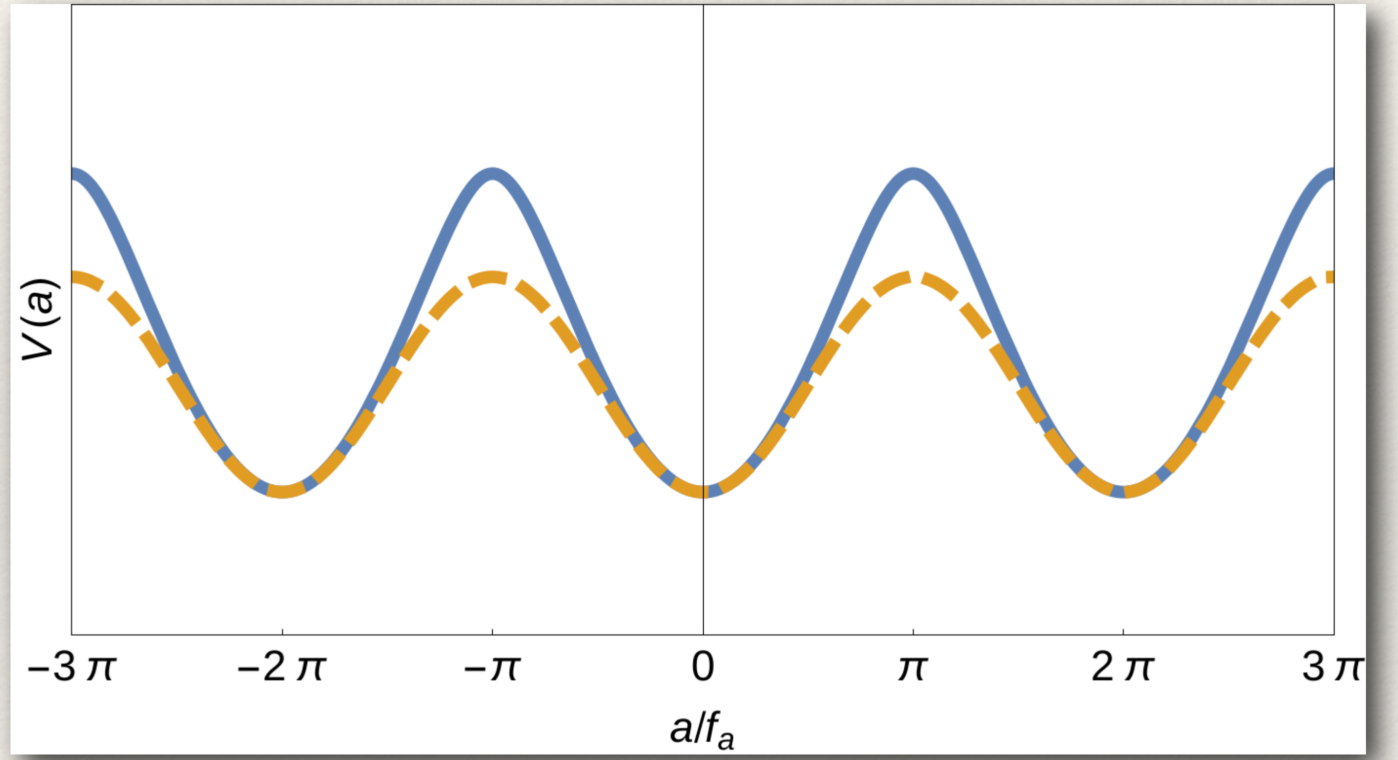
$$\mathcal{L}_{p^2} \supset 2B_0 \frac{f_\pi^2}{4} \langle U M_a^\dagger + M_a U^\dagger \rangle$$

where $U = e^{i\Pi/f_\pi}$, $\Pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$

Free to shift axion into the first generation only

Potential

$$V(a,\pi^0) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}$$



Not a cosine

Immediately gives $m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}$

Mass at NLO

Just have to compute within CPT, including higher order terms

$$\mathcal{L}_{p^4} \supset \frac{l_7}{8} \left\langle \left(D^\mu U + D^\mu U^\dagger \right) 2B_0 \left(M_a + M_a^\dagger \right) \right\rangle$$

Gives

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} \left[1 + 2 \frac{m_\pi^2}{f_\pi^2} \left(h_1^r - h_3^r - l_4^r + \frac{m_u^2 - 6 m_u m_d + m_d^2}{(m_u + m_d)^2} l_7^r \right) \right]$$

Turns out there are no loop contributions at this order, beyond those reabsorbed in tree-level factor

Related to 3-flavour calculation via low energy constants

Determining low energy constants

$$l_7^r = \frac{m_u + m_d}{m_s} \frac{f_\pi^2}{8m_\pi^2} - 36L_7 - 12L_8^r + \frac{\log(m_\eta^2/\mu^2) + 1}{64\pi^2} + \frac{3\log(m_K^2/\mu^2)}{128\pi^2} \quad = 7(4) \cdot 10^{-3},$$

$$h_1^r - h_3^r - l_4^r = -8L_8^r + \frac{\log(m_\eta^2/\mu^2)}{96\pi^2} + \frac{\log(m_K^2/\mu^2) + 1}{64\pi^2} \quad = (4.8 \pm 1.4) \times 10^{-3}$$

$$z \equiv \frac{m_u^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})}{m_d^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})} = 0.48(3)$$

$$\boxed{m_a = 5.70(6)(4)\ \mu\mathrm{eV} \left(\frac{10^{12}\mathrm{GeV}}{f_a}\right) = 5.70(7)\ \mu\mathrm{eV} \left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)}$$

First error from z, second from low energy constants

Potential

$$V(a)^{\rm NLO} = -m_\pi^2\big(\tfrac{a}{f_a}\big) f_\pi^2 \left\{ 1 - 2\frac{m_\pi^2}{f_\pi^2}\left[l_3^r + l_4^r - \frac{(m_d-m_u)^2}{(m_d+m_u)^2} l_7^r - \frac{3}{64\pi^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)\right]\right.$$
$$\left. + \frac{m_\pi^2\big(\tfrac{a}{f_a}\big)}{f_\pi^2}\left[h_1^r - h_3^r + l_3^r + \frac{4m_u^2m_d^2}{(m_u+m_d)^4}\frac{m_\pi^8 \sin^2\big(\tfrac{a}{f_a}\big)}{m_\pi^8\big(\tfrac{a}{f_a}\big)} l_7^r - \frac{3}{64\pi^2}\left(\log\left(\frac{m_\pi^2\big(\tfrac{a}{f_a}\big)}{\mu^2}\right) - \frac{1}{2}\right)\right]\right\}$$

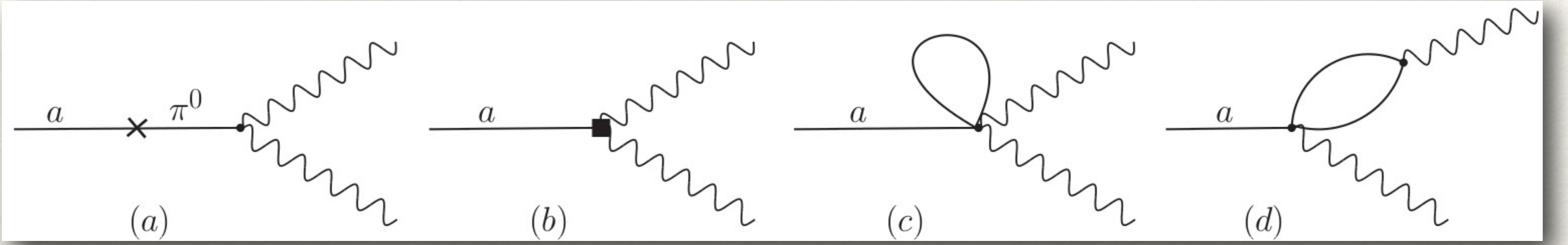
Similar calculation: 1-loop + NLO Lagrangian + rewriting physical constants

In passing, easy to extract domain wall tension to sub-% level

$$\sigma = 2 f_a \int_0^\pi d\theta \, \sqrt{2[V(\theta) - V(0)]}$$

$$= 8.97(5)\, m_a f_a^2$$

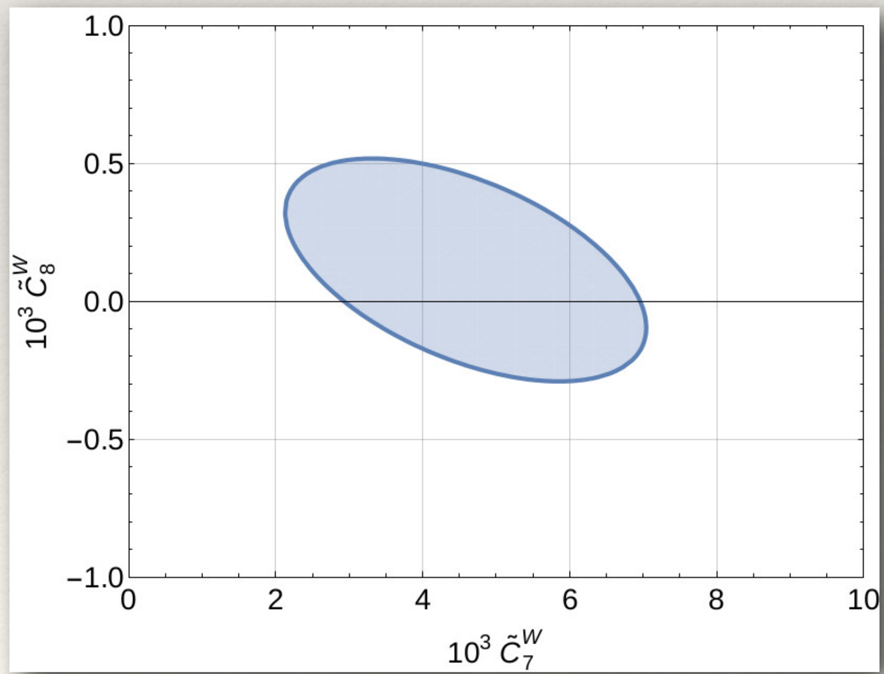
Coupling to Photons

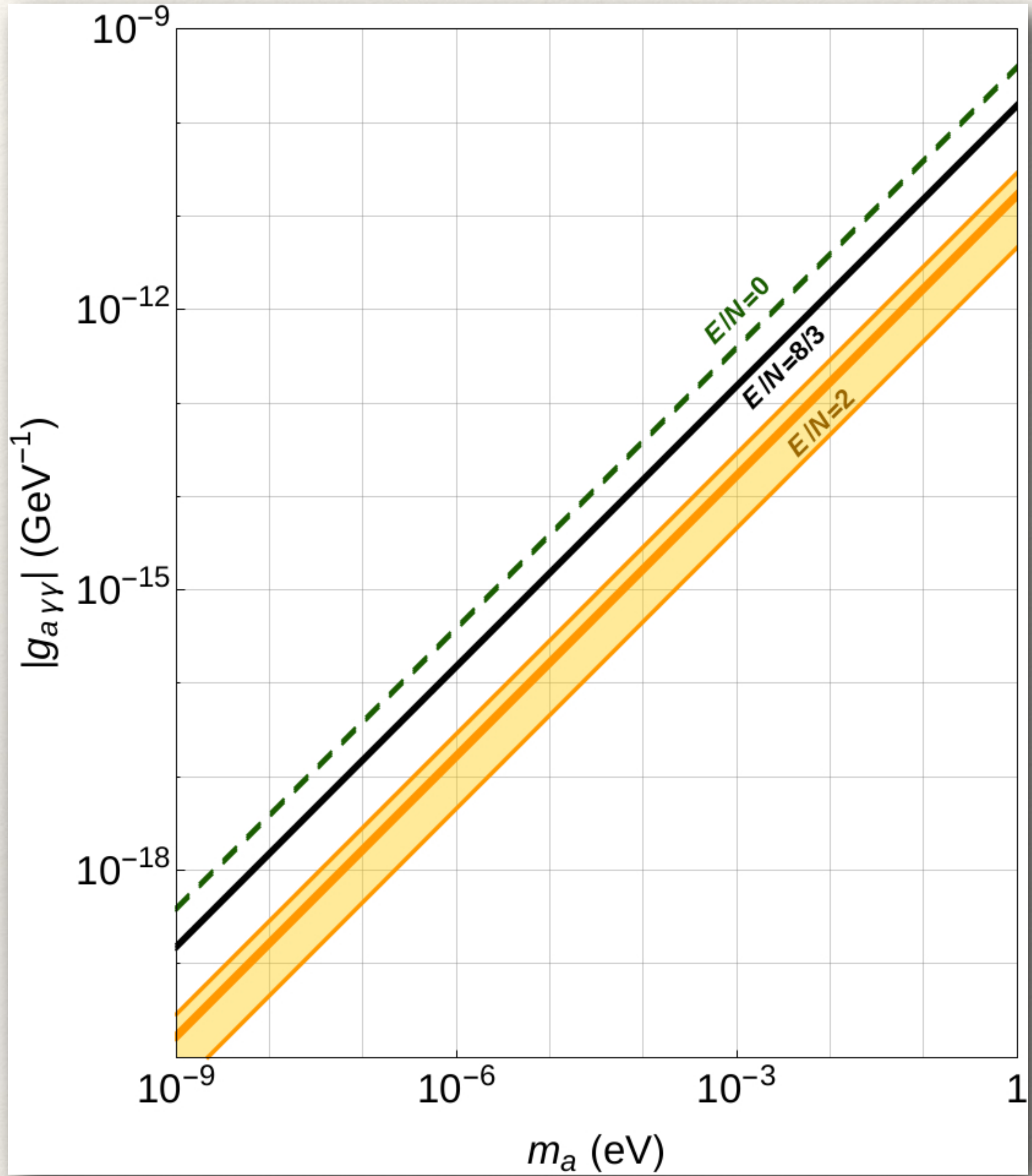


$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\frac{E}{N} - \frac{2}{3}\frac{4m_d + m_u}{m_d + m_u} + \frac{m_\pi^2}{f_\pi^2}\frac{8m_u m_d}{(m_u+m_d)^2}\left[\frac{8}{9}\left(5\tilde{c}_3^W + \tilde{c}_7^W + 2\tilde{c}_8^W\right) - \frac{m_d - m_u}{m_d + m_u} l_7^r\right]\right\}$$

New low energy constants to determine

Harder to get these precisely





$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left[\frac{E}{N} - 1.92(4)\right]$$

$$= \left[0.203(3)\frac{E}{N} - 0.39(1)\right]\frac{m_a}{\mathrm{GeV}^2}$$

Possible to get cancellation for particular UV models

(We also do couplings to nucleons, get similar precision)

Finite Temperature

As T increases QCD gets weaker, and axion mass decreases

Might think that precision of calculations will improve, but actually while $T \lesssim 10^5\,\mathrm{GeV}$ the opposite is true!

Important for axion relic abundance (also interesting in its own right)

Low Temperature

Compute temperature dependence of mass with CPT

Temperature dependence only from non-local contributions

But one loop correction is only from local NLO couplings, so

$$\frac{m_a^2(T)}{m_a^2} = \frac{\chi_{top}(T)}{\chi_{top}} \overset{\text{NLO}}{=} \frac{m_\pi^2(T) f_\pi^2(T)}{m_\pi^2 f_\pi^2} = \frac{\langle \bar{q} q \rangle_T}{\langle \bar{q} q \rangle}$$

$$= 1 - \frac{3}{2} \frac{T^2}{f_\pi^2} J_1\left[\frac{m_\pi^2}{T^2}\right], \qquad J_1[\xi] = \frac{1}{\pi^2} \frac{\partial}{\partial \xi} \int_0^\infty dq\, q^2 \log\left(1 - e^{-\sqrt{q^2+\xi}}\right).$$

Effects of heavy states suppressed by $e^{m/T}$

Ratio m/T_c not huge, and many new states appear, so breaks down at crossover

High Temperatures

At high enough temperatures, instanton calculation (Gross, Pisarski, Yaffe)

$$f_a^2 m_a^2(T) \simeq 2 \int d\rho\, n(\rho, 0) e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \ldots}$$

Where $n(\rho, 0) \propto m_u m_d e^{-8\pi^2/g_s^2}$ is zero temperature instanton density

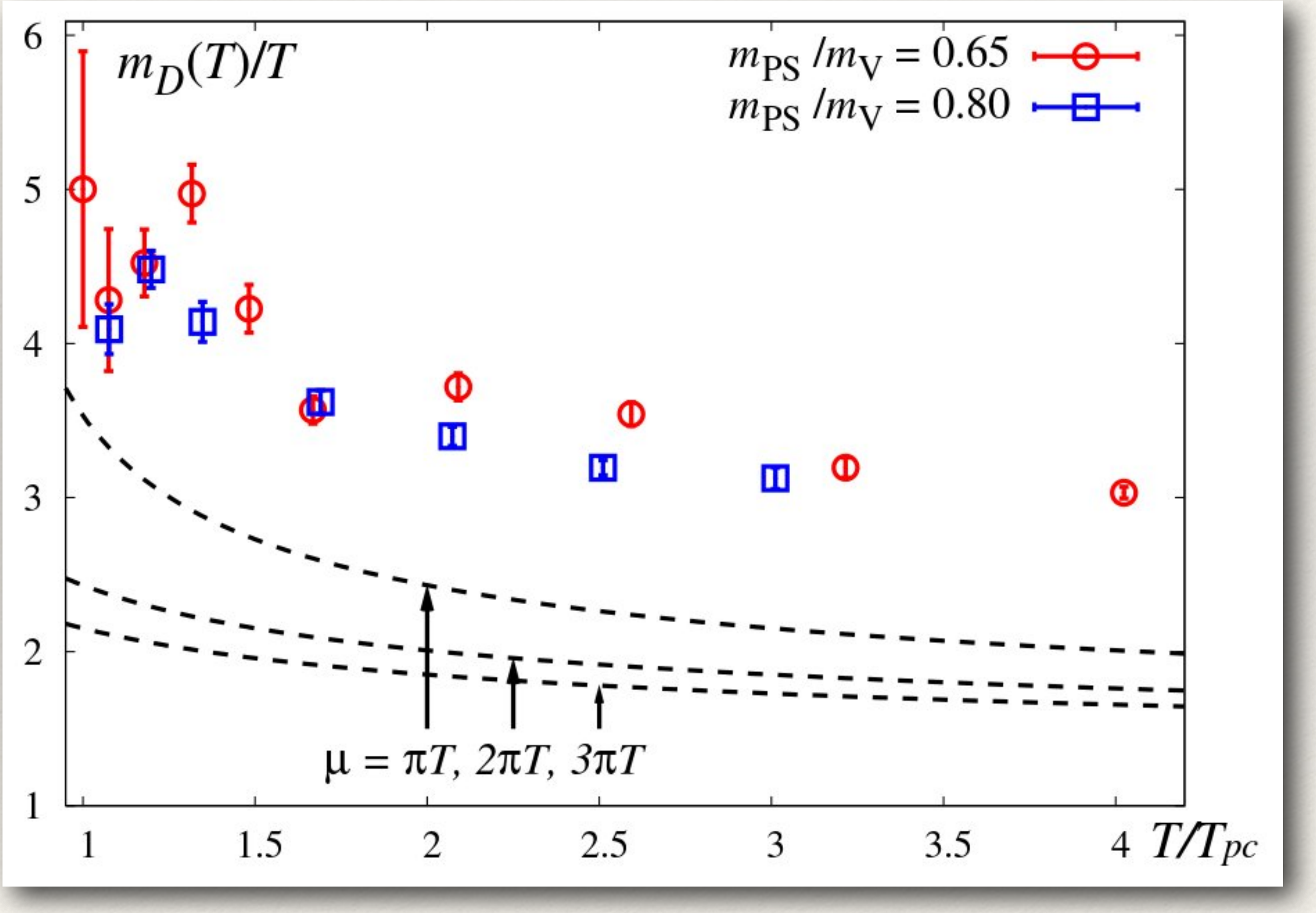
Integral is over instanton size ρ

Cut off by screening from leading order Debye mass

$$m_{D1}^2 = g_s^2 T^2 (1 + n_f/6)$$

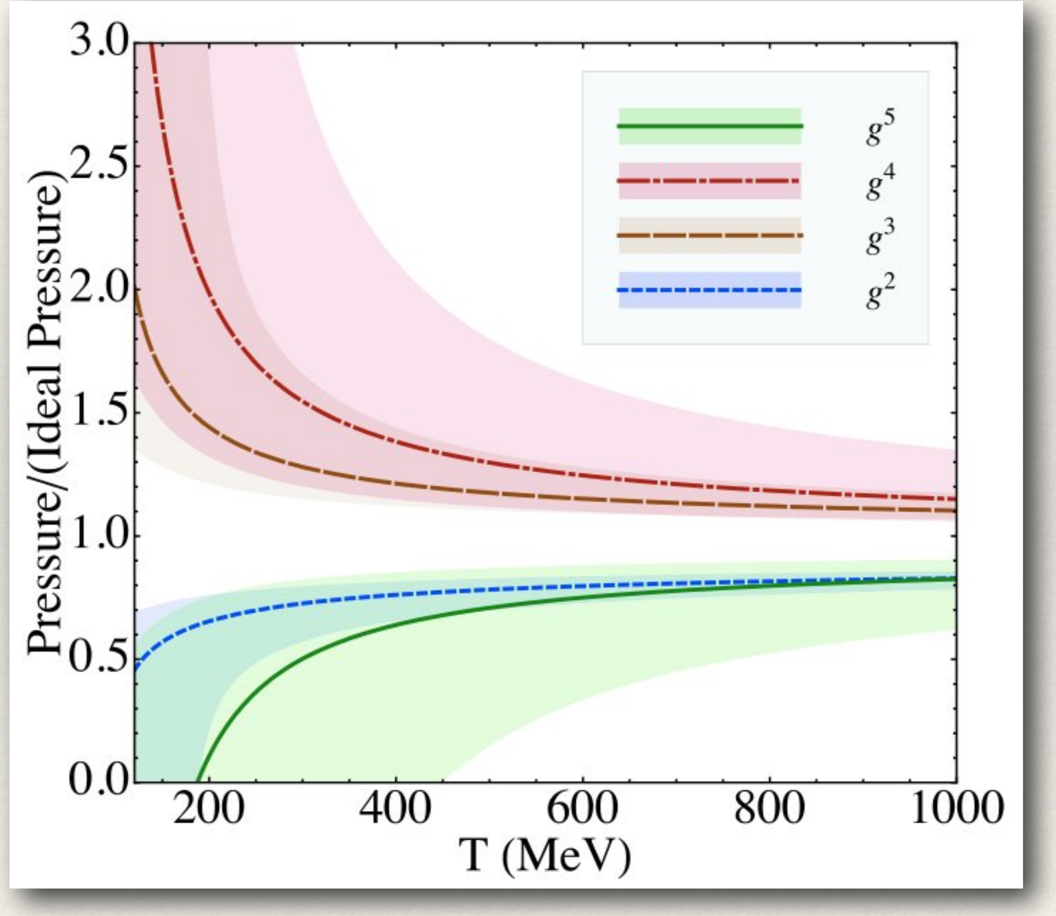
Finite T QCD convergence…

E.g. $$m_D = m_{D0} + \frac{N}{4\pi} g^2 T \log \frac{m_{D0}}{g^2 T} + c_N g^2 T + \mathcal{O}\left(g^3 T\right)$$



WHOT-QCD Collaboration

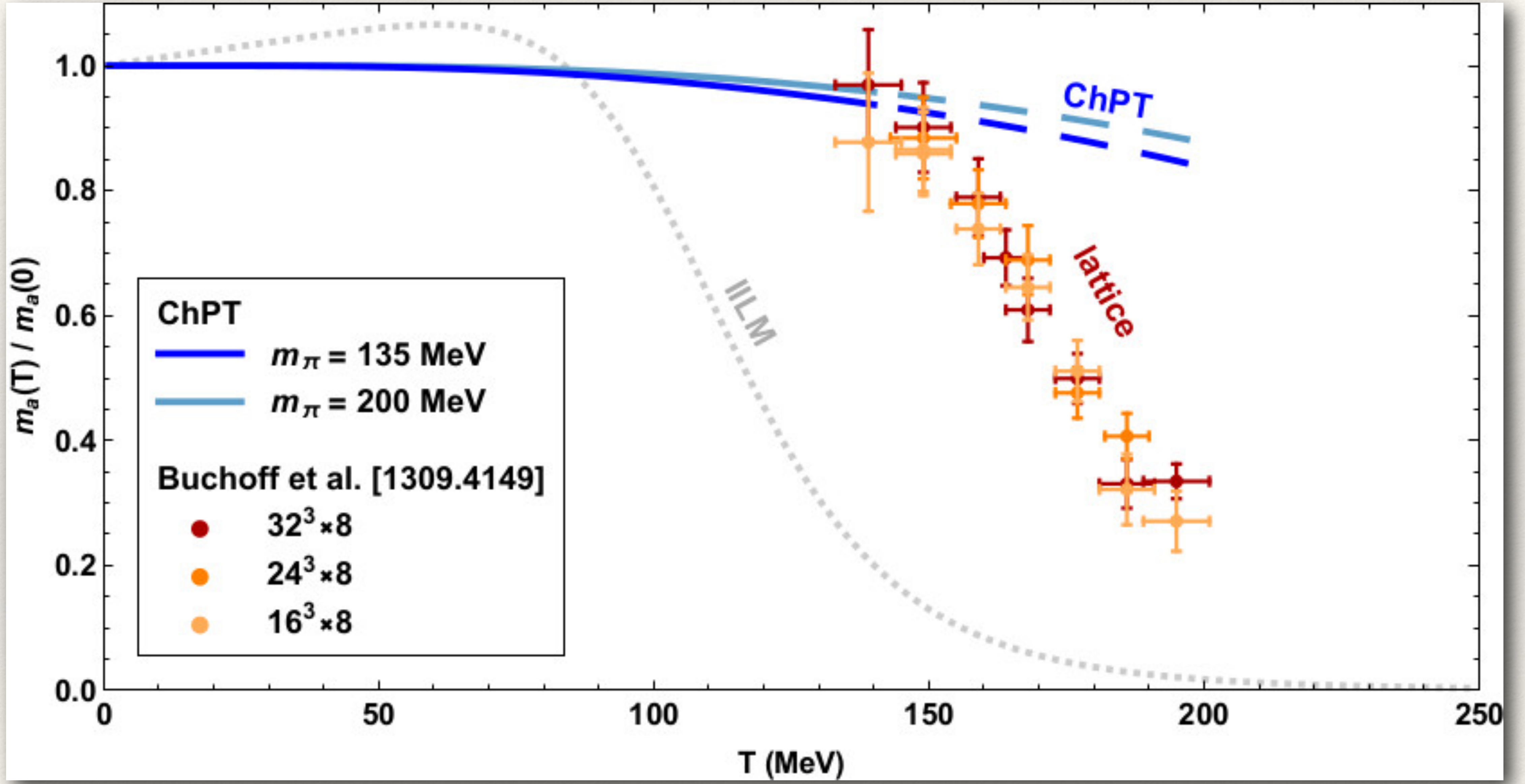
Debye mass



Andersen et al 1103.2528

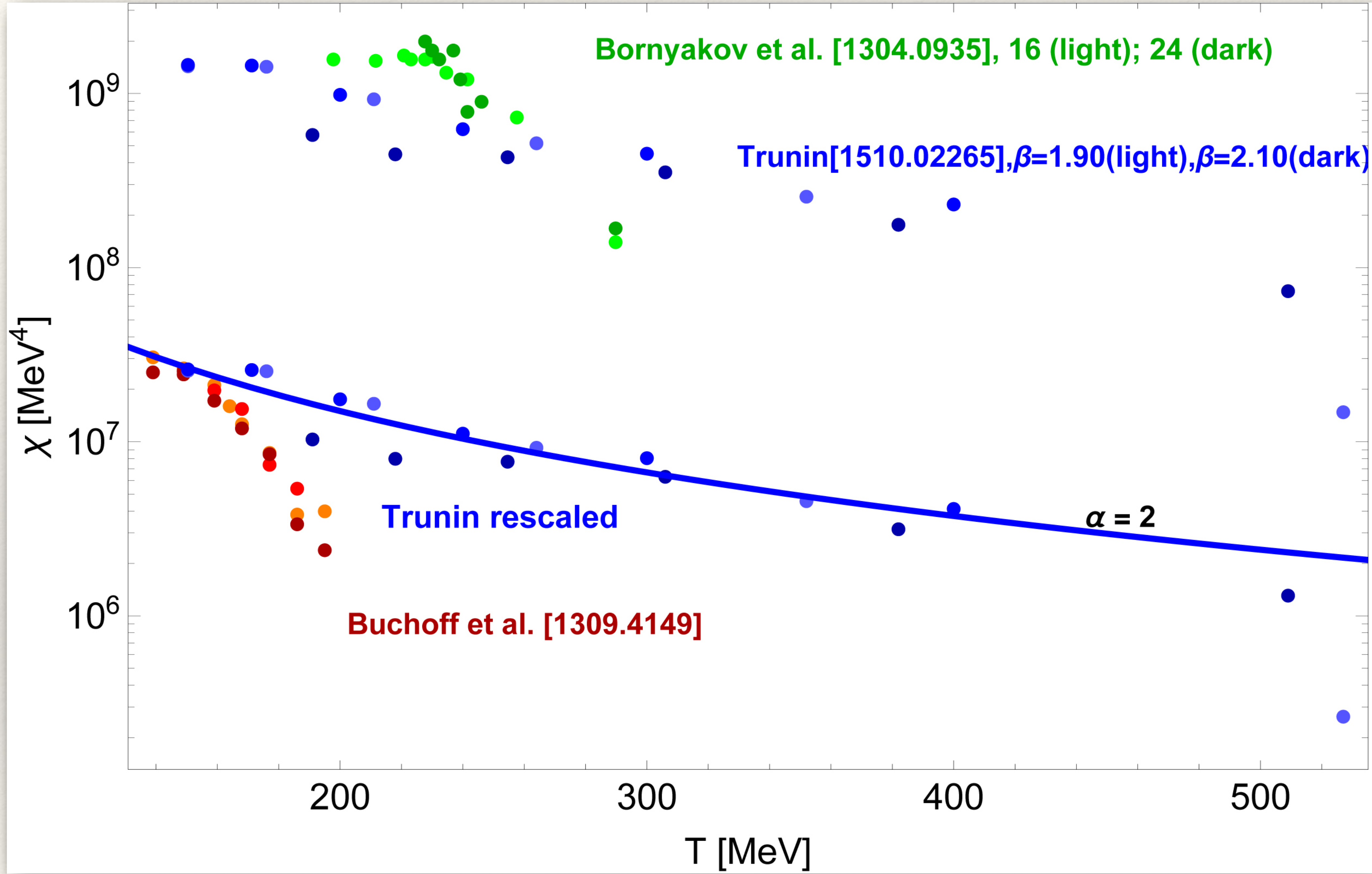
Pressure

Lattice



Looks good so far

But Turnin (1510.02265) looks very different



Seems strange: how does it transition to the instanton dominated region?

Axion Dark Matter

Evolution given by

$$\ddot{a} + 3H\dot{a} + m_a^2\,(T)\, f_a \sin\left(\frac{a}{f_a}\right) = 0\,.$$

At high temperatures axion field fixed:

- At constant value over observable universe if PQ breaking before inflation
- Randomly in $a \in f_a\,[0, 2\pi]$ if PQ broken after reheating

Axion starts oscillating when $m_a\,(T) \sim 3H$

Quickly reaches solution where comoving number density is an adiabatic invariant

Coherent oscillations of the axion act as cold dark matter

Misalignment

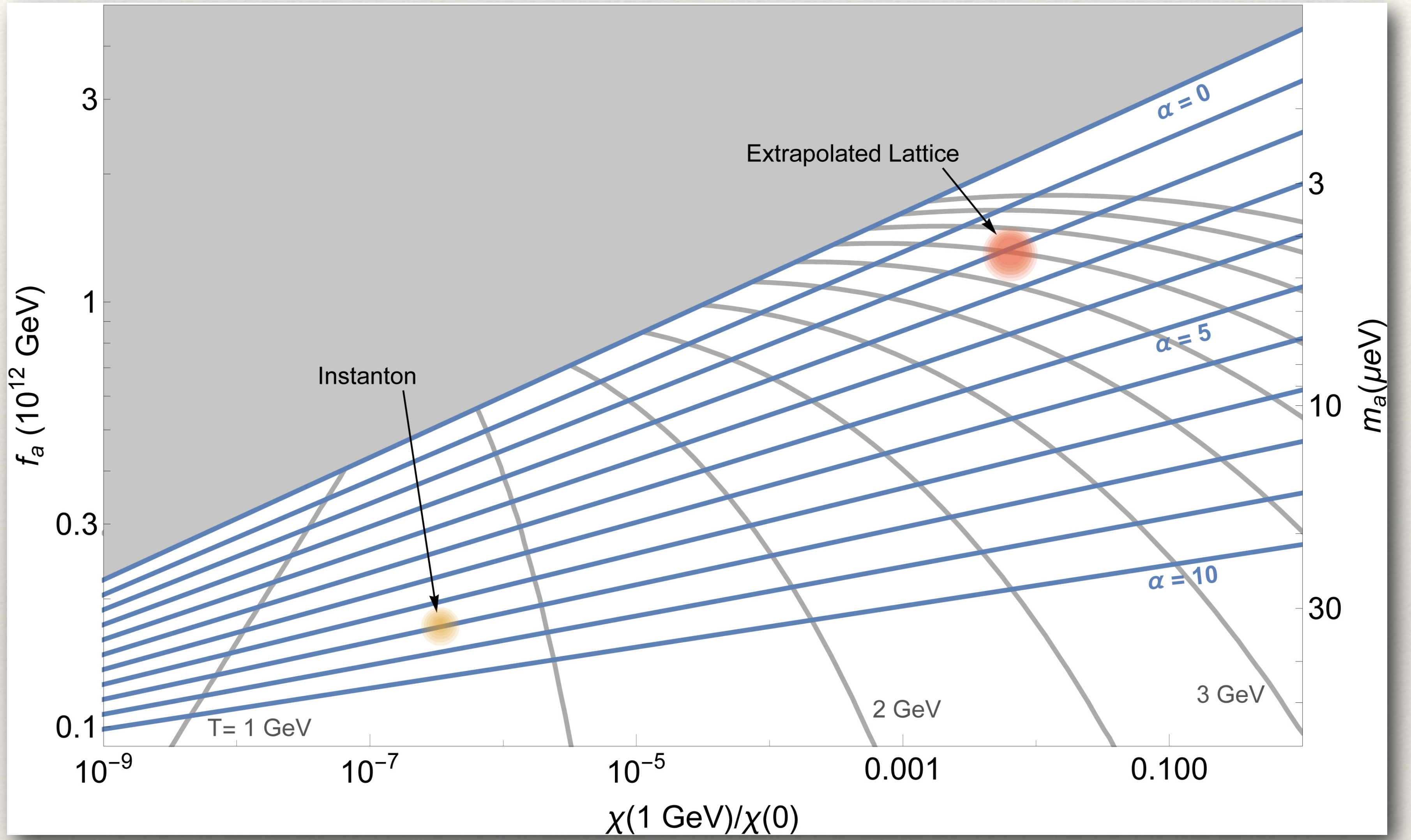
Straightforward to derive approximate analytic expressions for relic abundance

But most accurate just to do a numerical integration

To show on plots, fit axion mass temperature dependence with a power law

$$m_a^2(T) = m_a^2(1\ \mathrm{GeV}) \left(\frac{\mathrm{GeV}}{T}\right)^{\alpha}$$

Uncertainty on required f_a



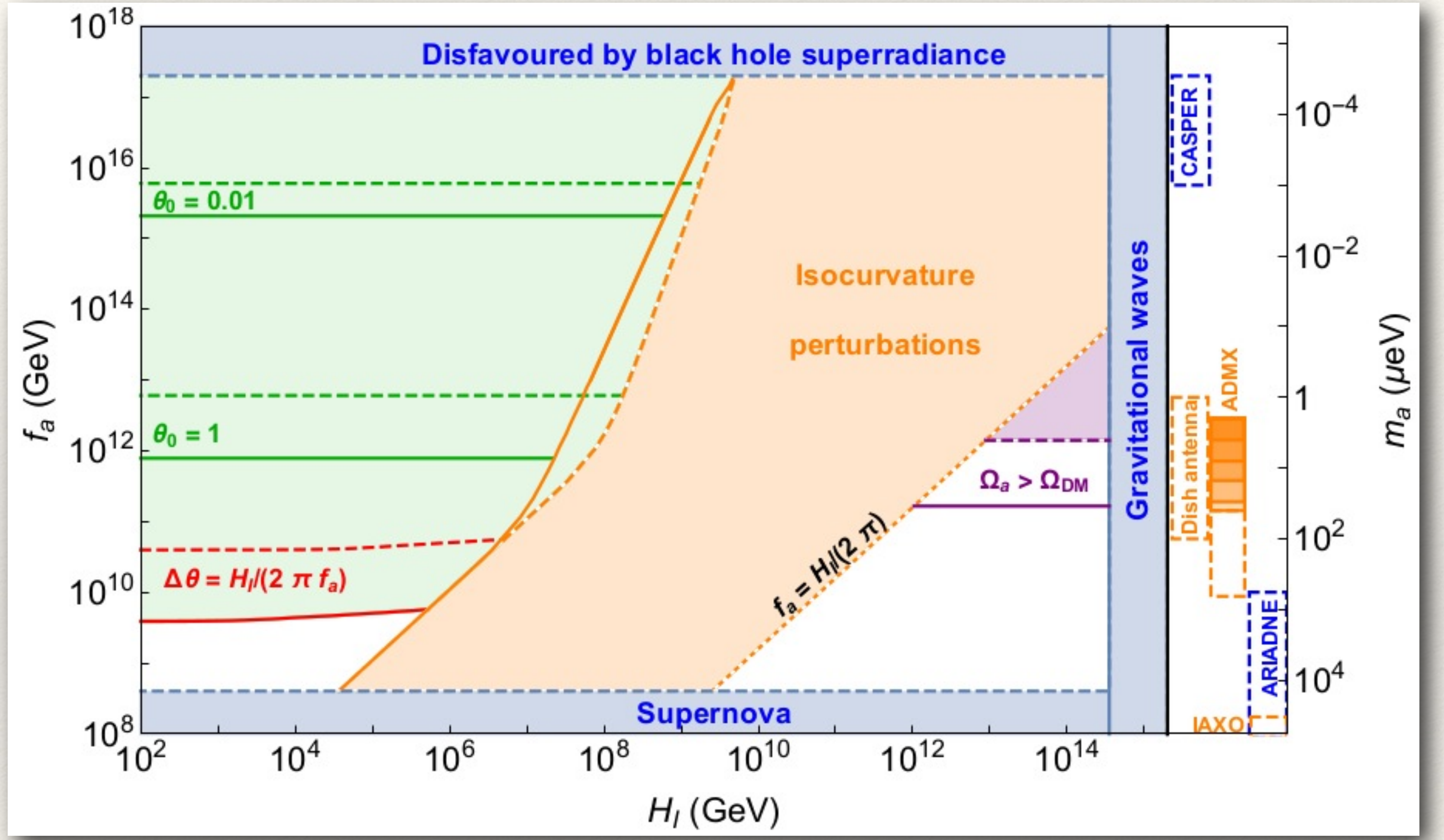
Starts oscillating near 1 GeV, value of power not very important

If axion mass has dropped a large amount already at 1 GeV, high precision

Oscillates at much higher temperature

Behaviour at T above 1 GeV (i.e. power in approximation) very important

Cosmological Parameter Space



Isocurvature limits assuming axion makes up all the dark matter

Summary: QCD Axion

- Zero and low temperature properties to % level accuracy
- Finite temperature physics very poorly known (but may improve with future lattice studies)
- Significant, but not enormous, effect on dark matter and cosmology
- Effect on string and domain wall contributions to axion relic density? *Work in progress*

The Relaxion

- Review the original model
- From finite temperature physics:
    - Strongly coupled hidden sector
    - Weakly coupled hidden sector
- (visible sector UV completions and supersymmetry)

Electroweak Relaxation

Interesting idea to solve the EW hierarchy problem (Graham, Kaplan, Rajendran) (closely related to previous proposal for cosmological constant)

Large number of vacua with different EW VEVs, cosmology selects one with small VEV

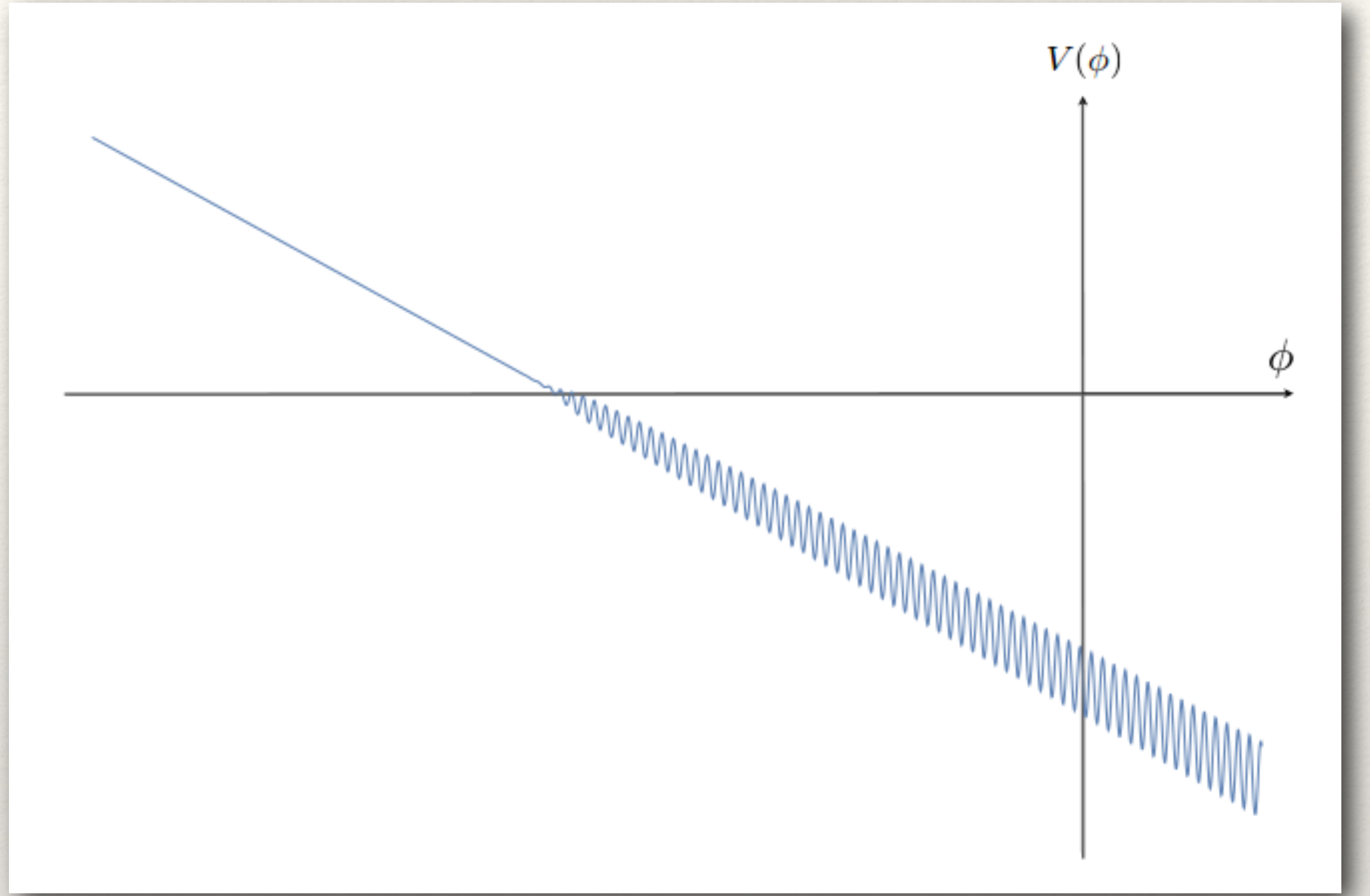
But still many model building challenges to be addressed

UV completion?

See:
Gupta, Komargodski, Perez, Ubaldi
arXiv:1509.00047

But, recall gauge mediated SUSY needs: supersymmetric moduli stabalisation, metastable dynamical SUSY breaking sector, mediating interactions, resolution of mu B mu, complications with gravitino relic, and still has tuning at least 1 in 100

$$\mathcal{L} \supset \left(M^2 - \epsilon M\phi\right)|h|^2 + V\left(\epsilon\phi\right) + \Lambda^3 h \cos\left(N\phi/f\right)$$



EW vacua ordered

Boundary between $\langle h\rangle = 0$ and $\langle h\rangle \neq 0$ is a special point in field space

Constraints

Stops rolling when

$$\epsilon M^3 \frac{f}{N} \sim \Lambda^3 \langle h \rangle$$

Relaxion doesn't dominate the vacuum energy

$$H > M^2/M_{pl}$$

Scans a large enough range of field space

$$n_e \gtrsim \frac{H^2}{\epsilon^2 M^2}$$

Rolls classically

$$H < \left(\epsilon M^3\right)^{1/3}$$

End up needing 10^{40} e-folds of inflation!

Many of these are because it is linked to inflation

Thermal Relaxation

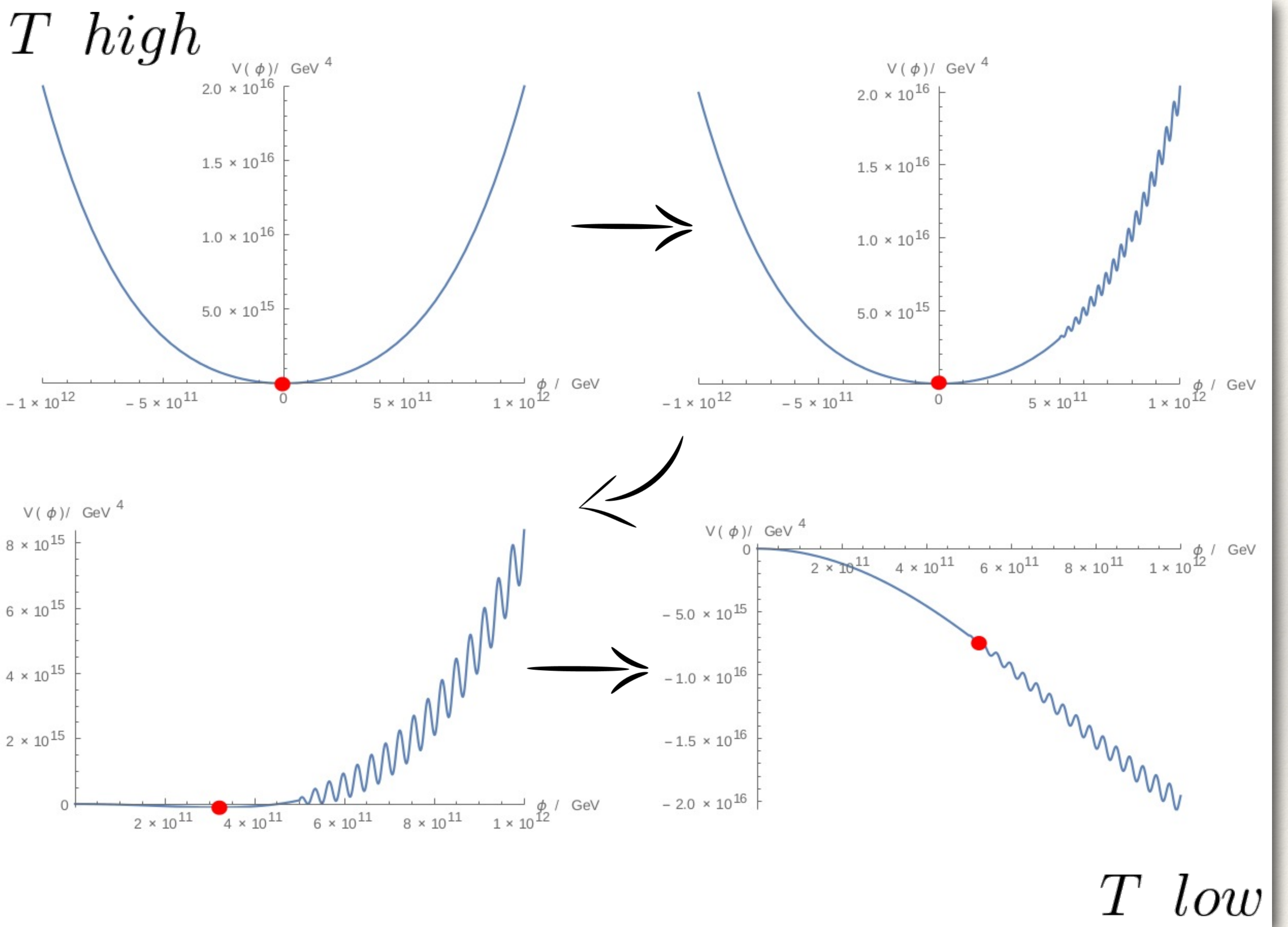
Aim: remove link with inflation, everything happens after inflation is finished

Change the way the scanning occurs

Scalar always close to the minimum of its potential at a given temperature

But the potential changes because the temperature is changing

Evolution is adiabatic rather than slow roll



T high
V(ϕ)/ GeV 4
ϕ / GeV
V(ϕ)/ GeV 4
ϕ / GeV
V(ϕ)/ GeV 4
ϕ / GeV
V(ϕ)/ GeV 4
ϕ / GeV
T low

Strong Coupling Model

ϕ couples anomalously to two gauge groups, the one dependent on h has a large anomaly coefficient

$$\mathcal{L} = M^2 |h|^2 + \lambda_h |h|^4 + \text{S.M.} - \epsilon M \phi |h|^2$$
$$+ V(\epsilon\phi) + \Lambda_a^3 \langle h \rangle \cos\left(N\frac{\phi}{f}\right) - \Lambda_b^4 \left(\frac{\phi}{f}\right)^2,$$

Typical field range of $\phi \sim M/\epsilon$ must be $< f$

Strong coupling generically dependent on temperature as

$$\Lambda_{b,T} \sim \begin{cases} \Lambda_b \left(\frac{\Lambda_b}{T_{hid}}\right)^n, & T_{hid} > \Lambda_b \\ \Lambda_b, & \text{otherwise} \end{cases}$$

Evolution of phi

For significant evolution of ϕ need a hidden sector temperature $\sim M$

Need different reheat temperatures in two sectors

Interactions of ϕ with the hidden sector weak enough that it doesn't thermalise

Consistent to reheat hidden sector above M since this is only the cutoff of the visible sector, and above this scale no shift symmetry breaking couplings

Consider example potential

$$V\left(\epsilon\phi\right)=\epsilon^{2}M^{2}\phi^{2}+\epsilon^{4}\phi^{4},$$

$$V_{T}=-\Lambda_{b}^{4}\left(\frac{\Lambda_{b}}{T}\right)^{n}\left(\frac{\phi}{f}\right)^{2}.$$

Provided hidden sector reheat temperature is high enough ϕ settles to minimum before potential starts changing

Minimum of potential evolves with temperature as

$$\phi_{min}=\frac{M}{\sqrt{2}\epsilon}\sqrt{\frac{\Lambda_{b,T}^{4}}{f^{2}\epsilon^{2}M^{2}}-1},$$

So need

$$\Lambda_{b}^{2}>f\epsilon M\gtrsim M^{2}.$$

Rate of interaction between the axion and the hidden sector

$$\frac{\Gamma_{hid}}{H} \sim \frac{T_{hid} M_{pl}}{f^2},$$

Evolution is close to adiabatic: time dependent potential for ϕ can be approximated

$$V(\phi) \sim \epsilon^2 M^2 \left(\phi - \dot{\phi}_{min} t\right)^2 .$$

For energy gained by ϕ to be small need $\dot{\phi}_{min} \lesssim T_{vis}^2$

$$\frac{M^3}{\epsilon M_{pl}} \lesssim (100\,\mathrm{GeV})^2 ,$$

Developing a Higgs VEV

Order 1 probability ϕ will move m_h^2 in the right direction, e.g

$$m_h^2 = \frac{1}{2}M^2 - \epsilon\phi M.$$

For ϕ to stop evolving when Higgs has the correct VEV need

$$\Lambda_a^3 \langle h \rangle \frac{N}{f} = \frac{\Lambda_b^4 \langle \phi \rangle}{f^2}$$

$$\Rightarrow \Lambda_a^3 \langle h \rangle N \sim M^4, \quad \text{when} \quad \langle h \rangle \sim 250\,\mathrm{GeV}$$

Other constraints: Vacua close enough together,
Rate of tunnelling to deeper ones exponentially slow

There are viable points in parameter space

Strongest limiting factor is $\Lambda_a \lesssim 100\,\mathrm{GeV}$

Possible to push to $M \sim 100\,\mathrm{TeV}$, but safer with the limits is a point

$$M = 10^4\,\mathrm{GeV}, \qquad \epsilon = 10^{-8}, \qquad f = 10^{12}\,\mathrm{GeV},$$
$$N = 10^8, \qquad \Lambda_a = 100\,\mathrm{GeV}.$$

N worryingly large, other parameters reasonable

Weakly Coupled Hidden Sector

Strong coupling model reasonable, but nice to have a fully calculable example

Hidden sector states ψ

$$\mathcal{L} \supset \epsilon\, \phi \psi_1 \psi_2,$$

Generates a thermal potential for ϕ

$$V_T \sim \epsilon^2 \phi^2 T^2.$$

Similar to before except a term in the potential now disappears as T drops (rather than turning on) e.g. zero temperature potential

$$V(\epsilon\phi) = -\epsilon^2 M^2 \phi^2 + \epsilon^4 \phi^4,$$

which has vacua at large field values. The evolution of the minimum with temperature is

$$\phi_{min} = \frac{M}{\sqrt{2}\epsilon}\sqrt{1 - \frac{T^2}{M^2}},$$

Constraints very similar to the previous model.

Need for a UV Completion

Still need a hidden sector temperature $\gtrsim M$ for ϕ to be displaced

But hidden sector cutoff can't be much higher than M , otherwise radiatively generate a too large potential for ϕ : i.e. $V(\epsilon\phi)$ with mass parameter Λ_{hidden}

Solution: supersymmetry in the hidden sector, broken at (or below) M

$$\mathcal{L} = \int d^2\theta \, \epsilon \, \Phi \Psi_1 \Psi_2.$$

Non-renormalisation theorems protect zero T potential , but finite T breaks SUSY

Supersymmetry

After introducing broken SUSY to the hidden sector, reasonable to consider a supersymmetric UV completion of the visible sector at cutoff M as well

Need to reconsider selection of EW scale

Simplest possibility is for ϕ to modify the visible sector μ term, e.g. as

$$\mathcal{L} \supset \int d^2\theta \, \left(M H_u H_d - \epsilon \Phi H_u H_d \right) .$$

Selecting the EW Scale

EW symmetry breaking requires

$$\left(|\mu|^2 + m^2_{Hu}\right)\left(|\mu|^2 + m^2_{Hd}\right) - B^2_\mu < 0.$$

Also, no run-away direction along a D-flat direction

$$2B_\mu < 2\,|\mu|^2 + m^2_{Hu} + m^2_{Hd}.$$

Even so need to make some assumptions about the other parameters for selection to work

For example

One possibility is $m^2_{Hu} < 0$ and $m^2_{Hd} > 0$ at the scale M

e.g. taking $m^2_{Hu} = -M^2/4$, m^2_{Hd} smaller, and evolution of ϕ to decrease the μ parameter

$$v \simeq \frac{2}{\sqrt{g^2 + g'^2}} \sqrt{-m^2_{Hu} - |\mu_{eff}|^2}$$

$$\simeq \frac{2}{\sqrt{g^2 + g'^2}} \sqrt{-\epsilon\phi' M},$$

where $\phi' = \phi - \frac{M}{2\epsilon}$ and $\mu_{eff} = M - \epsilon\phi$

Summary: Relaxion

- EW relaxation is interesting, especially given the lack of LHC signals
- Thermal version shows it is not intrinsically linked to a long period of inflation
- Still theoretical problems and models require multiple sectors
- Improvements may be possible?

Thank you!

Backups

Already required $\dot{\phi} \ll 100\,\mathrm{GeV}^2$ so safe from rolling over barriers

Vacua sufficiently close together

$$\epsilon M \frac{f}{N} < (100\,\mathrm{GeV})^2\,.$$

Rate of tunneling to deeper ones must be tiny

$$\left(\frac{f}{N}\right)^4 \gg \Lambda_a^3 \left(\epsilon M \frac{f}{N}\right)^{1/2}\,.$$

A hope for future work

Current versions all have a coincidence problem: new strong coupling scale Λ_a , which has to be close to the EW scale

Suppose we could build a theory with much larger Λ_a (e.g. non-EW breaking term)

And say that the visible sector temperature while evolving is $\sim 100\,\mathrm{GeV}$

Then finite temperature potential means that a Higgs VEV first develops when

$$m_h^2 \lesssim -100\,\mathrm{GeV}$$

If ϕ is trapped immediately, then the Higgs VEV could be selected by the temperature, rather than the stopping condition we had before

Couplings to Matter

Safest thing to do is use an effective theory well below QCD mass gap,

Non-relativistic nucleons $$\frac{\partial_\mu a}{2 f_a} c_N \bar{N} \gamma^\mu \gamma_5 N$$

We perform running from UV and matching

Need matrix elements $s^\mu \Delta q \equiv \langle p | \bar{q} \gamma^\mu \gamma_5 q | p \rangle_Q$: use neutron decay and lattice

$$c_p^{\text{KSVZ}} = -0.48(3)\,,$$
$$c_n^{\text{KSVZ}} = 0.03(3)\,,$$

$$c_p^{\text{DFSZ}} = -0.622 + 0.434 \sin^2\beta \pm 0.024\,,$$
$$c_n^{\text{DFSZ}} = 0.249 - 0.415 \sin^2\beta \pm 0.024\,.$$

Temperature the axion starts oscillating at:

$$T_a = 10^{\frac{6.4}{4+\alpha}} \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{-2/(4+\alpha)} \left(\frac{\chi\,(1\,\mathrm{GeV})}{\chi\,(0)} \right)^{1/(4+\alpha)} \mathrm{GeV}$$

Required relation for misalignment axion to provide all the dark matter

$$f_a = 4.3\times 10^{12}\,\mathrm{GeV}\times 10^{-0.036(\alpha-7)+0.0027(\alpha-7)^2} \times \theta_0^{\frac{-2\alpha-8}{\alpha+6}} \left(\frac{\chi\,(1\,\mathrm{GeV})}{\chi\,(0)} \right)^{\frac{1}{\alpha+6.}} .$$

A good field basis

The Lagrangian contains a mixing with the pion

$$\mathcal{L}_{p^2} \supset 2B_0 \frac{f_\pi}{4f_a} a \langle \Pi \{Q_a, M_q\} \rangle$$

Can be removed by choosing $Q_a = \frac{M_q^{-1}}{\langle M_q^{-1} \rangle}$ (Georgi, Kaplan, Randall)

Shifts $g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a} \left[\frac{E}{N} - \frac{2}{3} \frac{4m_d + m_u}{m_d + m_u} \right]$