Thermalization and hydrodynamization in weakly coupled nonabelian plasmas

Aleksi Kurkela

AK, Zhu PRL 115 (2015) 18, 182301
AK, Lu PRL 113 (2014) 18, 182301
AK, Moore JHEP 1111 (2011) 120
AK, Moore JHEP 1112 (2011) 044

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Motivation

What:

How far-from-equilibrium gauge fields reach equilibrium at weak coupling $\lambda = 4\pi N_c \alpha_s$?

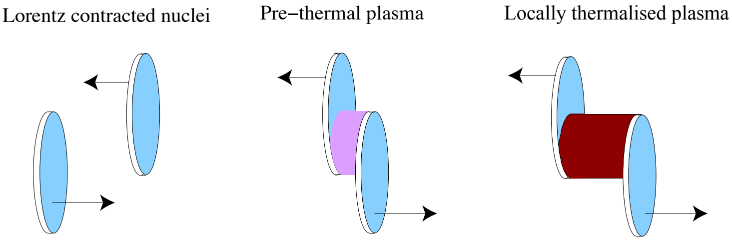
Why:

- Cosmology: pre/reheating, Parametric resonance, decay products etc.
- Cosmology: phase transitions, electro-weak
- Heavy-ion collisions: at least in the limit of large $\sqrt{s}$

Here:

Mainly application to HIC, but several generic systems

HIC motivation



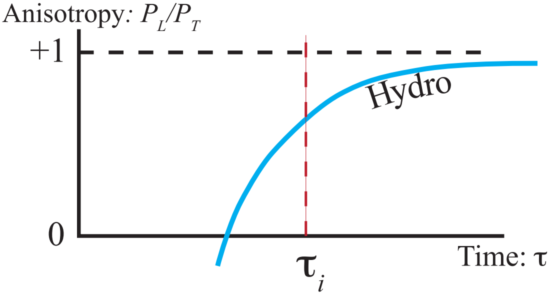
- Phenomenological success of describing HIC using fluid dynamics

$$\partial_\mu T^{\mu\nu} = 0$$

- Gradient expansion around local thermal equilibrium

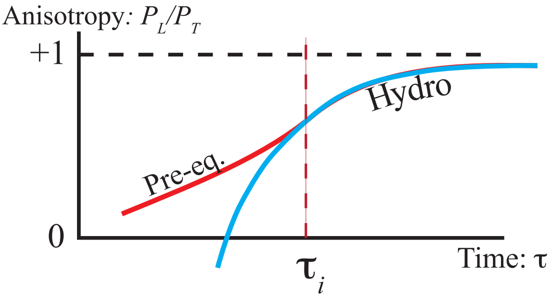
$$T^{\mu\nu} = T^{\mu\nu}_{\text{Thermal equilibrium}} - \eta(\epsilon)\sigma^{\mu\nu} - \zeta(\epsilon)\{g^{\mu\nu} + u^\mu u^\nu\}(\nabla \cdot u) + \ldots$$

HIC motivation



- Strong anisotropy $P_L/P_T \ll 1$, sign of large corrections
- At early times *pre-equilibrium* evolution
- Hydro simulations start at *intialization time* τ_i

HIC motivation

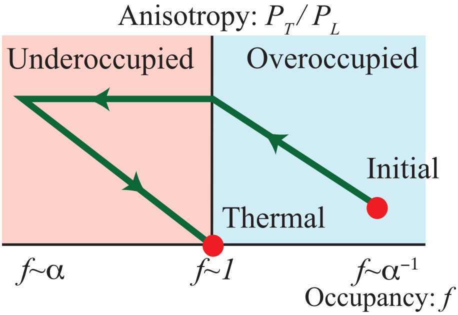


- If prethermal evolution converges smoothly to hydro, independence of unphysical τ_i

- Explicit example: Strong coupling $\mathcal{N} = 4$ SYM
  Chesler, Yaffe PRL 106 (2011) 021601; van der Schee et al. PRL 111 (2013) 22, 222302

This has proven to be challenging in QCD, even at weak coupling

Bottom-up thermalization at weak coupling



- Saturation: Initial condition overoccupied

McLerran, Venugopalan PRD49 (1994) 2233-2241 , PRD49 (1994) 3352-3355 ; Gelis et. al Int.J.Mod.Phys. E16 (2007) 2595-2637 , Ann.Rev.Nucl.Part.Sci. 60 (2010) 463-489

$$f(Q_s) \sim 1/\alpha_s, \qquad Q_s \sim 2\text{GeV}$$

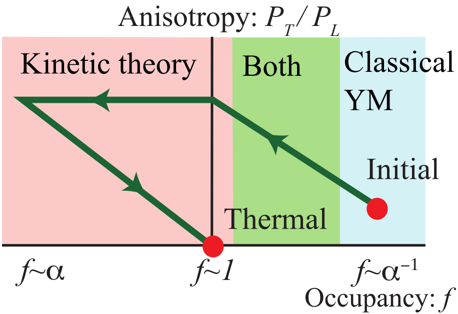
- Expansion makes system underoccupied before thermalizing

Baier et al Phys.Lett. B502 (2001) 51-58; AK, Moore JHEP 1111 (2011) 120

$$f(Q_s) \ll 1$$

- Underoccupied system thermalizes through *radiative breakup*

Bottom-up thermalization at weak coupling

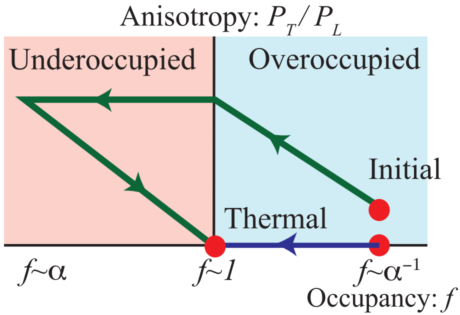


- Degrees of freedom:
  - $f \gg 1$: Classical Yang-Mills theory (CYM)
  - $f \ll 1/\alpha_s$: (Semi-)classical particles, Eff. Kinetic Theory (EKT)

- Transmutation of fields to particles: Field-particle duality
  Son, Mueller PLB582 (2004) 279-287; Jeon PRC72 (2005) 014907; Mathieu et al EPJ. C74 (2014) 2873 ; AK, Moore, Lu, York PRD89 (2014) 7, 074036

$$1 \ll f \ll 1/\alpha_s$$

Outline

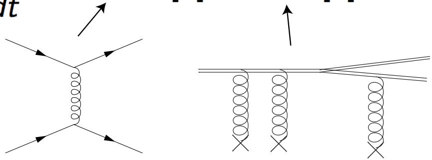


- Isotropic overoccupied: Transmutation of d.o.f's reheating?
- Isotropic underoccupied: Radiative break-up inflaton decay?
- Application to HIC: effect of longitudinal expansion

Effective kinetic theory of Arnold, Moore, Yaffe

JHEP 0301 (2003) 030

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



- Soft and collinear divergences lead to nontrivial matrix elements

soft: screening, Hard-loop; collinear: LPM, ladder resum

- Based on spectral function being mass quasiparticle pole:

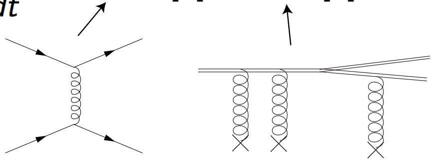
$$p^2 \gg m_D^2 \equiv 2N_c g^2 \int_{\mathbf{p}} f(p)/p$$

- No free parameters; LO accurate in the $\alpha \to 0$, $\alpha f \to 0$ limit.
- Used for LO transport coefficients in QCD, jet energy loss

Arnold et al. JHEP 0305 (2003) 051; Moore, York PRD79 (2009) 054011; Ghiglieri, Teaney 1502.03730; AK, Wiedemann PLB740 (2015) 172-178; Iancu, Wu 1506.07871

$2 \leftrightarrow 2$ scattering, screening

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



$$C_{2\leftrightarrow 2}[f] = \int_{k,p',k'} |M|^2 \left[f_p f_k (1 + f_{p'})(1 + f_{k'}) - f_{p'} f_{k'} (1 + f_p)(1 + f_k)) \right]$$

- Naively $|M|^2$ diverges as $1/q^4$. Dynamically regulated by screening

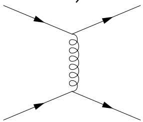
$$\frac{1}{q^4} \Rightarrow \frac{1}{(q^2 + \Pi(\omega, q, m_D)^2} \Rightarrow \frac{1}{(q^2 + \tilde{m}^2)^2}$$

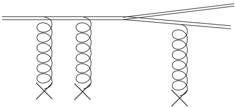
with carefully chosen $\tilde{m}^2 = e^{5/6} 2^{-3/2} m_D$ isotropic case

AK, Lu, Moore, York PRD89 (2014) 7, 074036

$1 \leftrightarrow 2$ splitting, soft radiation

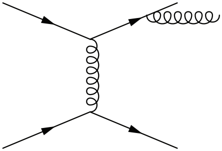
$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$





$$C_{1\leftrightarrow 2} \sim \int dp\ \gamma^{p}_{k,p-k}\,[f_p(1+f_k)(1+f_{p-k}) - f_k f_{p-k}(1+f_p)]$$

IR divergence in the elastic scattering makes soft scattering rate large
Each time a particle undergoes a soft scattering, has g^2 chance to split



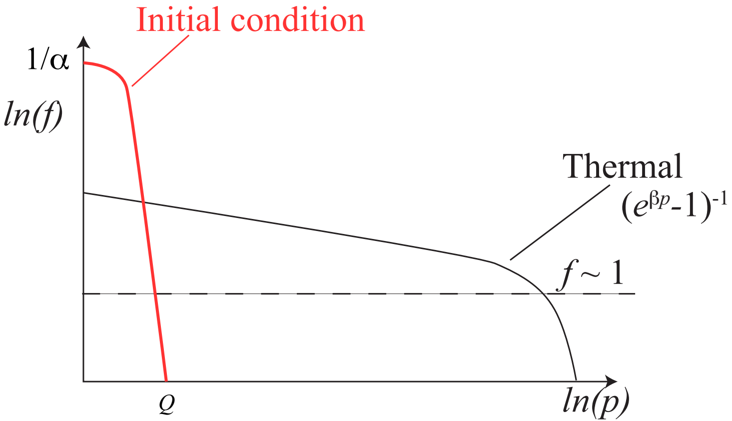
$$\Gamma_{\rm split} \sim \alpha_s \Gamma_{\rm soft}(1+f_{\rm final}) \gtrsim \Gamma_{\rm hard}$$

As important for under-, more important for underoccupied

Overoccupied cascade

AK, Moore JHEP 1112 (2011) 044

What happens if you have too many soft gluons, $f \sim 1/\alpha$.
No longitudinal expansion.



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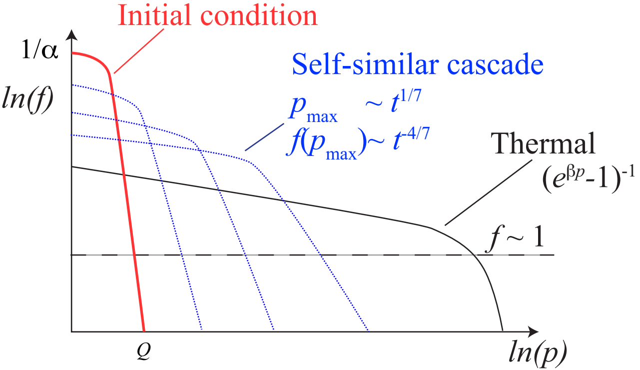
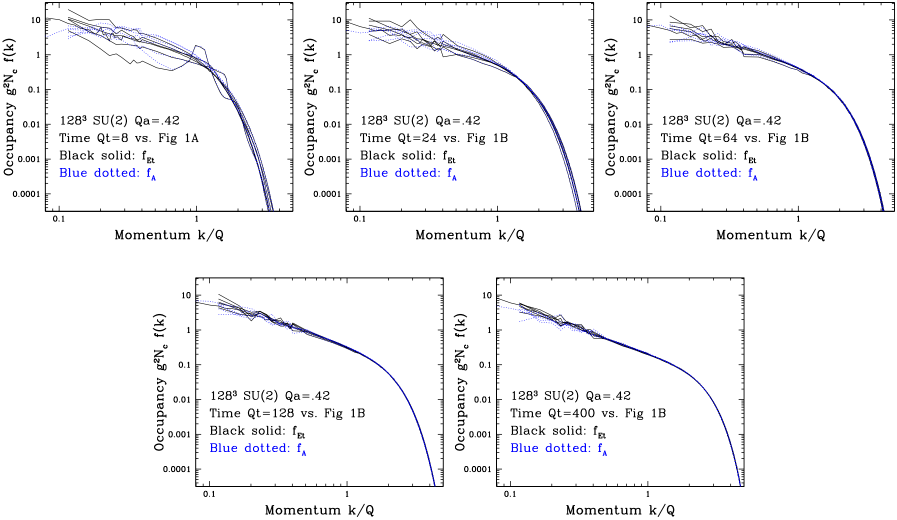


$$\tau_{\rm init} \sim \sigma n(1+f) \sim \left(\frac{Q}{T}\right)^7 \frac{1}{\alpha^2 T} \ll \frac{1}{\alpha^2 T} \sim \tau_{\rm them.}$$

Overoccupied cascade

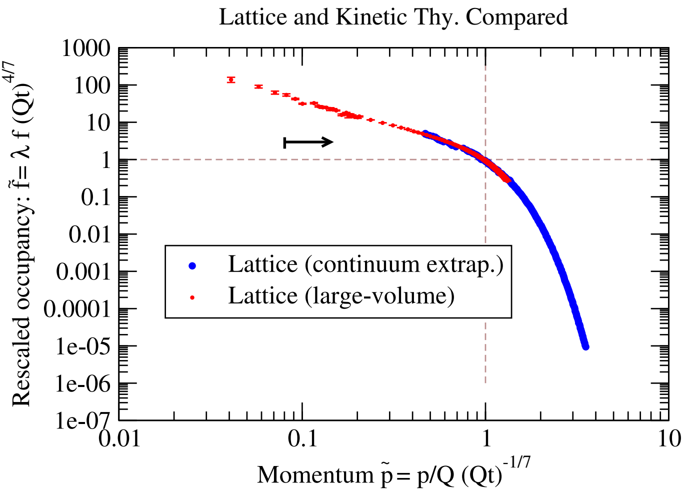
AK, Moore PRD86 (2012) 056008

Scaling reached in scattering time of the initial condition



Overoccupied cascade

AK, Lu, Moore, PRD89 (2014) 7, 074036



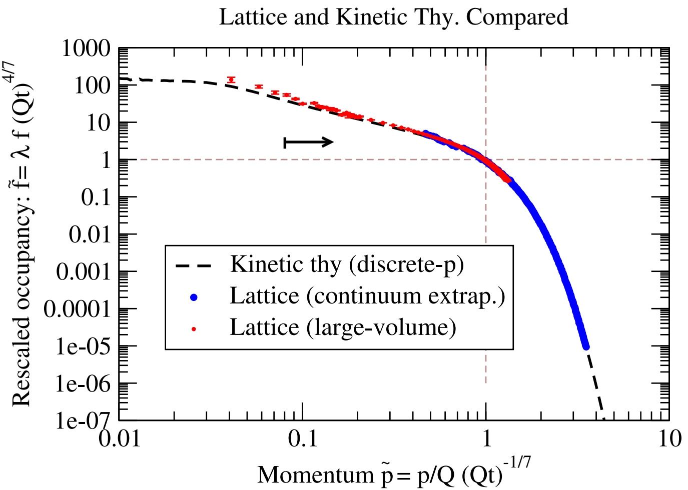
Form of cascade from classical lattice simulation,

$$1 \ll f \lesssim 1/\alpha$$

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6, Qt=2000, $\tilde{m} = 0.08$

Overoccupied cascade

AK, Lu, Moore, PRD89 (2014) 7, 074036

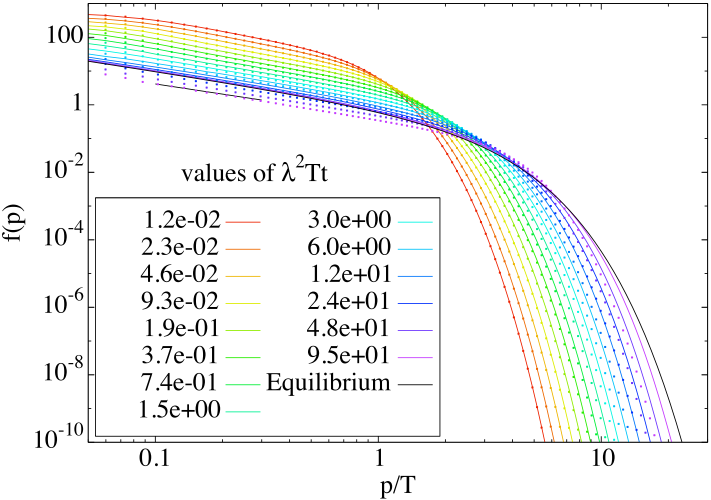


Same system, very different degrees of freedom

$$1 \lesssim f \ll 1/\alpha$$

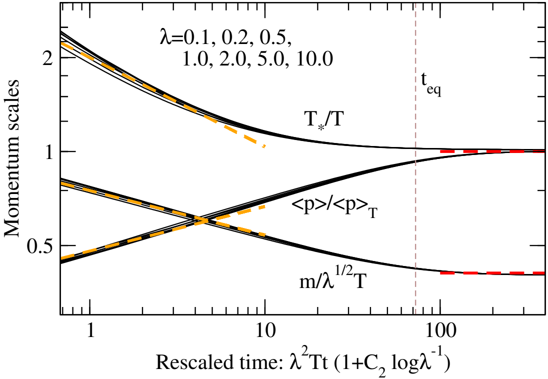
Numerical demonstration of field-particle duality

Ending of the overoccupied cascade AK, Lu PRL 113 (2014) 18, 182301



Thermal equilibrium reached once $f \sim 1, p \sim T$ (or $t \sim \frac{1}{\alpha^2 T}$).

Ending of the overoccupied cascade AK, Lu PRL 113 (2014) 18, 182301



$$m^2 = \lambda \int_{\mathbf{p}} \frac{f(p)}{p}$$

$$T_* = \frac{\lambda}{2} \int_{\mathbf{p}} f(p)[1+f(p)]/m^2$$

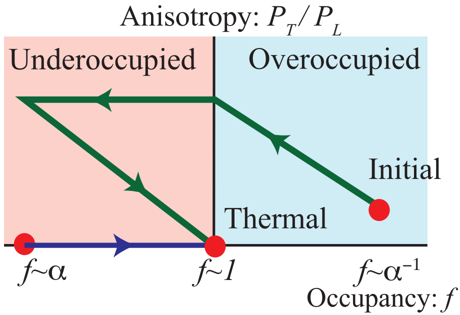
$$\langle p \rangle = \frac{1}{n} \int_{\mathbf{p}} p\, f(p)$$

Therm. time through the approach of $\langle p \rangle - \langle p \rangle_T \sim \exp(-t/t_{\mathrm{eq}})$

$$t_{\mathrm{eq}} \approx \frac{72.}{1 + 0.12 \log \lambda^{-1}} \frac{1}{\lambda^2 T}$$

$\lambda = 4\pi N_c \alpha$

Outline



- Isotropic overoccupied: Transmutation of d.o.f's reheating?
- Isotropic underoccupied: Radiative break-up inflaton decay?
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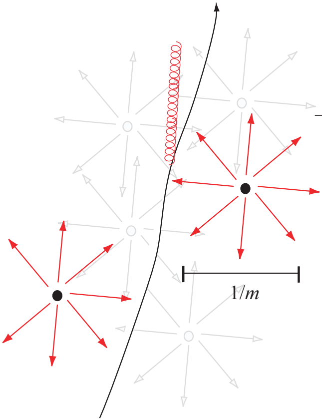
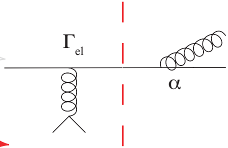
Underoccupied cascade: Formation of thermal bath

What happens to collection of few energetic particles:



- Soft modes quick to emit



$$\Gamma_{\rm el} \sim \alpha^2 \frac{n}{m_D^2} \sim \alpha^2 \frac{\int_{\mathbf{p}} f}{\alpha \int_{\mathbf{p}} f/p}$$

$$n_{\rm soft} \sim \alpha\, \Gamma_{\rm el}\, t$$

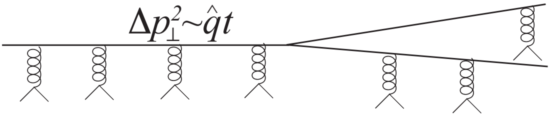
- Low-p: easy to thermalize
- Can dominate the dynamics

scattering, screening, . . .

⇒ Few energetic “jets” propagating in thermal bath

Underoccupied cascade: Radiational breakup

What happens to jet in medium:

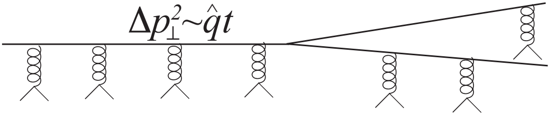


- In vacuum: on-shell splitting kin. disallowed
- In medium:
  - frequent soft scatterings with medium, mom. diffusion: $\Delta p^2 \sim \hat{q}t$
  - Scatterings lead to virtuality: $P^2 \sim \hat{q}t$
  - Now offshell particle may split collinearly: $t_f \sim Q/P^2 \sim \sqrt{Q/\hat{q}}$
  - Splitting time (per particle) $t_{\text{split}}(Q) \sim \frac{1}{\alpha} t_f \sim \frac{1}{\alpha}\sqrt{\frac{Q}{\hat{q}}}$

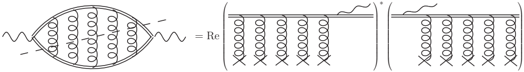
QED: Landau, Pomeranchuk, Migdal 1953.
QCD: Baier Dokshitzer Mueller Peigne Schiff hep-ph/9607355

Underoccupied cascade: Radiational breakup

What happens to jet in medium:



- Formation time t_f longer than scattering time
- Particle still off-shell when scatters: Interference, no Boltzmann

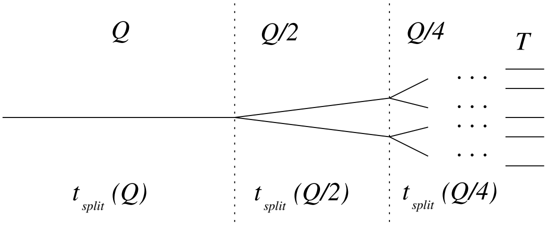


$\Rightarrow$ Effective in-medium rate equation, LPM suppression

QED: Landau, Pomeranchuk, Migdal 1953.
QCD: Baier Dokshitzer Mueller Peigne Schiff Nucl.Phys. B483 (1997) 291-320

Underoccupied cascade: Radiational breakup

What happens to jet in medium:



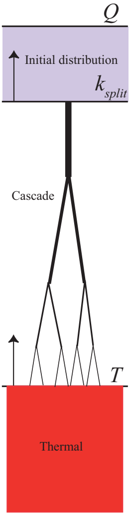
- Successive splittings happen in faster times scales:

$$t_{\text{quench}}(Q) \sim t_{\text{split}}(Q) + t_{\text{split}}(Q/2) + t_{\text{split}}(Q/4) + \ldots \sim t_{\text{split}}(Q)$$

- Once the parton has had time to split once it cascades its energy to IR

Bottom-up thermalization

AK, Moore JHEP 1112 (2011) 044



- Hard particles scatter with soft thermal bath

$$\hat{q} \sim \alpha^2 T^3$$

- Scales below $k_{\rm split}$ have had time to undergo radiational breakup

$$t_{\rm quench}(k_{\rm split}) \sim t \Rightarrow k_{\rm split} \sim \alpha^2 \hat{q} t^2$$

- "Falling" particles heat the soft thermal bath

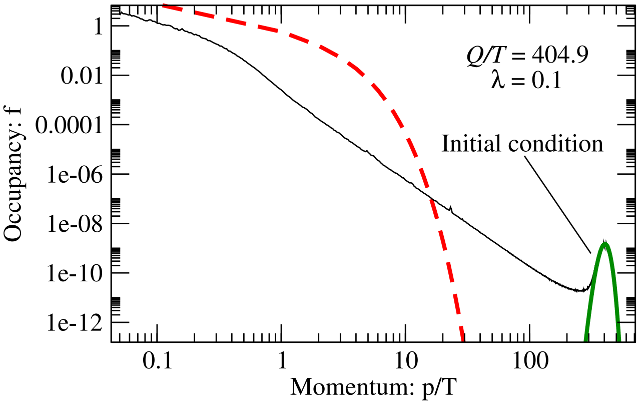
$$T^4 \sim k_{\rm split} \int d^3p f(p)$$

- Thermalization when hardest scale Q gets eaten $k_{\rm split}(\tau_0) \sim Q$ at

$$\tau_0 \sim t_{\rm quench}(Q) \sim \frac{1}{\alpha}\sqrt{\frac{Q}{\hat{q}}} \sim \left(\frac{Q}{T_{\rm final}}\right)^{1/2} \frac{1}{\alpha^2 T_{\rm final}}$$

Radiational breakup

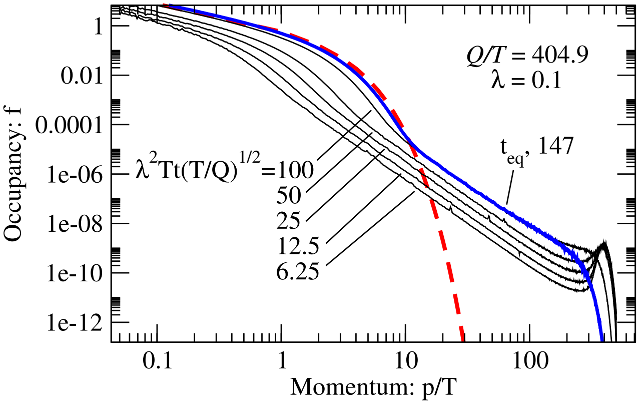
AK, Lu, PRL 113 (2014) 18, 182301



- Start with an underoccupied initial condition $p \sim Q$
- after a very short time, an IR bath is created (1$\leftrightarrow$ 2–processes)

Radiational breakup

AK, Lu, PRL 113 (2014) 18, 182301



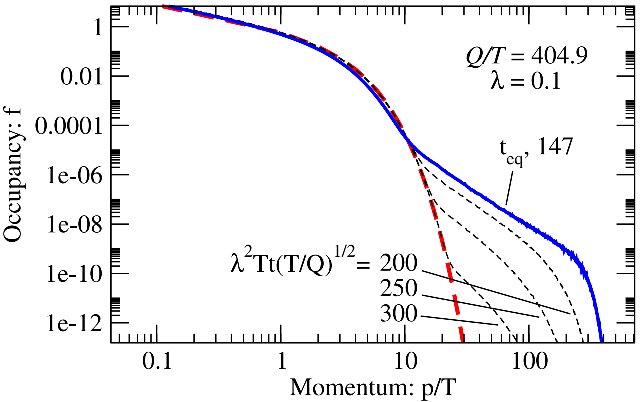
- More energy flows to the IR, temperature increases, "Bottom-up"
- $t_{\rm eq} \sim$ time to quench a jet Q in a thermal bath carrying the energy of the jets $T \sim \epsilon^{1/4}$

AK, Moore JHEP 1112 (2011) 044

$$t_{\rm eq} \sim (Q/T)^{1/2} \frac{1}{\lambda^2 T}$$

Radiational breakup

AK, Lu, PRL 113 (2014) 18, 182301

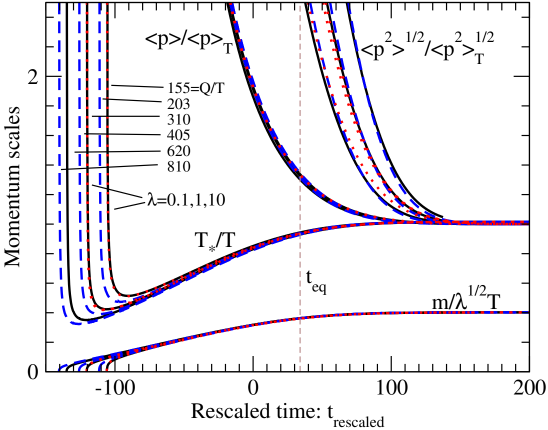


- Hardest scales reach equilibrium last.

Close resemblance to Blaizot, Iancu, Mehtar-tani for jets PRL 111 (2013) 052001

Scaling analysis

AK, Lu PRL 113 (2014) 18, 182301



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Therm. time through the approach of $\langle p \rangle - \langle p \rangle_T \sim \exp(-t/t_{\rm eq})$

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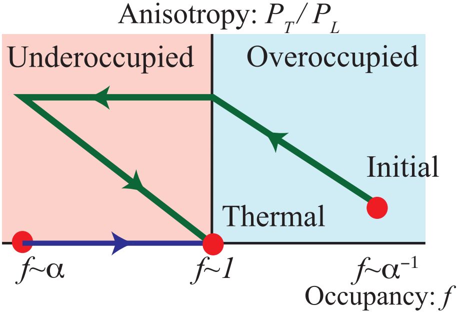
Outline



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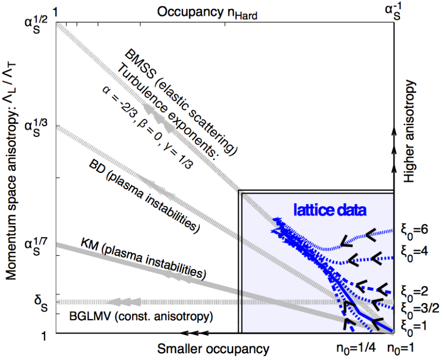
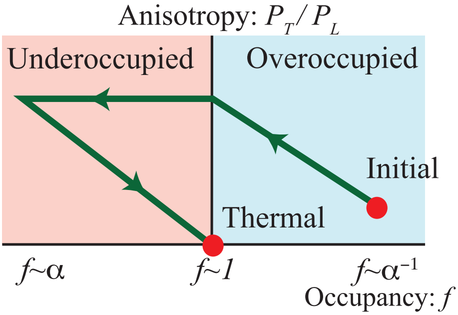
Early classical evolution



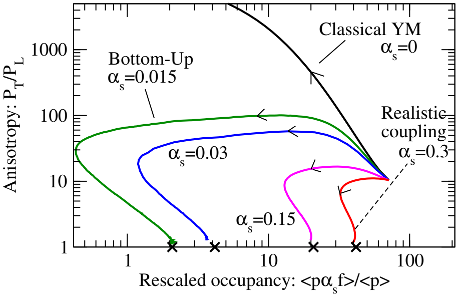


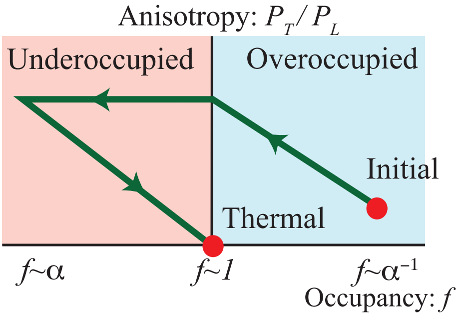
Berges et al. Phys.Rev. D89 (2014) 7, 074011

- Numerical demonstration of classical/overoccupied part of the diagram
- Classical theory never thermalises or isotropizes
- Before $f \sim 1$, must switch to kinetic theory

Route to equilibrium in EKT

AK, Zhu, PRL 115 (2015) 18, 182301

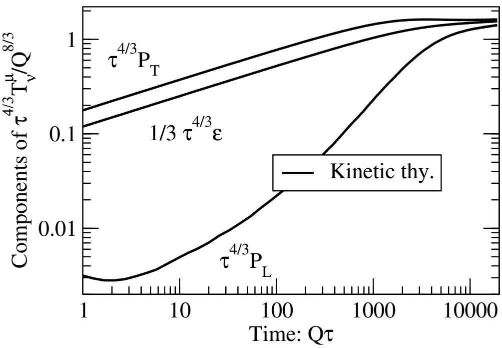




- Initial condition ($f \sim 1/\alpha$) from classical field theory calculation
  Lappi PLB703 (2011) 325-330
- In the classical limit ($\alpha \to 0, \alpha f$ fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as α grows

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

$$\alpha = 0.03$$



- Kinetic theory converges to hydro smoothly and automatically

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

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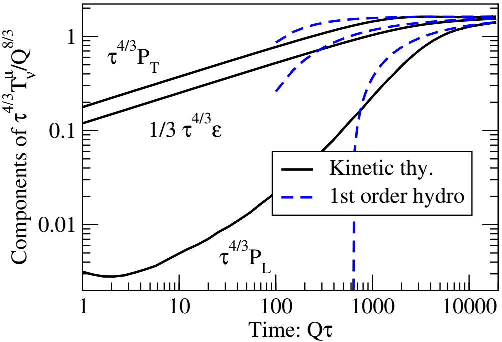


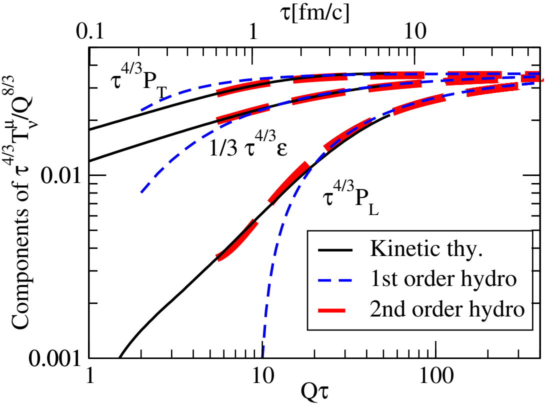
- Kinetic theory converges to hydro smoothly and automatically
- Hydro prediction fixed by perturbative η/s

Arnold et al. JHEP 0305 (2003) 051

$$P_L = \frac{1}{3}\epsilon - \frac{4\eta}{3\tau}$$

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

$$\alpha = 0.3$$



- For realistic couplings, hydrodynamics reached around $\lesssim$ 1fm/c. Consistent with phenomenology.
- Hydro seems to give a good description even when $P_L/P_T \sim 1/5$

Caveats

- Fermions
- Transverse dynamics, preflow
- Plasma instabilities, anisotropic screening
  AK, Moore, JHEP 1112 (2011) 044 , JHEP 1111 (2011) 120
  - Numerically small effect
    Berges et al. Phys.Rev. D89 (2014) 7, 074011
- Improved initial CYM simulations for initial condition of EKT

and

- Potentially large NLO corrections
  Caron-Huot, Moore PRL 100 (2008) 052301, Ghiglieri et al. JHEP 1305 (2013) 010, JHEP 1412 (2014) 029, 1502.03730, 1509.07773

Qualitative $\Rightarrow$ Quantitative

Conclusions

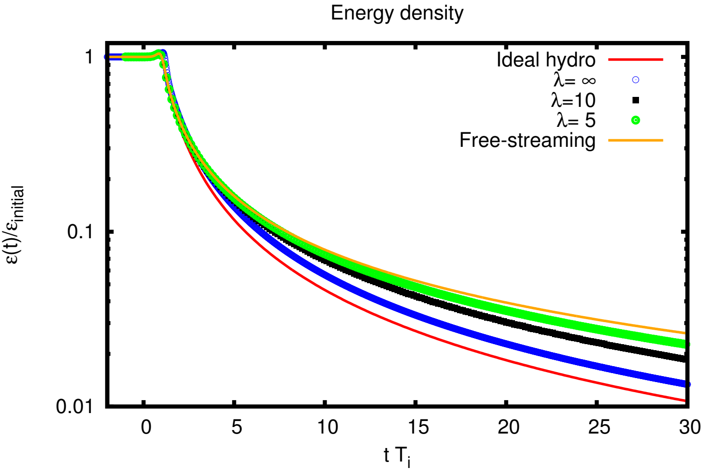
- In general: The approach to equilibrium of wide class of systems can be followed using combination of classical Yang-Mills theory and effective kinetic theory simulations.

- For HIC: weak coupling thermalization extrapolated to realistic couplings shows agreement with hydro around

$$\tau_{\rm i} \sim 1{\rm fm/c}$$

Weakly or strongly coupled thermalization?

Apples to apples comparison of weak and strong coupling



Under construction, AK, Romatschke, van der Schee

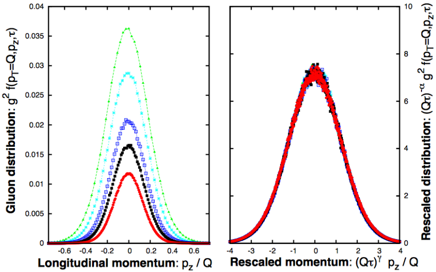
Backup sildes

Comparison between CYM and EKT: Expanding

In non-pert classical regime $1 \ll f \ll 1/\alpha_s$

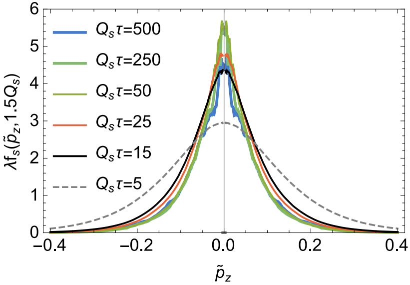
$$f(p_z, p_\perp, \tau) = (Q_s\tau)^{-2/3} f_S((Q_s\tau)^{1/3} p_z, p_\perp),$$

CYM $\alpha_s f \ll 1$ limit but $f \gg 1$



Berges et al. Phys.Rev. D89 (2014) 7, 074011

EKT $f \gg 1$ limit but $f \ll 1/\alpha_s$



AK, Zhu 1506.06647

Effective $C_{1\leftrightarrow 2}$ matrix element revisited

$$\gamma^{p'}_{p,k} \sim \underbrace{\frac{p'^4 + p^4 + k^4}{p'^3 p^3 k^3}}_{\text{DGLAP split-kernel}} \int \frac{d^2h}{(2\pi)^2} \mathbf{h} \cdot \mathrm{Re}\mathbf{F}(\mathbf{h}; p', p, k)$$

$$2\mathbf{h} = i\delta E(\mathbf{h})\mathbf{F}(\mathbf{h}) + \frac{g^2 N_c}{2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left[T_* \left(\frac{1}{\mathbf{q}^2} - \frac{1}{\mathbf{q}^2 + m^2_{\text{screen}}} \right) \right]$$
$$\times \left(3\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} - p\mathbf{q}) - \mathbf{F}(\mathbf{h} - k\mathbf{q}) - \mathbf{F}(\mathbf{h} + p\mathbf{q}) \right)$$

Where sensitivity to the medium comes from

- δE is the difference of energies of one gluon with momentum p' compared the two with k, p': depends on effective masses
- Dependence on $p/T_*, m/T_*$: In praxis:
  - solve numerically, tabulate
  - Fitting with with correct asymptotics

$1 \leftrightarrow 2$ splitting, soft radiation

Soft limit, parametrically:

- Soft scattering rate $\Gamma_{soft} \sim \lambda T_* \sim \frac{\hat{q}}{m^2}$

$$T_* = \frac{1}{2} \int_{\mathbf{p}} f_p(1+f_p) / \int_{\mathbf{p}} f(p)/p$$

Bose factors enchance, regulated by m^2

- Soft inelastic rate, Bethe-Heitler:

$$\frac{d\Gamma_{\rm BH}}{dp'} \sim \lambda^2 T_*/p'$$

- Collision kernel related to the rate $\gamma \sim p^2 \frac{d\Gamma}{dp'} \sim \lambda T_* p^2/p'$
- Constant of proportionality analytically:

$$\lim_{p' \to 0} \gamma(p; p', p-p') = \frac{\mathcal{Q}(m^2/m_D^2)}{4(2\pi)^4} \lambda^2 T_* \frac{p^2}{p'}$$

$$\mathcal{Q}(m_\infty^2/m_D^2) \equiv 8 \int_{p_\perp, q_\perp} \left[\frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_D^2} \right] \left(\frac{\mathbf{p}_\perp}{m_\infty^2 + p_\perp^2} - \frac{\mathbf{p}_\perp - \mathbf{q}_\perp}{m_\infty^2 + (\mathbf{p}_\perp - \mathbf{q}_\perp)^2} \right.$$

$1 \leftrightarrow 2$ splitting, deep LPM limit

Arnold, Dogan 0804.3359

- Hard collinear radiation suppressed by formation time

$$t_{\text{form}}(p') \sim \sqrt{\frac{p'}{\lambda T_* m^2}}$$

- The rate bounded from above by

$$\gamma \sim p^2 \frac{d\Gamma_{\text{hard}}}{dp'} \sim \lambda p^2 / t_{\text{form}} \sim \lambda^{3/2} p^2 \sqrt{T_* m^2 / p'^3}$$

- Prefactor to NLL by Arnold

log related to the UV div. of $\hat{q}$

$$\gamma(p, p', p - p') = \frac{\sqrt{2}\lambda}{4(2\pi)^5} m^2 \hat{\mu}^2(1, x, 1-x) \frac{1 + x^4 + (1-x)^4}{x^2(1-x)^2}$$

$$\hat{\mu}^2 = \frac{\lambda^{1/2} T_*}{\sqrt{2} m} \left[\frac{1}{\pi} x_1 x_2 x_3 \frac{p}{T_*}\right]^{1/2} \left[\sum_{i=1}^{3} (x_i^2) \ln(\xi \hat{\mu}^2 / x_i^2)\right]^{1/2},$$

With $\xi = 9.09916$