Dark matter bound state formation

with Kallia Petraki and Michael Wiechers
1505.00109

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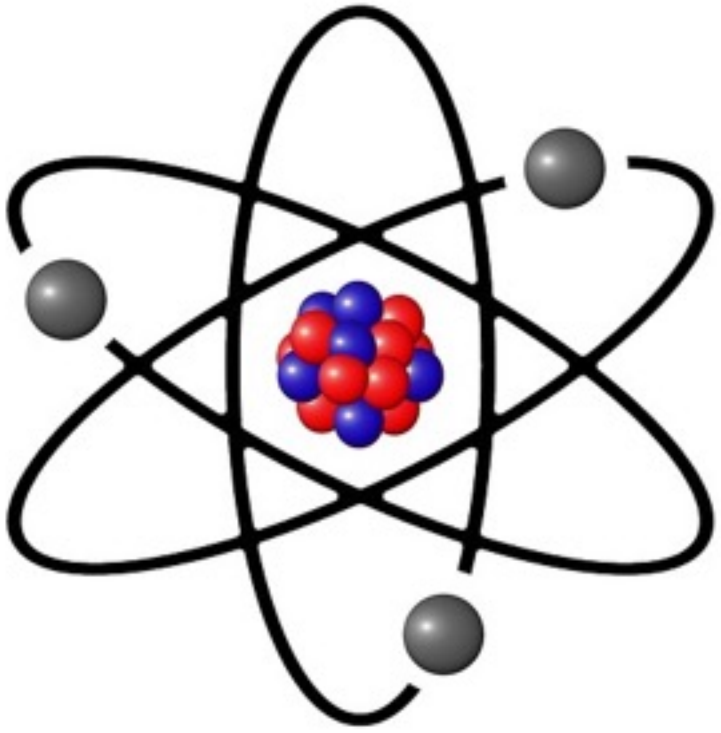
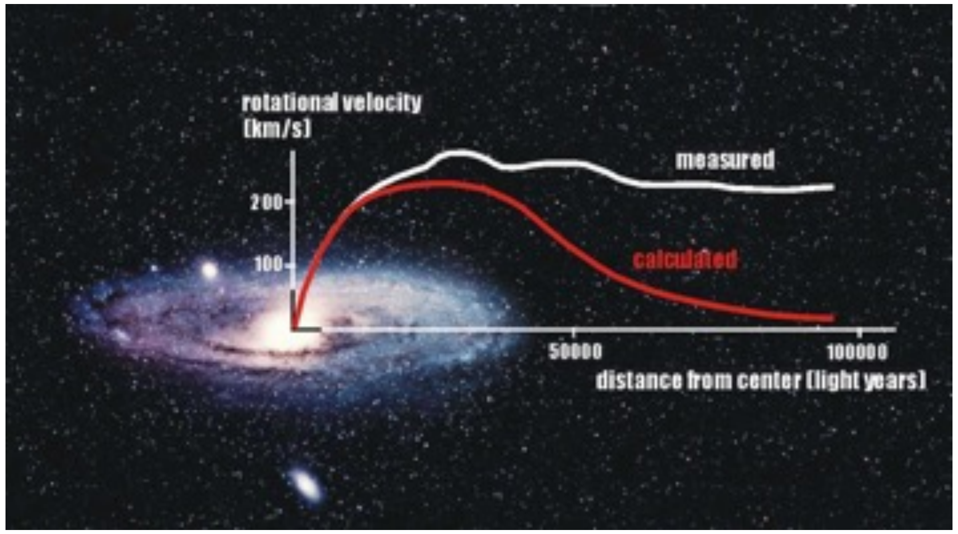
Nikhef, Amsterdam



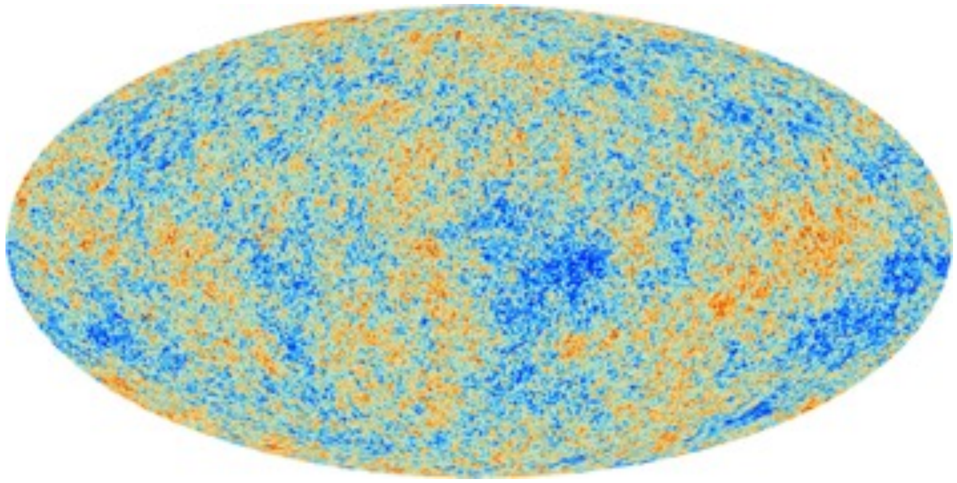
DESY
January 2016

Evidence for dark matter

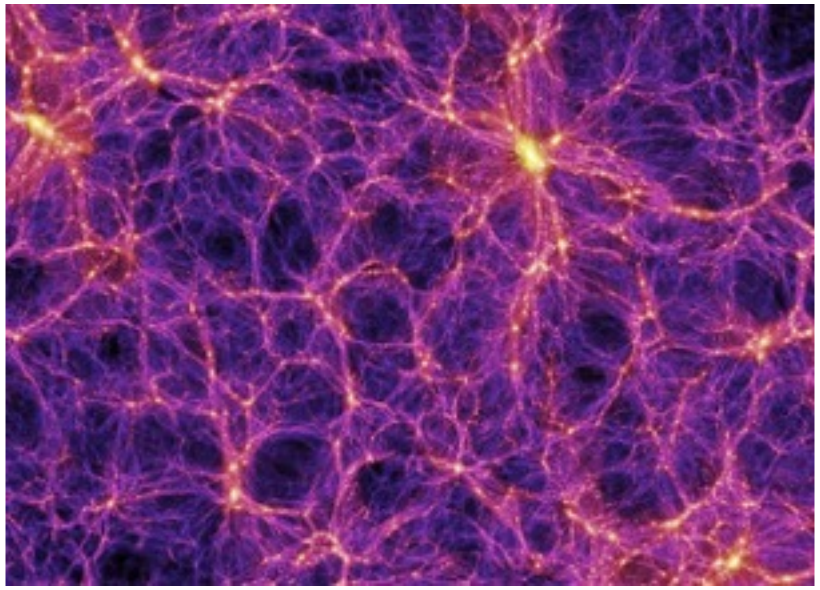


galaxy/cluster rotation curves

gravitational lensing

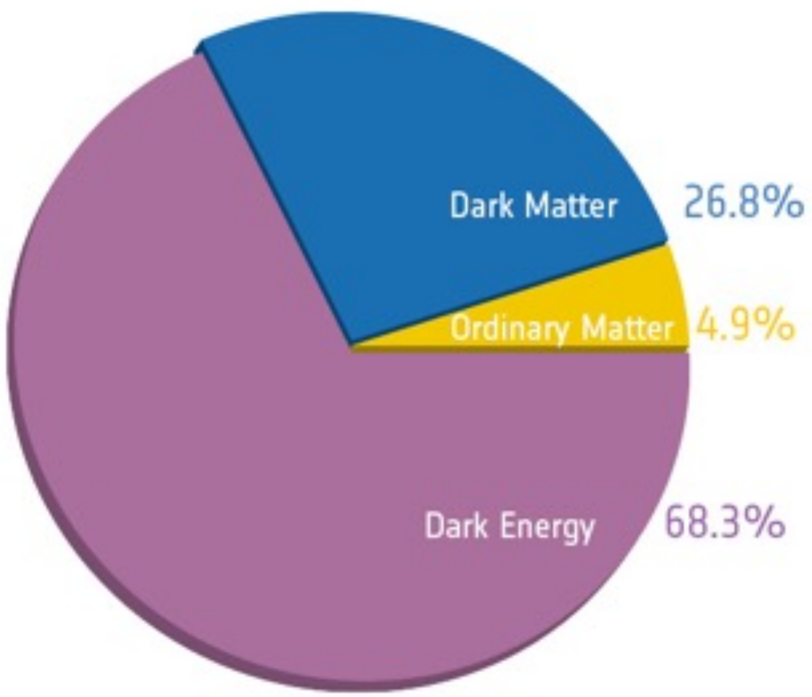


cosmic microwave background



structure formation

Dark matter: what do we know?



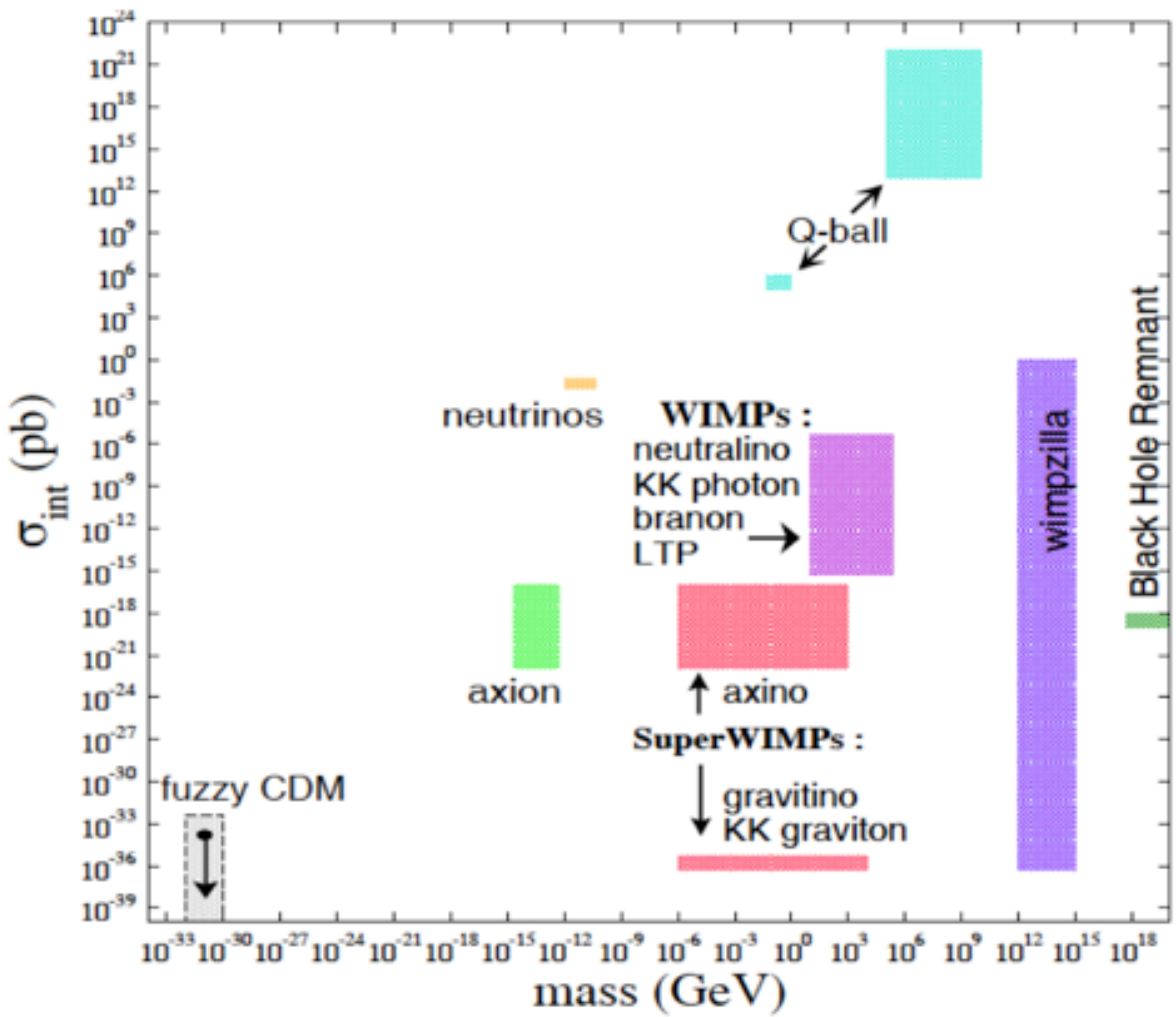
Gravitational interactions

- 26.8% energy budget
- pressureless fluid

Microphysics

- non-relativistic particles, condensate, deviations from GR?
- dark
- non-baryonic
- weak self-interactions
- (nearly) stable

Dark matter candidates



Dark matter

Minimalistic approach

- add single particle

Embed in beyond the SM physics

- typically lots of particles and interactions

bound states

Overview

- bound states: why interesting?

- QFT derivation of (non-confining) bound states formation

- results

Bound states — when important?

DM with long range interactions

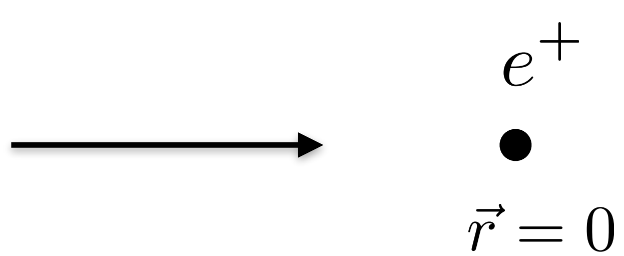
bound states exist $\Leftrightarrow$ Sommerfeld enhancement

Sommerfeld enhancement

Hisano et al ’04, Cirelli et al ’07, etc.

DM with long range interactions:

$$e^- e^+ \to \gamma\gamma$$



$$\sigma \propto |\psi(0)|^2$$

non-relativistic regime

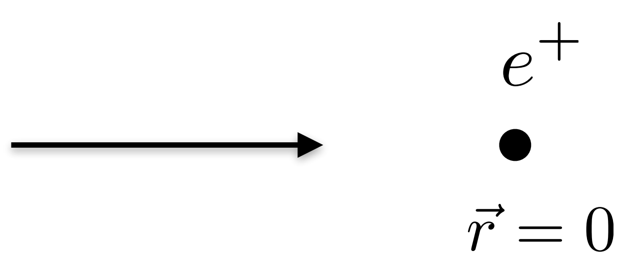
Sommerfeld enhancement

Hisano et al '04, Cirelli et al '07, etc.

DM with long range interactions:

1. $v^2 m_e < \alpha^2 m_e$
2. $1/m_\gamma > 1/(\alpha m_e)$

$$e^- e^+ \to \gamma\gamma$$



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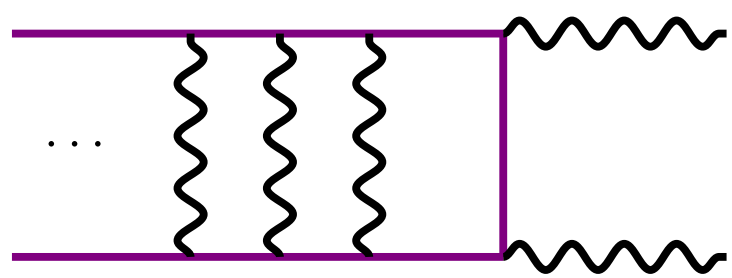
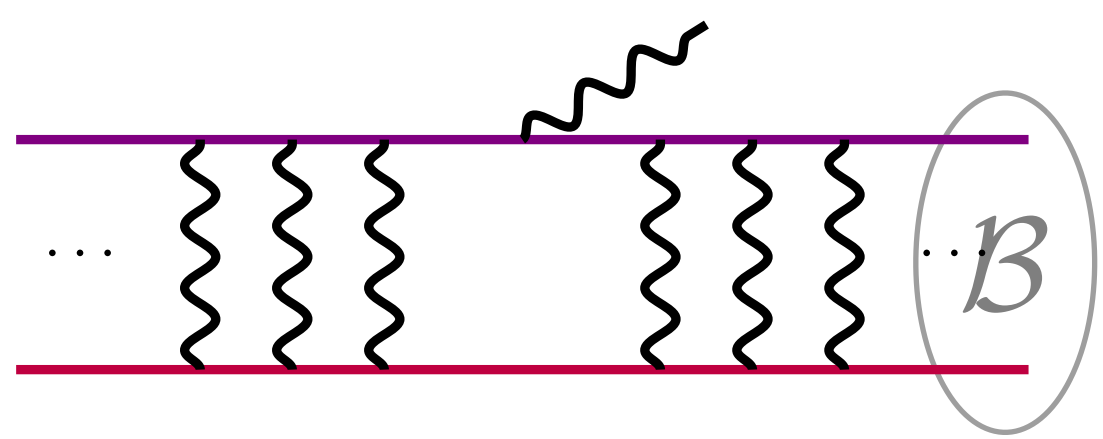


$e^- e^+ \to \mathcal{B}\gamma$



Bound states — when important?

DM with long range interactions: $\alpha \gtrsim m_\chi/m_\phi$

- self-interacting DM — spergel & steinhardt '00, etc.
- asymmetric DM — Davoudiasl & Mohapatra '12, etc.
- atomic DM — Kaplan et al '09, etc.
- 10 TeV WIMP — Hisano et al '03, Cirelli et al '07, etc.
- ...

Bound states — why important?

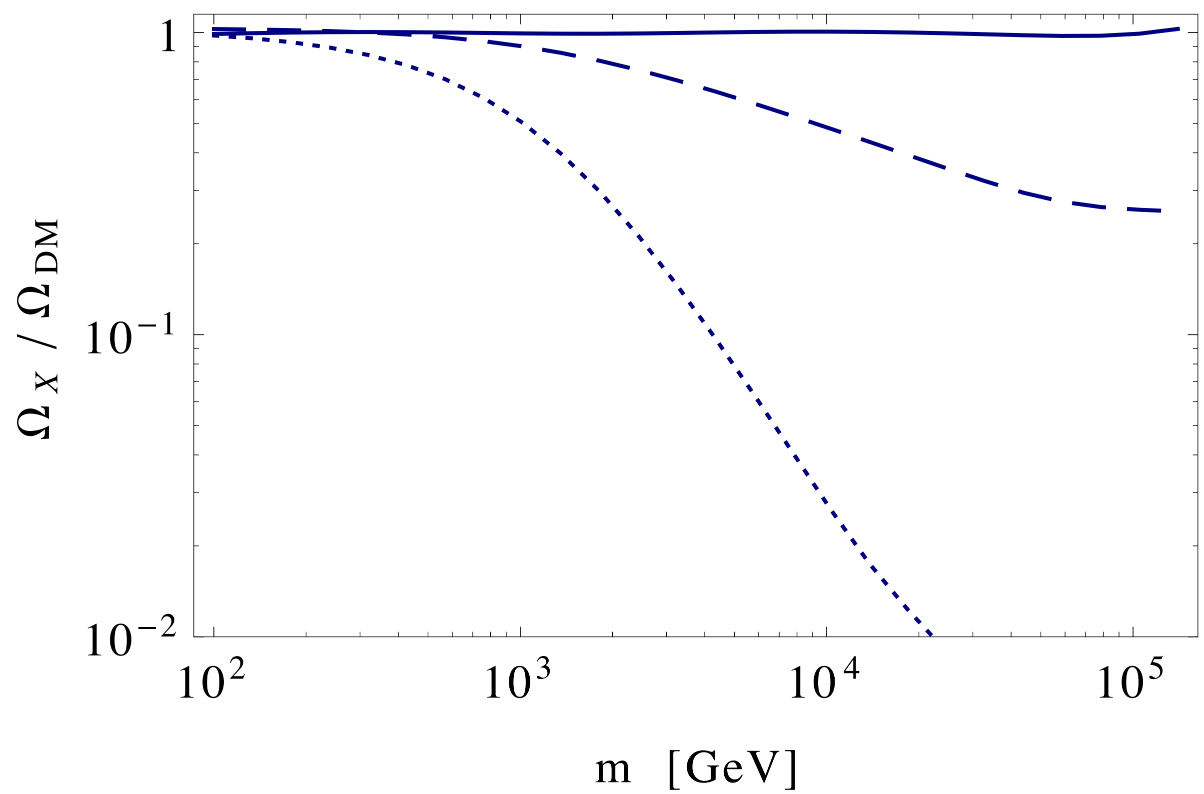
Impact of bound states:

Bound states — why important?

Impact of bound states:

- relic density: unstable bound states extra annihilation channel

von Harling & Petraki ’14



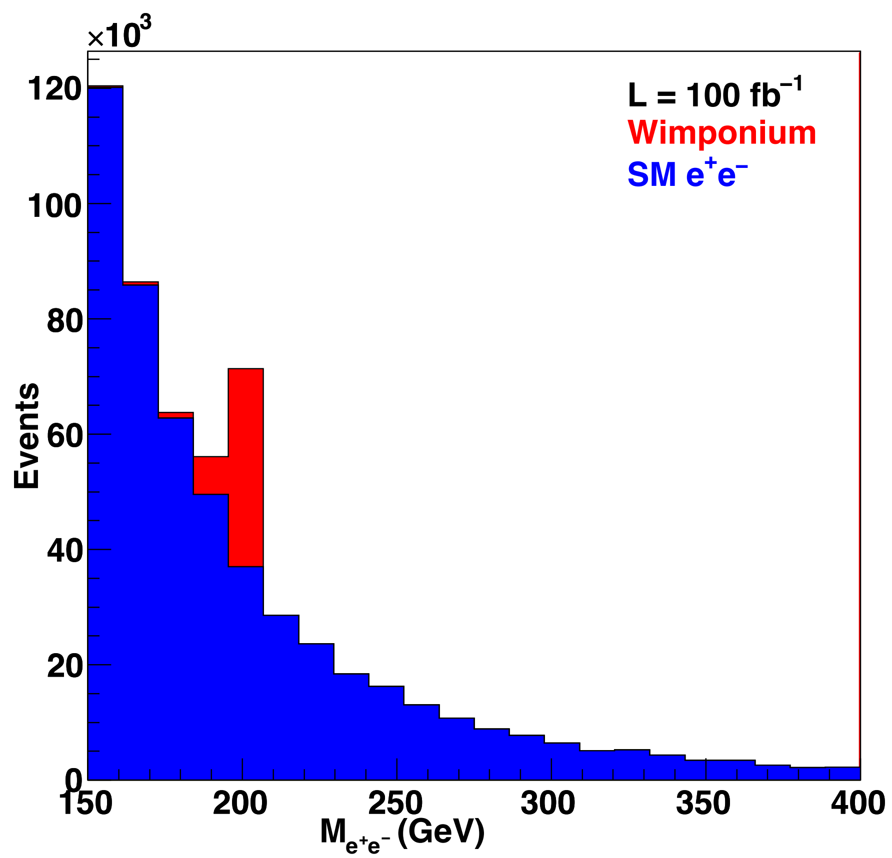
The following processes take place during the DM freeze-out: (i) annihilation without any Sommerfeld enhancement (dotted), (ii) Sommerfeld-enhanced annihilation only (dashed), (iii) Sommerfeld-enhanced annihilation and BSF (solid).

Bound states — why important?

Impact of bound states:

- relic density: unstable bound states extra annihilation channel
  von Harling & Petraki '14

- WIMPonium at the LHC
  Shepherd, Tait, and Zaharijas '09



Sample reconstructed invariant mass of the l^+l^- for the Standard Model (blue) and a vector WIMPonium ${}^3S_1$ state signal for a mass of 200 GeV, universal Λf = M, and $\alpha_\chi = 0.2$ (red) at the LHC for an integrated luminosity of 100 fb^{-1}

Bound states — why important?

Impact of bound states:

- relic density: unstable bound states extra annihilation channel
  von Harling & Petraki '14
- WIMPonium at the LHC
  Shepherd, Tait, and Zaharijas '09
- indirect & direct detection experiments
  Pearce & Kusenko '13, Lah and Braaten '13, etc.
- kinetic decoupling of DM from radiation
  Cyr-Racine et al '14
- self-scattering in halos
  Cline et al '12, Cyr-Racine & Sigurdson '12, etc.
- ...

Overview

- bound states: why interesting?

- QFT derivation of (non-confining) bound states formation

- results

Bound state formation

Quantum mechanics vs. Quantum field theory

Sommerfeld ‘31

Bethe & Salpeter ‘57

Akhiezer & Merenkov ‘96

- modern approach

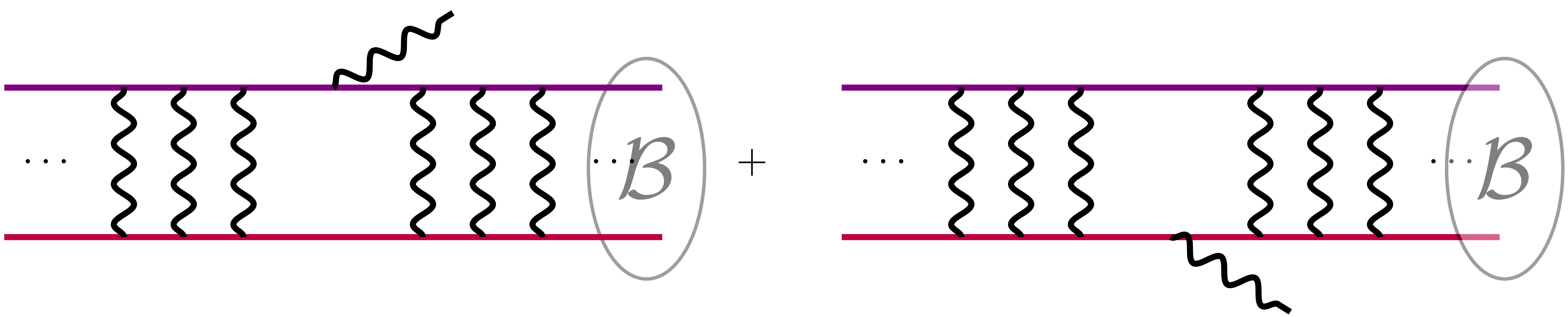
- systematic relativistic and radiative corrections

Hryczuk & Iengo ‘12

- generalization to non-abelian interactions

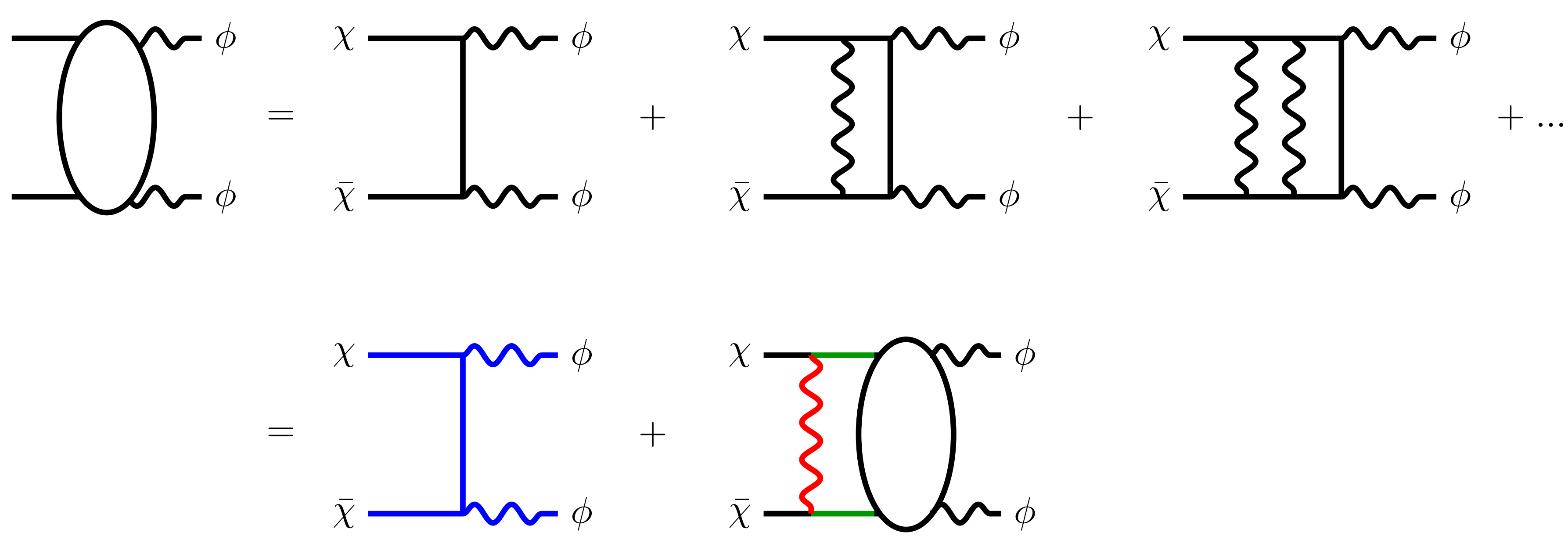
Feynman diagram

$$\chi_1 + \chi_2 \to \mathcal{B} + \phi$$



Sommerfeld enhancement from QFT

Sommerfeld enhancement in QFT in $\chi\bar{\chi} \to \phi\phi$ Iengo '09, Cassel '09



$\mathcal{O}(\alpha)$

typical Bohr exchange momentum and energy

$$|\vec{q}| \sim \alpha m_\chi \quad \& \quad q^0 \sim \frac{1}{2}\alpha^2 m_\chi$$

Bethe-Salpeter equation

$$\mathcal{A}(p,p') = \mathcal{A}_0(p,p') + \int \frac{\mathrm{d}^4 q}{(2\pi)^4} D_\phi(p-q) D_\chi(q) D_\chi(-q) A(q,p')$$

Sommerfeld enhancement from QFT

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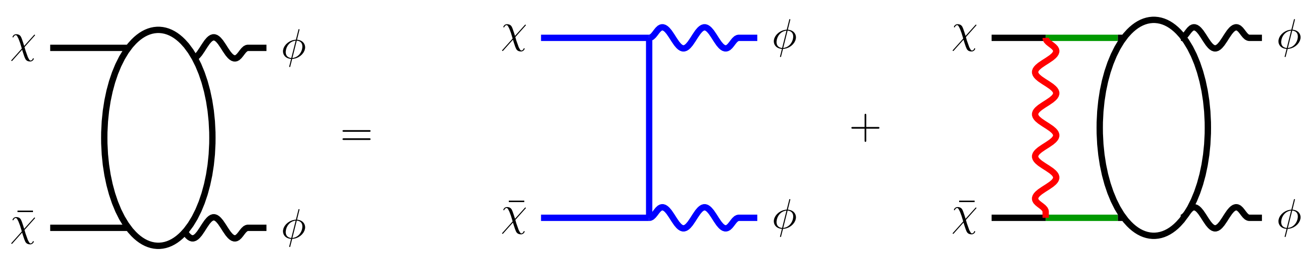

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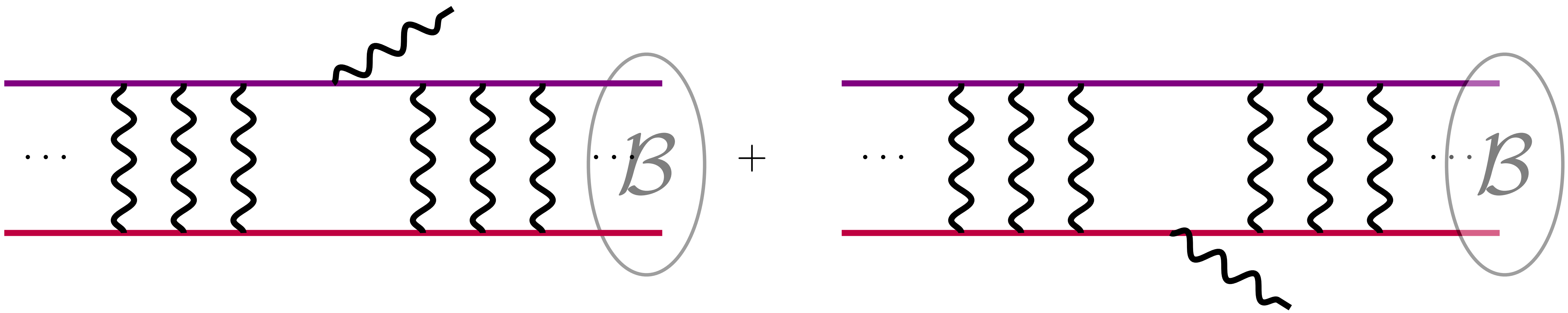
$$\phi \sim \int dq^0 D_\chi D_\chi \mathcal{A}$$

$$V \sim \int d^3q\, D_\phi\, e^{i\vec{q}.\vec{r}}$$

End result in non-relativistic limit

$$\mathcal{A}(p,p') = \int \frac{d^3 q}{(2\pi)^3} \phi_{\vec{p}}(q) \mathcal{A}_0(q,p')$$

Bound state formation — Challenge



- double summation
- ‘missing’ propagator
- amplitude vanishes on-shell

Bound state formation — Solution

LSZ-reduction

n-point function $\Longleftrightarrow$ *n*-point amputated amplitude

pole & branch cut
structure

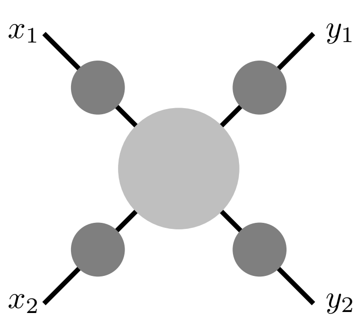
Bound state formation — Solution

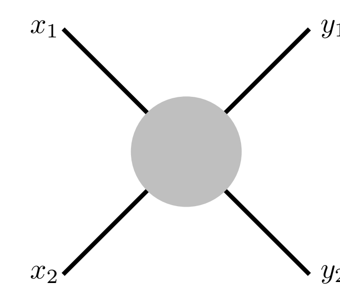
LSZ-reduction

n-point function $\Longleftrightarrow$ *n*-point amputated amplitude

pole & branch cut structure

example:





$$\left(\Pi_i \int d^4x_i e^{ip_i x_i}\right)\left(\Pi_j \int d^4y_j e^{ik_j y_j}\right) \langle\Omega|T\{\phi(x_1)\phi(x_2)\phi(y_1)\phi(y_2)\}|\Omega\rangle \sim$$

$$\left(\Pi_i \frac{i}{p_i^2 - m^2 + i\epsilon}\right)\left(\Pi_j \frac{i}{k_j^2 - m^2 + i\epsilon}\right)(\sqrt{Z})^4 \langle \mathbf{p_1 p_2}|S|\mathbf{k_1 k_2}\rangle$$

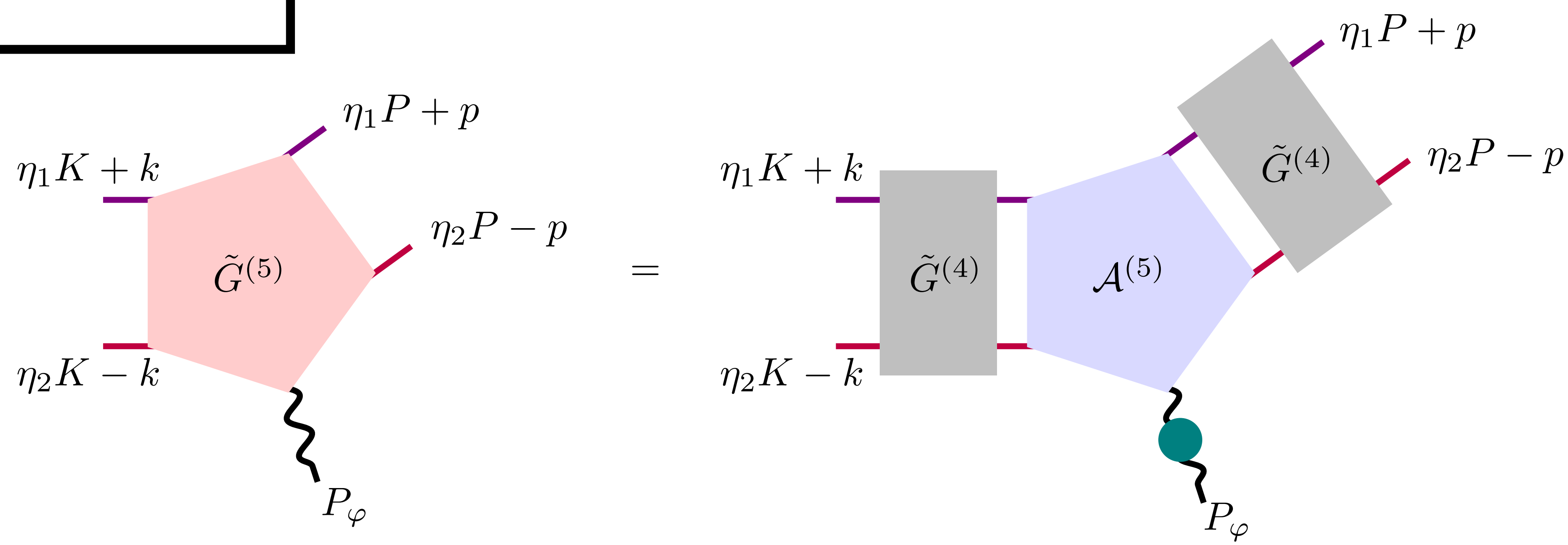
Bound state formation — Solution

LSZ-reduction

n-point function ⟺ *n*-point amputated amplitude

pole & branch cut structure

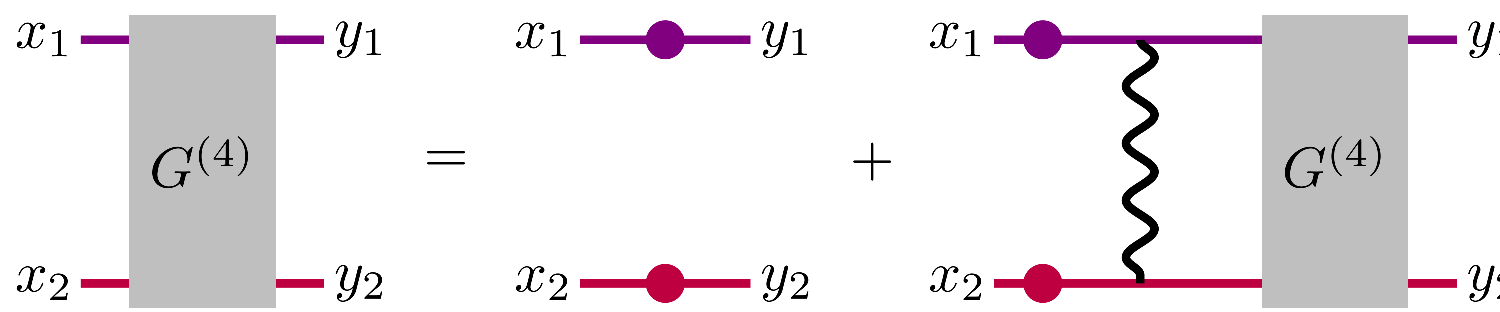
$$\chi_1 + \chi_2 \to \mathcal{B} + \phi$$



$$\tilde{G}^{(5)} \sim \tilde{S}_\phi \int d^4k\, d^4k' \tilde{G}^{(4)} \delta^4(K - P - P_\phi) i\mathcal{A}^{(5)} \tilde{G}^{(4)}$$

Bound state formation — Solution

$$G^{(4)} = \langle \Omega | T(\chi_1 \chi_2 \chi_1^\dagger \chi_2^\dagger) | \Omega \rangle =$$



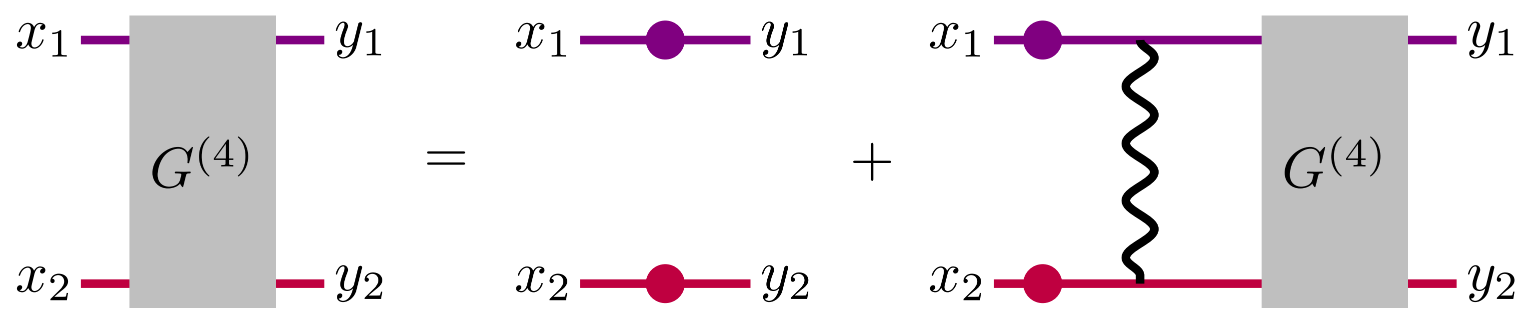
Bethe-Salpeter equation

insert completeness relation

$$\mathbf{1} = \sum_n \int \frac{\mathrm{d}^3 Q}{(2\pi)^3 2\omega_{\vec{Q},n}} |\mathcal{B}_{\vec{Q},n}\rangle\langle\mathcal{B}_{\vec{Q},n}| + \int \frac{\mathrm{d}^3 q}{(2\pi)^3 2\omega_q} \frac{\mathrm{d}^3 Q}{(2\pi)^3 2\omega_Q} |\mathcal{U}_{\vec{Q},\vec{q}}\rangle\langle\mathcal{U}_{\vec{Q},\vec{q}}|$$

Bound state formation — Solution

$$G^{(4)} = \langle \Omega | T(\chi_1 \chi_2 \chi_1^\dagger \chi_2^\dagger) | \Omega \rangle =$$



Bethe-Salpeter equation

non-relativistic schrödinger equation for bound state wave function

$$\left[-\frac{\nabla^2}{2\mu} + V(\mathbf{r}) \right] \psi_n(\mathbf{r}) = \mathcal{E}_n \psi_n(\mathbf{r})$$

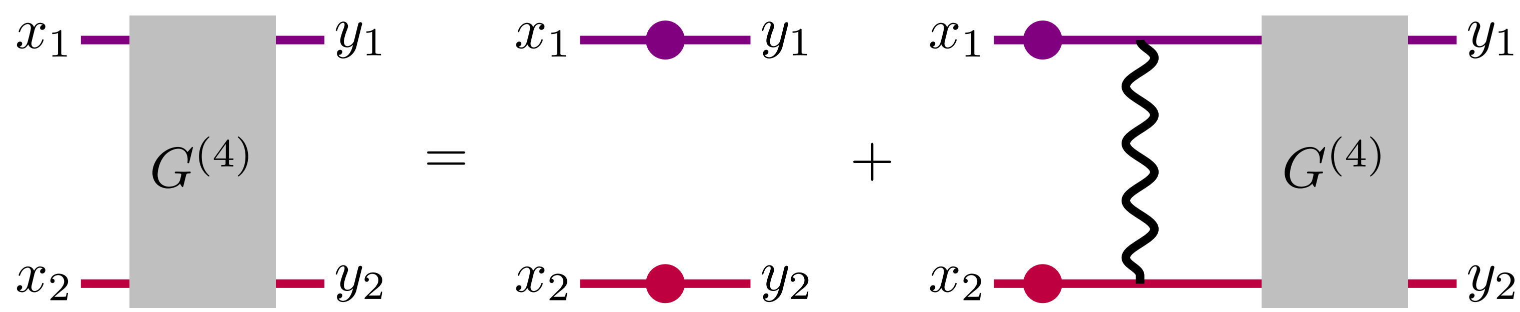
$$\tilde{\psi}_n(\mathbf{p}) \propto \int dp^0 \, \langle \Omega | T(\chi_1 \chi_2) | \mathcal{B}_{\mathbf{Q},n} \rangle$$

4-pnt function:

$$\int dp^0 dp'^0 \tilde{G}_{\mathcal{B}}^{(4)} \propto \frac{\tilde{\psi}_n(\mathbf{p}) \tilde{\psi}_n^*(\mathbf{p}')}{(Q^0 - \omega_{\mathbf{Q},n} + i\epsilon)}$$

Bound state formation — Solution

$$G^{(4)} = \langle \Omega | T(\chi_1 \chi_2 \chi_1^\dagger \chi_2^\dagger) | \Omega \rangle =$$



Bethe-Salpeter equation

non-relativistic Schrödinger equation for scattering state wave function

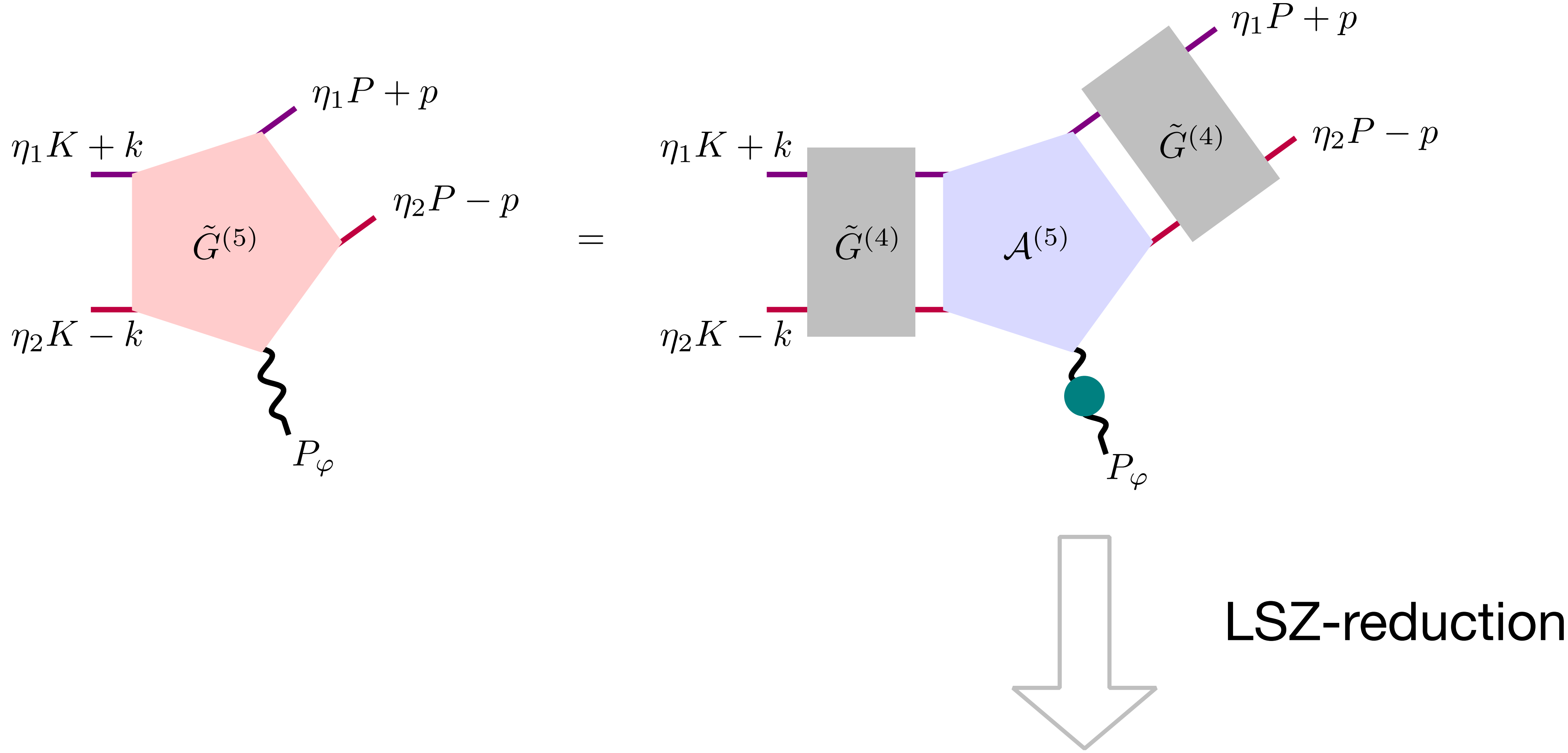
$$\left[-\frac{\nabla^2}{2\mu} + V(\mathbf{r})\right] \phi_{\mathbf{q}}(\mathbf{r}) = \mathcal{E}_{\mathbf{q}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$\tilde{\phi}_{\mathbf{q}}(\mathbf{p}) \propto \int dp^0 \, \langle \Omega | T(\chi_1 \chi_2) | \mathcal{U}_{\mathbf{Q},\mathbf{q}} \rangle$$

4-pnt function:

$$\int dp^0 dp'^0 \tilde{G}^{(4)}_{\mathcal{U}}(p, p') \propto \frac{\tilde{\phi}_{\mathbf{q}}(\mathbf{p}) \tilde{\phi}^*_{\mathbf{q}}(\mathbf{p}')}{(Q^0 - \omega_{\mathbf{Q},\mathbf{q}} + i\epsilon)}$$

On shell result



$$\mathcal{M}_{\mathbf{k}\to n} \sim \int d^3p\, d^3q\, \tilde{\psi}_n^*(\mathbf{p})\phi_{\mathbf{k}}(\mathbf{q})\mathcal{A}^{(5)}\big|_{\text{on shell}}$$

Overview

- bound states: why interesting?

- QFT derivation of (non-confining) bound states formation

- results

Vector mediation

Gauge field ϕ_μ plus two complex scalars χ^+, χ^-

$$\mathcal{L} = |D_\mu \chi^+|^2 + |D_\mu \chi^-|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - m_+^2 |\chi^+|^2 - m_-^2 |\chi^-|^2$$

$$D_\mu \chi^\pm = (\partial_\mu \mp i g \phi_\mu) \chi^\pm$$

Coulomb interaction $m_\phi = 0$

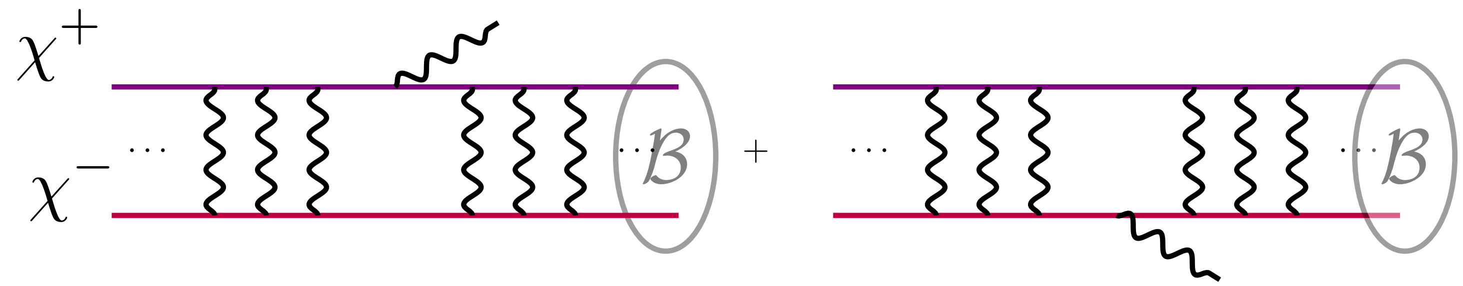
Yukawa interaction $m_\phi \neq 0$

- Higgs mechanism
- Stückelberg mechanism

Vector mediation

Coulomb limit

$$\mathcal{M}^{\mu}_{(\chi^-\chi^+\to\mathcal{B}_n\phi)} = -\sqrt{2\mu}\int d^3p\, d^3q\, g p^{\mu}\tilde{\psi}^*_n(\mathbf{p})\Big[\tilde{\phi}_{\mathbf{k}}(\mathbf{p}+\mathbf{P}_\phi/\mathbf{2}) + \tilde{\phi}_{\mathbf{k}}(\mathbf{p}-\mathbf{P}_\phi/\mathbf{2})\Big]$$



Result

$$\sigma v_{\rm rel} = \frac{2^7\pi\alpha^2}{3\mu^2\zeta^2}2\pi\zeta^3$$

$S_{l=1}$

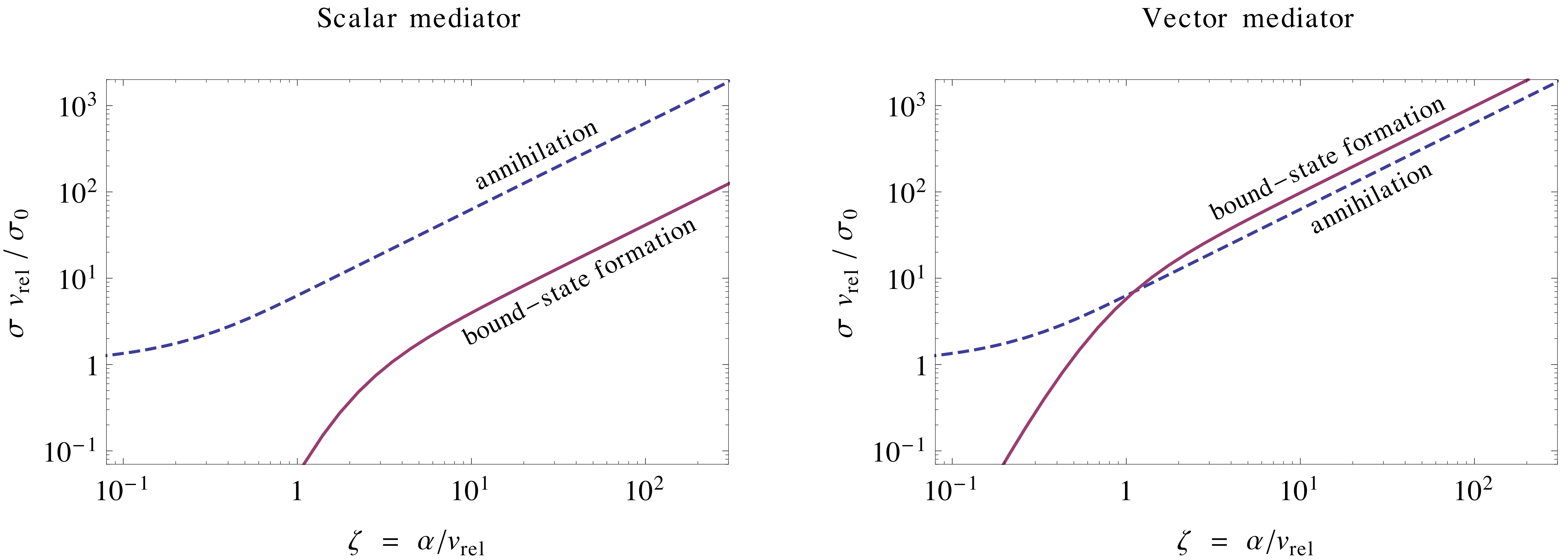
$$\phi_{\mathbf{k}}(\mathbf{r}) = \sum_l \phi_l(|\mathbf{r}|)P_l(\mathbf{k}.\mathbf{r})$$

$$\zeta = \alpha/v_{\rm rel}$$

$$\alpha = \frac{g^2}{4\pi}$$

Bound state formation

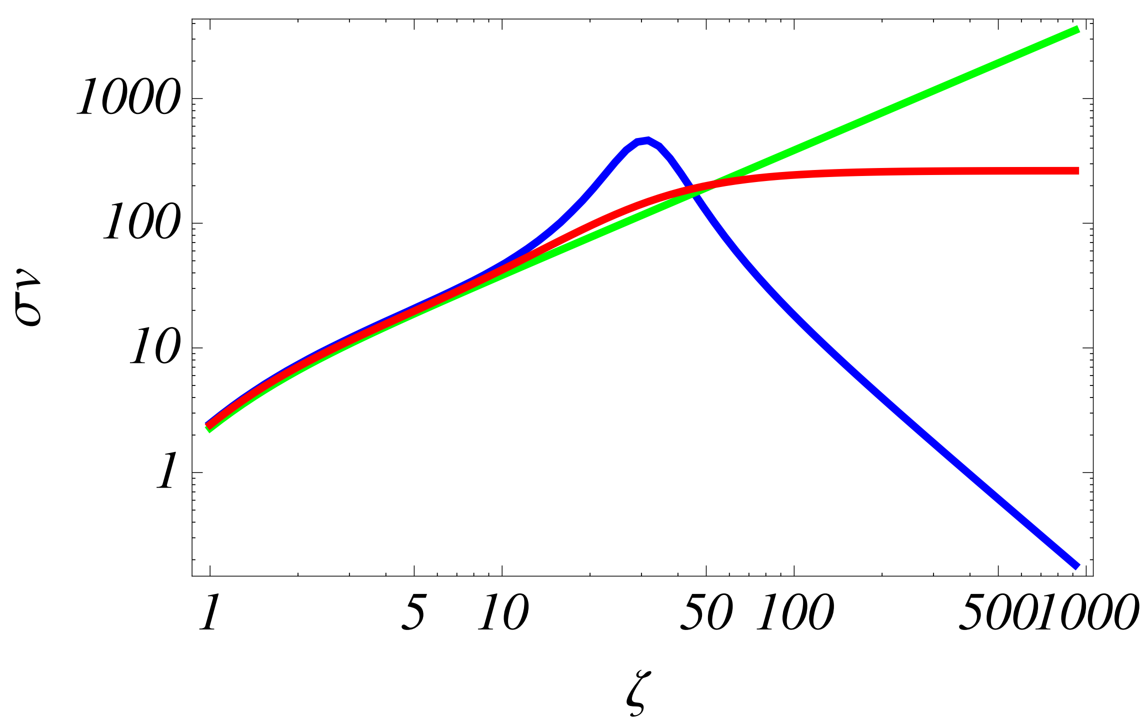
Coulomb potential

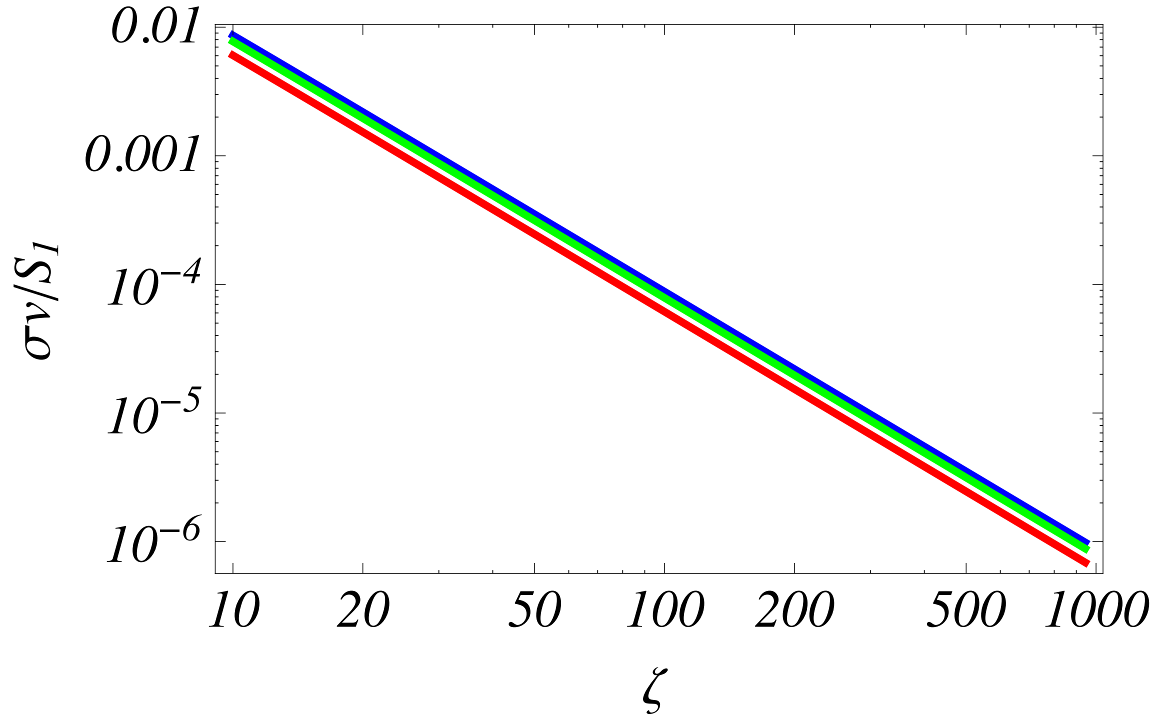


Bound state formation

Yukawa potential — vector mediation $\sigma \sim S_1 \sigma_0$

$\mu\alpha/m_\phi = 2.62$





blue: exact, green: Huelten potential/approx, red: Coulomb potential

Conclusions

- bound states: why interesting?

DM with long range interactions, e.g. heavy WIMP

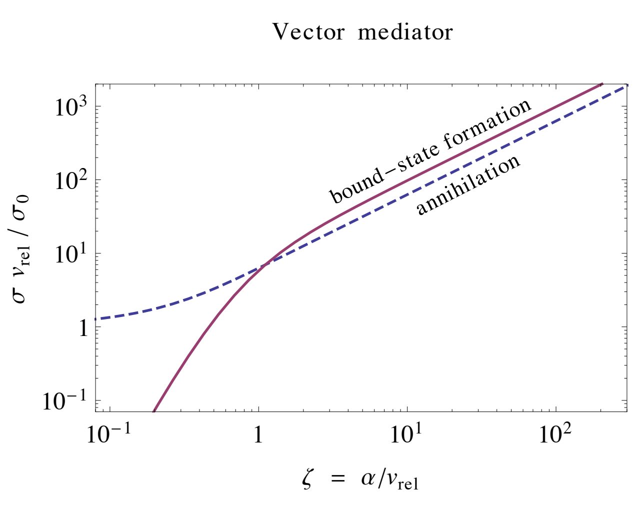
- QFT derivation of (non-confining) bound states formation

Schrödinger equation for bound and scattering state wave function

cross section: $\sigma \sim S_l (\sigma_0)_l$

- results

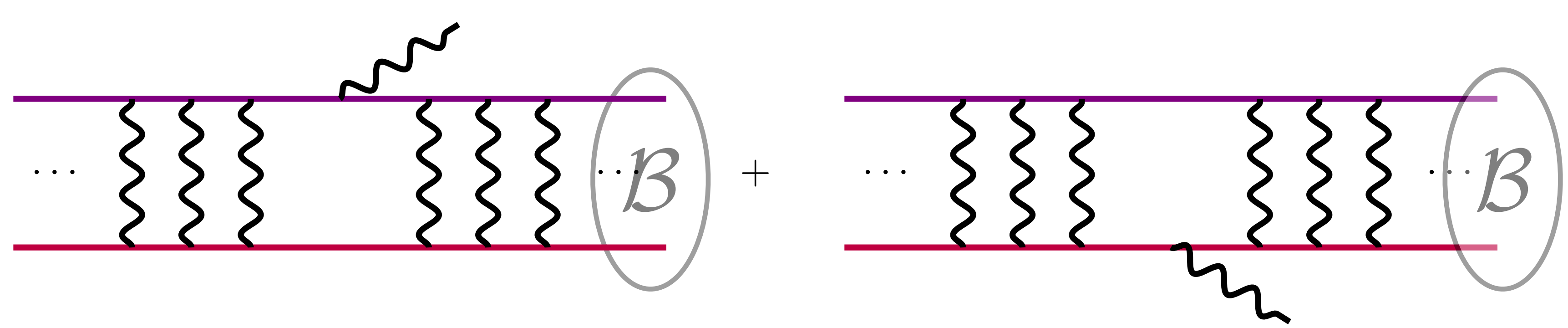
vector mediation: same order as annihilation rate



Bound state formation — Result

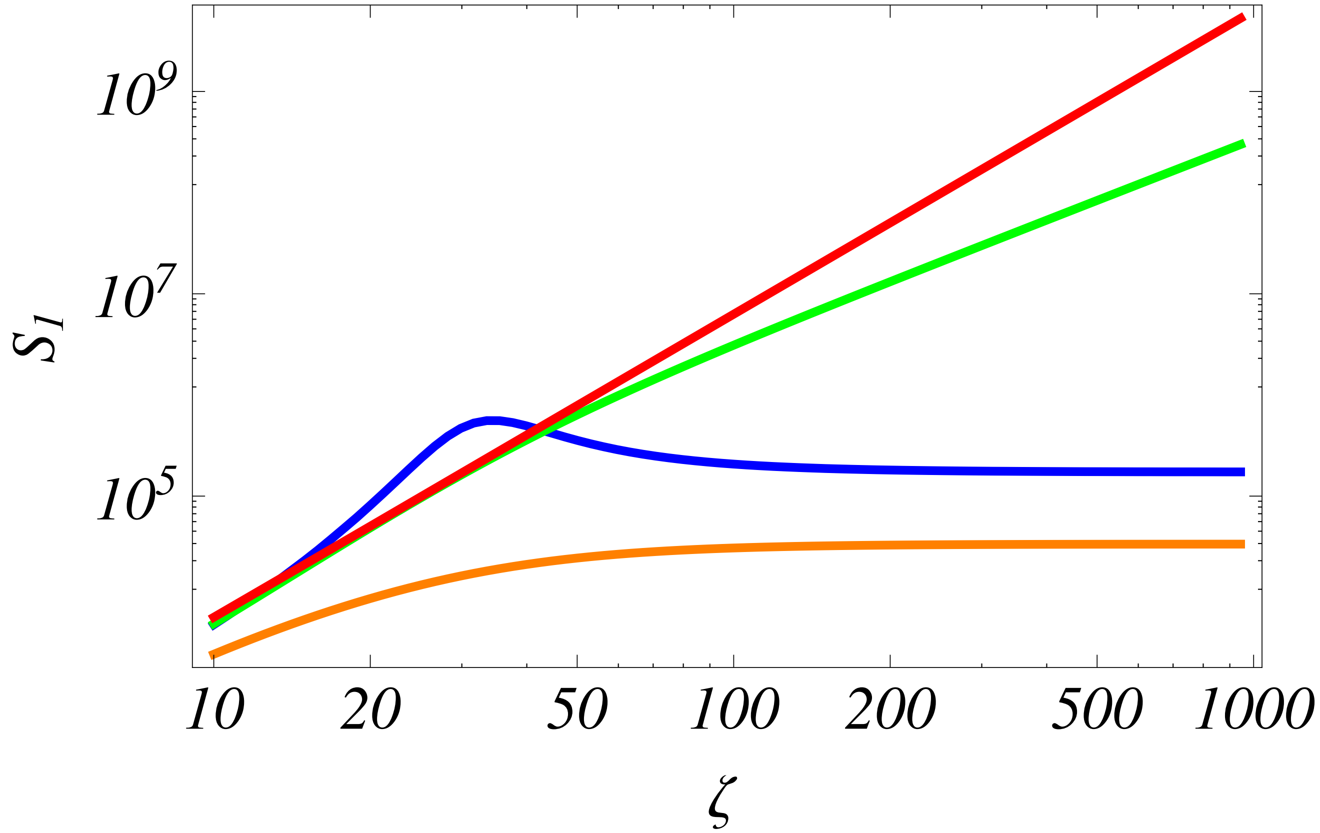
-scalar mediator and real scalar DM $\mathcal{L}_{\rm int} = -gm\phi\chi^2$

$$\mathcal{M}_{k\to n} = -2gm^{3/2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left(1+\frac{\vec{p}^2}{2m^2}\right)\psi_n^*\left[\phi_k(\vec{p}+\frac{1}{2}P_\phi)+\phi_k(p-\frac{1}{2}P_\phi)\right]$$



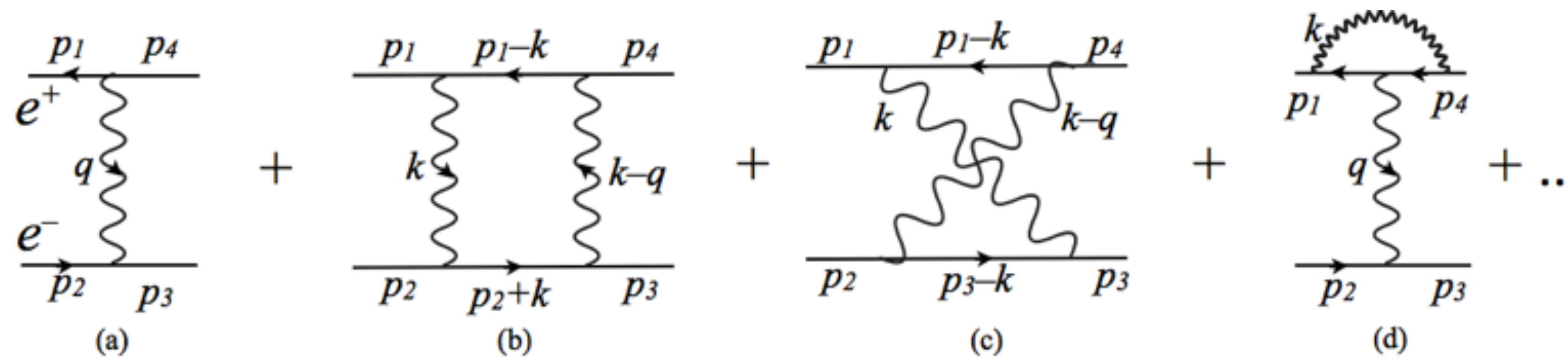
Sommerfeld enhancement

$\mu\alpha/m_\phi = 2.62$



blue: exact, green: Huelten potential/approx, red: Coulomb potential, orange: Cassel improvement

Ladder diagrams



typical Bohr exchange momentum and energy $|\vec{q}| \sim \alpha m_\chi \quad \& \quad q^0 \sim \frac{1}{2}\alpha^2 m_\chi$

$$\mathcal{A}[(a)] \sim \alpha/|\vec{q}|^2 \sim 1/\alpha$$

$$\mathcal{A}[(b)] \sim \alpha^2 \int d^4k D_\phi^2 D_\chi^2 \sim \alpha\alpha^5\alpha^{-8} = 1/\alpha$$

$\mathcal{A}[(c,d)] \sim \alpha$ leading term cancels