Lattice QCD+QED: Towards a Quantitative Understanding of the Stability of Matter

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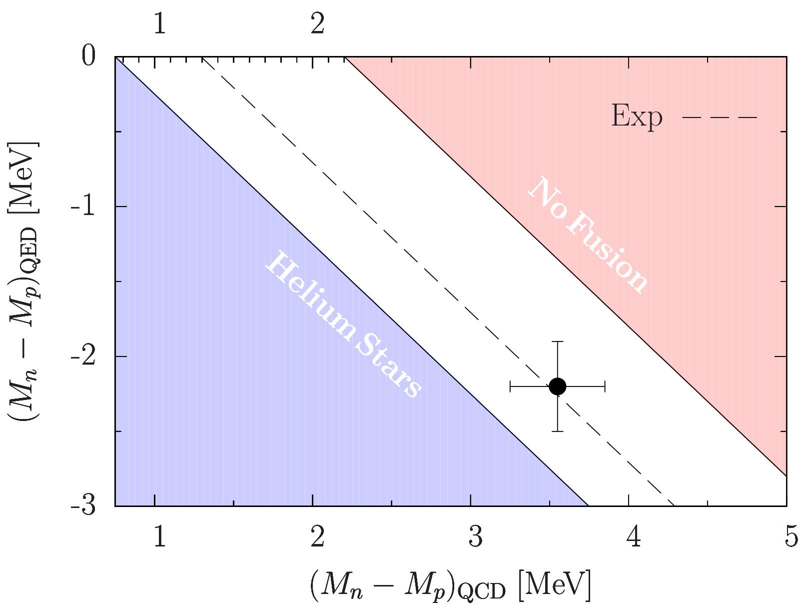
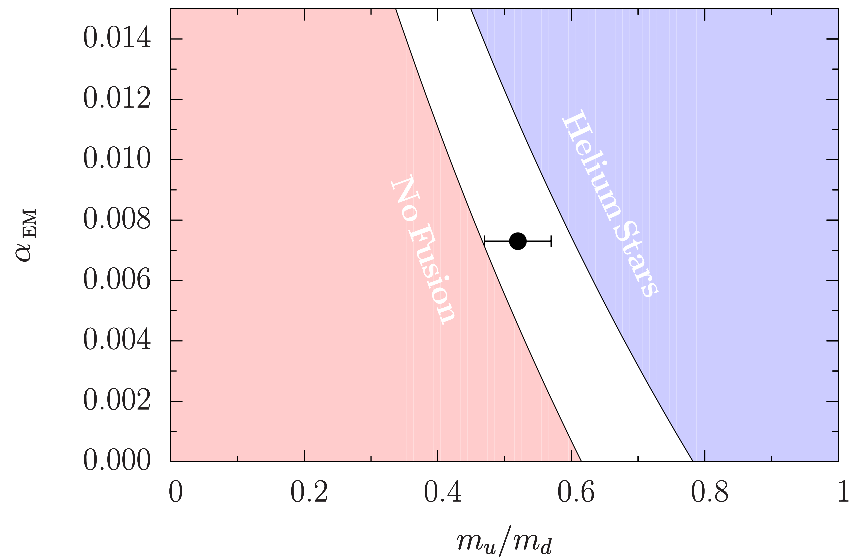
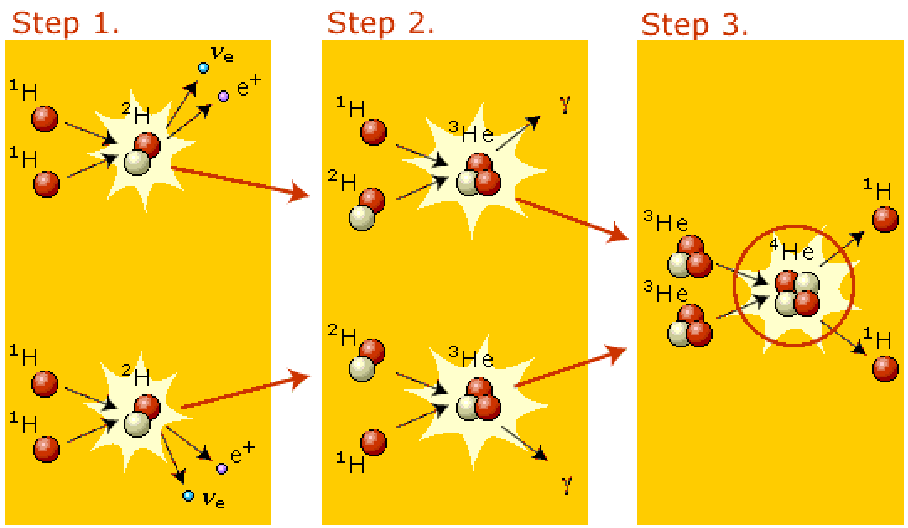
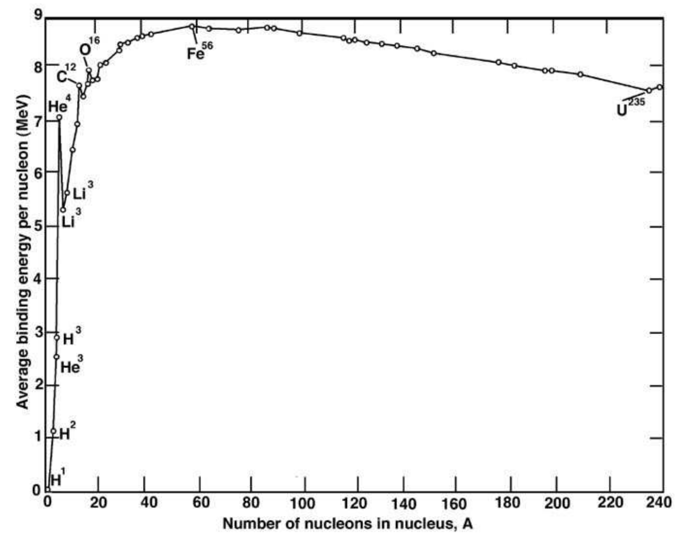
The Challenge

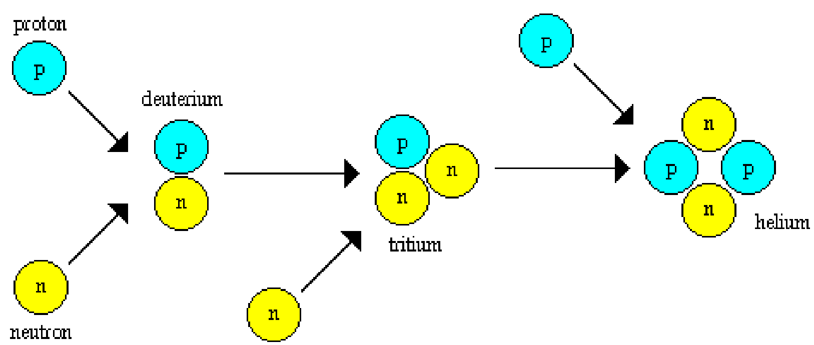




Having an analytic expression for the nucleon mass as a function of quark masses and $\alpha_{\rm EM}$, we can visualize the allowed region







If $m_u/m_d \gtrsim 0.8$ even, protons would decay spontaneously to neutrons

- The neutron – proton mass difference is one of the most consequential quantities of physics. It is extremely fine tuned for the stability of matter as we know it and the existence of our Universe. This calls for a calculation from first principles

- Lattice Gauge Theory is the method of choice. Lattice calculations are now reaching a level of precision, where it is possible to address isospin breaking effects

- These effects have two sources, the mass difference of u and d quarks, and electromagnetic interactions

- Both effects are of the same order of magnitude and cannot be separated unambiguously due to the nonperturbative nature of the strong interactions, which makes a direct calculation from QCD + QED necessary

Other issues

- We would like to be sure that $m_u > 0$, since this empowers the P- and T-violating θ parameter

- The ratio m_u/m_d determines the axion coupling and plays a vital role in pinning down the underlying parameters if and when axions are observed

- There is the prospect of making precise predictions for appropriate isospin violating processes

- Lattice calculations of hadronic processes are approaching O(1%) precision. At this level electromagnetic corrections must be included in the calculation

Outline

Lattice QCD + QED

Vacuum Structure

Flavor Physics and Spectroscopy

Isospin Splittings

Conclusions

Lattice QCD + QED

BMW

arXiv:1406.4088

- Octet baryons

QCDSF

arXiv:1311.4554
arXiv:1508.06401
arXiv:1509.00799

- Vacuum structure
- Octet mesons
- Octet baryons
- Light quark masses

With

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P.E.L. Rakow (Liverpool), A. Schiller (Leipzig), H. Stüben (Hamburg),
R. Young (Adelaide), J. Zanotti (Adelaide)

Action

$$S = S_G + S_{QED} + S_F^u + S_F^d + S_F^s$$

$$S_G = \frac{6}{g^2} \sum_{x,\mu<\nu} \frac{1}{3} \mathrm{Tr} \left\{ c_0 \left[1 - U_{\mu\nu}(x)\right] + c_1 \left[1 - R_{\mu\nu}(x)\right] \right\}$$

$$S_{QED} = \frac{1}{2e^2} \sum_{x,\mu<\nu} \left(A_\mu(x) + A_\nu(x+\mu) - A_\mu(x+\nu) - A_\nu(x)\right)^2$$ noncompact

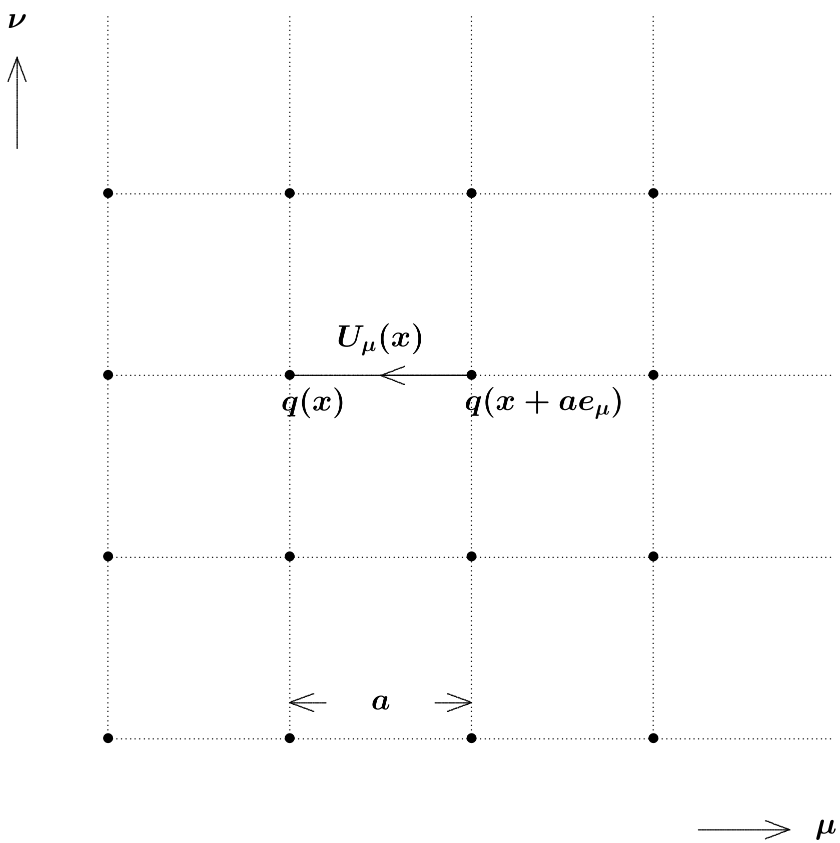
$$S_F^q = \sum_x \Big\{ \sum_\mu \Big[\overline{q}(x) \frac{\gamma_\mu - 1}{2} e^{-ie_q A_\mu(x)} \tilde{U}_\mu(x) q(x+\hat{\mu})$$

$$- \overline{q}(x) \frac{\gamma_\mu + 1}{2} e^{ie_q A_\mu(x-\hat{\mu})} \tilde{U}_\mu^\dagger(x-\hat{\mu}) q(x-\hat{\mu}) \Big]$$

$$+ \frac{1}{2\kappa_q} \overline{q}(x) q(x) - \frac{1}{4} c_{SW} \sum_{\mu\nu} \overline{q}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) q(x) \Big\}$$

Lattice spacing a implicit

$$e_u = \frac{2}{3}\,,\ e_d = e_s = -\frac{1}{3}$$



$$a\,\Lambda_L = \exp\{-2b_0/g^2\}$$

The simulation

- Generate a sequence of field configurations $\{U_\mu^{(i)}, A_\mu^{(i)} | i = 1, \cdots, N\}$ with probability

$$\int \Pi\, \mathcal{D}q\, \mathcal{D}\bar{q} \exp\{-S_F^q\} \exp\{-S_G - S_A\}$$

$$= \det(\mathcal{M}) \exp\{-S_G - S_A\}$$

$\uparrow$

$10^8 \times 10^8$ matrix

- Compute observable

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_i^N \mathcal{O}(U_\mu^{(i)}, A_\mu^{(i)})$$

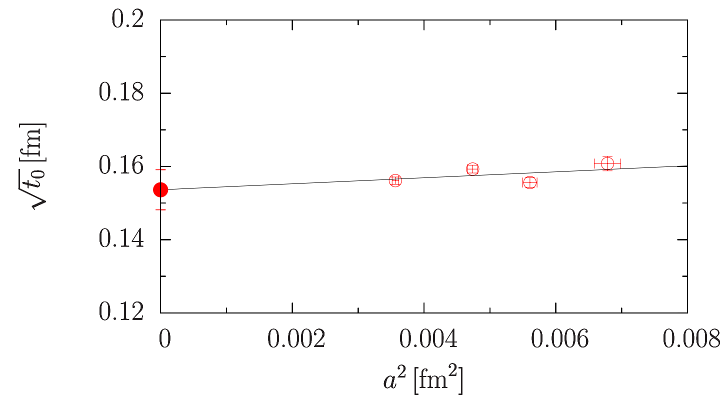
At the end of the calculation $L \to \infty,\ a \to 0$

Volumes

$24^3 \times 48 \qquad 32^3 \times 64 \qquad 48^3 \times 96$

Couplings

$$\beta \equiv \frac{6}{g^2} = 5.50 \quad \Rightarrow \quad a = 0.068(2) \text{ fermi}$$



Hopping parameters κ fixed at symmetric point

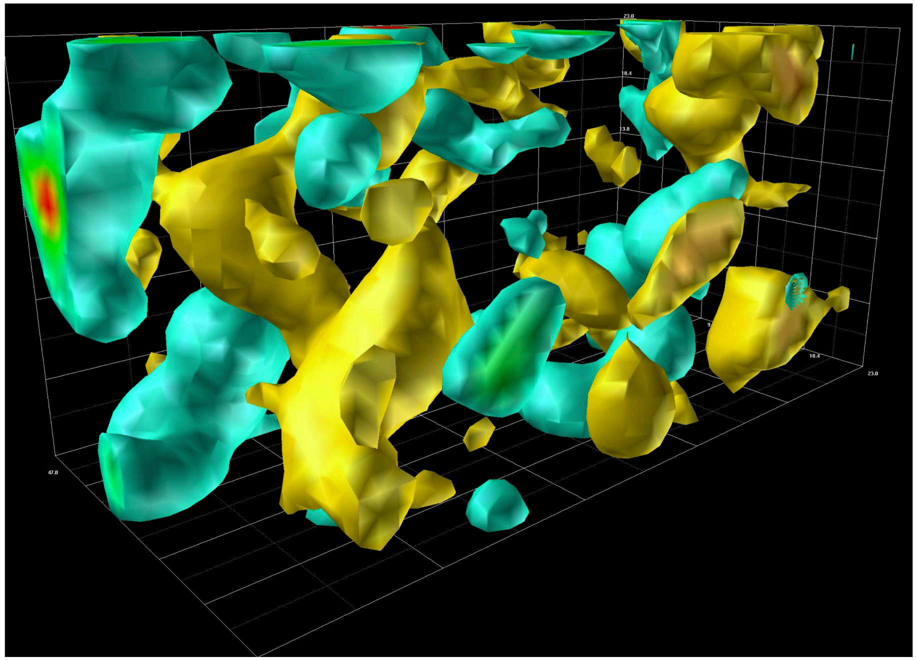
$$\bar{\kappa}_u = 0.124382\,, \;\; \bar{\kappa}_d = \bar{\kappa}_s = 0.121703$$

Valence quark masses μ_q ranging from $M_{\rm PS}/M_N = 0.22$ to 0.5

$$\alpha_{\rm EM} \equiv \frac{e^2}{4\pi} = 0.10 \quad \longrightarrow \quad \alpha_{\rm EM} = 1/137$$

Quark sea flavor blind ($i.e.$ flavor singlet)

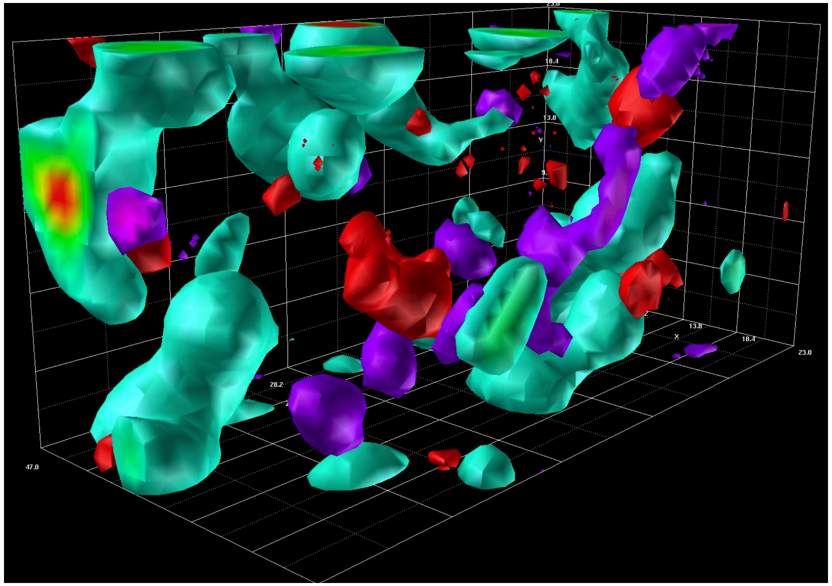
Vacuum Structure



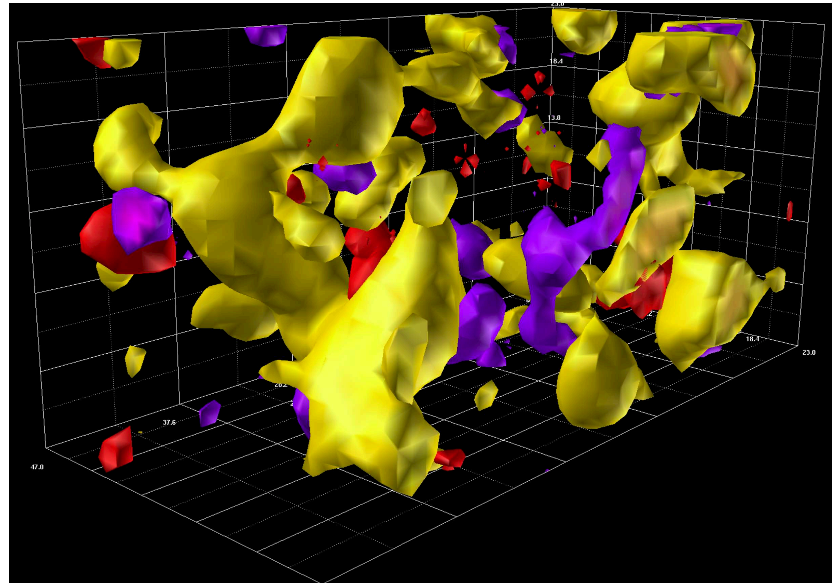
Density of QCD (aqua) and QED (yellow) actions

↑

Electromagnetic field strength
repelled by chromoelectric one



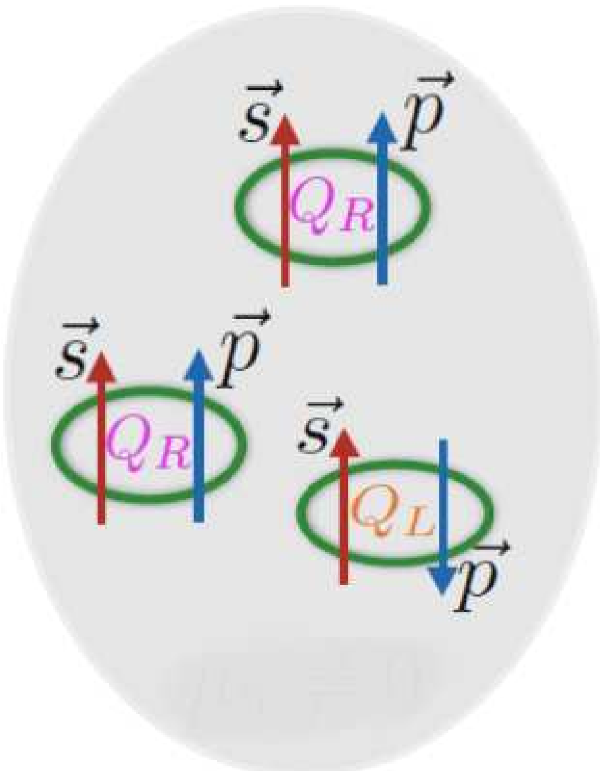
Density of positive (red) and negative (purple) charge compared with QCD action density



Density of positive (red) and negative (purple) charge compared with QED action density

(Charvetto)

Chiral Magnetic Effect



Instanton

$\Longrightarrow$

Excess of right-handed quarks due to chiral anomaly

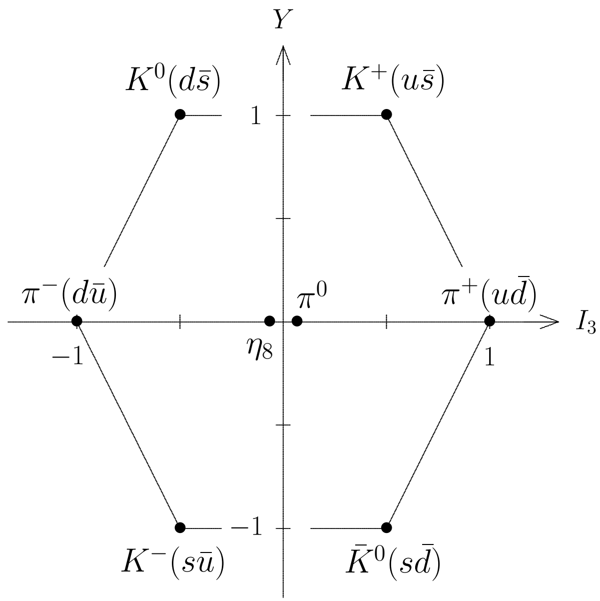
$$\downarrow$$

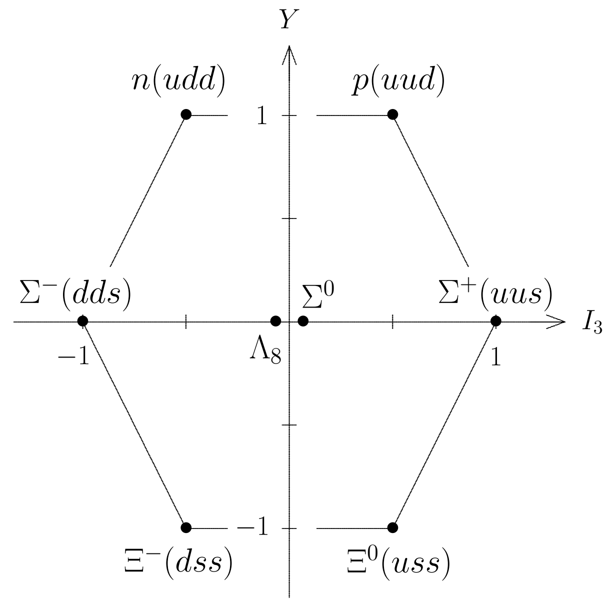
$$\vec{p} \,||\, \vec{s}$$

$$\vec{p} \propto \vec{J}\,,\ \vec{s} \propto \vec{B}$$

We find evidence for $\vec{J}\vec{B}$ to be correlated with position of instanton

Flavor Physics and Spectroscopy





$\longleftarrow \quad < 1\% \quad \longrightarrow$

Strategy

arXiv:1102.5300

- QCD interactions are flavor blind. The only difference between flavors comes from the quark mass matrix
- In lattice calculations one can vary the quark masses freely, which helps to illuminate the pattern of flavor symmetry breaking
- One has the best theoretical understanding when all quark masses are equal, because one can use the full power of flavor SU(3)
- We interpolate between the symmetric point $\mu_u = \mu_d = \mu_s$ and the physical point by keeping the sum of the quark masses $(\mu_u + \mu_d + \mu_s)/3 \equiv \bar{m}$ fixed at its physical value, which is particularly instructive
- The symmetry of the electromagnetic current is similar to the symmetry of the quark mass matrix
- The simplifications from keeping $\bar{m}$ = constant in the mass expansion are analog to the simplifications from the identity $e_u + e_d + e_s = 0$. We thus can read off the QED corrections from the mass expansion changing masses to charges

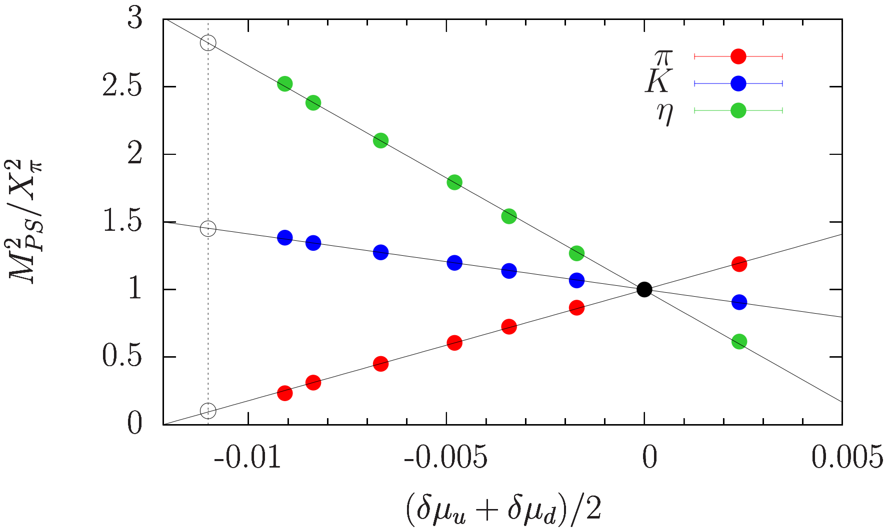
QCD

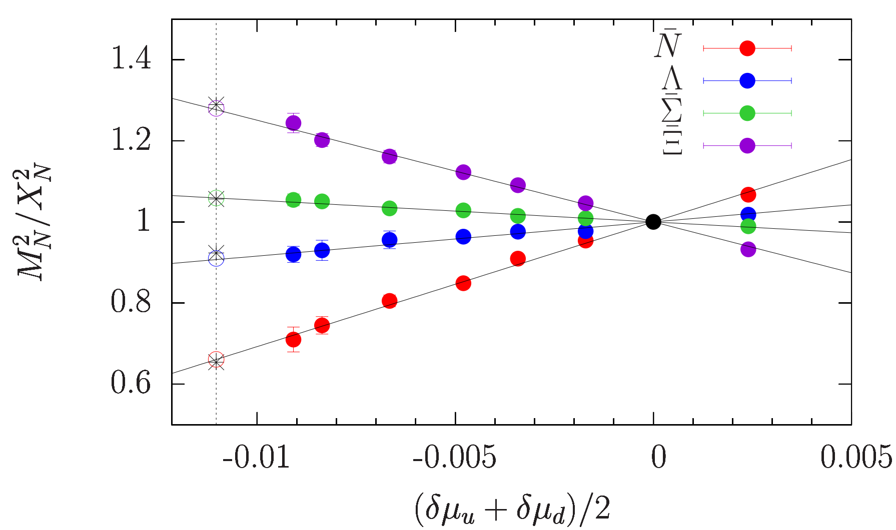
Gell-Mann–Okubo

1 **8** **10**, **27**

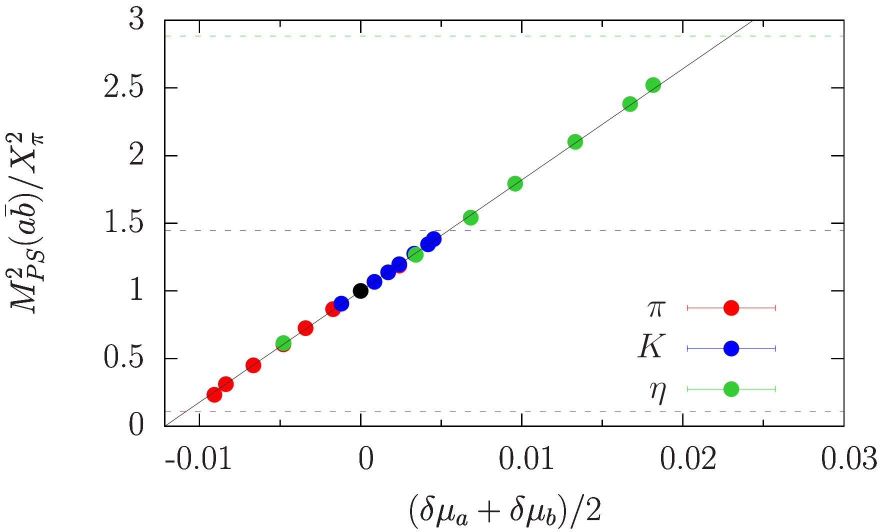
$$M^2(a\bar{b}) = M_0^2 + \alpha\,(\delta\mu_a + \delta\mu_b) + \cdots \qquad \delta\mu_q = \mu_q - \bar{m}$$

$$M^2(aab) = M_0^2 + \alpha_1(2\delta\mu_a + \delta\mu_b) + \alpha_2\,(\delta\mu_a - \delta\mu_b) + \cdots \qquad \delta\mu_u + \delta\mu_d + \delta\mu_s = 0$$





$$X_\pi^2 = (M^2_{K^0} + M^2_{K^+} + 2M^2_{\pi^0} - M^2_{\pi^+})/3 \qquad X_N^2 = (M_n^2 + M_p^2 + M^2_{\Sigma^-} + M^2_{\Sigma^+} + M^2_{\Xi^-} + M^2_{\Xi^0})/6$$

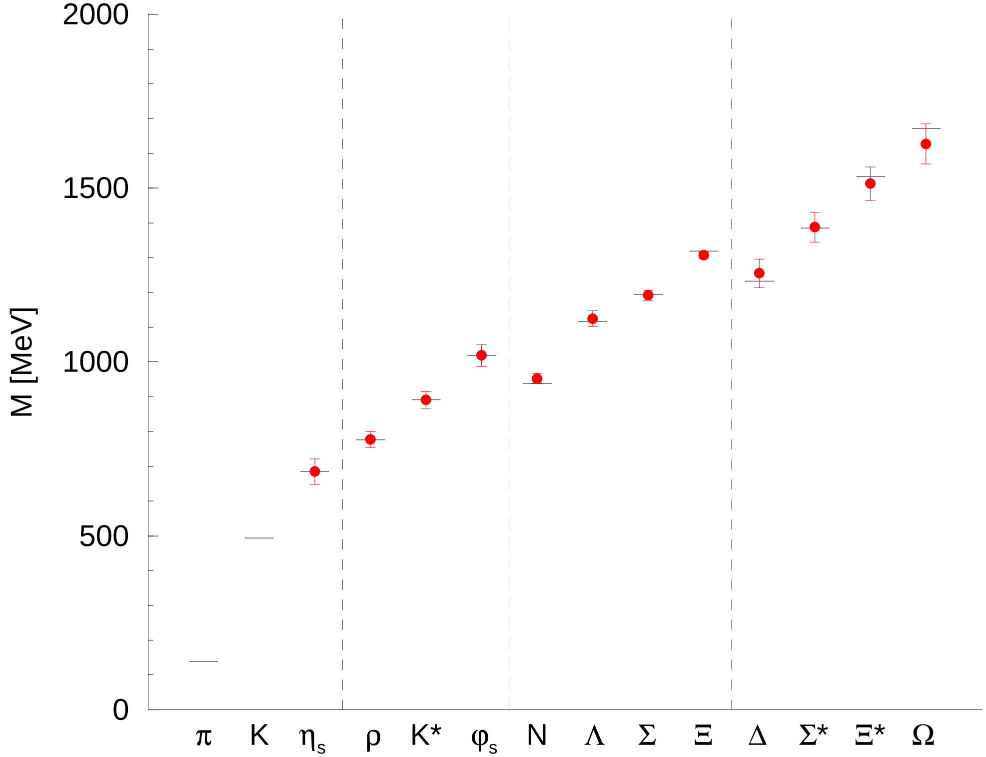


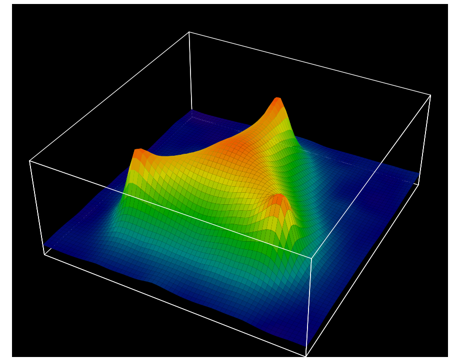
$$\delta\mu_q = \mu_q - \bar{m}$$

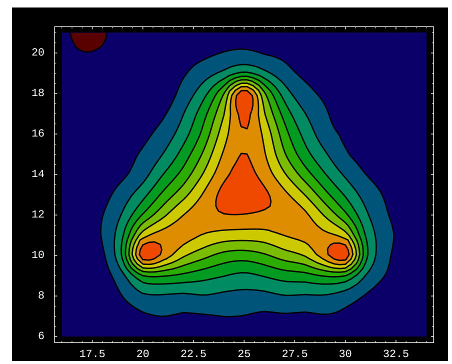
arXiv:1102.5300

Spectrum

80% of mass







Dual superconductor

QCD + QED

$$M^2(a\bar{b}) = M_0^2 + \alpha\,(\delta\mu_a + \delta\mu_b) + \beta_1^{\mathrm{EM}}\,(e_a - e_b)^2$$

Dashen scheme

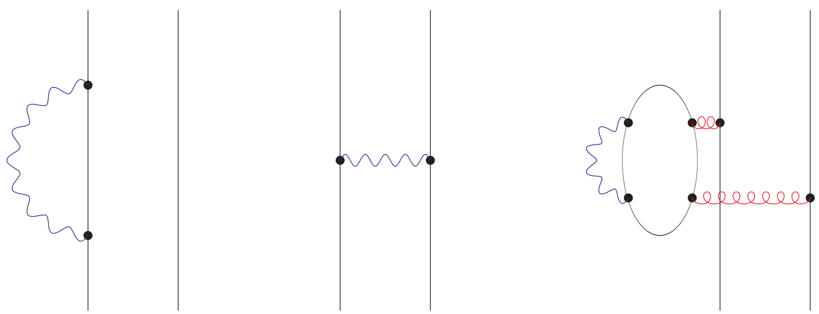
$$+\,\gamma_1^{\mathrm{EM}}\,(e_a - e_b)^2\,(\delta\mu_a + \delta\mu_b) + \gamma_2^{\mathrm{EM}}\,(e_a^2 - e_b^2)\,(\delta\mu_a - \delta\mu_b)$$

$$M_{\pi^0}^2 = M_0^2 + \alpha\,(\delta\mu_u + \delta\mu_d)$$

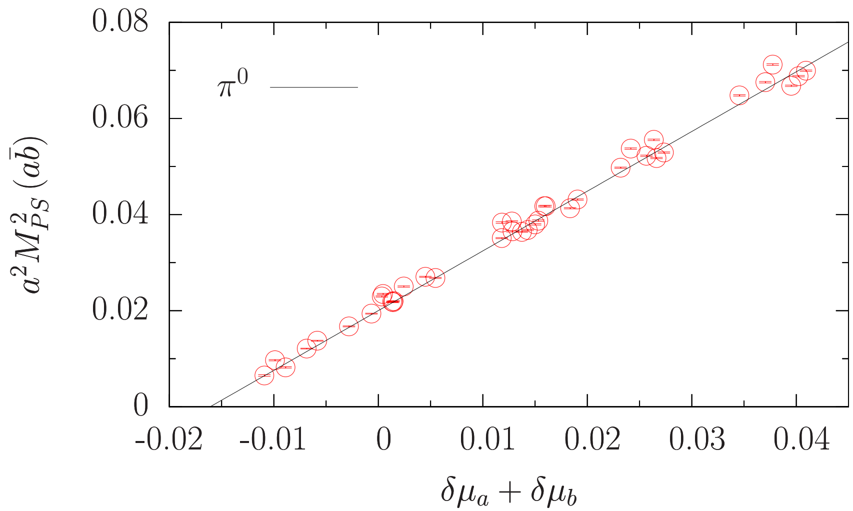
$$M_{K^0}^2 = M_0^2 + \alpha\,(\delta\mu_d + \delta\mu_s)$$

↑

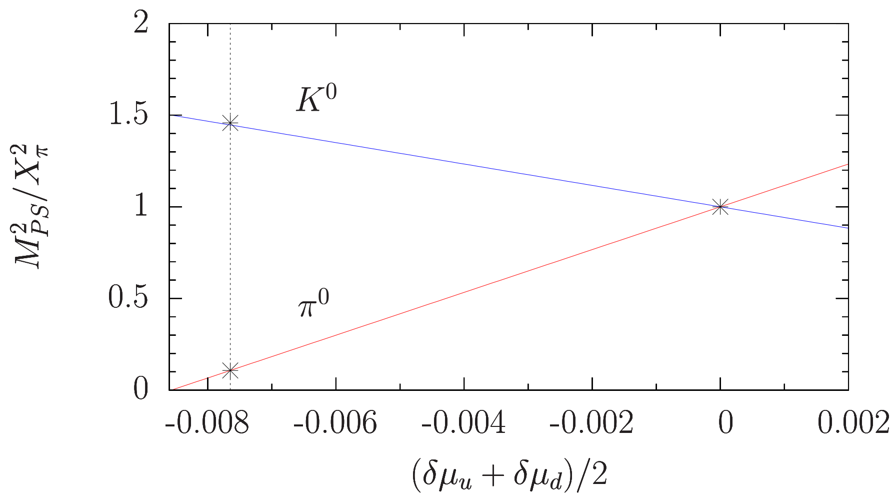
Used to fix physical quark masses μ_u, μ_d



$48^3 \times 96$

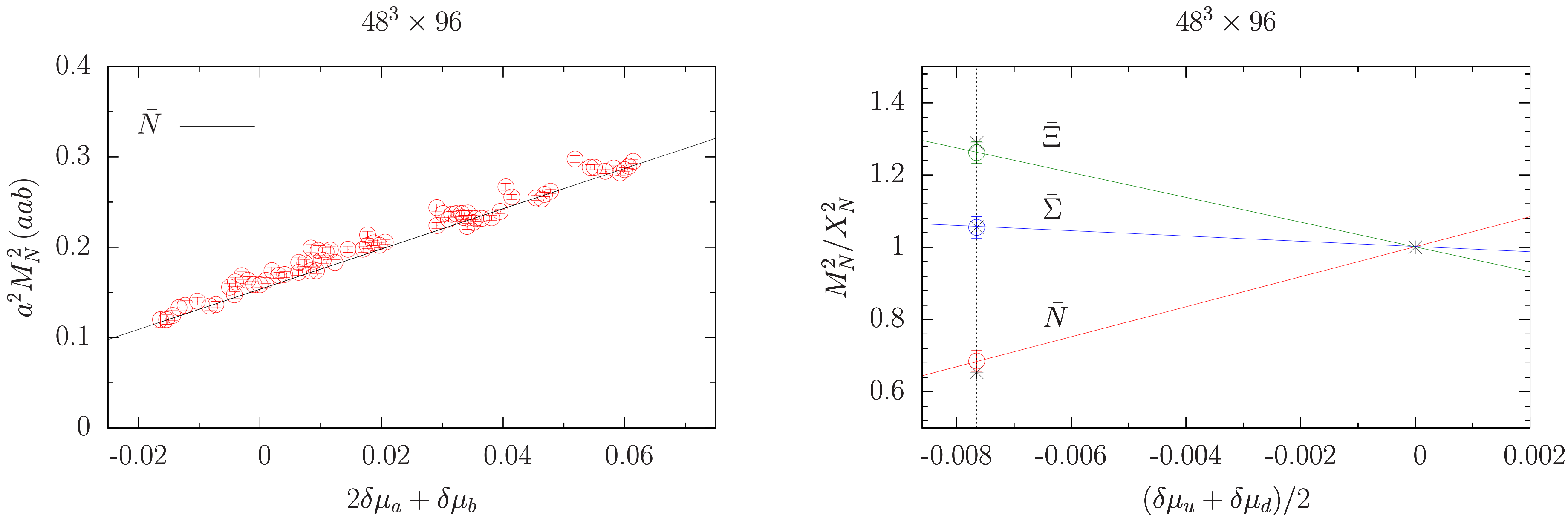


$$e_q = -\frac{1}{3},\ 0,\ \frac{2}{3}$$



Fit

$$M^2(aab) = M_0^2 + \alpha_1\,(2\delta\mu_a + \delta\mu_b) + \alpha_2\,(\delta\mu_a - \delta\mu_b)$$
$$+\,\beta_1^{\mathrm{EM}}\,(2e_a^2 + e_b^2) + \beta_2^{\mathrm{EM}}\,(e_a - e_b)^2 + \beta_3\,(e_a^2 - e_b^2)$$



Isospin Splittings

Quark Masses

$$M^2_{\pi^0} = M^2_0 + \alpha(\delta\mu_u + \delta\mu_d) = \alpha(\mu_u + \mu_d)$$

$$M^2_{K^0} = M^2_0 + \alpha(\delta\mu_d + \delta\mu_s) = \alpha(\mu_d + \mu_s)$$

$$\mu_u + \mu_d + \mu_s = \text{constant}$$

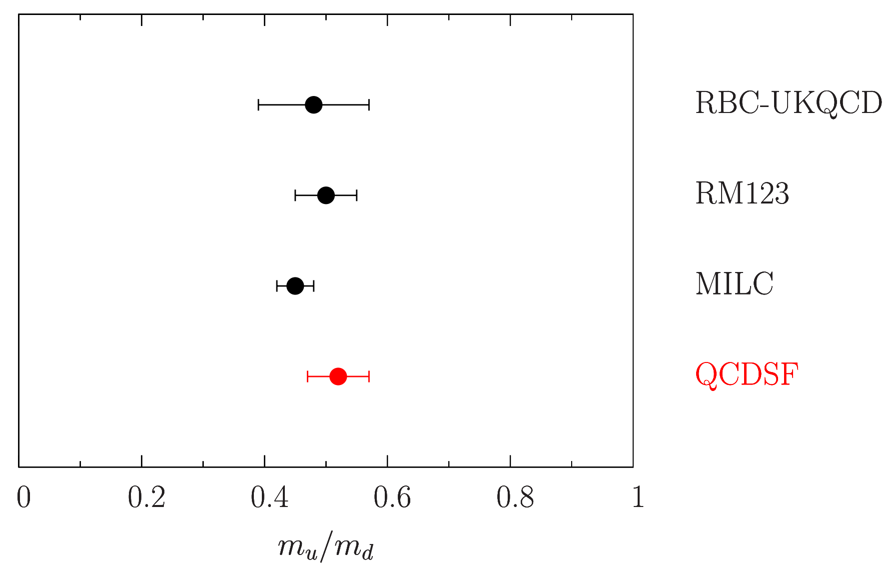
$$m_q = Z_m^{\overline{\rm MS}}(2\ \text{GeV})\, \Delta Z_D^{\overline{\rm MS}} \mu_q$$

$$m_u = 2.49(14)\ \text{MeV}$$

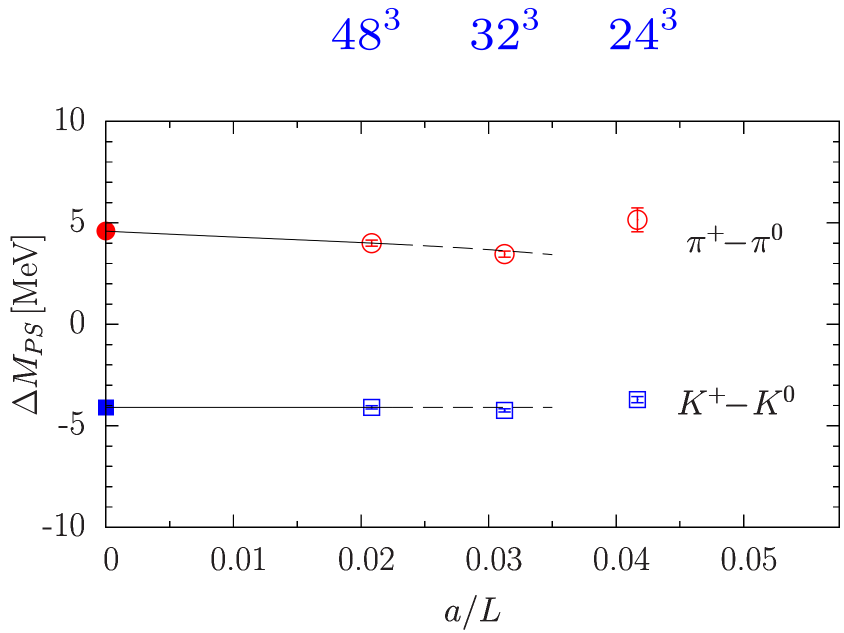
$$m_d = 4.80(27)\ \text{MeV}$$

$$m_s = 94.5(52)\ \text{MeV}$$

$$\frac{m_u}{m_d} = 0.52(2)\,, \quad \frac{m_s}{m_d} = 19.7(9)$$



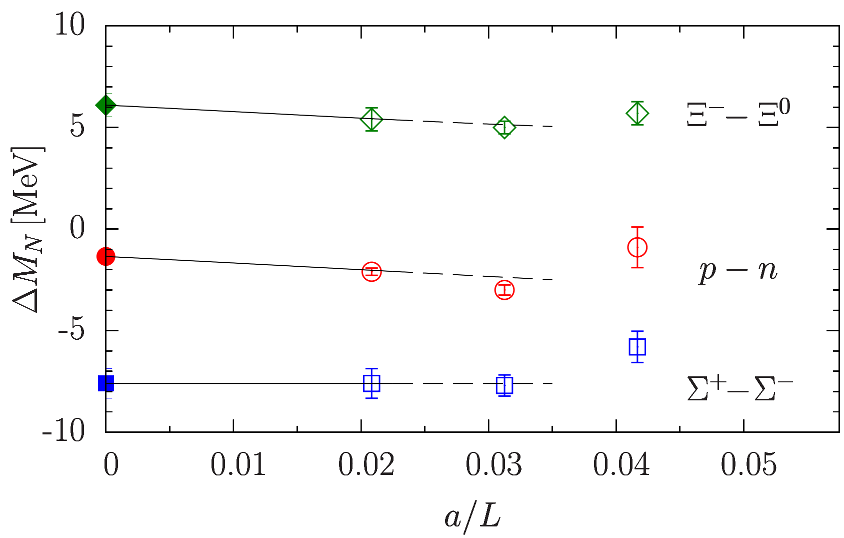
Meson Octet



$$\Delta M_{\pi^+} = \frac{\alpha_{\mathrm{EM}}}{2L} c_1 \left(1 + \frac{2}{M_{\pi^+} L}\right) + \frac{2\pi\alpha_{\mathrm{EM}}}{3L^3} \left(1 + \frac{4\pi}{M_{\pi^+} L} c_{-1}\right) \langle r^2 \rangle_{\pi^+} + \cdots$$

Davoudi & Savage

Baryon Octet

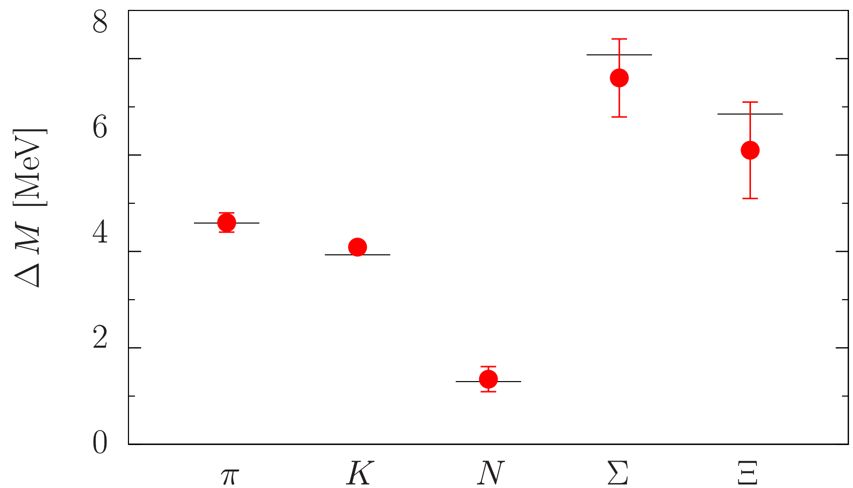


$$\Delta M_p = \frac{\alpha_{\rm EM}}{2L} c_1 \left(1 + \frac{2}{M_p L}\right) + \frac{2\pi\alpha_{\rm EM}}{3L^3}\left(1 + \frac{4\pi}{M_{\pi^+} L} c_{-1}\right)\langle r^2\rangle_p + \cdots$$

Davoudi & Savage

Splittings

ΔM	QCD + QED	QED	Experiment
$M_{\pi^+} - M_{\pi^0}$		4.60(20)	4.59
$M_{K^0} - M_{K^+}$	4.09(10)	−1.66(6)	3.93
$M_n - M_p$	1.35(18)(8)	−2.20(28)(10)	1.30
$M_{\Sigma^-} - M_{\Sigma^+}$	7.60(73)(8)	−0.63(8)(6)	8.08
$M_{\Xi^-} - M_{\Xi^0}$	6.10(55)(45)	1.26(16)(13)	6.85

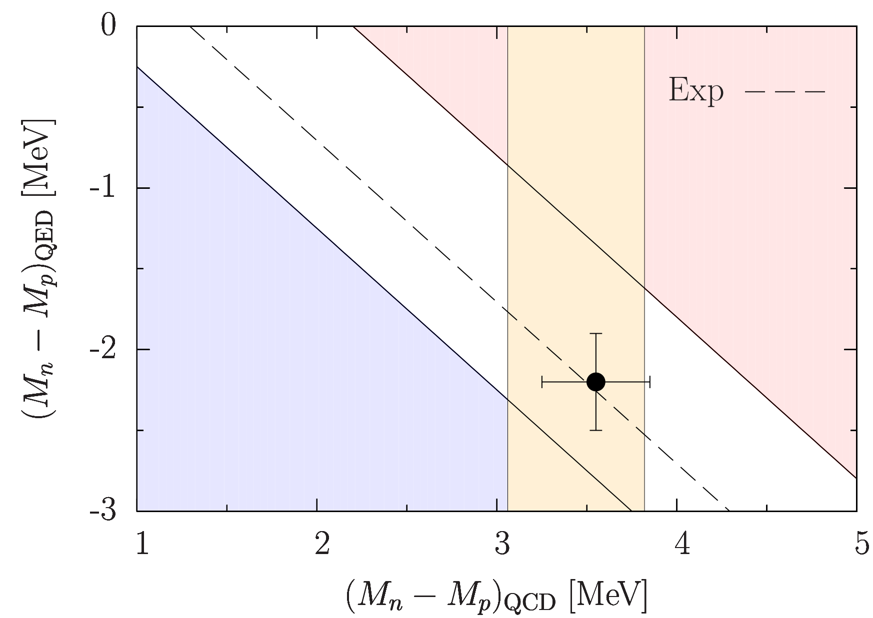


$$M_n - M_p + M_{\Sigma^+} - M_{\Sigma^-} + M_{\Xi^-} - M_{\Xi^0} = 0 \propto \underline{\mathbf{10}}$$

Coleman–Glashow

QCD vs QED

arXiv:1508.05916



Dashen scheme $\simeq \overline{\mathrm{MS}}$

Analytic Solution?

Renormalization Group

Tells us how the bare parameters of the theory must behave to keep the physics constant as the cut-off is varied

$$\frac{\partial \ \ln \ m_u/m_d}{\partial \ \ln \ \mu} = 0 - \frac{e^2}{8\pi^2} + \frac{e^2}{12\pi^2}\left(4\,\frac{m_u^2}{\mu^2} - \frac{m_d^2}{\mu^2}\right) \qquad \mu = 1/a$$

QCD QED

$$\Uparrow$$

$$\gamma_q^{\mathrm{EM}} = -\frac{3e_q^2}{8\pi^2}\left[1 - \frac{m_q^2}{\mu^2}\ln\left(1 + \frac{\mu^2}{m_q^2}\right)\right]$$

Solution

$$\frac{m_u}{m_d} = \frac{1}{2}\,\mu^{-\frac{e^2}{8\pi^2}} = \boxed{\frac{1}{2}} - \frac{e^2}{16\pi^2}\ln\,\mu + O(e^4)$$

Conclusions

- Flavor and isospin symmetry breaking of hadron masses follow a very simple pattern, made visible by systematic lattice simulations of QCD + QED

- So far we have investigated isospin breaking of pseudoscalar meson and octet baryon masses. That allowed us to look simultaneously at both sources of isospin breaking, the quark mass differences and electromagnetic interactions, which are of comparable importance

- The stability of matter, and the existence of the Universe as we know it, largely hinges on the ratio of up to down quark mass

- From a broader perspective, we can look forward to a better understanding of the QCD vacuum and the mechanism of confinement and chiral symmetry breaking

- With increased computer power it will be possible to improve on the precision of the calculation, which is still limited