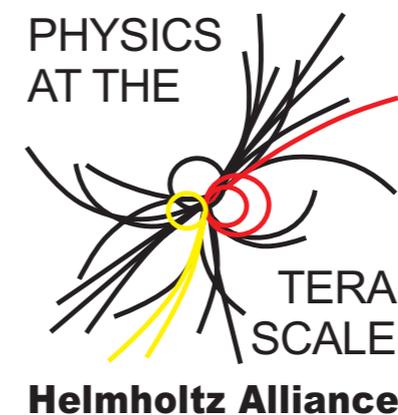
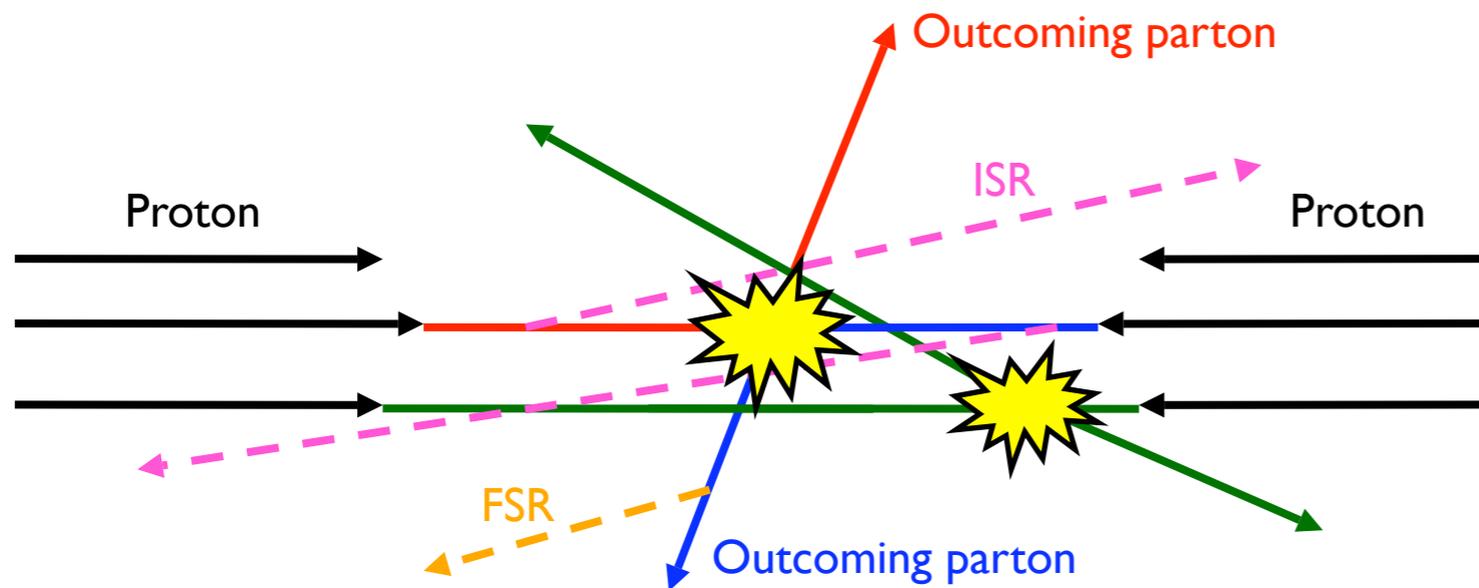


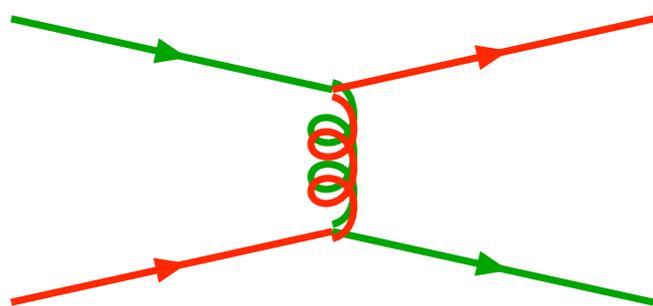
# Multiple Interactions and Parton Rescattering with Pythia

Florian Bechtel (Hamburg University)  
Analysis Center: MC Group Meeting  
September 25<sup>th</sup> 2008



# What are MI?

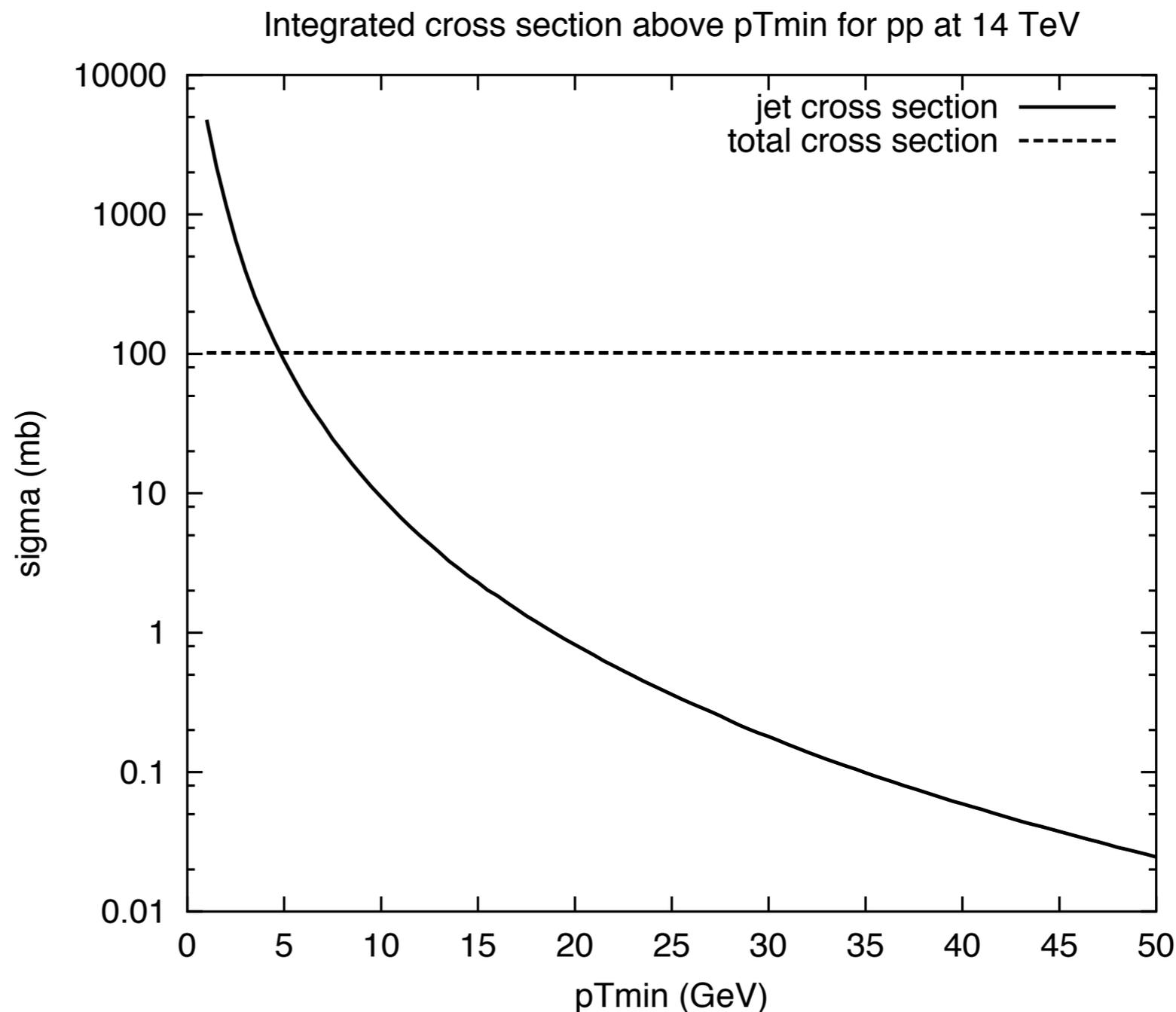
Cross section for  $2 \rightarrow 2$  interactions is dominated by  $t$ -channel gluon exchange, so diverges like  $d\hat{\sigma}/dp_{\perp}^2 \approx 1/p_{\perp}^4$  for  $p_{\perp} \rightarrow 0$ .



integrate QCD  $2 \rightarrow 2$

- $qq' \rightarrow qq'$
- $q\bar{q} \rightarrow q'\bar{q}'$
- $q\bar{q} \rightarrow gg$
- $qg \rightarrow qg$
- $gg \rightarrow gg$
- $gg \rightarrow q\bar{q}$

with CTEQ 5L PDF's



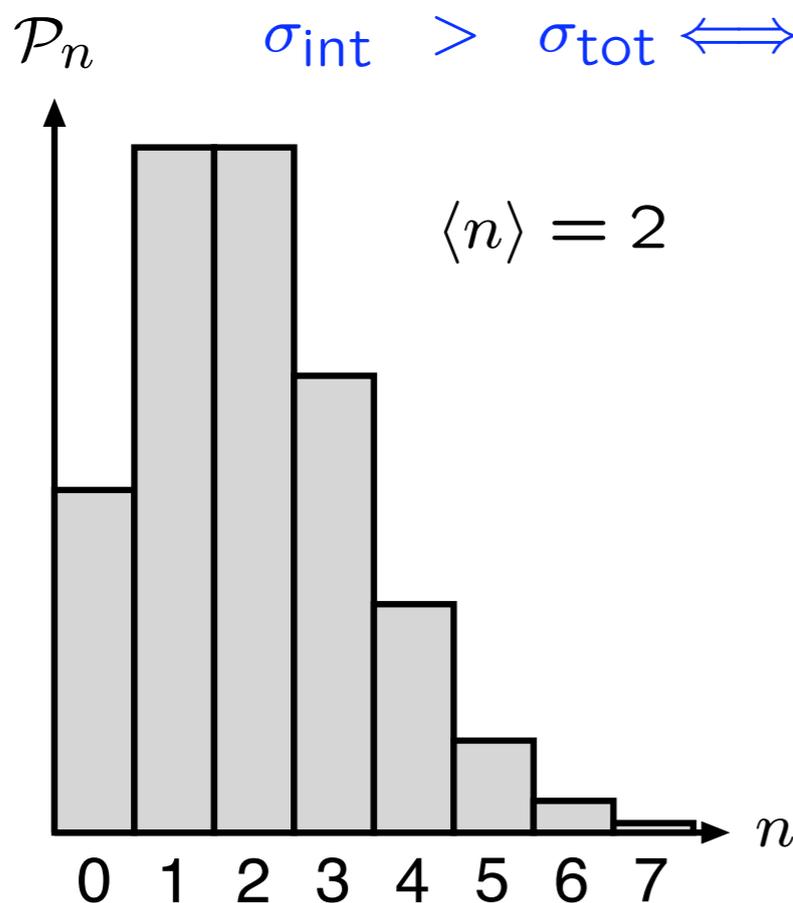
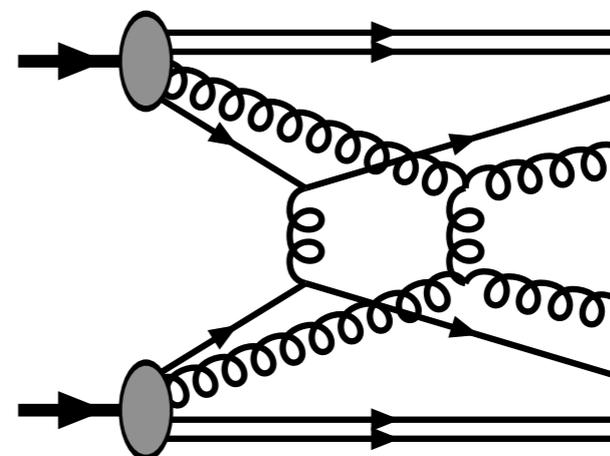
$$\sigma_{\text{int}}(p_{\perp \text{min}}) = \iiint_{p_{\perp \text{min}}} dx_1 dx_2 dp_{\perp}^2 f_1(x_1, p_{\perp}^2) f_2(x_2, p_{\perp}^2) \frac{d\hat{\sigma}}{dp_{\perp}^2}$$

Half a solution to  $\sigma_{\text{int}}(p_{\perp \text{min}}) > \sigma_{\text{tot}}$ : many interactions per event

$$\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$$

$$\sigma_{\text{int}} = \sum_{n=0}^{\infty} n \sigma_n$$

$$\sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1$$



If interactions occur independently  
then **Poissonian statistics**

$$\mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

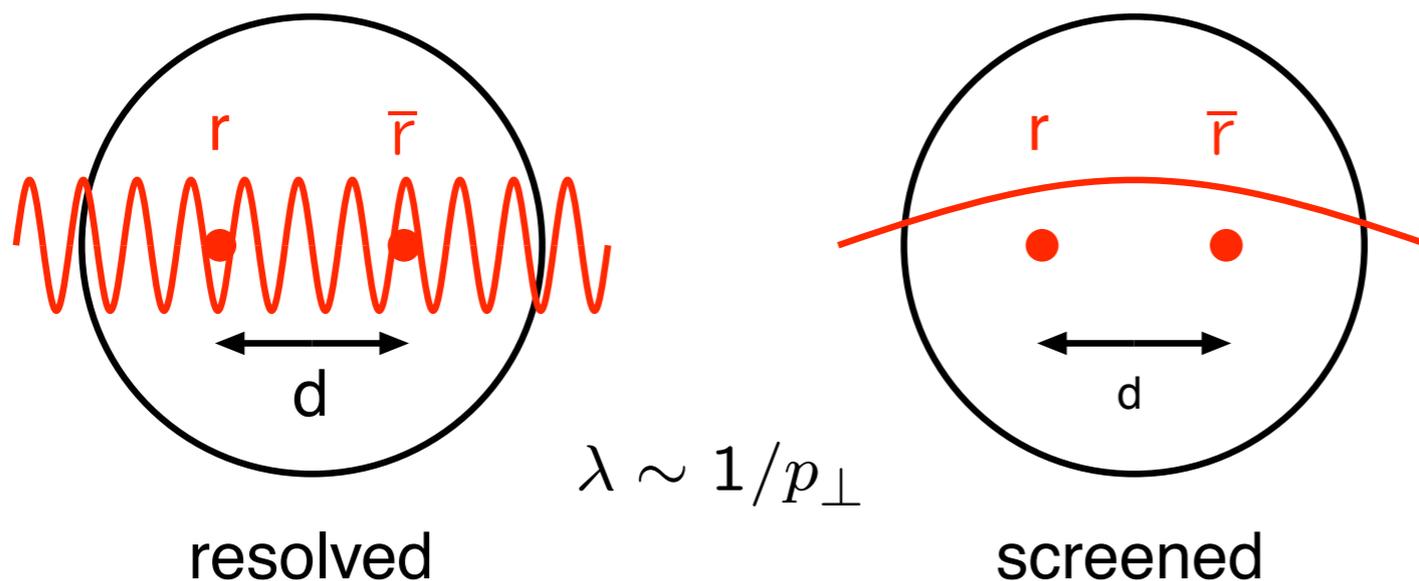
but energy–momentum conservation  
 $\Rightarrow$  large  $n$  suppressed

Other half of solution:  
perturbative QCD not valid at small  $p_{\perp}$  since  $q, g$  not asymptotic states  
(confinement!).

Naively breakdown at

$$p_{\perp \text{min}} \simeq \frac{\hbar}{r_p} \approx \frac{0.2 \text{ GeV} \cdot \text{fm}}{0.7 \text{ fm}} \approx 0.3 \text{ GeV} \simeq \Lambda_{\text{QCD}}$$

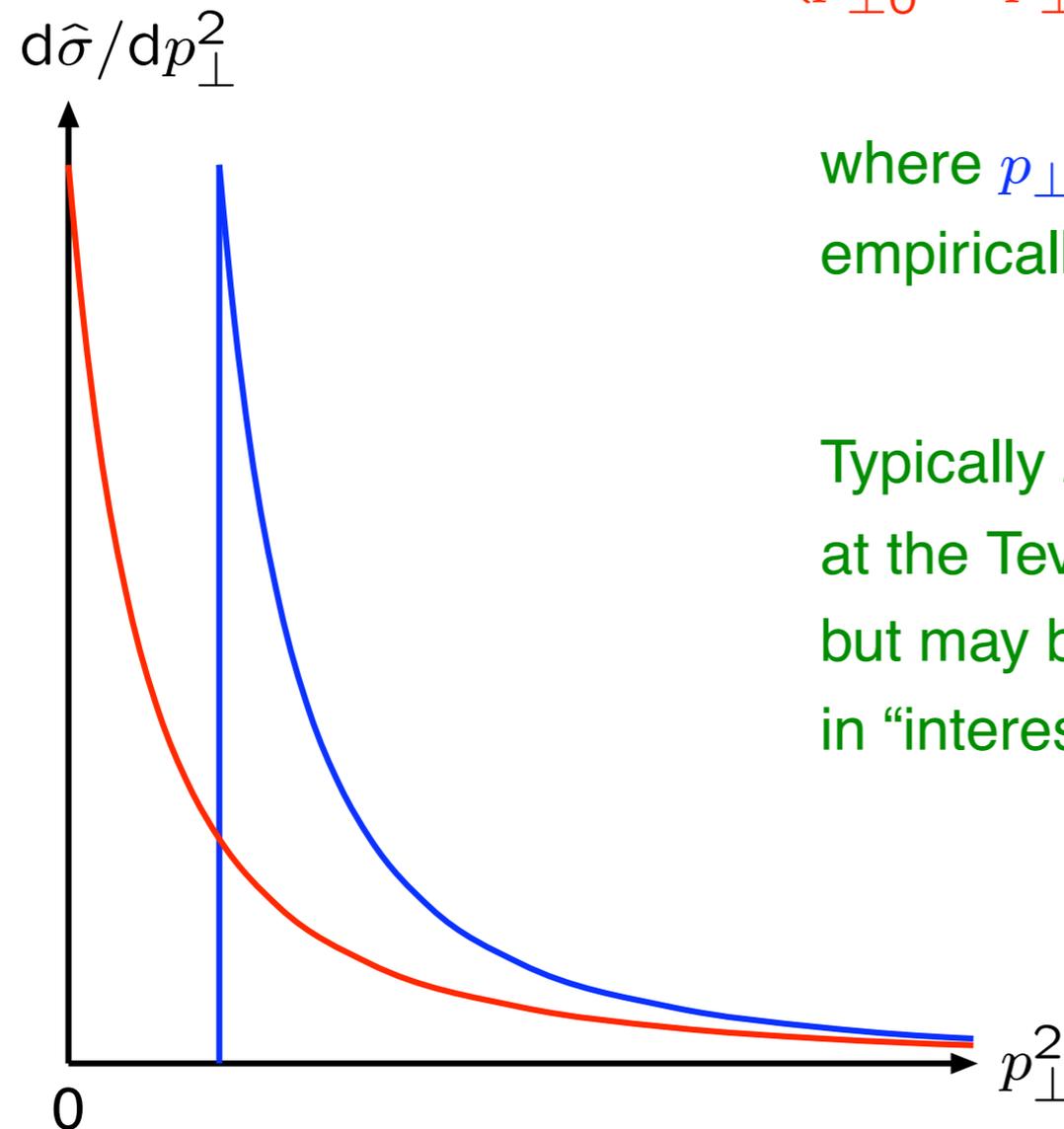
... but better replace  $r_p$  by (unknown) colour screening length  $d$  in hadron



so modify

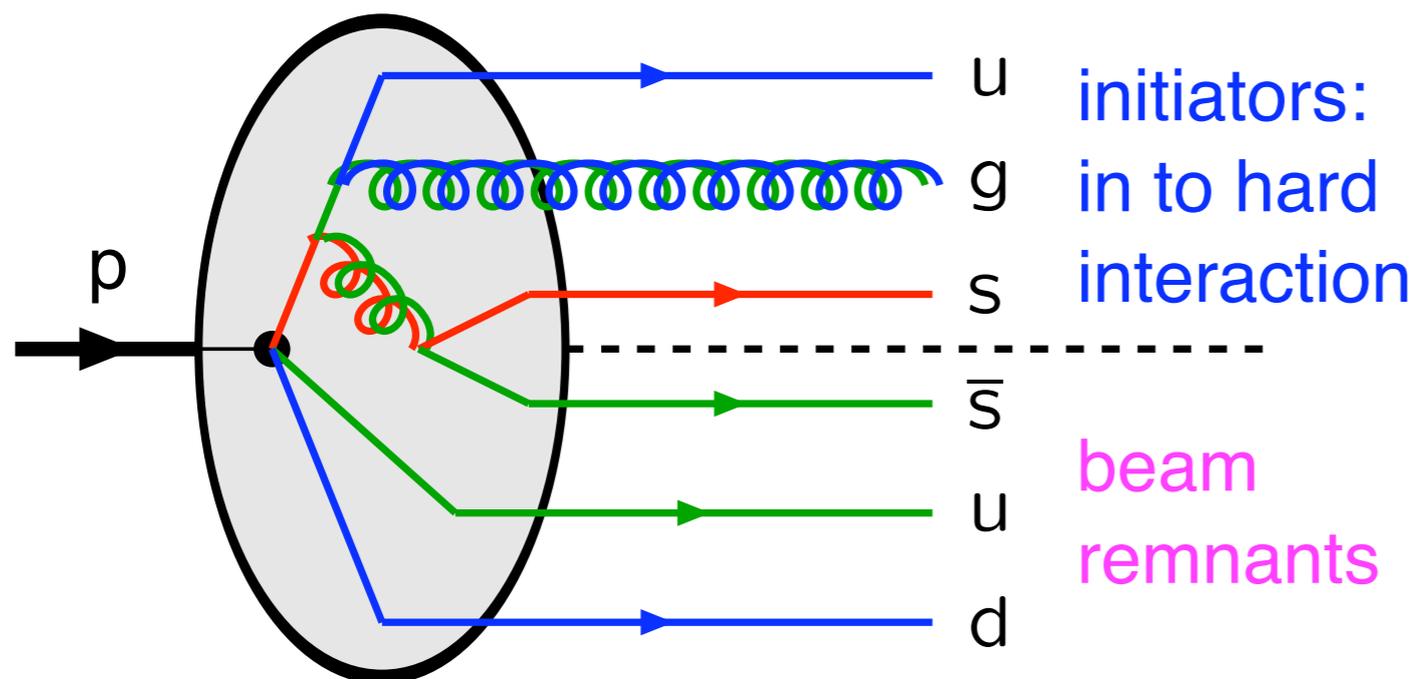
$$\frac{d\hat{\sigma}}{dp_{\perp}^2} \propto \frac{\alpha_S^2(p_{\perp}^2)}{p_{\perp}^4} \rightarrow \frac{\alpha_S^2(p_{\perp}^2)}{p_{\perp}^4} \theta(p_{\perp} - p_{\perp\min}) \quad (\text{simpler})$$

$$\text{or} \rightarrow \frac{\alpha_S^2(p_{\perp 0}^2 + p_{\perp}^2)}{(p_{\perp 0}^2 + p_{\perp}^2)^2} \quad (\text{more physical})$$



where  $p_{\perp\min}$  or  $p_{\perp 0}$  are free parameters,  
empirically of order **2 GeV**

Typically 2 – 3 interactions/event  
at the Tevatron, 4 – 5 at the LHC,  
but may be more  
in “interesting” high- $p_{\perp}$  ones.



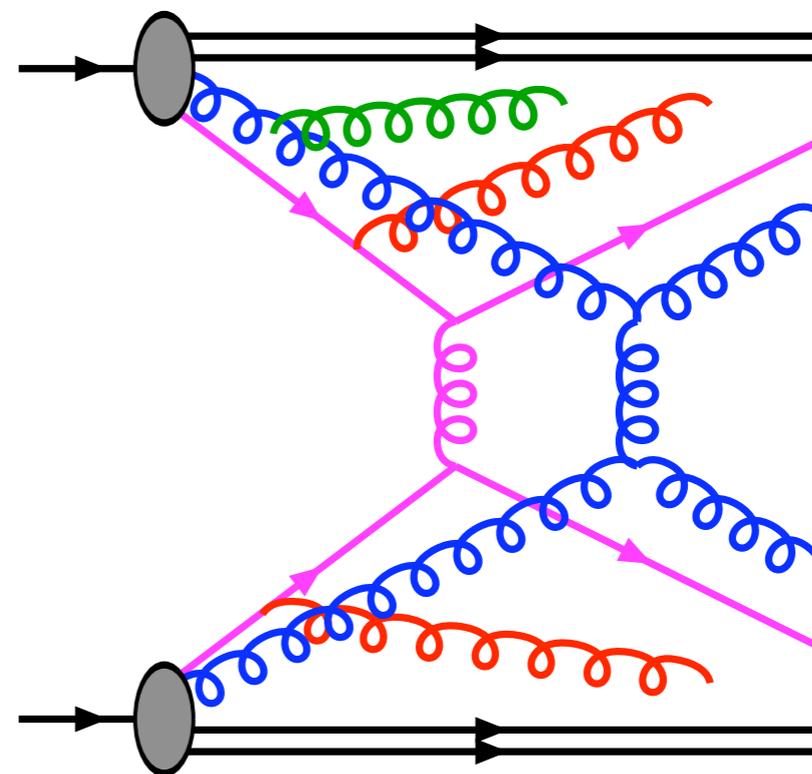
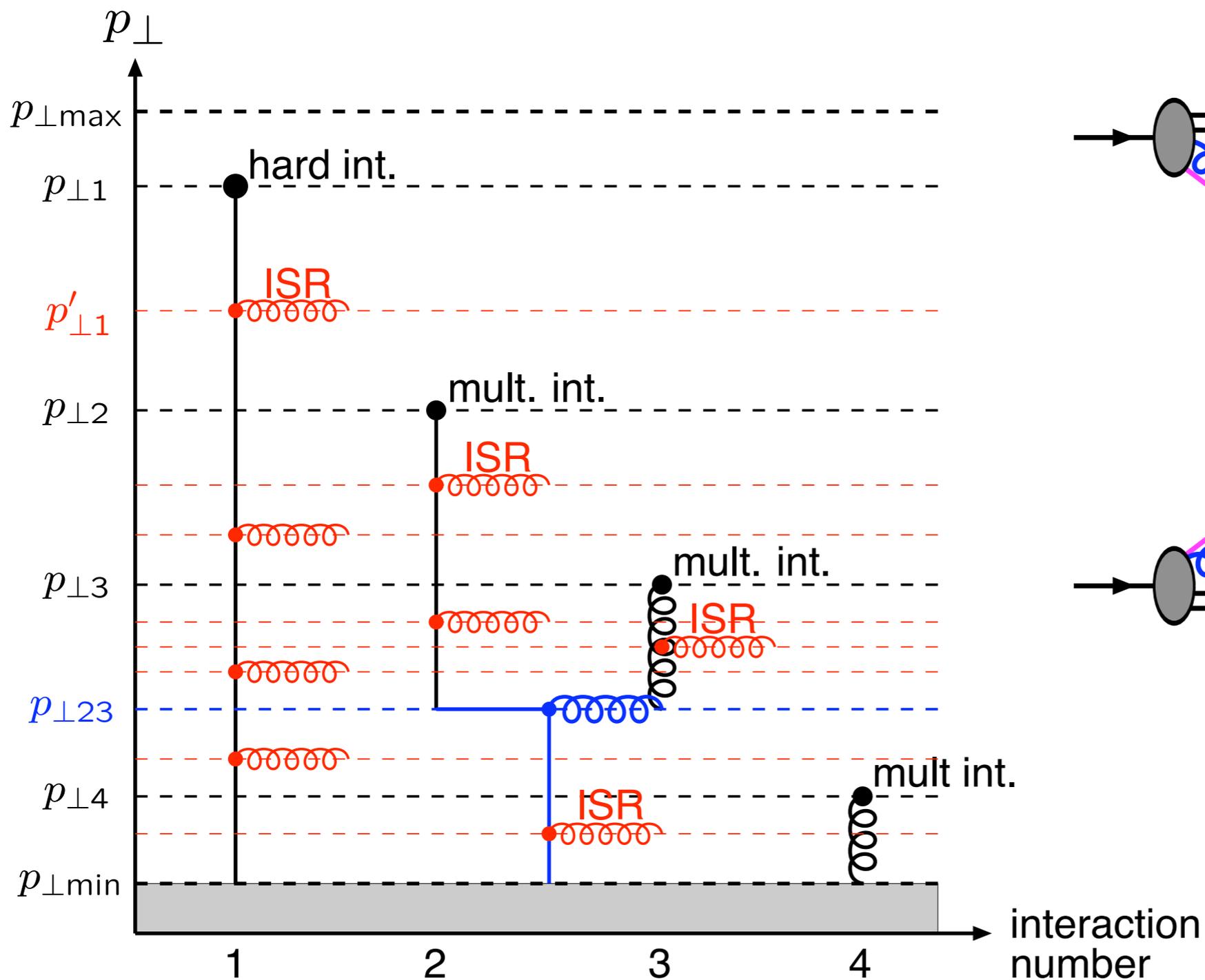
Need to assign:

- correlated flavours
- correlated  $x_i = p_{zi}/p_{ztot}$
- correlated primordial  $k_{\perp i}$
- correlated colours
- correlated showers

● **PDF after preceding MI/ISR activity:**

- 0) Squeeze range  $0 < x < 1$  into  $0 < x < 1 - \sum x_i$  (ISR:  $i \neq i_{\text{current}}$ )
- 1) Valence quarks: scale down by number already kicked out
- 2) Introduce companion quark  $q/\bar{q}$  to each kicked-out sea quark  $\bar{q}/q$ , with  $x$  based on assumed  $g \rightarrow q\bar{q}$  splitting
- 3) Gluon and other sea: rescale for total momentum conservation

# Interleaved MI



# MI: New Evolution Equation

	time	evolution	probability
FSR	forwards	$p_{\perp} \searrow 0$	normal & local
ISR	backwards	$p_{\perp} \searrow 0$	conditional
MI	simultaneous	$p_{\perp} \searrow 0$	conditional

ISR + MI: PDF competition  $\Rightarrow$  interleaving (PYTHIA 6.3)

FSR: previously at end, now also interleaved (PYTHIA 8.1):

$$\frac{d\mathcal{P}}{dp_{\perp}} = \left( \frac{d\mathcal{P}_{\text{MI}}}{dp_{\perp}} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp_{\perp}} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp_{\perp}} \right) \times \exp \left( - \int_{p_{\perp}}^{p_{\perp i-1}} \left( \frac{d\mathcal{P}_{\text{MI}}}{dp'_{\perp}} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp'_{\perp}} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp'_{\perp}} \right) dp'_{\perp} \right)$$

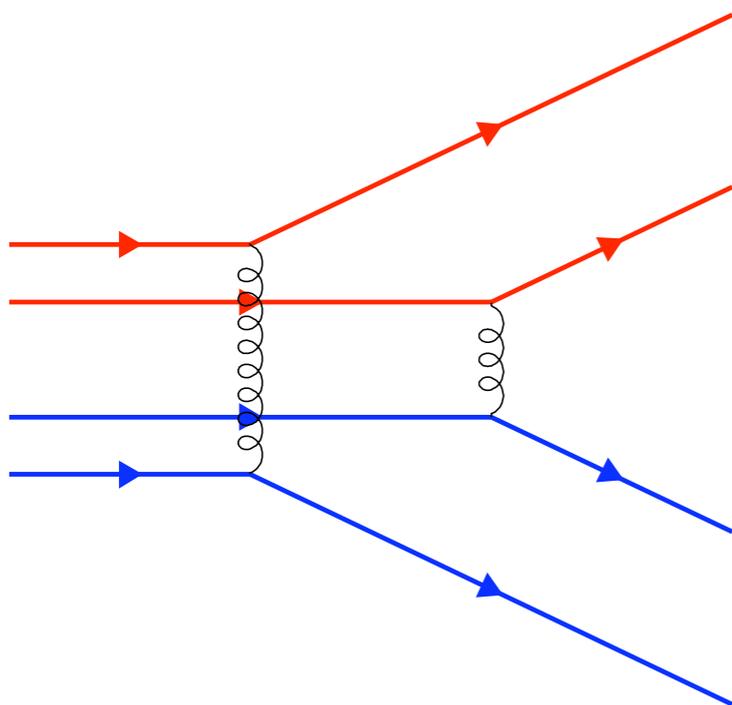
“resolution evolution”

Monte Carlo: winner takes all

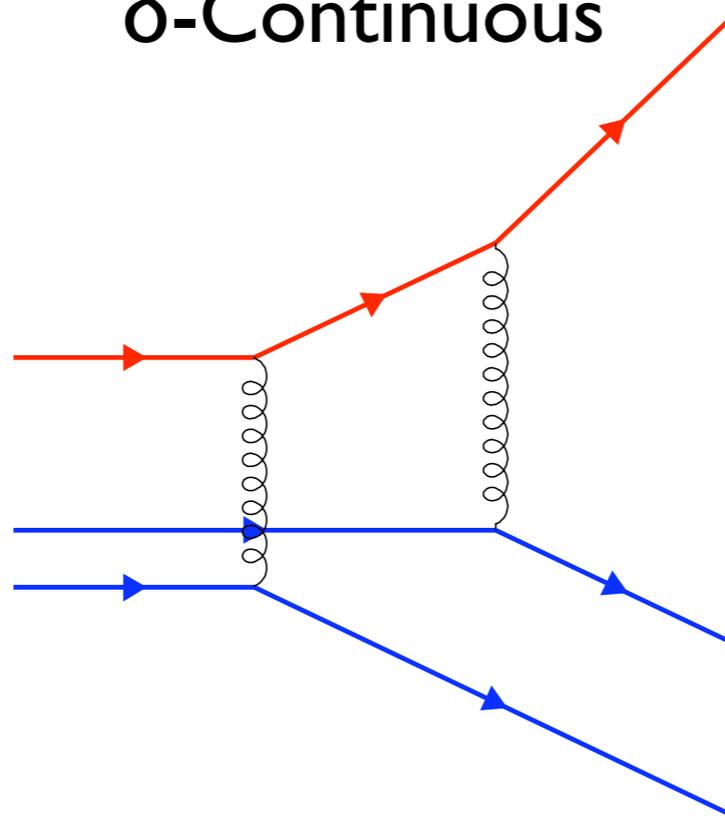
+ many other assumptions/models

## Distinguish 3 types of multiple interactions:

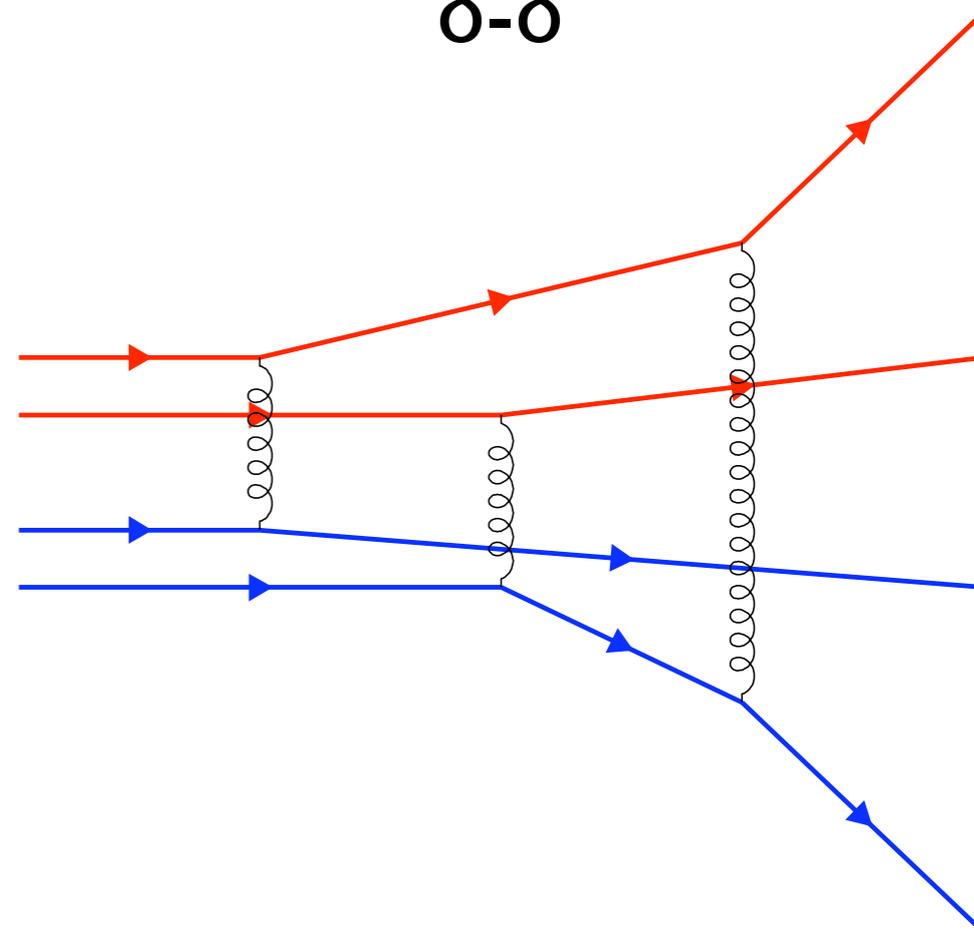
Continuous-  
Continuous



$\delta$ -Continuous



$\delta$ - $\delta$



MI Scattering type reflected in PDFs  $\mathbf{f}(\mathbf{x}, \mathbf{p}_\perp^2)$  :

$$\sigma_{\text{int}}(p_{\perp \text{min}}) = \iiint_{p_{\perp \text{min}}} dx_1 dx_2 dp_{\perp}^2 \mathbf{f}_1(\mathbf{x}_1, \mathbf{p}_{\perp}^2) \mathbf{f}_2(\mathbf{x}_2, \mathbf{p}_{\perp}^2) \frac{d\hat{\sigma}}{dp_{\perp}^2}$$

## “Radioactive decay” problem:

Looking for probability that *something* (nucleus decay, parton branch, multiple interaction) happens at time  $t$  provided that it did not happen at earlier times  $t'$

$$\mathcal{P}(t) = f(t) \underbrace{\exp \left\{ - \int_0^t f(t') dt' \right\}}_{\rightarrow \text{Sudakov}}$$

## Use-case: Interleaved Evolution (Pythia)

- For each step: Ask for  $p_T$  of next ISR, FSR, MI
- Choose interaction with largest  $p_T$

**Case 1:**  $f(t)$  has primitive function  $F(t)$  with known inverse

$$\int_0^t \mathcal{P}(t') dt' = 1 - \exp \left\{ - \int_0^t f(t') dt' \right\} = 1 - \mathcal{R} \Leftrightarrow t = F^{-1}(F(0) - \ln \mathcal{R})$$

**Case 2:**  $f(t) \leq g(t)$  with primitive function  $G(t)$  with known inverse

→ use **“Veto-Algorithm”**

1. start with  $i = 0$  and  $t_0 = 0$ ;
2. add 1 to  $i$  and select  $t_i = G^{-1}(G(t_{i-1}) - \ln R)$ , i.e. according to  $g(t)$ , but with the constraint that  $t_i > t_{i-1}$ ,
3. compare a (new)  $R$  with the ratio  $f(t_i)/g(t_i)$ ; if  $f(t_i)/g(t_i) \leq R$ , then return to point 2 for a new try;
4. otherwise  $t_i$  is retained as final answer.

# $\delta$ - $\delta$ -Type Rescattering

$$\sigma_{\text{int}}(p_{\perp \text{min}}) = \iiint_{p_{\perp \text{min}}} dx_1 dx_2 dp_{\perp}^2 \delta(\mathbf{x}_1 - \mathbf{x}_1^0, \mathbf{p}_{\perp}^2) \delta(\mathbf{x}_2 - \mathbf{x}_2^0, \mathbf{p}_{\perp}^2) \frac{d\hat{\sigma}}{dp_{\perp}^2}$$

$$\frac{d\hat{\sigma}}{dp_{\perp}^2} = \frac{d\hat{\sigma}}{d\hat{t}} \left| \frac{d\hat{t}}{dp_{\perp}^2} \right| = \frac{d\hat{\sigma}}{d\hat{t}} \frac{1}{\sqrt{1 - \frac{4p_{\perp}^2}{\hat{s}}}} \propto \frac{1 - \frac{x}{2}}{x^2 \sqrt{1 - x}}$$

$$\frac{d\hat{\sigma}}{d\hat{t}} \propto \frac{N}{2} \left( \frac{1}{\hat{t}^2} + \frac{1}{\hat{u}^2} \right) = 2N \left( \frac{2 - x}{4p_{\perp}^4} \right) \text{ where } x \equiv \frac{4p_{\perp}^2}{\hat{s}}$$

**Veto Algorithm:**  $\frac{d\hat{\sigma}}{dp_{\perp}^2} \propto \frac{1 - \frac{x}{2}}{x^2 \sqrt{1 - x}} \leq \frac{1}{x^2} + \frac{1}{2\sqrt{1 - x}}$

pick  $x_1$  according to  $\frac{1}{x^2} \exp\left(-\int_x^{x_{\text{max}}} \frac{1}{x'^2} dx'\right)$

pick  $x_2$  according to  $\frac{1}{2\sqrt{1 - x}} \exp\left(-\int_x^{x_{\text{max}}} \frac{1}{2\sqrt{1 - x'}} dx'\right)$

**Winner-takes-all:**  $x = \max\{x_1, x_2\}$

# $\delta$ -Cont.-Type Rescattering

$$\sigma_{\text{int}}(p_{\perp \text{min}}) = \int \int \int_{p_{\perp \text{min}}} dx_1 dx_2 dp_{\perp}^2 \delta(\mathbf{x}_1 - \mathbf{x}_1^0, \mathbf{p}_{\perp}^2) f_2(x_2, p_{\perp}^2) \frac{d\hat{\sigma}}{dp_{\perp}^2}$$

Variable substitution with  $y_3, y_4$  (rapidities of the two produced partons):

$$\int \frac{dx_1}{x_1} \frac{dx_2}{x_2} (d\hat{t} + d\hat{u}) = \int dy_3 dy_4 dp_{\perp}^2$$

**Result:**  $\frac{d\hat{\sigma}}{dp_{\perp}^2} = 2 \int d\Delta y x_2 f_2(x_2) \frac{d\hat{\sigma}}{d\hat{t}}$  with  $\Delta y = \frac{1}{2}(y_3 - y_4)$

Study limits and pick  $\Delta y$  flat in allowed interval:

$$\Delta y_{\text{max}} = \frac{1}{2} \ln \frac{E + p_z}{p_{\perp}} \leq \ln \frac{\sqrt{\hat{s}_{\text{max}}}}{p_{\perp}}$$

**Solve for  $x_2$ :**  $x_2 = \frac{4p_{\perp}^2 \cosh^2 \Delta y}{sx_1^0}$

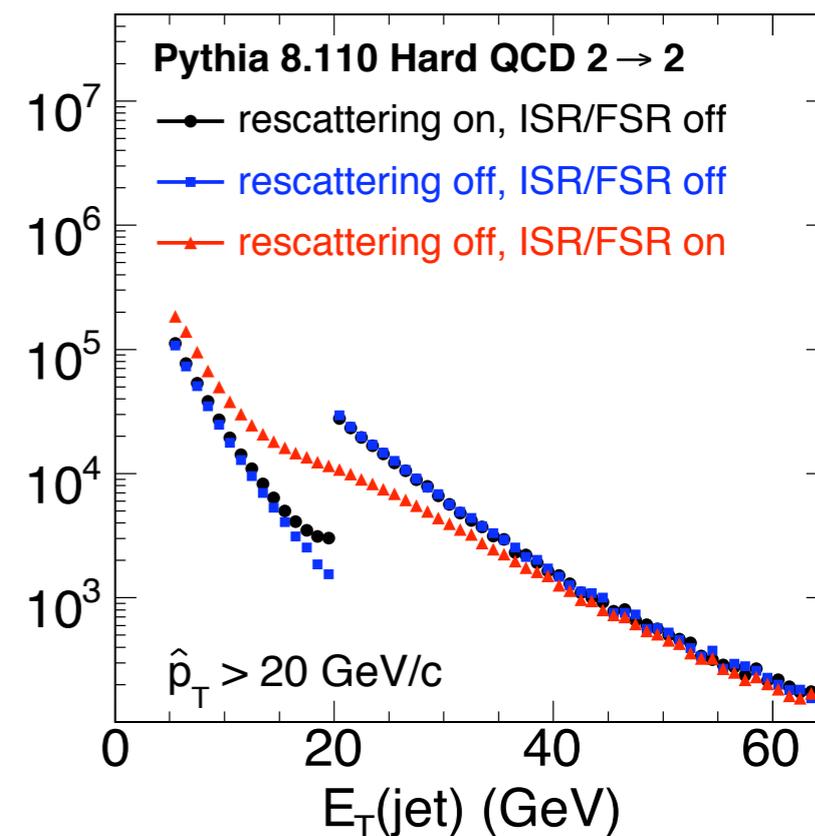
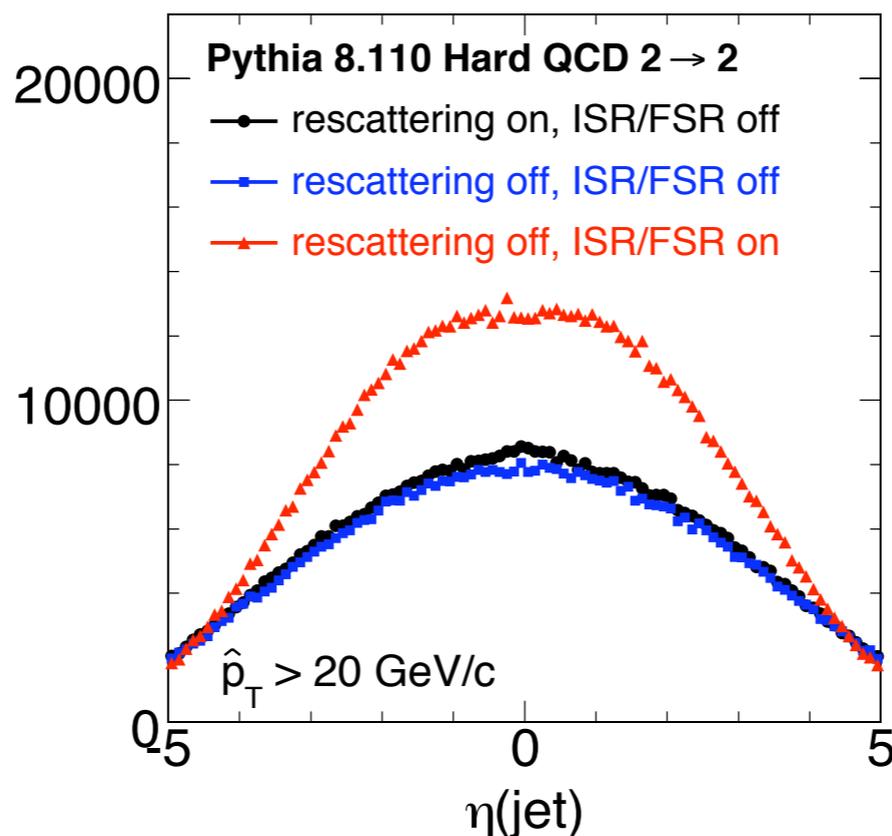
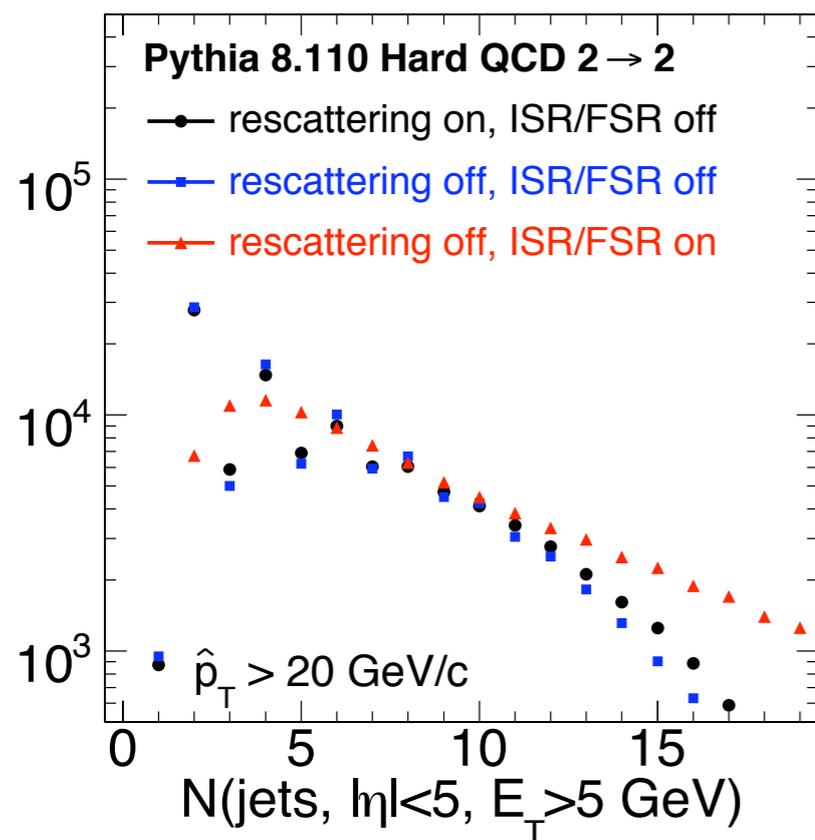
**NB: Studies done before Rescattering + PS implemented**

Compare three settings:

Pythia with rescattering

without rescattering

Influence of parton shower



**Parton level rescattering effects ~ few percent contribution**

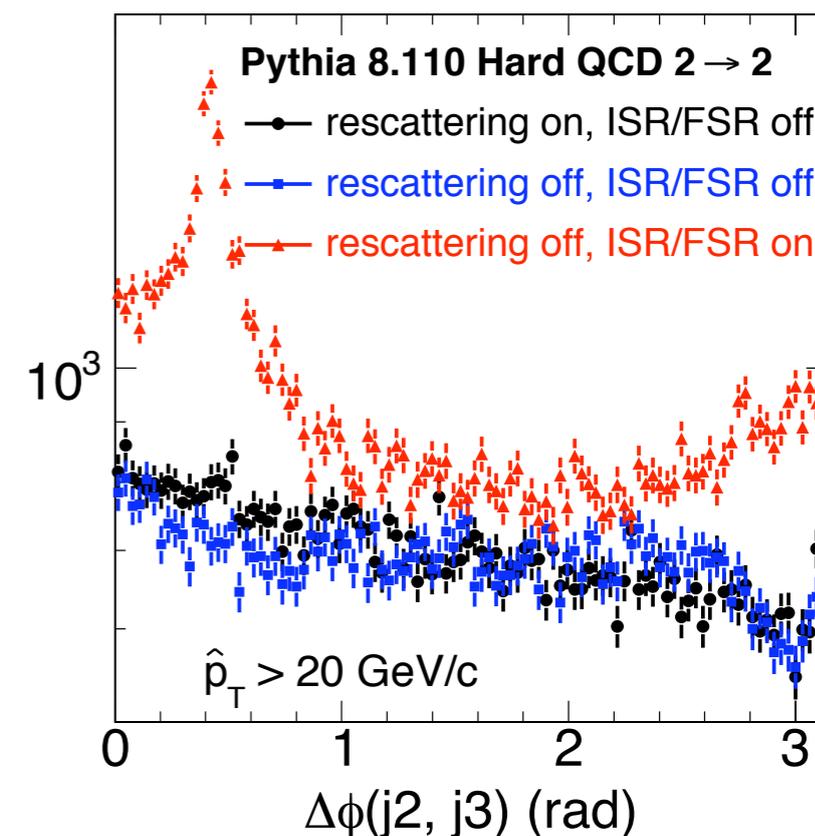
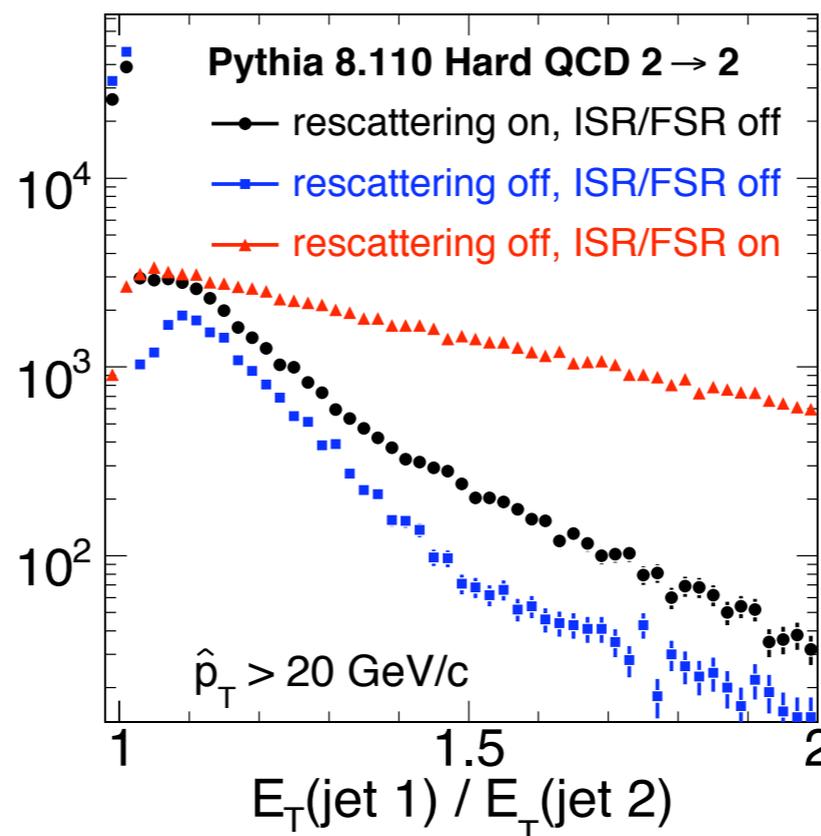
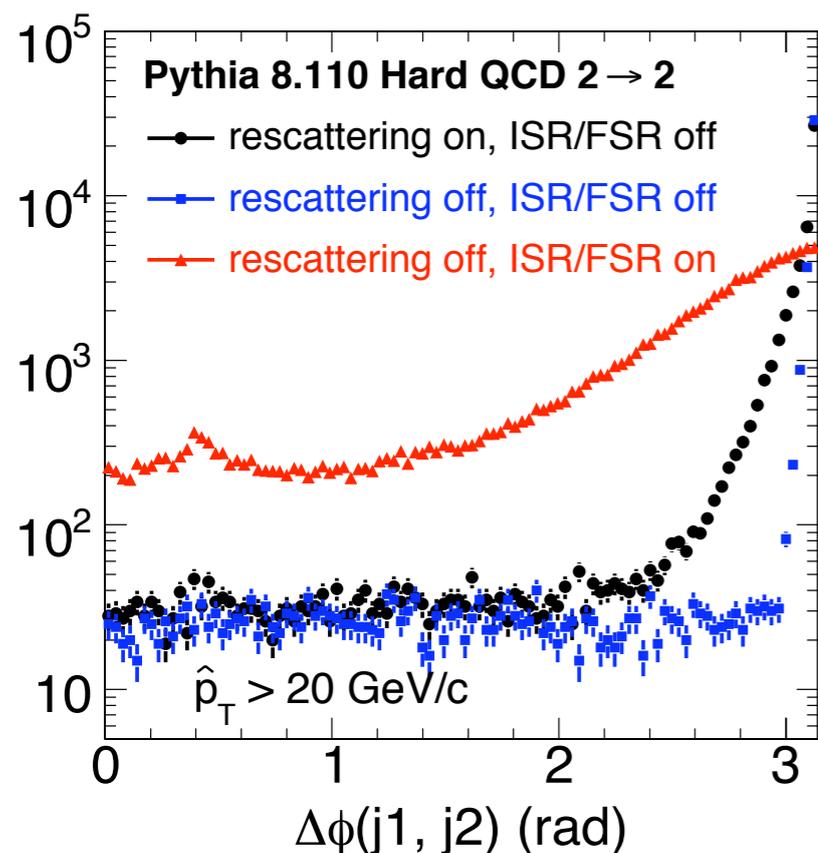
**NB: Studies done before Rescattering + PS implemented**

Compare three settings:

Pythia with rescattering

without rescattering

Influence of parton shower



**Parton level rescattering effects ~ few percent contribution**

- ▶ **Pythia models multiple interactions in great detail**
- ▶ **Rescattering option available in Pythia 8**
- ▶ **Validation ongoing**
- ▶ **Percent level effects, may be ~10-20% in some distributions**

# BACKUP



T. Sjöstrand, M. van Zijl, PRD36 (1987) 2019: first models for event properties based on perturbative multiple interactions

(1) Simple scenario:

no longer used (no impact-parameter dependence)

(2) More sophisticated scenario:

still in frequent use (Tune A, Tune DWT, ATLAS tune, ...)

- Is only a model for nondiffractive events, i.e. for  $\sigma_{\text{nd}} \simeq (2/3)\sigma_{\text{tot}}$
- Smooth turn-off at  $p_{\perp 0}$  scale
- Require  $\geq 1$  interaction in an event
- Interactions generated in ordered sequence  $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > \dots$  by “Sudakov” trick (what happens “first”?)

$$\frac{d\mathcal{P}}{dp_{\perp i}} = \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp_{\perp}} \exp \left[ - \int_{p_{\perp}}^{p_{\perp(i-1)}} \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp'_{\perp}} dp'_{\perp} \right]$$

- After each interaction rescaled new PDF's for momentum conservation
- Leads to  $n_{\text{int}}$  narrower than Poissonian, except that ...

- Hadrons are extended,  
e.g. double Gaussian (“hot spots”):

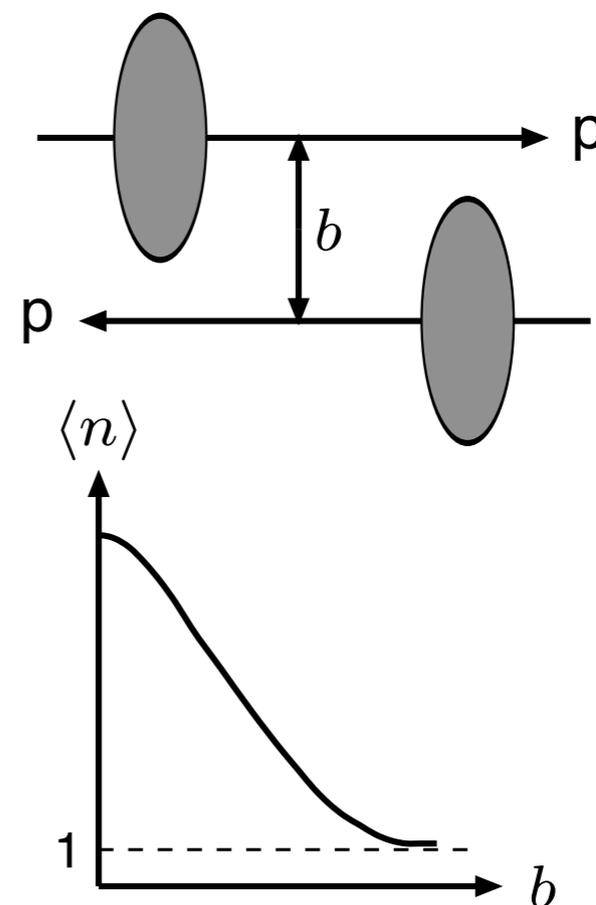
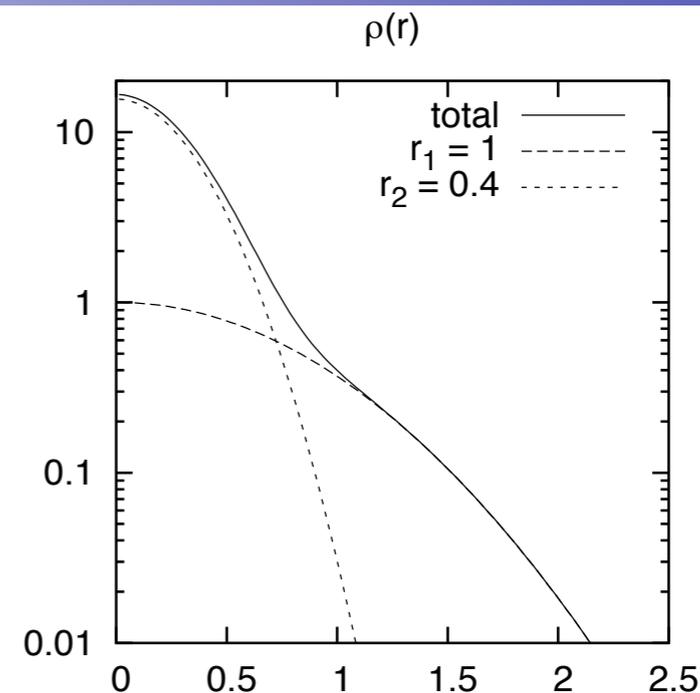
$$\rho_{\text{matter}}(r) = N_1 \exp\left(-\frac{r^2}{r_1^2}\right) + N_2 \exp\left(-\frac{r^2}{r_2^2}\right)$$

where  $r_2 \neq r_1$  represents “hot spots”

- Events are distributed in impact parameter  $b$
- Overlap of hadrons during collision

$$\mathcal{O}(b) = \int d^3\mathbf{x} dt \rho_{1,\text{matter}}^{\text{boosted}}(\mathbf{x}, t) \rho_{2,\text{matter}}^{\text{boosted}}(\mathbf{x}, t)$$

- Average activity at  $b$  proportional to  $\mathcal{O}(b)$   
 $\Rightarrow$  central collisions normally more active  
 $\Rightarrow \mathcal{P}_n$  broader than Poissonian
- Time-consuming  $(b, p_{\perp})$  generation
- Problems if many valence quarks kicked out  
 $\Rightarrow$  Simplify after first interaction:  
 only  $gg$  or  $q\bar{q}$  outgoing, no showers, ...



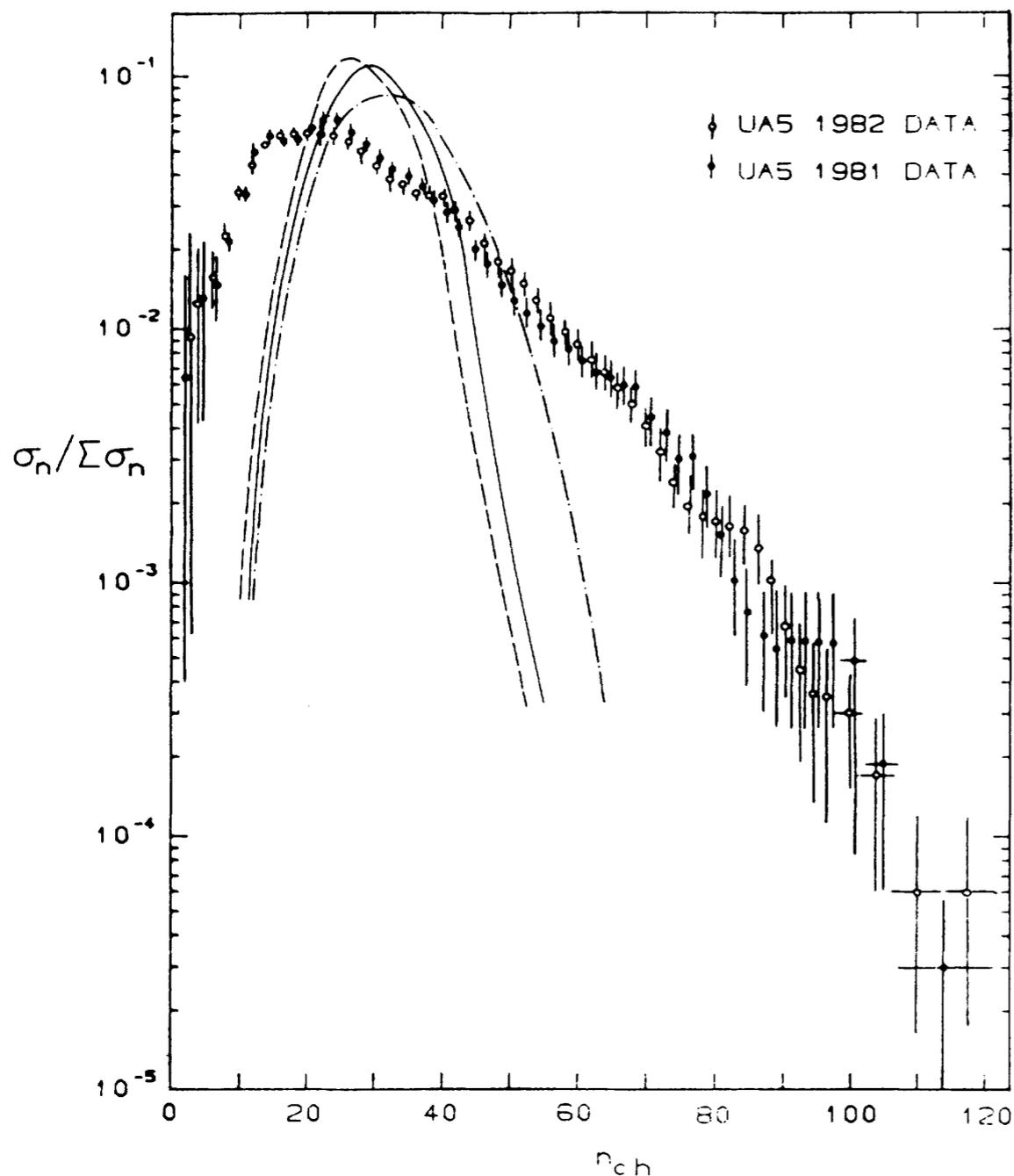


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low  $p_T$  only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

without multiple interactions

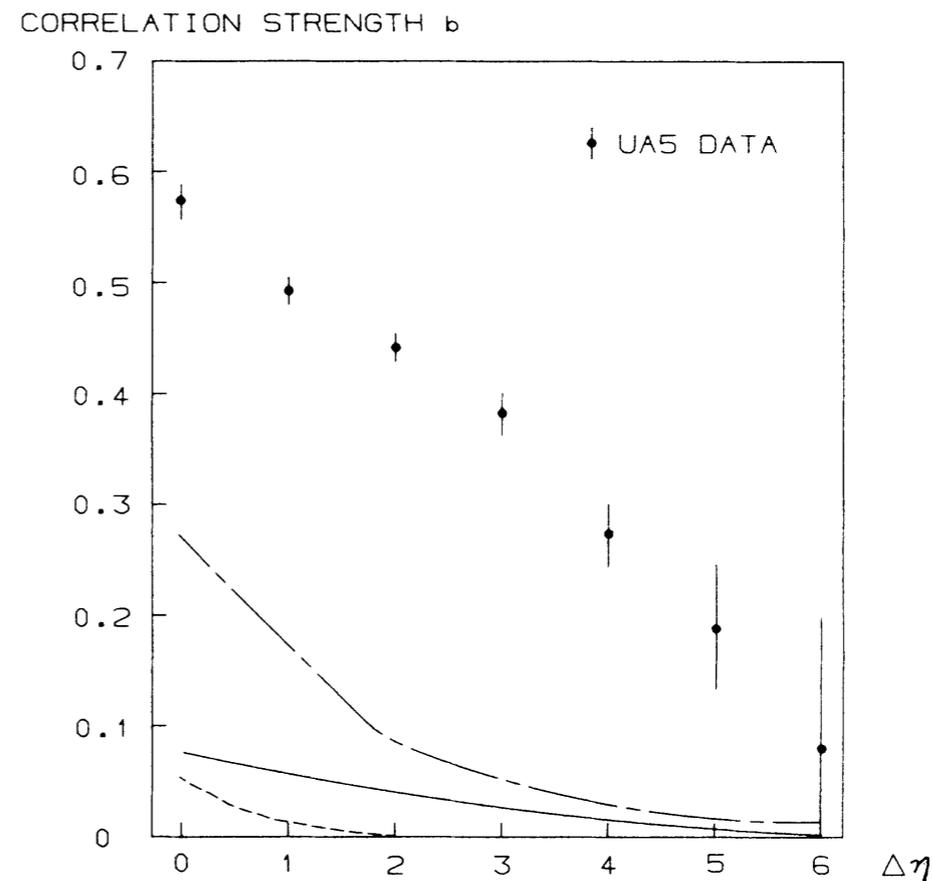


FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.

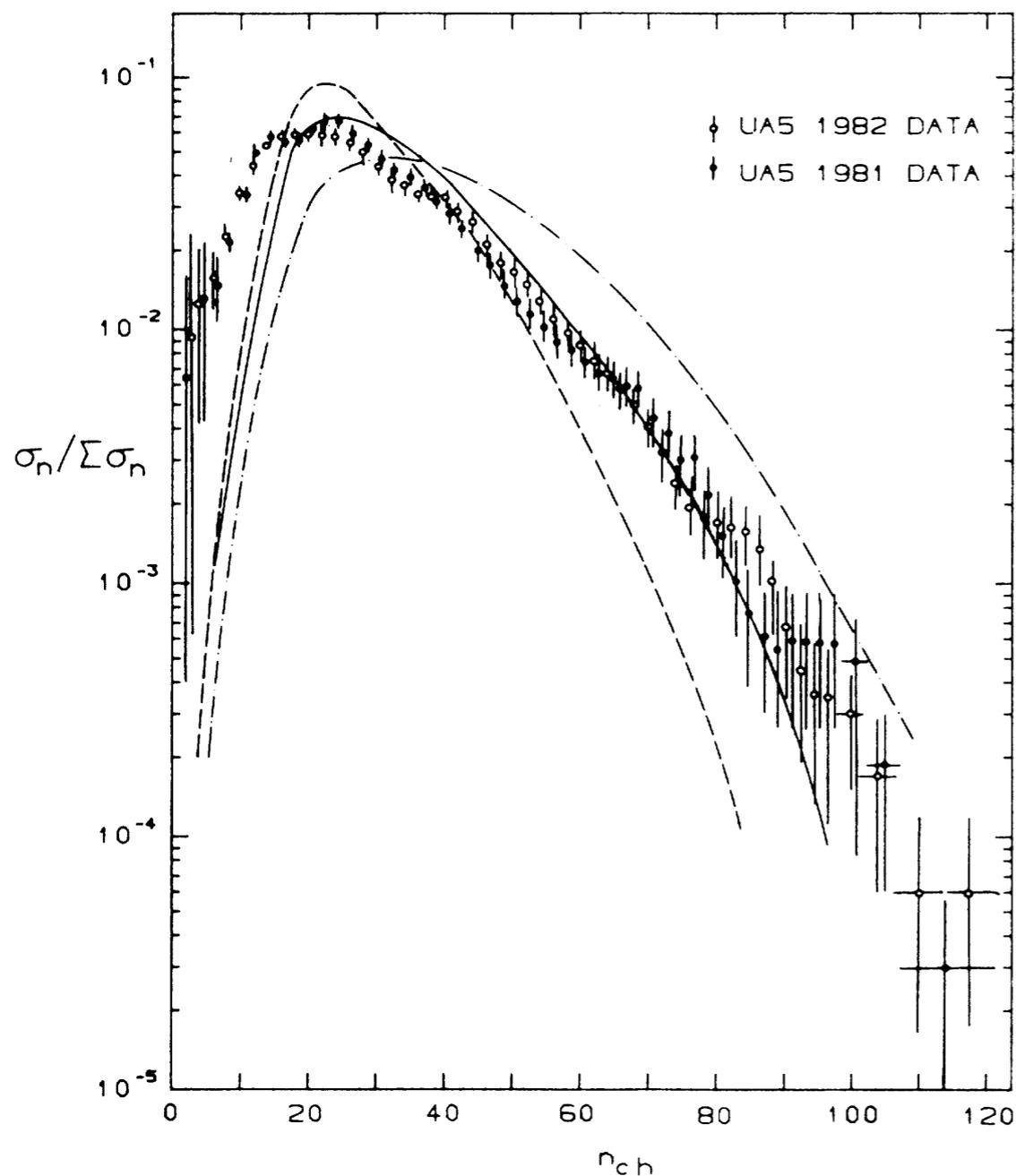


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line,  $p_{Tmin}=2.0$  GeV; solid line,  $p_{Tmin}=1.6$  GeV; dashed-dotted line,  $p_{Tmin}=1.2$  GeV.

with multiple interactions

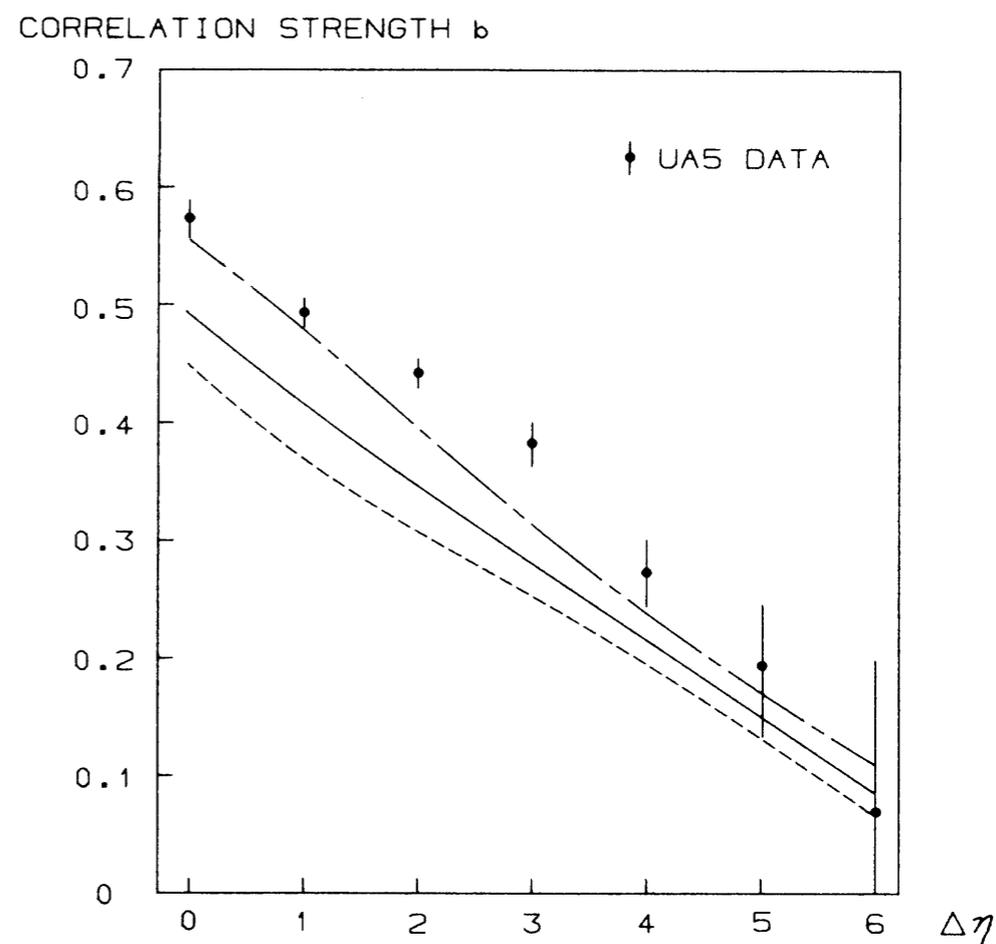


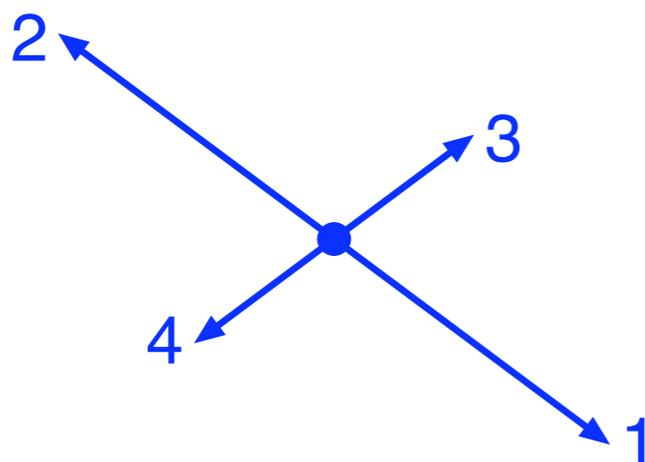
FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.

# Direct Observation of MI

Four studies: AFS (1987), UA2 (1991), CDF (1993, 1997)

Order 4 jets  $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > p_{\perp 4}$  and define  $\varphi$   
as angle between  $p_{\perp 1} \mp p_{\perp 2}$  and  $p_{\perp 3} \mp p_{\perp 4}$  for AFS/CDF

## Double Parton Scattering

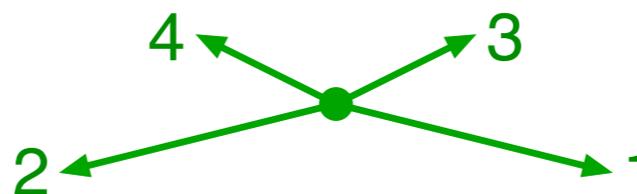


$$|p_{\perp 1} + p_{\perp 2}| \approx 0$$

$$|p_{\perp 3} + p_{\perp 4}| \approx 0$$

$d\sigma/d\varphi$  flat

## Double BremsStrahlung



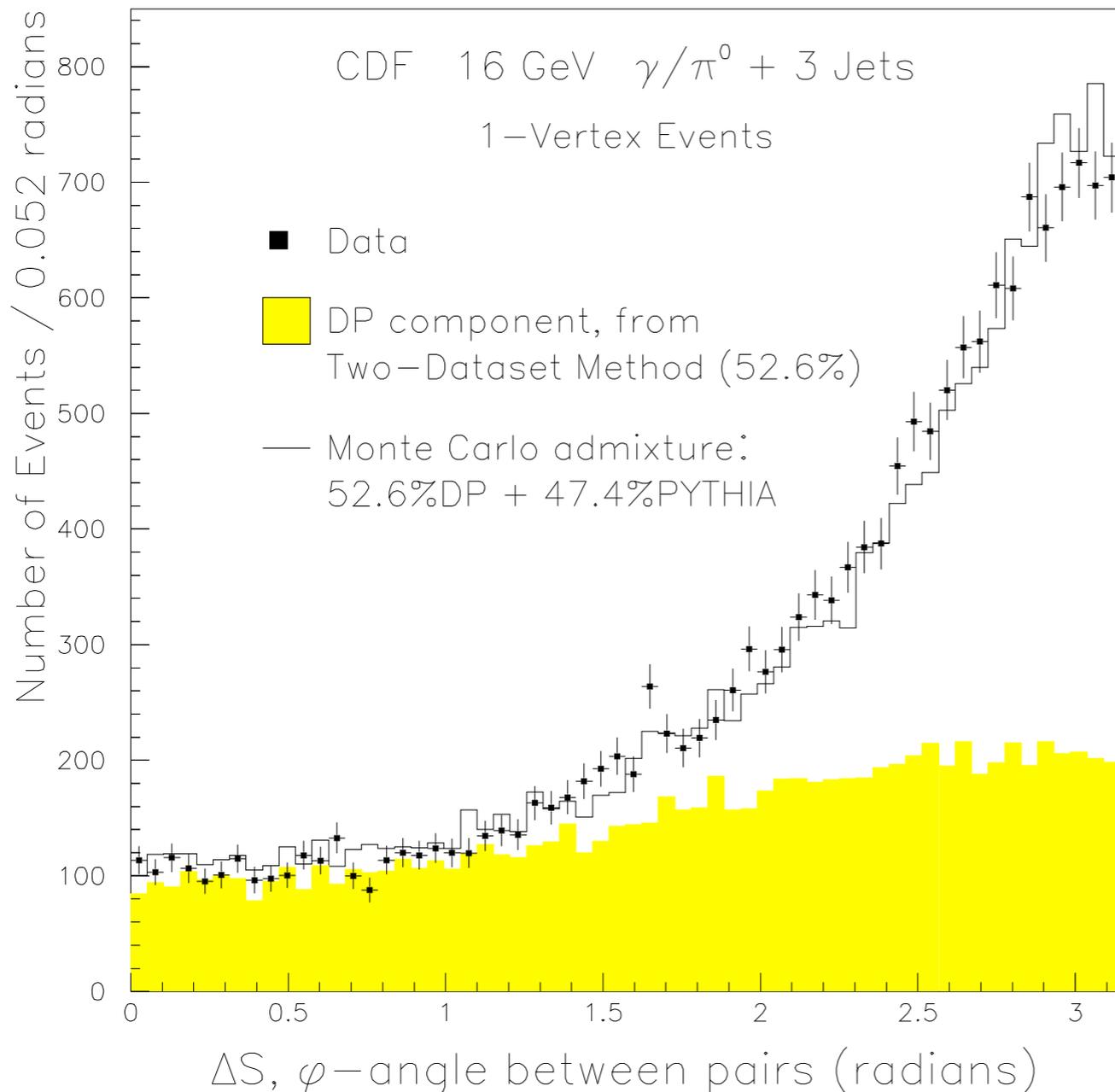
$$|p_{\perp 1} + p_{\perp 2}| \gg 0$$

$$|p_{\perp 3} + p_{\perp 4}| \gg 0$$

$d\sigma/d\varphi$  peaked at  $\varphi \approx 0/\pi$  for AFS/CDF

AFS 4-jet analysis (pp at 63 GeV): observe 6 times Poissonian prediction,  
with impact parameter expect 3.7 times Poissonian,  
but big errors  $\Rightarrow$  low acceptance, also UA2

# CDF: Photon + 3 Jets



CDF 3-jet + prompt photon analysis

Yellow region = double parton scattering (DPS)

The rest = PYTHIA showers

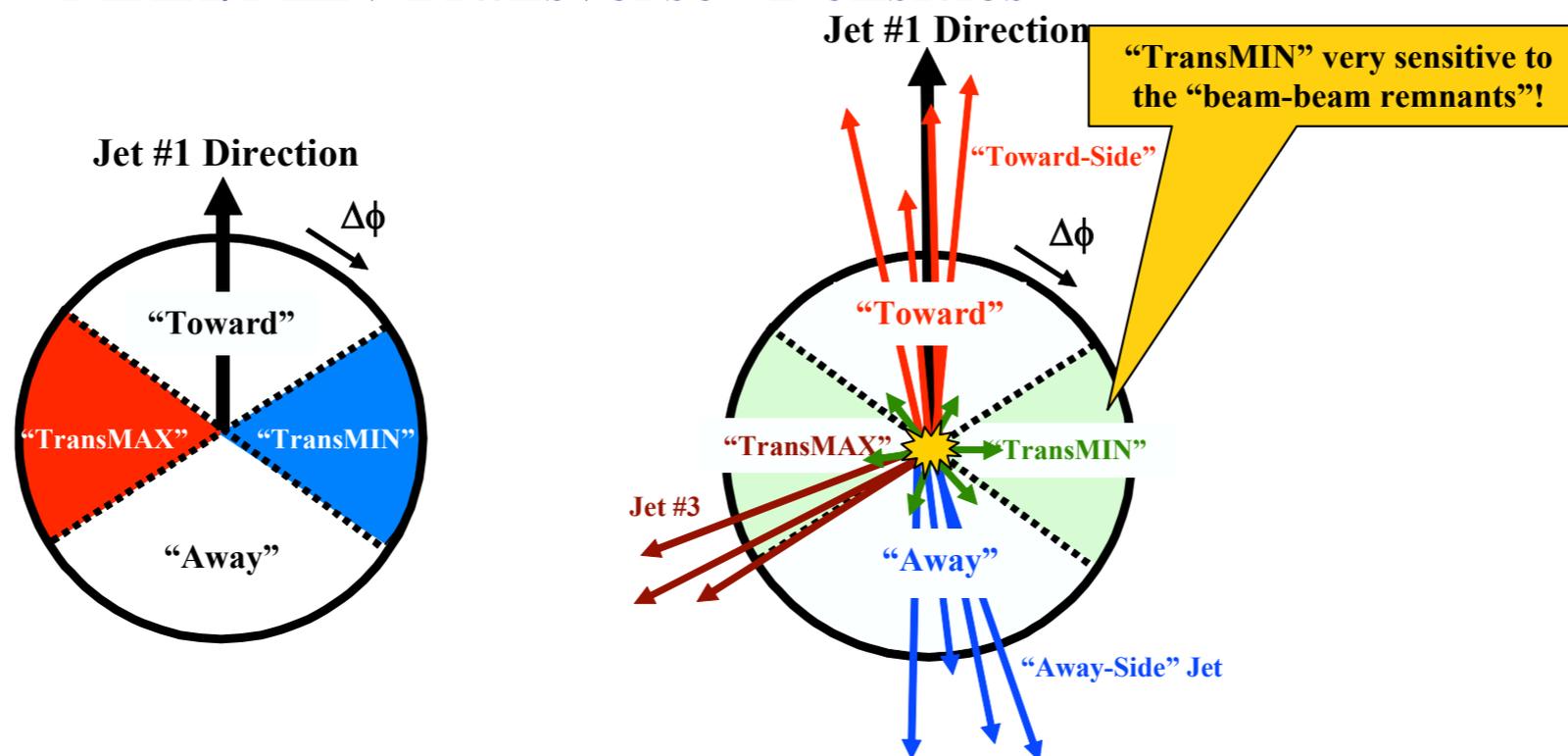
$$\sigma_{\text{DPS}} = \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}} \quad \text{for } A \neq B \quad \implies \sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb}$$

**Strong enhancement relative to naive expectations!**

Events with hard scale (jet,  $W/Z$ , ...) have more underlying activity!  
 Events with  $n$  interactions have  $n$  chances that one of them is hard,  
 so “trigger bias”: hard scale  $\Rightarrow$  central collision  
 $\Rightarrow$  more interactions  $\Rightarrow$  larger underlying activity.  
 Centrality effect saturates at  $p_{\perp\text{hard}} \sim 10$  GeV.

Studied in detail by Rick Field, comparing with CDF data:

## “MAX/MIN Transverse” Densities



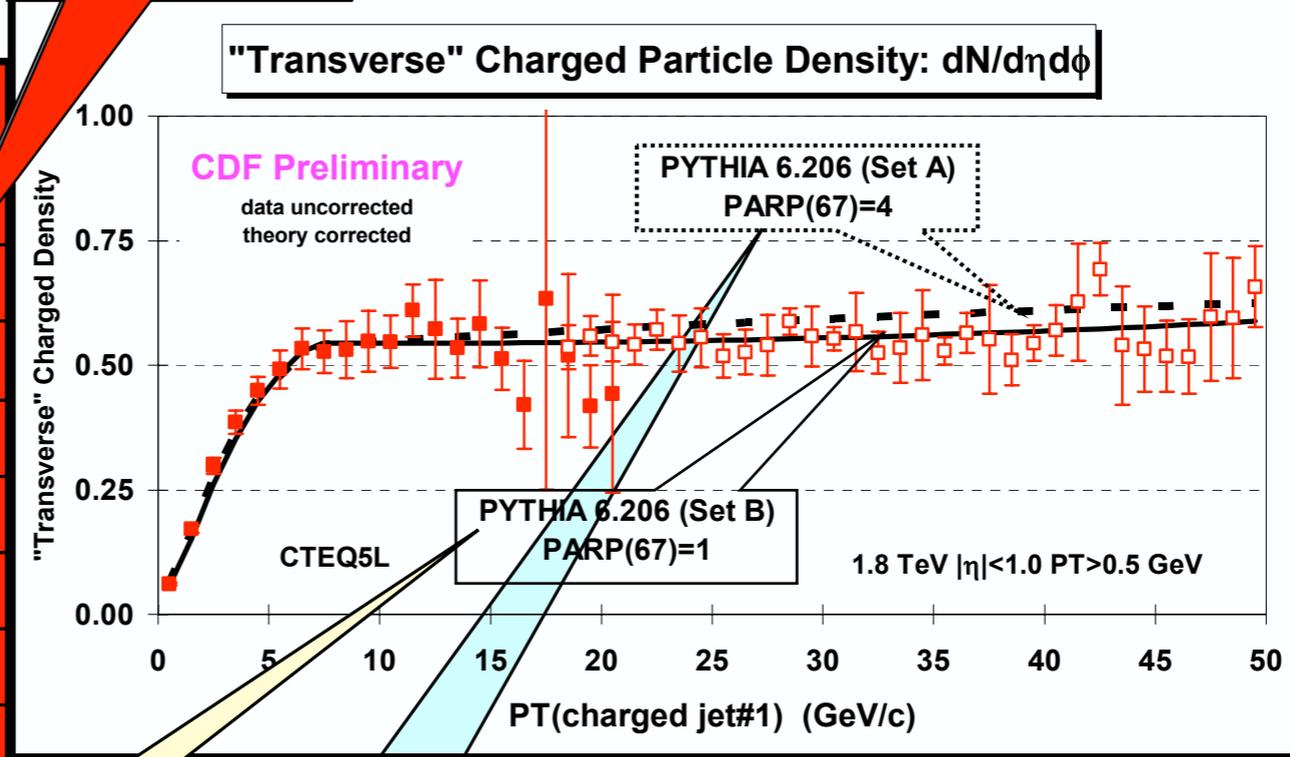
- Define the **MAX and MIN “transverse” regions** on an event-by-event basis with MAX (MIN) having the largest (smallest) density.

## Tuned PYTHIA 6.206

**PYTHIA 6.206 CTEQ5L**

Parameter	Tune B	Tune A
MSTP(81)	1	1
MSTP(82)	4	4
PARP(82)	1.9 GeV	2.0 GeV
PARP(83)	0.5	0.5
PARP(84)	0.4	0.4
PARP(85)	1.0	0.9
PARP(86)	1.0	0.95
PARP(89)	1.8 TeV	1.8 TeV
PARP(90)	0.25	0.25
PARP(67)	1.0	4.0

**Tune A CDF  
Run 2 Default!**



Plot shows the “**Transverse**” charged particle density versus  $P_T(\text{chgjet\#1})$  compared to the QCD hard scattering predictions of two **tuned** versions of **PYTHIA 6.206** (CTEQ5L, **Set B** (PARP(67)=1) and **Set A** (PARP(67)=4)).

**New PYTHIA default  
(less initial-state radiation)**

**Old PYTHIA default  
(more initial-state radiation)**

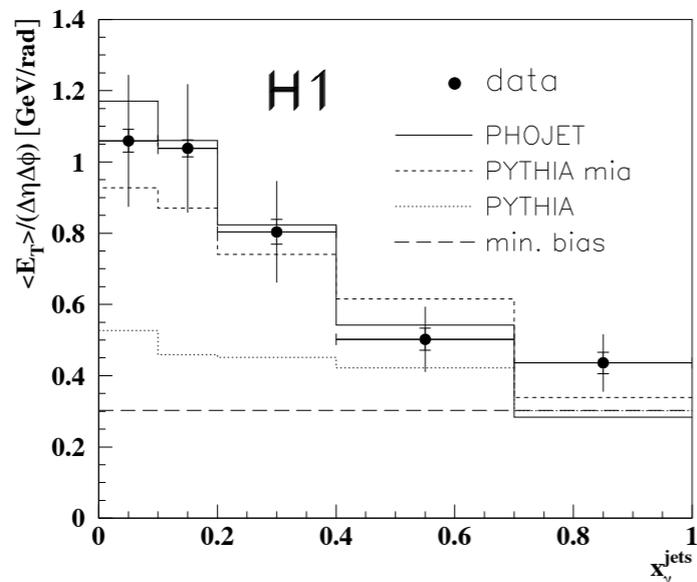
CERN July 31, 2003

Rick Field - Florida/CDF

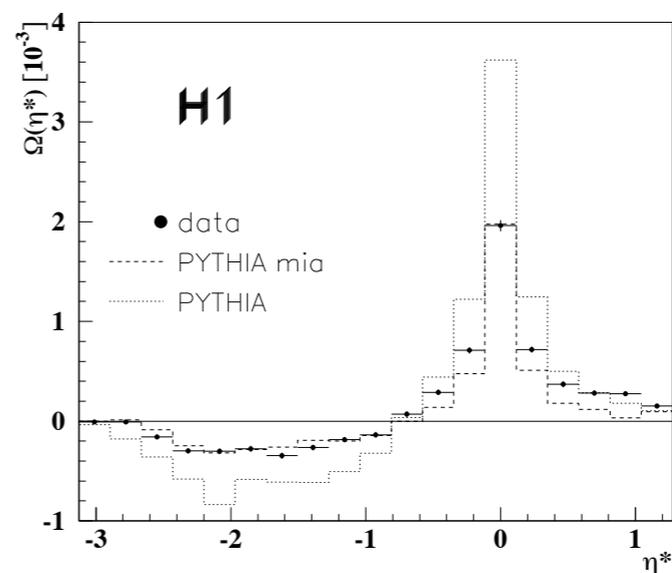
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Multiple interactions also preferred by HERA photoproduction data:

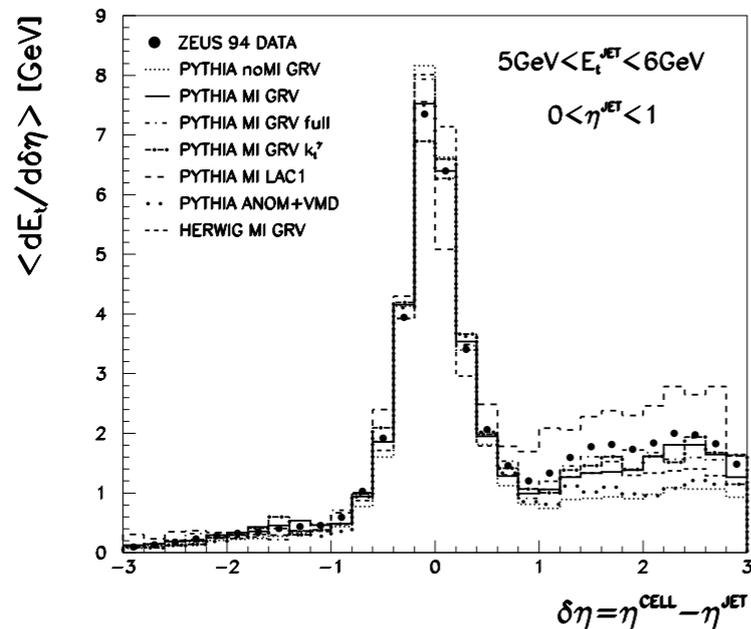
underlying activity in  
photoproduction vs. DIS



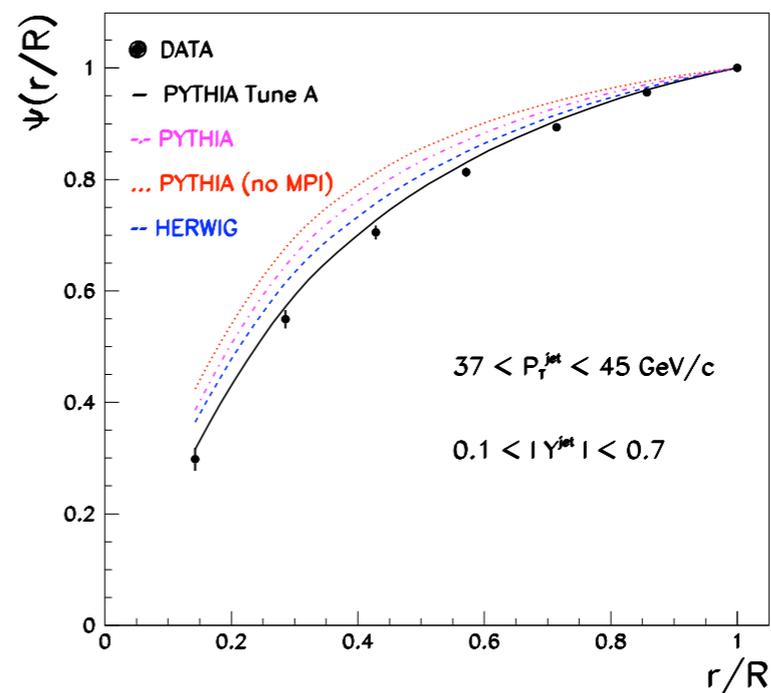
(anti)correlations in  
energy flow around jet



ZEUS 1994 Preliminary



CDF II Preliminary



## Issues requiring further thought and study:

- Multi-parton PDF's  $f_{a_1 a_2 a_3 \dots}(x_1, Q_1^2, x_2, Q_2^2, x_3, Q_3^2, \dots)$
- Close-packing in initial state, especially small  $x$
- Impact-parameter picture and  $(x, b)$  correlations  
e.g. large- $x$  partons more central!, valence quarks more central?
- Details of colour-screening mechanism
- Rescattering: one parton scattering several times
- Intertwining: one parton splits in two that scatter separately
- Colour sharing: two FS–IS dipoles become one FS–FS one
- Colour reconnection: required for  $\langle p_{\perp} \rangle (n_{\text{charged}})$
- Collective effects (e.g. QGP, cf. Hadronization above)
- Relation to diffraction: eikonalization, multi-gap topologies, ...

## Action items:

- Vigorous experimental program at LHC
- Study energy dependence: RHIC (pp)  $\rightarrow$  Tevatron  $\rightarrow$  LHC
- Develop new frameworks and refine existing ones

Much work ahead!

■ About the network

■ Monte Carlo schools

■ Short-term studentships

■ The projects

■ The teams

■ Meetings

■ Publications

■ Contact info

## About the network

*MCnet is a European Union funded Marie Curie Research Training Network dedicated to developing the next generation of Monte Carlo event generators and providing training of a wide selection of its user base, particularly through funded short-term 'residencies' and Annual Schools.*

Monte Carlo event generators are central to high energy particle physics. They are used by almost all experimental collaborations to plan their experiments and analyze their data, and by theorists to simulate the complex final states of the fundamental interactions that may signal the presence of new physics. This network includes all authors of current general purpose event generators, and the main objectives are:

- to train a large section of our user base, using short-term residences of Early Stage Researchers and annual highlevel schools on the physics and techniques of event generators;
- to train the next generation of event generator authors through dedicated studentships and a small number of focused Experienced Researcher contract positions;
- to develop the next generation of event generators intended for use throughout the LHC data analysis era and well into the ILC era;
- to play a central rôle in the analysis of early LHC data and the discovery of new particles and interactions there.

## Latest news

30/06/08 - The next deadline for applications to **short-term studentships** has been set to October 6th 2008.

## Older news items

MCnet is funded by



[internal MCnet wiki](#)

[www.montecarlonet.org](http://www.montecarlonet.org)

## ▶ Event Generators

- Herwig++, Ariadne, Sherpa, Pythia8

## ▶ Frameworks

- CEDAR (general-purpose facilities to do HEP analysis)
- ThePEG (implement event generators)
- Rivet (generator-independent framework for comparison with data)

## ▶ Training

- Monte Carlo schools (Durham 07, CTEQ 08, Lund 09, Karlsruhe 10)
- **Short-term studentships: to be held at any MCnet node (CERN, Durham, Cambridge, Karlsruhe, Lund, London) for 3 - 6 months**

## ▶ **Who can apply?**

- Ph.D. students in experimental or theoretical particle physics whose research would benefit from a short integration into one of the teams

## ▶ **How to apply?**

- <http://www.montecarlonet.org/index.php?p=Residency/application>
- CV, research plan, letter of reference

## ▶ **Research proposal**

- “Students propose a research project that fits in with their more long-term PhD projects, while maximizing the benefit of working in an MCnet group.”

- *The underlying event in proton-proton collisions and its relation to multiple parton-parton interactions*, Florian Bechtel, **Lund**, four months from March 2008.
- *Minimal Walking Technicolor in Sherpa*, Jonathan Ferland, **Durham**, three months from February 2008.
- *Simulation of top quark production events*, Alexander Flossdorf, **CERN**, four months from February 2008.
- *Building a New Exotic MC Tool*, Noam Hod, **UCL**, four months from April 2008.
- *Z/W + jets production at the LHC: a comparison among different Monte Carlo generators*, Piergiulio Lenzi, **UCL**, three months from January 2008.
- *The importance of the bPDF input to MC generator predictions on neutral Higgs production processes*, Peter Steinbach, **UCL**, four months from May 2008.

**3 students from German institutes!**

# Lund

