Combined measurement and QCD analysis of the inclusive $e^{\pm} p$ scattering cross section at HERA

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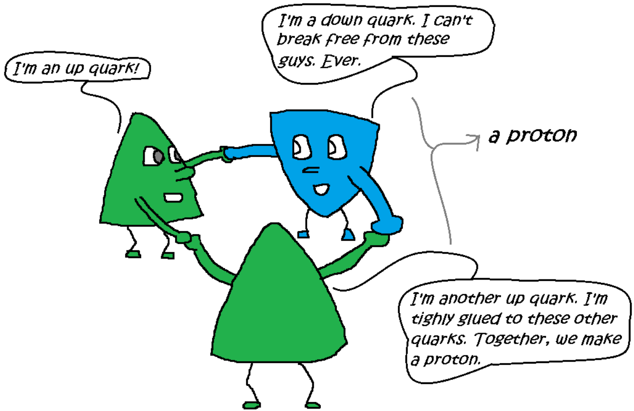
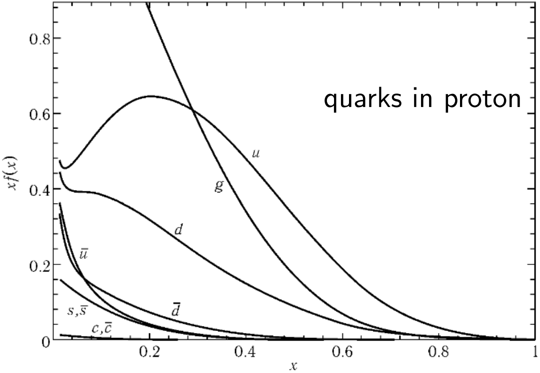
Picture of proton



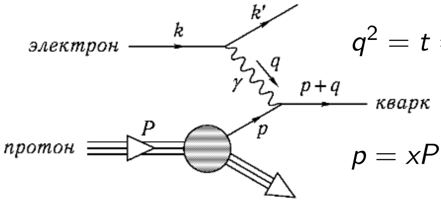
Structure functions



$f_q(x)$ is probability density function to find quark(gluon) q inside composite particle (today, **proton**) which carries x-part of momentum. Obviously,

$< x_q >= \int x f_q(x) \mathrm{d}x,$ then $xf(x)$ gives filling about average momentum.

Kinematics of DIS. Neutral current



$$q^2 = t = -Q^2$$

$$p = xP$$

$$0 \approx (p+q)^2 = 2p \cdot q + q^2 = 2xP \cdot q - Q^2, \quad x = \frac{Q^2}{2Pq}$$

$$y = \frac{2P \cdot q}{s}, \quad \text{so} \quad y = \frac{Q^2}{xs}, \quad Y^{\pm}(Q^2) = 1 \pm (1 - y(Q^2))^2.$$

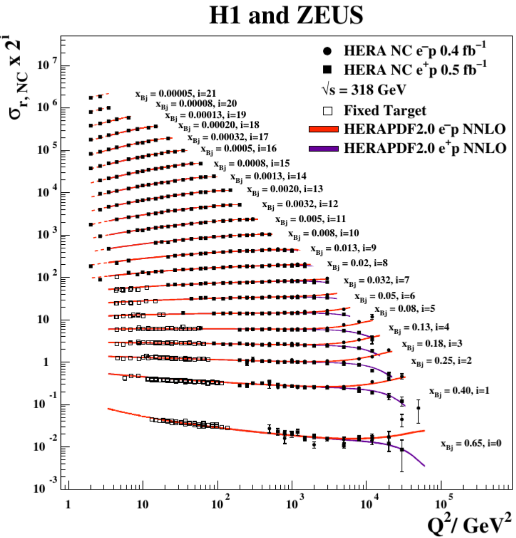
Scattering:

$$\frac{1}{4}\sum_{\text{spin}} |M_{e\mu\to e\mu}|^2 = \frac{8e^4}{t^2}\frac{s^2+u^2}{4} \quad \Rightarrow \quad \frac{\mathrm{d}^2\sigma_{\text{NC}}}{\mathrm{d}x\mathrm{d}Q^2} = \sum_i f_i(x) Q_i^2 \cdot \frac{2\pi\alpha^2}{Q^4} Y^+(Q^2).$$

First look to results

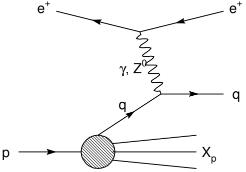
$$\sigma_{r,NC} = \frac{\mathrm{d}^2\sigma_{\mathrm{NC}}}{\mathrm{d}x\mathrm{d}Q^2}\frac{Q^4}{2\pi\alpha^2 Y^+}$$

"Bjorken scaling": $\sigma_{r,NC}$ almost doesn't depend on Q^2.



More sophisticated calculations, NC

Take into account Z-boson $F^{\gamma Z}$, F^{Z}, and longitudinal photons(Z) F_L, they have:



$$\sigma^{\pm}_{r,\mathrm{NC}} = \frac{\mathrm{d}^2\sigma^{e^{\pm}p}_{\mathrm{NC}}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2} \cdot \frac{Q^4 x_{\mathrm{Bj}}}{2\pi\alpha^2 Y_+} = \tilde{F}_2 \mp \frac{Y_-}{Y_+} x\tilde{F}_3 - \frac{y^2}{Y_+}\tilde{F}_{\mathrm{L}}$$

F_2 is main term. F_3 correction for Z boson. F_L is longitudinal virtual exchange ($F_L = 0$ for QPM). All kinematical factors are taken apart for Fs.

$$\begin{aligned}
\tilde{F}_2 &= F_2 - \kappa_Z v_e \cdot F_2^{\gamma Z} + \kappa_Z^2 (v_e^2 + a_e^2) \cdot F_2^Z\,, \\
\tilde{F}_{\mathrm{L}} &= F_{\mathrm{L}} - \kappa_Z v_e \cdot F_{\mathrm{L}}^{\gamma Z} + \kappa_Z^2 (v_e^2 + a_e^2) \cdot F_{\mathrm{L}}^Z\,, \\
x\tilde{F}_3 &= -\kappa_Z a_e \cdot xF_3^{\gamma Z} + \kappa_Z^2 \cdot 2 v_e a_e \cdot xF_3^Z\,,
\end{aligned}$$

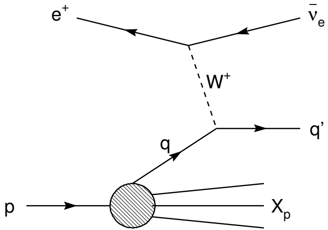
Finally, F_2, F_3 are the functions of PDFs $f(x)$.

$$\begin{aligned}
(F_2, F_2^{\gamma Z}, F_2^Z) &\approx [(e_u^2, 2e_u v_u, v_u^2 + a_u^2)(xU + x\bar{U}) + (e_d^2, 2e_d v_d, v_d^2 + a_d^2)(xD + x\bar{D})]\,, \\
(xF_3^{\gamma Z}, xF_3^Z) &\approx 2[(e_u a_u, v_u a_u)(xU - x\bar{U}) + (e_d a_d, v_d a_d)(xD - x\bar{D})]\,,
\end{aligned}$$

More sophisticated calculations, CC, LO

The kinematics part is different to NC:

$$\sigma_{r,\mathrm{CC}}^{\pm} = \frac{2\pi x_{\mathrm{Bj}}}{G_F^2}\left[\frac{M_W^2 + Q^2}{M_W^2}\right]^2 \frac{\mathrm{d}^2\sigma_{\mathrm{CC}}^{e^\pm p}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2}$$



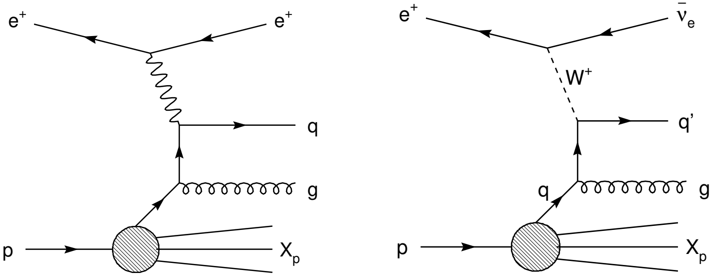
$$\sigma_{r,\mathrm{CC}}^{\pm} = \frac{Y_+}{2} W_2^{\pm} \mp \frac{Y_-}{2} x W_3^{\pm} - \frac{y^2}{2} W_{\mathrm{L}}^{\pm}$$

$$W_2^+ \approx x\bar{U} + xD\,, \qquad xW_3^+ \approx xD - x\bar{U}\,, \qquad W_2^- \approx xU + x\bar{D}\,, \qquad xW_3^- \approx xU - x\bar{D}\,.$$

The CC depend on $y(Q^2)$ even at QPM.

$$\sigma_{r,\mathrm{CC}}^{+} \approx (x\bar{U} + (1-y)^2 xD)\,, \qquad \sigma_{r,\mathrm{CC}}^{-} \approx (xU + (1-y)^2 x\bar{D})\,.$$

Even more sophisticated calculations, NLO



Examples of NLO diagrams of NC and CC. The current order of calculation is NNLO (2015).

DGLAB at QED

Dokshitzer–**Gribov**–**Lipatov**–Altarelli–Parisi

- Some diagrams gives corrections with singularity
- cancellation takes place (collinear photons and infrared divergence) until we use parton distribution functions
- Non-cancelled singularities change DF depending on scale.

$$P_{e\leftarrow e}(z) = \frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z),$$

$$P_{\gamma\leftarrow e}(z) = \frac{1+(1-z)^2}{z},$$

$$P_{e\leftarrow\gamma}(z) = z^2 + (1-z)^2,$$

$$P_{\gamma\leftarrow\gamma}(z) = -\frac{2}{3}\delta(1-z).$$

$$\frac{d}{d\ln Q} f_\gamma(x,Q) = \frac{\alpha}{\pi}\int_x^1 \frac{dz}{z}\left\{P_{\gamma\leftarrow e}(z)\left[f_e(\tfrac{x}{z},Q) + f_{\bar{e}}(\tfrac{x}{z},Q)\right] + P_{\gamma\leftarrow\gamma}(z) f_\gamma(\tfrac{x}{z},Q)\right\},$$

$$\frac{d}{d\ln Q} f_e(x,Q) = \frac{\alpha}{\pi}\int_x^1 \frac{dz}{z}\left\{P_{e\leftarrow e}(z) f_e(\tfrac{x}{z},Q) + P_{e\leftarrow\gamma}(z) f_\gamma(\tfrac{x}{z},Q)\right\},$$

$$\frac{d}{d\ln Q} f_{\bar{e}}(x,Q) = \frac{\alpha}{\pi}\int_x^1 \frac{dz}{z}\left\{P_{e\leftarrow e}(z) f_{\bar{e}}(\tfrac{x}{z},Q) + P_{e\leftarrow\gamma}(z) f_\gamma(\tfrac{x}{z},Q)\right\}.$$

Demonstration of DGLAB at QED

Let's start from scale $Q_0^2 = m_e^2$: no photons, no positrons.

$$f_e(x, Q_0^2) = \delta(1-x), \quad f_{\bar{e}}(x, Q_0^2) = 0, \quad f_\gamma(x, Q_0^2) = 0.$$

Then for any scale Q:

$$\int \mathrm{d}x [f_e(x) - f_{\bar{e}}(s)] = 1, \text{amount of } \bar{e} \text{ is equal to amount of appeared } e$$

$$\int \mathrm{d}x \, x [f_e(x) + f_{\bar{e}}(s) + f_\gamma(s)] = 1, \text{ total momentum is shared by } e, \bar{e}, \gamma.$$

Last equations are called **sum rules**.

DGLAB at QCD

Dokshitzer–Gribov–Lipatov–**Altarelli–Parisi**

$$\frac{d}{d\ln Q} f_g(x,Q) = \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{dz}{z} \Big\{ P_{g\leftarrow q}(z) \sum_f \left[f_f(\tfrac{x}{z},Q) + f_{\bar{f}}(\tfrac{x}{z},Q) \right] + \\ + P_{g\leftarrow g}(z) f_g(\tfrac{x}{z},Q) \Big\},$$

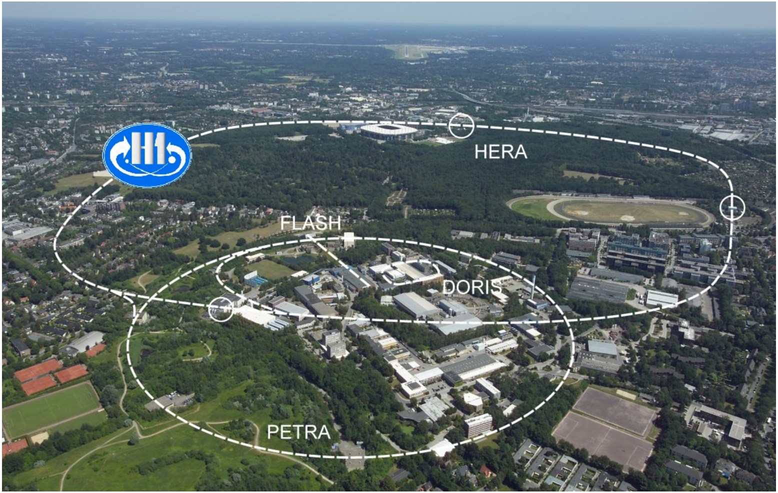
$$\frac{d}{d\ln Q} f_f(x,Q) = \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{dz}{z} \Big\{ P_{q\leftarrow q}(z) f_f(\tfrac{x}{z},Q) + P_{q\leftarrow g}(z) f_g(\tfrac{x}{z},Q) \Big\},$$

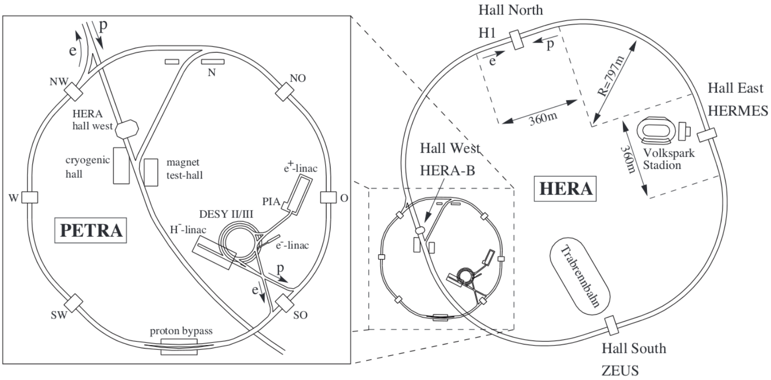
$$\frac{d}{d\ln Q} f_{\bar{f}}(x,Q) = \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{dz}{z} \Big\{ P_{q\leftarrow q}(z) f_{\bar{f}}(\tfrac{x}{z},Q) + P_{q\leftarrow g}(z) f_g(\tfrac{x}{z},Q) \Big\}.$$

$f_g(x, Q^2)$ is gluon DF, $f_f(x, Q^2)$ is parton DF.

Experiments

HERA is only existing electron/proton ring collider.



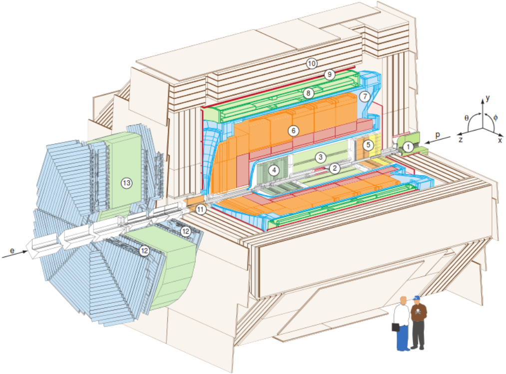


- 6.3 km, 25 m uder the ground.
- Four collision points (H1, ZEUS, HERMES, HERA-B).
- proton energy is 820(920) GeV for HERA-I (HERA-II) stages
- electron energy is 27.6 GeV.

H1: General view

Main parts:

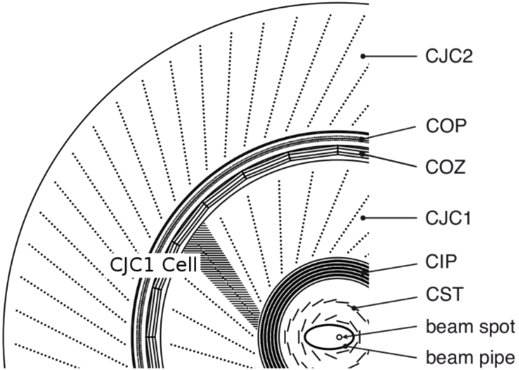
- Central Tracking Detector (CTD, $[15, 165]^\circ$).
- Forward traching detector (FTD, $[5, 25]^\circ$).
- Backward proportioanl chamber.
- Silicon Trackers



1. Beam pipe and beam magnets
2. Silicon tracking detector
3. Central tracking detector
4. Forward tracking detector
5. Spacal calorimeter (em and had)
6. Liquid Argon calorimeter (em and had)
7. Liquid Argon cryostat
8. Superconductiong coil
9. Muon chambers
10. Instrumented iron (streamer tube detectors)
11. Plug calorimeter
12. Forward muon detector
13. Muon toroid magnet

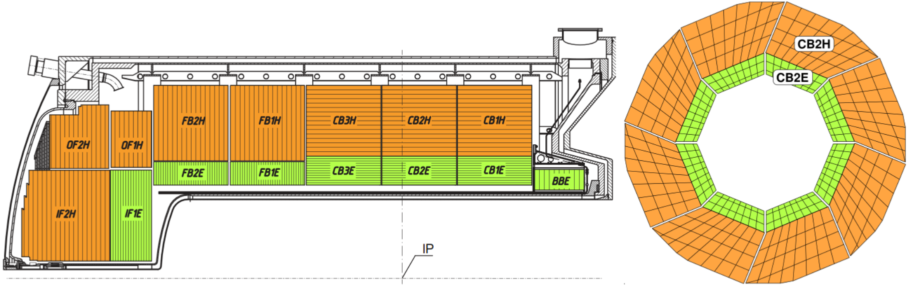
H1: Central Tracking Detector

- central silicon trackers (CST, 5 cm to IP),
- two drift chanbers CJC1, 2 (720, 1920 gold wires, ethane + argon gas mixture), 8 bit flash-ADC, 104 MHz.
- two thin proportional chambers (CIP, COP - triggers)
- thin drift chamber COZ(wires perpendicular to beam axis)



The interaction point is measures with precision 12 μm$\times$22 μm ($r\phi \times z$).

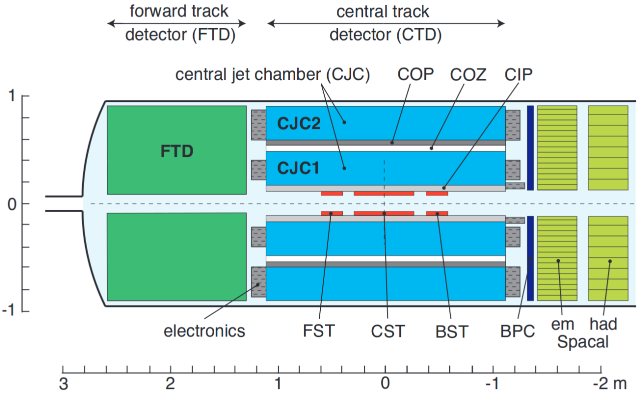
Liquid Argon Calorimeter (LAr)



- non-compensating calorimeter (but NN)
- 44000 cells inside the cryostat, active matter samples with lead (stainless steel).
- inner (outer) layers serve as electromagnetic (hadronic) parts.
- $\sigma_{\text{em}}(E)/E = 11\%/\sqrt{E} \oplus 0.15\,\text{GeV}/E \oplus 0.6\%$.
- $\sigma_{\text{had}}(E)/E = 55\%/\sqrt{E} \oplus E \oplus 1.6\%$.

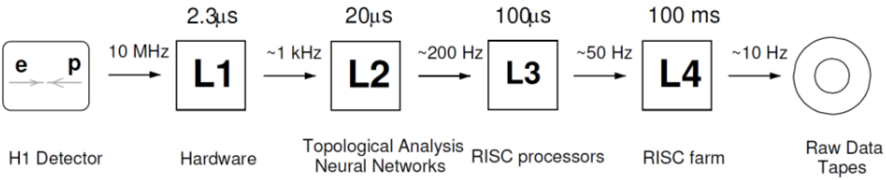
Spaghetti Calorimeter (SpaCla)

is a lead/scintillating-fiber calorimeter (cover $[153, 177]^{\circ}$). Aimed for presice measurement of scattered electron.

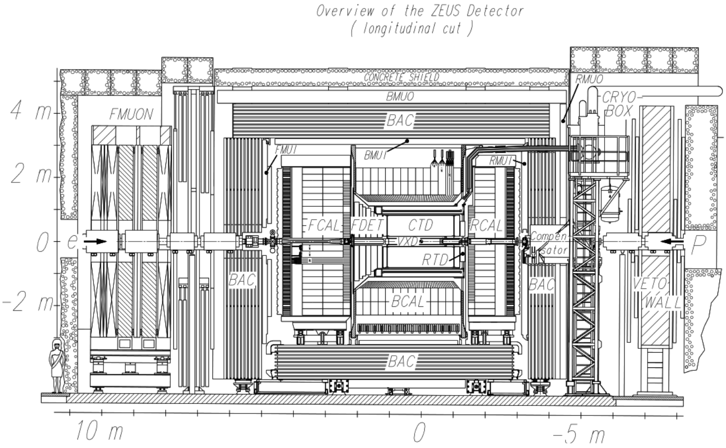


- Just fibers (0.5 cm) embedded into lead matrix guiding the light to PMT,
- $\sigma_{\text{em}}(E)/E = 7\%/\sqrt{E} \oplus 1\%$.

trigger


2.3µs
20µs
100µs
100 ms
e
p
10 MHz
L1
~1 kHz
L2
~200 Hz
L3
~50 Hz
L4
~10 Hz
H1 Detector
Hardware
Topological Analysis
Neural Networks
RISC processors
RISC farm
Raw Data
Tapes

ZEUS: General view



- 10 $\times$ 12 $\times$ 19 m, 3600 tons
- Central part: superconductive solenoid magnet (1.4 T), CTD (DC).
- Forward detector:
- Calorimetry: uranium scintilator calorimeter + BAC (after absorber),
- muon system.

- E-method (only electron is measured)

$$y_e = 1 - \frac{E'_e(1-\cos\Theta_e)}{2E_e}, \quad Q^2_e = \frac{P^2_{T,e}}{1-y_e}, \quad x_e = \frac{Q^2_e}{sy_e}.$$

- Σ-method (all hadrons are measured)

$$y_h = \frac{\sum_i (E_i - p_{z,i})}{2E_e}, \quad Q^2_h = \frac{|P_{T,h}|^2}{1-y_e}, \quad x_h = \frac{Q^2_h}{sy_h}.$$

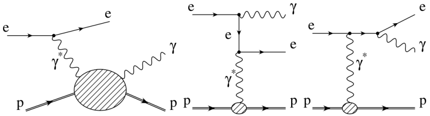
- Sigma method,
- $E-\Sigma$ method (Q^2_e, x_σ),
- Extendend $E-\Sigma$ method (takes radiative corrections at electron vertex),
- Double angle method (insensitive to hadronisation).

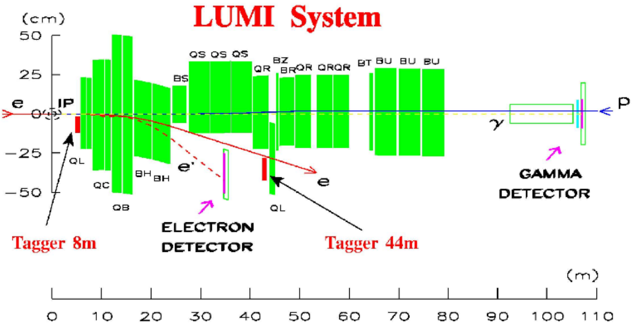
Table with data and used methods

Data Set		x Range		Q^2 Range GeV^2		$\mathcal{L}$ pb^{-1}	e^+/e^-	$\sqrt{s}$ GeV	x,Q^2 Reconstruction Method Equation	Reference
H1 svx-mb	95-00	5×10^{-6}	0.02	0.2	12	2.1	e^+p	301-319	10,14,16	[1]
H1 low Q^2	96-00	2×10^{-4}	0.1	12	150	22	e^+p	301-319	10,14,16	[2]
H1 NC	94-97	0.0032	0.65	150	30000	35.6	e^+p	301	15	[3]
H1 CC	94-97	0.013	0.40	300	15000	35.6	e^+p	301	11	[3]
H1 NC	98-99	0.0032	0.65	150	30000	16.4	e^-p	319	15	[4]
H1 CC	98-99	0.013	0.40	300	15000	16.4	e^-p	319	11	[4]
H1 NC HY	98-99	0.0013	0.01	100	800	16.4	e^-p	319	10	[5]
H1 NC	99-00	0.0013	0.65	100	30000	65.2	e^+p	319	15	[5]
H1 CC	99-00	0.013	0.40	300	15000	65.2	e^+p	319	11	[5]
ZEUS BPC	95	2×10^{-6}	6×10^{-5}	0.11	0.65	1.65	e^+p	301	10	[6]
ZEUS BPT	97	6×10^{-7}	0.001	0.045	0.65	3.9	e^+p	301	10, 15	[7]
ZEUS SVX	95	1.2×10^{-5}	0.0019	0.6	17	0.2	e^+p	301	10	[8]
ZEUS NC	96-97	6×10^{-5}	0.65	2.7	30000	30.0	e^+p	301	18	[9]
ZEUS CC	94-97	0.015	0.42	280	17000	47.7	e^+p	301	11	[10]
ZEUS NC	98-99	0.005	0.65	200	30000	15.9	e^-p	319	17	[11]
ZEUS CC	98-99	0.015	0.42	280	30000	16.4	e^-p	319	11	[12]
ZEUS NC	99-00	0.005	0.65	200	30000	63.2	e^+p	319	17	[13]
ZEUS CC	99-00	0.008	0.42	280	17000	60.9	e^+p	319	11	[14]

Determination of luminosity

- Bethe-Heitler scattering (small angle, 100 *mu*b)
- QED Compton scattering (wide angles, 50 b)





Photon taggers 100 m down the *e*-line

Methods of combining

$$\chi^2_{\text{exp}}(\boldsymbol{m},\boldsymbol{b}) = \sum_i \frac{\left[m^i - \sum_j \gamma^i_j m^i b_j - \mu^i\right]^2}{\delta^2_{i,\text{stat}}\,\mu^i\left(m^i - \sum_j \gamma^i_j m^i b_j\right) + \left(\delta_{i,\text{uncor}}\,m^i\right)^2} + \sum_j b_j^2 \,.$$

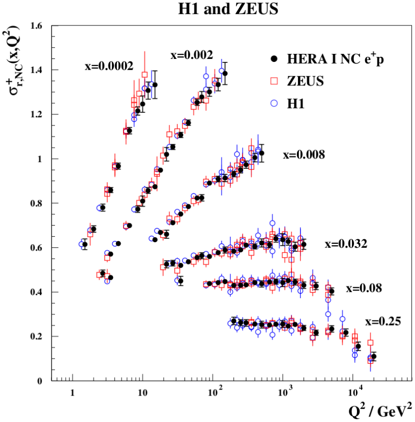
μ_i is the measured value, γ_i^j relative corrected systematic uncertenty, m_i is presicted value, $\delta_{i,\text{stat}}$, $\delta_{i,\text{uncor}}$ is statistical and uncorrected systematic uncertenties, b_j are the shift of correlated systematic error sources.

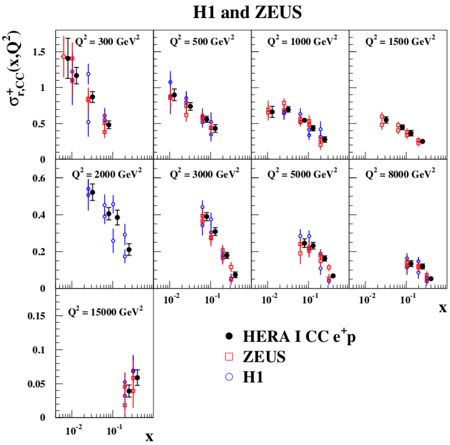
The sources of common systematic uncertinties for ZEUS and H1

- **theoretical uncertinties to BH cross section**
- photoproduction background,
- hadron energy scale,
- other minor 10 has been studied.

Combined results

Interpolation to the common grid (x, Q^2) is done multiplying the measured value to theoretically calculated $\mathrm{d}^2\sigma_{\mathrm{th}}/\mathrm{d}x\,\mathrm{d}Q^2$ (iterative procedure).





QCD analysis

Flaxible parametrisation of PDF at scale $Q_0^2 = 1.9\,\mathrm{GeV}^2$,

$$xf(x) = Ax^B(1-x)^C(1+\epsilon\sqrt{x}+Dx+Ex^2).$$

$$\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g}, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1+E_{u_v}x^2\right), \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}, \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}$$

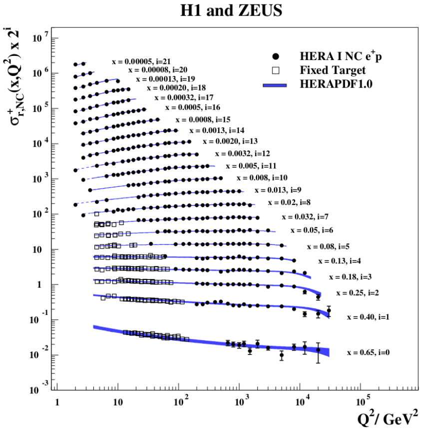
	A	B	C	E
xg	6.8	0.22	9.0	
xu_v	3.7	0.67	4.7	9.7
xd_v	2.2	0.67	4.3	
$x\bar{U}$	0.113	−0.165	2.6	
$x\bar{D}$	0.163	−0.165	2.4	

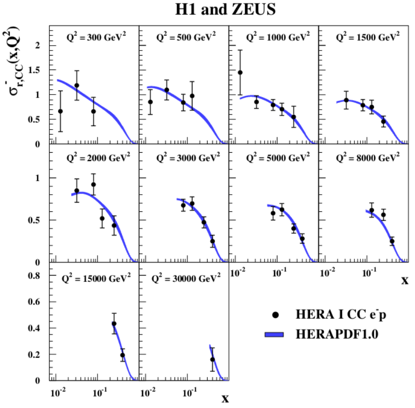
Noticable

- No heavy quarks at Q_0^2 scale (u, d, s only).
- Constant relative contribution of s quark $x\bar{s} = f_s x\bar{D}$.
- Heavy quarks appear from DGLAP at higher scale (QCDNUM).
- Constrains: normalisation, $B_{\bar{U}} = B_{\bar{D}}$, $xd_v > x\bar{d}$.

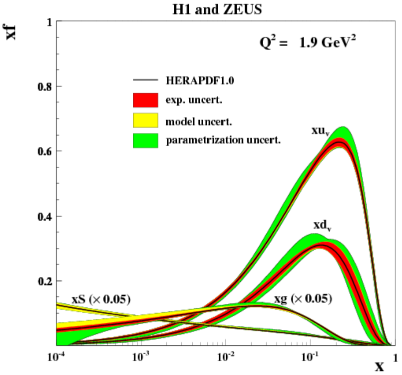
Global fit

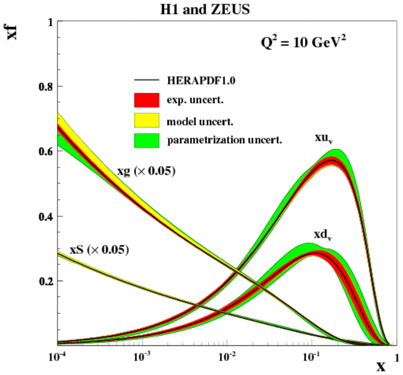
- hessian method, offset method, quadrature combination.



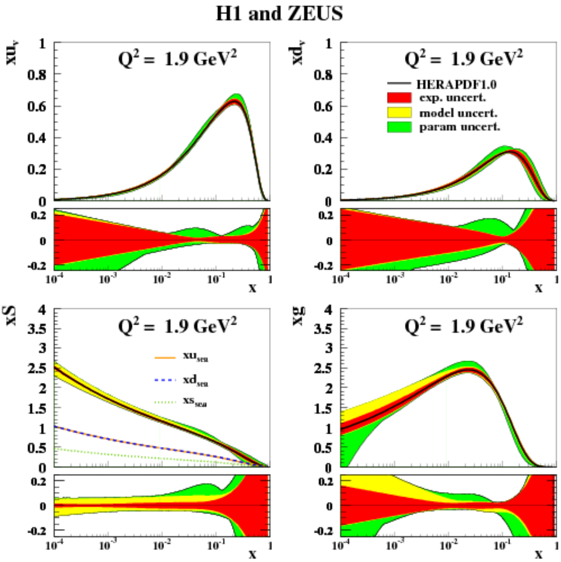


Parton distribution functions

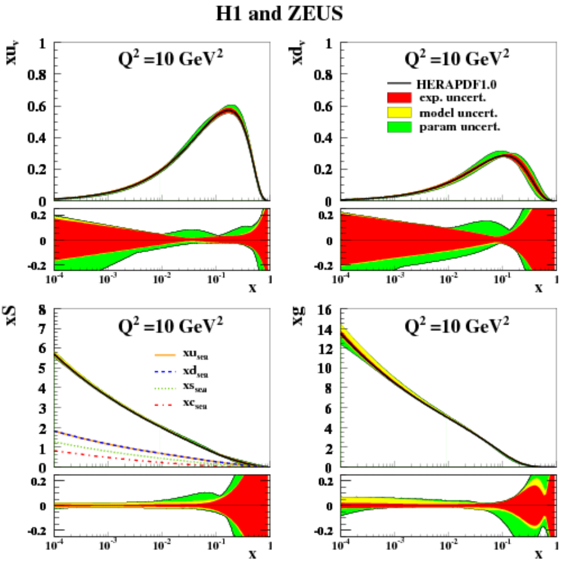




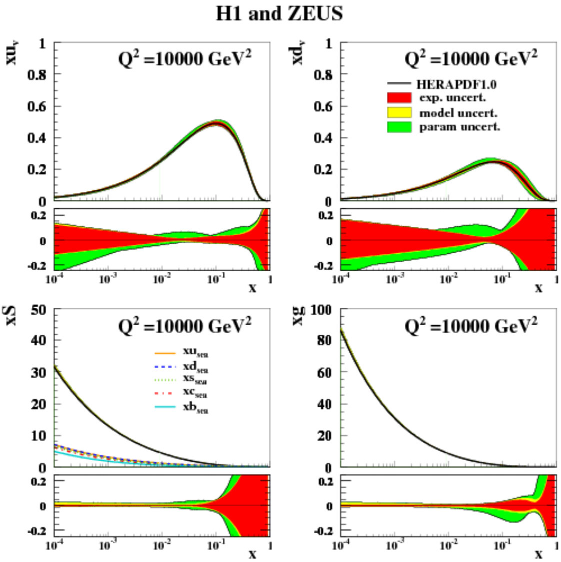
Evolution: $Q^2 = 1.9\,\mathrm{GeV}^2$



Evolution: $Q^2 = 10\,\mathrm{GeV}^2$



Evolution: $Q^2 = 10000\,\text{GeV}^2$



Update 2015, HERAPDF 2.0

- NNLO

Conclusion

- H1 and ZEUS experiments collected $1\,\text{fb}^{-1}$ at period 1994-2008 years.
- Two different configuration HERAI($E_p = 820\,\text{GeV}$) and HERAII ($E_p = 920\,\text{GeV}$),
- Wide kinematics region were covered $6 \cdot 10^{-6} < x < 0.65$, $0.045 < Q^2 < 30000\,\text{GeV}^2$.
- NLO, NNLO analysis has been performed, the new set to parton distribution functions is calculated.

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