The (super)conformal bootstrap program

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DESY Theory Fellows Workshop

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Conformal Field Theories

Conformal Group

- $P_\mu \, , \qquad M_{\mu\nu} \, ,$

Conformal Field Theories

Conformal Group

- $P_\mu\,, \qquad M_{\mu\nu}\,, \qquad D\,,$

Conformal Field Theories

Conformal Group

- $P_\mu\,, \qquad M_{\mu\nu}\,, \qquad D\,, \qquad K_\mu$

Conformal Field Theories

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- $P_\mu\,, \qquad M_{\mu\nu}\,, \qquad D\,, \qquad K_\mu$

Conformal field theory defined by

Set of local operators $\mathcal{O}$ and their correlation functions

Conformal Field Theories

Conformal Group

- $P_\mu\,, \qquad M_{\mu\nu}\,, \qquad D\,, \qquad K_\mu$

Conformal field theory defined by

Set of local operators $\mathcal{O}$ and their correlation functions

- Conformal primaries
  $[K_\mu, \mathcal{O}(0)] = 0$
- Conformal Descendant
  $[P_{\mu_1}, \ldots [P_{\mu_n}, \mathcal{O}(0)]] = 0$

Conformal Bootstrap

Operator Product Expansion

$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x,\partial)\mathcal{O}_k(0)$

Conformal Bootstrap

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$\rightarrow$ Finite radius of convergence

Conformal Bootstrap

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- $\rightarrow$ Finite radius of convergence
- $\rightarrow$ $n-$point function by recursive use of the OPE until $\langle \mathbb{1} \rangle = 1$

Conformal Bootstrap

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CFT data

$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\}$, and

Conformal Bootstrap

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Conformal Bootstrap

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CFT data

$\{\mathcal{O}_{\Delta,\ell,\ldots}(x)\}$, and $\{\lambda_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$

CFT data strongly constrained

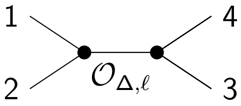
- Unitarity
- Associativity of the operator product algebra

Conformal Bootstrap

Crossing Symmetry

$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle =$

$$\sum_{\mathcal{O}_{\Delta,\ell}}$$



Conformal Bootstrap

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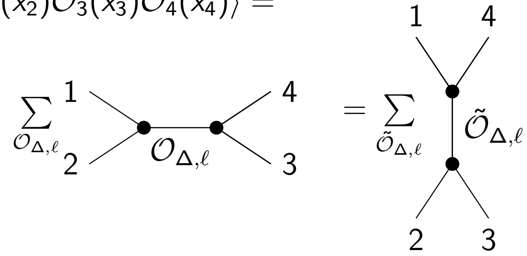

$\rightarrow$ Solve crossing equations for *all* four-point functions

Conformal Bootstrap

- "Solve" theory from crossing symmetry
  [Polyakov '74]

Conformal Bootstrap

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- Success story in $2d$

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[Rattazzi, Rychkov, Tonni, Vichi ’08]

- Solving $\Rightarrow$ constraining

Conformal Bootstrap

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- Solving $\Rightarrow$ constraining
  $\rightarrow$ Guess for the spectrum

Conformal Bootstrap

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- Success story in $2d$
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[Rattazzi, Rychkov, Tonni, Vichi '08]

- Solving $\Rightarrow$ constraining
  - $\rightarrow$ Guess for the spectrum
  - $\rightarrow$ Can it ever define a consistent CFT?

The Superconformal Bootstrap

Add Supersymmetry

$\rightarrow$ Conformal families organized in superconformal families

The Superconformal Bootstrap

Add Supersymmetry

- $\rightarrow$ Conformal families organized in superconformal families
- $\rightarrow$ For $\mathcal{N} \geqslant 2$ in $4d$, or $\mathcal{N} = (2,0)$ in $6d$

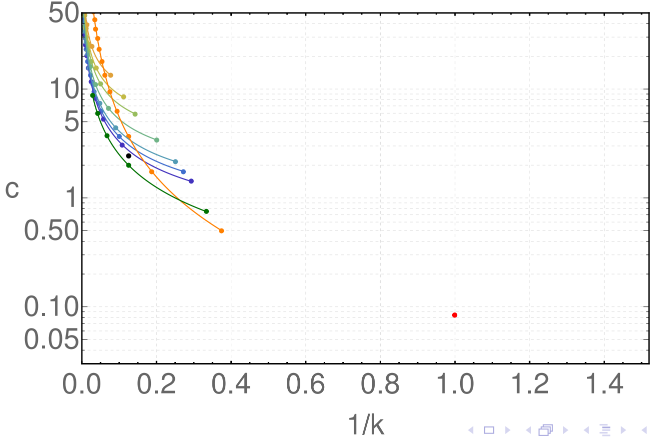
  There is a solvable subsector of the crossing equations

  [Beem, ML, Liendo, Peelaers, Rastelli, van Rees '13]

  [Beem, Rastelli, van Rees '14]

What is the space of consistent SCFTs?

$4d$ $\mathcal{N} = 2$ **SCFTs with** $SU(2)$ **flavor symmetry**



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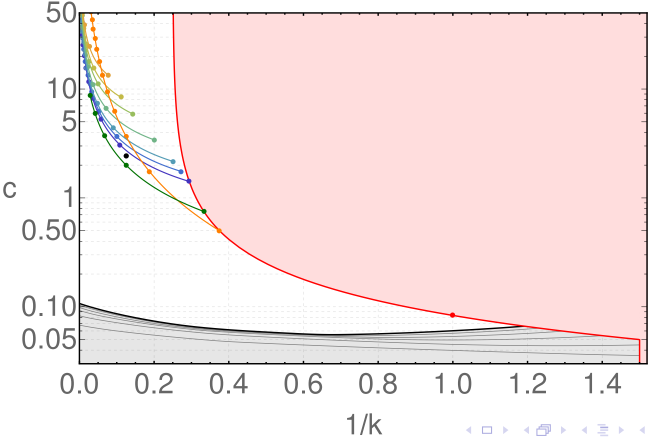
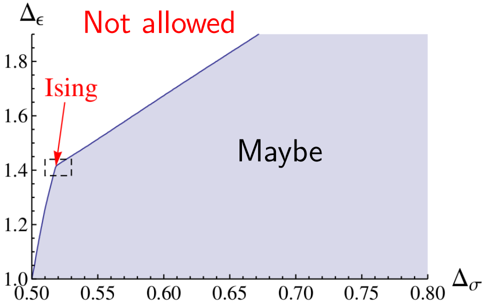


Solving the $3d$ Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



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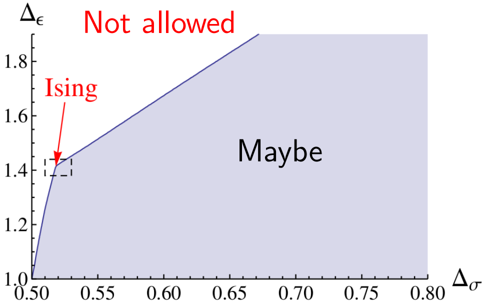
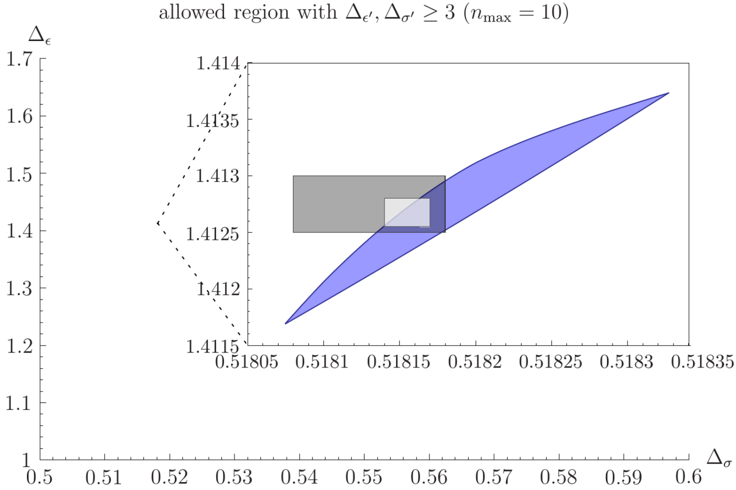


$\rightarrow$ $3d$ Ising lives at “kink”

$3d$ **Ising Model**

[Kos, Poland, Simmons-Duffin 1406.4858]



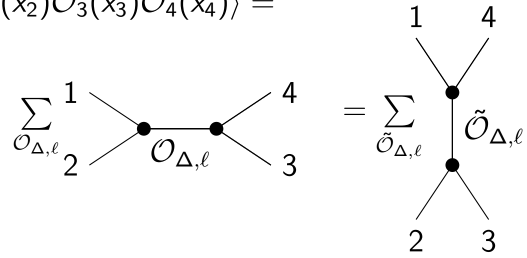
Thank you!

Backup slides

Conformal Bootstrap

Crossing Symmetry

$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle =$



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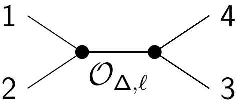
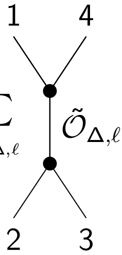


$= \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}}$



$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_3\mathcal{O}_4\mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(u,v) =$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

Conformal Bootstrap

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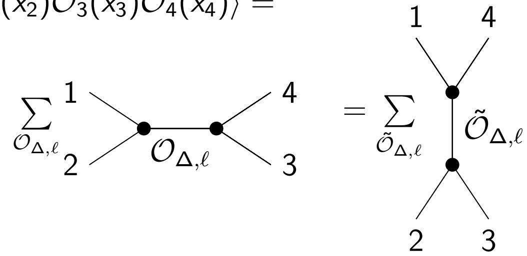

$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}}x_{34}^{2\Delta_{\mathcal{O}}}}\sum_{\mathcal{O}_{\Delta,\ell}}\lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_{\Delta,\ell}}\lambda_{\mathcal{O}_3\mathcal{O}_4\mathcal{O}_{\Delta,\ell}}g_{\Delta,\ell}(u,v) =$$
$$\frac{1}{x_{14}^{2\Delta_{\mathcal{O}}}x_{23}^{2\Delta_{\mathcal{O}}}}\sum_{\tilde{\mathcal{O}}_{\Delta,\ell}}\lambda_{\mathcal{O}_1\mathcal{O}_4\tilde{\mathcal{O}}_{\Delta,\ell}}\lambda_{\mathcal{O}_2\mathcal{O}_3\tilde{\mathcal{O}}_{\Delta,\ell}}g_{\Delta,\ell}(v,u)$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2x_{14}^2}{x_{13}^2x_{24}^2} = (1-z)(1-\bar{z})$

Conformal Bootstrap

Sum rule: identical scalars ϕ

$\rightarrow$ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

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$$1 = \sum_{\substack{\mathcal{O}_{\Delta_\ell} \neq \mathbb{1} \\ \mathcal{O} \in \phi\phi}} \lambda^2_{\phi\phi\mathcal{O}} \underbrace{\frac{u^{\Delta_\phi} g_{\Delta,\ell}(v,u) - v^{\Delta_\phi} g_{\Delta,\ell}(u,v)}{v^{\Delta_\phi} - u^{\Delta_\phi}}}_{F_{\Delta,\ell}}$$

$\rightarrow$ Guess for the spectrum

$\rightarrow$ Can it ever define a consistent CFT?

Conformal Bootstrap

Sum rule: identical scalars ϕ

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→ Guess for the spectrum

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- Find Functional Ψ such that
  - ↪ $\psi \cdot 1 < 0$ ($\mathbb{1}$)
  - ↪ $\psi \cdot F_{\Delta,\ell}(u,v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum

Conformal Bootstrap

Sum rule: identical scalars ϕ

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→ Spectrum is inconsistent ⇒ rule out CFT

Conformal Bootstrap

Sum rule

- Truncate

$$\psi = \sum_{m,n}^{m,n \leqslant \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n |_{z=\bar{z}=\frac{1}{2}}$$

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$\rightarrow$ Increase $\Lambda \Rightarrow$ bounds get stronger

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$\rightarrow$ Always true bounds