

The (super)conformal bootstrap program

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DESY Theory Fellows Workshop

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Conformal Field Theories

Conformal Group

► P_μ , $M_{\mu\nu}$,

Conformal Field Theories

Conformal Group

► P_μ , $M_{\mu\nu}$, D ,

Conformal Field Theories

Conformal Group

► P_μ , $M_{\mu\nu}$, D , K_μ

Conformal Field Theories

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► P_μ , $M_{\mu\nu}$, D , K_μ

Conformal field theory defined by

Set of local operators \mathcal{O} and their correlation functions

Conformal Field Theories

Conformal Group

► P_μ , $M_{\mu\nu}$, D , K_μ

Conformal field theory defined by

Set of local operators \mathcal{O} and their correlation functions

- Conformal primaries

$$[K_\mu, \mathcal{O}(0)] = 0$$

- Conformal Descendant

$$[P_{\mu_1}, \dots [P_{\mu_n}, \mathcal{O}(0)]] = 0$$

Conformal Bootstrap

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial) \mathcal{O}_k(0)$$

Conformal Bootstrap

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial) \mathcal{O}_k(0)$$

→ Finite radius of convergence

Conformal Bootstrap

Operator Product Expansion

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- Finite radius of convergence
- n -point function by recursive use of the OPE until $\langle \mathbb{1} \rangle = 1$

Conformal Bootstrap

Operator Product Expansion

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CFT data

$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\}$, and

Conformal Bootstrap

Operator Product Expansion

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- Finite radius of convergence
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CFT data

$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\}$, and $\{\lambda_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$

Conformal Bootstrap

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial) \mathcal{O}_k(0)$$

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CFT data

$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\}$, and $\{\lambda_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$

CFT data strongly constrained

- ▶ Unitarity
- ▶ Associativity of the operator product algebra

Conformal Bootstrap

Crossing Symmetry

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle =$$

$$\sum_{\mathcal{O}_{\Delta,\ell}} \begin{array}{c} 1 \\ \diagdown \\ \bullet \\ \diagup \\ 2 \end{array} \begin{array}{c} \text{---} \mathcal{O}_{\Delta,\ell} \text{---} \\ \bullet \end{array} \begin{array}{c} \diagup \\ 4 \\ \diagdown \\ 3 \end{array}$$

Conformal Bootstrap

Crossing Symmetry

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle =$$

The diagram shows the crossing symmetry of a four-point function. On the left, a sum over $\mathcal{O}_{\Delta,\ell}$ of a diagram with two vertices connected by a horizontal line. The left vertex has incoming lines 1 and 2, and the right vertex has outgoing lines 4 and 3. The horizontal line is labeled $\mathcal{O}_{\Delta,\ell}$. This is equal to a sum over $\tilde{\mathcal{O}}_{\Delta,\ell}$ of a diagram with two vertices connected by a vertical line. The top vertex has incoming lines 1 and 4, and the bottom vertex has outgoing lines 2 and 3. The vertical line is labeled $\tilde{\mathcal{O}}_{\Delta,\ell}$.

→ Solve crossing equations for *all* four-point functions

Conformal Bootstrap

- ▶ “Solve” theory from crossing symmetry
[Polyakov '74]

Conformal Bootstrap

- ▶ “Solve” theory from crossing symmetry
[Polyakov '74]
- ▶ Success story in $2d$

Conformal Bootstrap

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- ▶ Harder in $d > 2$

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[Rattazzi, Rychkov, Tonni, Vichi '08]

- ▶ Solving \Rightarrow constraining

Conformal Bootstrap

- ▶ “Solve” theory from crossing symmetry
[Polyakov '74]
- ▶ Success story in $2d$
- ▶ Harder in $d > 2$

[Rattazzi, Rychkov, Tonni, Vichi '08]

- ▶ Solving \Rightarrow constraining
→ Guess for the spectrum

Conformal Bootstrap

- ▶ “Solve” theory from crossing symmetry
[Polyakov '74]
- ▶ Success story in $2d$
- ▶ Harder in $d > 2$

[Rattazzi, Rychkov, Tonni, Vichi '08]

- ▶ Solving \Rightarrow constraining
 - \rightarrow Guess for the spectrum
 - \rightarrow Can it ever define a consistent CFT?

The Superconformal Bootstrap

Add Supersymmetry

→ Conformal families organized in superconformal families

The Superconformal Bootstrap

Add Supersymmetry

- Conformal families organized in superconformal families
- For $\mathcal{N} \geq 2$ in $4d$, or $\mathcal{N} = (2, 0)$ in $6d$

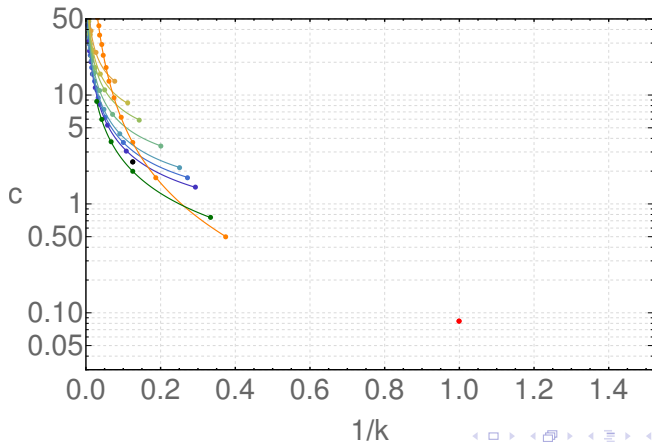
There is a solvable subsector of the crossing equations

[Beem, ML, Liendo, Peelaers, Rastelli, van Rees '13]

[Beem, Rastelli, van Rees '14]

What is the space of consistent SCFTs?

$4d$ $\mathcal{N} = 2$ SCFTs with $SU(2)$ flavor symmetry

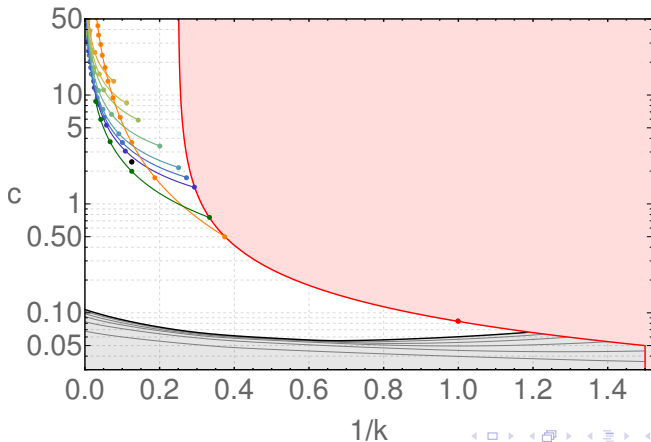


What is the space of consistent SCFTs?

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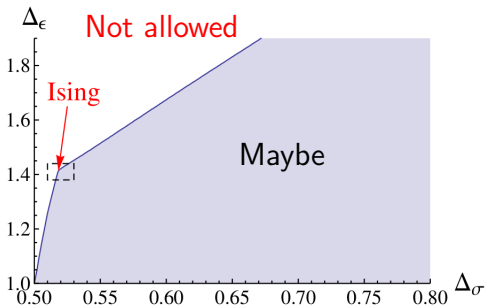
[Beem, ML, Liendo, Peelaers, Rastelli, van Rees '13]

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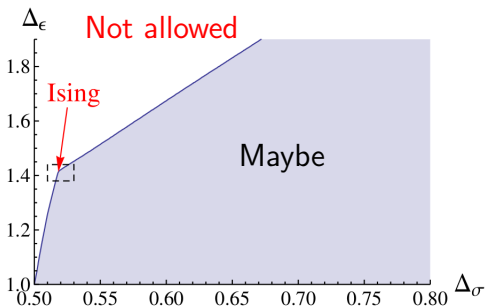
Solving the 3d Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



Solving the 3d Ising Model

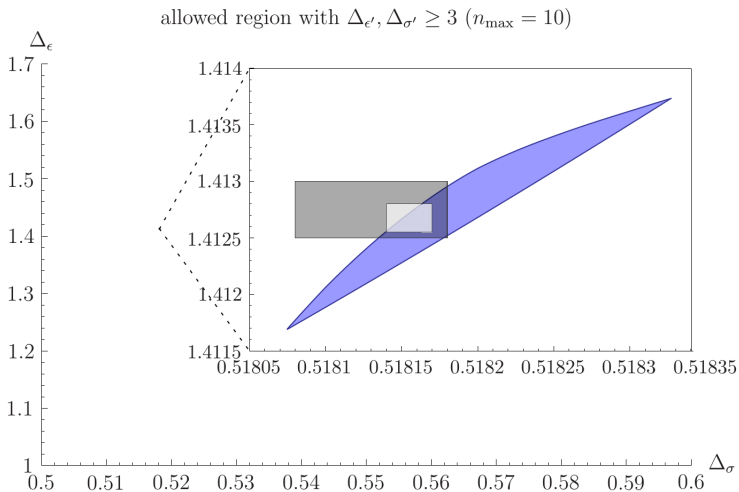
[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



→ 3d Ising lives at “kink”

3d Ising Model

[Kos, Poland, Simmons-Duffin 1406.4858]



Thank you!

Backup slides

Conformal Bootstrap

Crossing Symmetry

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle =$$

The diagrammatic equation shows the crossing symmetry of a four-point correlation function. On the left, a sum over operators $\mathcal{O}_{\Delta,\ell}$ is shown for a process where legs 1 and 2 meet at a vertex, legs 3 and 4 meet at another vertex, and these two vertices are connected by a horizontal line labeled $\mathcal{O}_{\Delta,\ell}$. On the right, the same sum is shown for a process where legs 1 and 4 meet at a top vertex, legs 2 and 3 meet at a bottom vertex, and these two vertices are connected by a vertical line labeled $\tilde{\mathcal{O}}_{\Delta,\ell}$. The two diagrams are set equal to each other.

$$\sum_{\mathcal{O}_{\Delta,\ell}} \begin{array}{c} 1 \\ \diagdown \\ \bullet \\ \diagup \\ 2 \end{array} \begin{array}{c} \text{---} \mathcal{O}_{\Delta,\ell} \text{---} \\ \bullet \end{array} \begin{array}{c} \diagdown \\ 4 \\ \diagup \\ 3 \end{array} = \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \begin{array}{c} 1 \\ \diagdown \\ \bullet \end{array} \begin{array}{c} \text{---} \tilde{\mathcal{O}}_{\Delta,\ell} \text{---} \\ \bullet \end{array} \begin{array}{c} \diagup \\ 4 \\ \diagdown \\ 3 \end{array}$$

Conformal Bootstrap

Crossing Symmetry

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle =$$

$$\sum_{\mathcal{O}_{\Delta,\ell}} \begin{array}{c} 1 \\ \diagdown \\ \bullet \\ \diagup \\ 2 \end{array} \begin{array}{c} \text{---} \mathcal{O}_{\Delta,\ell} \text{---} \\ \bullet \end{array} \begin{array}{c} \diagup \\ 4 \\ \diagdown \\ 3 \end{array} = \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \begin{array}{c} 1 \\ \diagdown \\ \bullet \\ \text{---} \tilde{\mathcal{O}}_{\Delta,\ell} \text{---} \\ \bullet \\ \diagup \\ 2 \end{array} \begin{array}{c} 4 \\ \diagdown \\ \bullet \\ \diagup \\ 3 \end{array}$$

$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(u, v) =$$

$$\text{where } \Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}, \quad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

Conformal Bootstrap

Crossing Symmetry

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle =$$

$$\sum_{\mathcal{O}_{\Delta, \ell}} \begin{array}{c} 1 \\ \diagdown \\ \bullet \\ \diagup \\ 2 \end{array} \begin{array}{c} \diagup \\ \bullet \\ \diagdown \\ \mathcal{O}_{\Delta, \ell} \end{array} \begin{array}{c} \diagdown \\ \bullet \\ \diagup \\ 3 \end{array} \begin{array}{c} 4 \\ \diagup \\ \bullet \\ \diagdown \end{array} = \sum_{\tilde{\mathcal{O}}_{\Delta, \ell}} \begin{array}{c} 1 \\ \diagdown \\ \bullet \\ \diagup \\ 2 \end{array} \begin{array}{c} \diagup \\ \bullet \\ \diagdown \\ \tilde{\mathcal{O}}_{\Delta, \ell} \end{array} \begin{array}{c} \diagdown \\ \bullet \\ \diagup \\ 3 \end{array} \begin{array}{c} 4 \\ \diagup \\ \bullet \\ \diagdown \end{array}$$

$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\mathcal{O}_{\Delta, \ell}} \lambda_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_{\Delta, \ell}} \lambda_{\mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_{\Delta, \ell}} g_{\Delta, \ell}(u, v) =$$

$$\frac{1}{x_{14}^{2\Delta_{\mathcal{O}}} x_{23}^{2\Delta_{\mathcal{O}}}} \sum_{\tilde{\mathcal{O}}_{\Delta, \ell}} \lambda_{\mathcal{O}_1 \mathcal{O}_4 \tilde{\mathcal{O}}_{\Delta, \ell}} \lambda_{\mathcal{O}_2 \mathcal{O}_3 \tilde{\mathcal{O}}_{\Delta, \ell}} g_{\Delta, \ell}(v, u)$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

Conformal Bootstrap

Sum rule: identical scalars ϕ

→ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

Conformal Bootstrap

Sum rule: identical scalars ϕ

→ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

$$1 = \sum_{\substack{\mathcal{O}_{\Delta_\ell} \neq \mathbb{1} \\ \mathcal{O} \in \phi\phi}} \lambda_{\phi\phi\mathcal{O}}^2 \underbrace{\frac{u^{\Delta_\phi} g_{\Delta,\ell}(v, u) - v^{\Delta_\phi} g_{\Delta,\ell}(u, v)}{v^{\Delta_\phi} - u^{\Delta_\phi}}}_{F_{\Delta,\ell}}$$

→ Guess for the spectrum

→ Can it ever define a consistent CFT?

Conformal Bootstrap

Sum rule: identical scalars ϕ

→ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

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→ Guess for the spectrum

→ Can it ever define a consistent CFT?

► Find Functional Ψ such that

$$\hookrightarrow \psi \cdot 1 < 0 \ (\mathbb{1})$$

$$\hookrightarrow \psi \cdot F_{\Delta,\ell}(u, v) \geq 0 \text{ for all } \{\Delta, \ell\} \text{ in spectrum}$$

Conformal Bootstrap

Sum rule: identical scalars ϕ

→ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

$$1 = \sum_{\substack{\mathcal{O}_{\Delta,\ell} \neq \mathbb{1} \\ \mathcal{O} \in \phi\phi}} \lambda_{\phi\phi\mathcal{O}}^2 \underbrace{\frac{u^{\Delta_\phi} g_{\Delta,\ell}(v, u) - v^{\Delta_\phi} g_{\Delta,\ell}(u, v)}{v^{\Delta_\phi} - u^{\Delta_\phi}}}_{F_{\Delta,\ell}}$$

→ Guess for the spectrum

→ Can it ever define a consistent CFT?

► Find Functional Ψ such that

$$\hookrightarrow \psi \cdot 1 < 0 \ (\mathbb{1})$$

$$\hookrightarrow \psi \cdot F_{\Delta,\ell}(u, v) \geq 0 \text{ for all } \{\Delta, \ell\} \text{ in spectrum}$$

→ Spectrum is inconsistent \Rightarrow rule out CFT

Conformal Bootstrap

Sum rule

- ▶ Truncate

$$\psi = \sum_{m,n}^{m,n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \Big|_{z=\bar{z}=\frac{1}{2}}$$

Conformal Bootstrap

Sum rule

- ▶ Truncate

$$\psi = \sum_{m,n}^{m,n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \Big|_{z=\bar{z}=\frac{1}{2}}$$

→ Increase $\Lambda \Rightarrow$ bounds get stronger

Conformal Bootstrap

Sum rule

- ▶ Truncate

$$\psi = \sum_{m,n}^{m,n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \Big|_{z=\bar{z}=\frac{1}{2}}$$

- Increase $\Lambda \Rightarrow$ bounds get stronger
- Always true bounds