Integrability & Scattering Amplitudes as a Key to Gauge Theory

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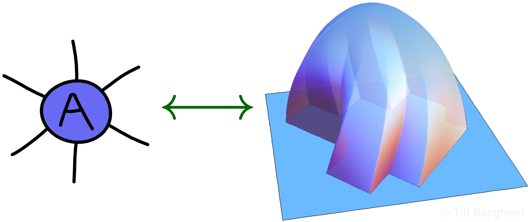
Motivation

Understand features of gauge theory

- At weak coupling
- At strong coupling
- Non-perturbatively

Two attack vectors:

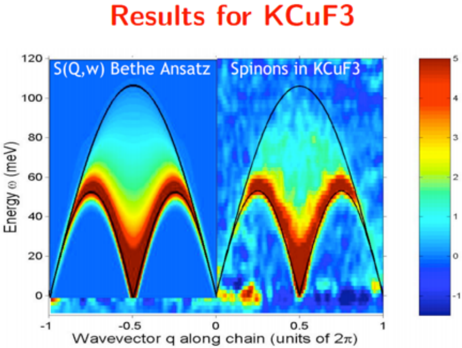
- AdS/CFT and integrability
- Structures in scattering amplitudes



Integrability

A miraculous property of certain models *in two dimensions*

Occurs for very specific systems, but sometimes really models reality:



Integrability in a Nutshell

In two dimensions:

$$\text{If} \quad Q_1 = \sum_j p_j\,, \quad Q_2 = \sum_j p_j^2 \quad \text{are conserved}$$

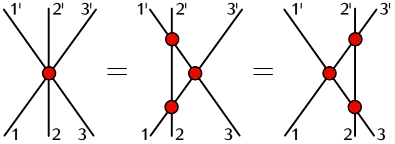
$$\Longrightarrow \quad \text{two momenta conserved:} \quad \{p_1, p_2\} = \{p'_1, p'_2\}$$

Further conserved charge:

$$Q_3 = \sum_j p_j^3 \quad \Longrightarrow \quad \{p_1, p_2, p_3\} = \{p'_1, p'_2, p'_3\}$$

Higher conserved charges $\Longrightarrow$ factorized scattering, $\{p_i\} = \{p'_i\}$

S-matrix obeys **Yang–Baxter equation**:



$\Rightarrow$ In two dimensions, an interacting theory can be **integrable** (solvable)

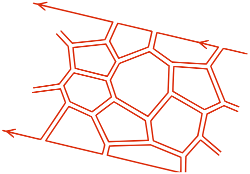
Gauge Theory

Simplest gauge theory: $\mathcal{N}=4$ super Yang–Mills theory in $d=4$

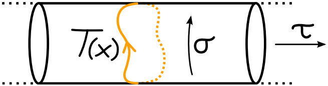
- Cancellations (susy), conformal symmetry, finiteness
- AdS/CFT $\longrightarrow$ string theory at strong coupling
- Planar limit: Integrability $\longrightarrow$ powerful techniques. Solvable?!

Why is planar $\mathcal{N}=4$ SYM integrable?

- Planar limit: Leading piece in t'Hooft large N_c expansion
- Two-dimensionality in color space



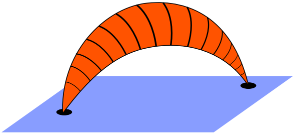
AdS/CFT: Dual to string theory in AdS
Worldsheet is two-dimensional
Family of flat connections
Higher charges: Holonomy $T(x)=\sum_j x^j Q_j$



→ Integrable charges form an infinite-dimensional **Yangian algebra**

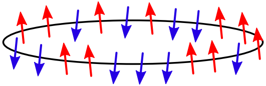
$\mathcal{N} = 4$ SYM: The Spectrum

AdS/CFT picture
for two-point functions:

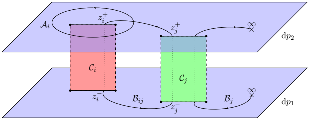


Spin chain of
single-trace operators:

$$\mathrm{Tr}(ZZ\phi Z \ldots Z\phi Z\phi) \sim$$



Large volume:
Weak coupling: Bethe ansatz
Strong coupling: Spectral curve

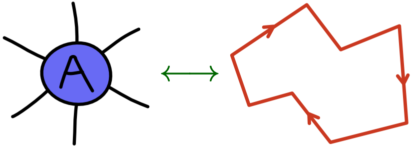


Finite-size corrections: Wrapping

Finite size, exact:
Thermodynamic Bethe Ansatz, FinLIE, Quantum spectral curve

$\Rightarrow$ Anomalous dimensions can now be computed (numerically)
for any operator at finite coupling

Scattering Amplitudes



Zoo of interesting structures

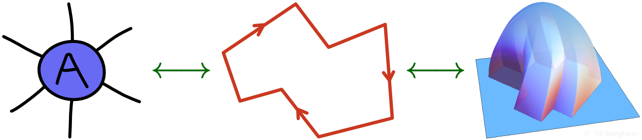
- Duality to polygon Wilson loops
- Dual superconformal symmetry
- Twistors / momentum twistors
- Grassmannian integral / on-shell recursion for trees & integrands
- Differential equations for loop integrals
- Amplitude bootstrap program

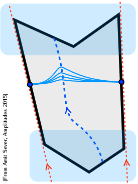
Relation to integrability only gradually uncovered

$\Longrightarrow$ **Room for fruitful interplay**

Integrability for Amplitudes

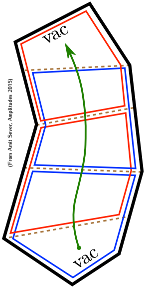
What is the map to 2d integrable system for scattering amplitudes?





Key: Look at Wilson loop as a flux tube

Decomposition: pentagon transition functions



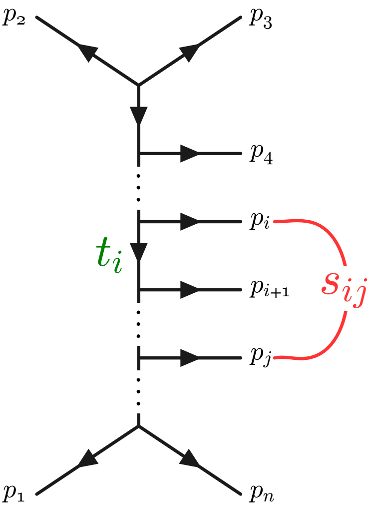
Flux tube states $\leftrightarrow$ large spin operators

$$\mathcal{O} = \mathrm{Tr}(ZDD\ldots DD\overset{p_1}{\overrightarrow{F}}DD\ldots DD\overset{p_2}{\overrightarrow{F}}DD\ldots DDZ)$$

Exact spectrum $E(p_i)$ and scattering phases $S(p_1, p_2)$ from integrability

Proposal for pentagon transitions: $P \sim \sqrt{S(p,q)/S(p,q^\gamma)}$

Physics: Regge Limit & BFKL



- Transverse momenta $\ll$ rapidities
- Hierarchy of rapidities:

$$s = s_{3,n} \gg s_{3,n-1}, s_{4,n} \gg \ldots$$
$$\cdots \gg s_{i,i+2} \gg s_i \gg -t_i$$

- Large logarithms: $\log(s_i) \gg 1$
- Double expansion in $\log(s_i)$ and coupling g_{YM}
- Very generic: Applies to general gauge theory
- Useful in practical computations

Physics: Regge Limit & BFKL

Resum perturbation theory, "Reggeization"

- Large-log approximation, but all orders in g_{YM}
- Schematically,

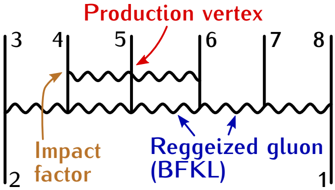
$$R_n \xrightarrow{\mathrm{MRL}} \sum \text{(Regge poles)} + \sum \text{(Regge cuts)}$$

- Sum over $\mathfrak{sl}(2)$ representations (ν, n):

$$\text{(Regge cut)} \sim \sum_{n=-\infty}^{\infty} \int d\nu \, \Phi^*_{\nu,n} \, s_i^{\omega(\nu,n)} \, \Phi_{\nu,n}$$

Ingredients:

- Energies $\omega(\nu, n)$
- Impact factors $\Phi_{\nu,n}$
- Production vertices (at higher points)



Energies $\omega(\nu, n)$ are eigenvalues of
BFKL Hamiltonian of $\mathfrak{sl}(2)$ spin chain $\rightarrow$ **integrable!**

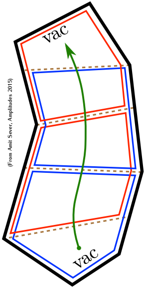
Apply Integrability!

Goal: Extract BFKL data (eigenvalues, impact factors, production vertices) from newly discovered structures in amplitudes;
make use of integrability!

- Six points: understood
  (analytic structure, BFKL data from Wilson loop OPE)
- A lot of structure beyond six points that needs to be understood
  - Analytic structure in different regions
  - BFKL data

Plan:

- *Step 1:* Extract MRL data from known amplitude data
  (from bootstrap: two loops, any n; three loops, $n = 7$)
- *Step 2:* Apply Wilson loop OPE to obtain all-order or even exact BFKL data beyond $n = 6$
- (*Step 3:* Data can be used for practical computations)



(From Amit Sever, Amplitudes 2015)