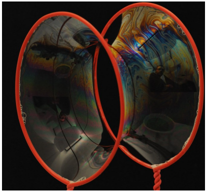
Beyond minimal surfaces: quantum strings for AdS/CFT

Lorenzo Bianchi

Universität Hamburg



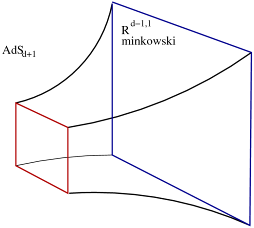
Hamburg
November 10^{th}, 2015

AdS/CFT correspondence

$\mathcal{N} = 4$ Super Yang-Mills.
$SU(N_c)$ gauge theory in 4d
Supersymmetric and conformal

Type *IIB* superstring
in $AdS_5 \times S^5$

$$\lambda = \frac{g^2 N_c}{4\pi}$$



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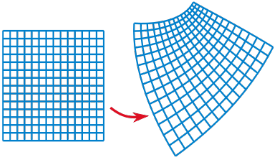

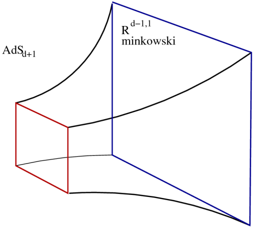

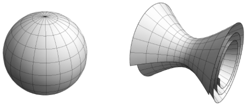

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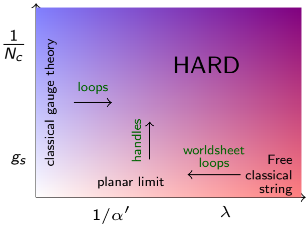

AdS/CFT integrability

$\mathcal{N}=4$ Super Yang-Mills.
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INTEGRABILITY
$(N_c \to \infty)$

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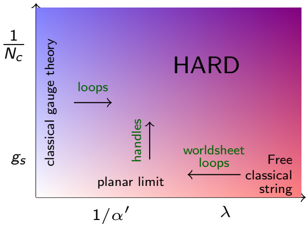

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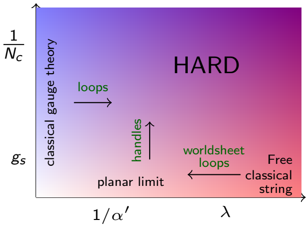

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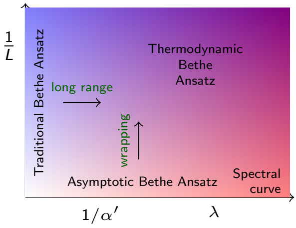
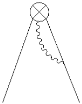


An example: cusp anomalous dimension

Twist-two operators

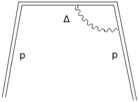
[Korchemsky, 1989; Korchemsky, Marchesini, 1993]



$$\Delta - S \sim 2 + f(\lambda) \log S$$

Wilson lines

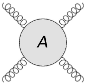
[Polyakov, 1980; Korchemsky, Radyushkin, 1987]



$$\langle W \rangle \sim \left(\frac{L}{\epsilon}\right)^{f(\lambda) \log(p\cdot\Delta)}$$

Scattering amplitudes

[Magnea, Sterman, 1990; Bern, Dixon, Smirnov, 2005]



$$\log \frac{A}{A^{(0)}} = \frac{2\, f(\lambda)}{\epsilon^2} + \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

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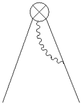

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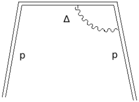

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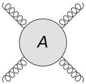

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In $\mathcal{N} = 4$ SYM [Beisert, Eden, Staudacher, 2006]

One can use the Asymptotic Bethe Ansatz to write down an all-loop integral equation (BES equation) for the cusp anomalous dimension $f(\lambda)$.

In $\mathcal{N} = 6$ Super Chern-Simons in 3d (ABJM [Aharony, Bergman, Jafferis, Maldacena, 2008]) [Gromov, Vieira, 2008]

$$f_{\rm ABJM}(\lambda) = \frac{1}{2}\, f_{\mathcal{N}=4}(\lambda_{\rm YM})\Big|_{\frac{\sqrt{\lambda_{\rm YM}}}{4\pi}\to h(\lambda)}$$

Perturbative procedure

How to compute the cusp anomalous dimension at strong coupling?

Step 1: The action [Metsaev, Tseytlin, 1998]

$$S = T \int d\tau d\sigma \gamma^{\alpha\beta} \partial_\alpha X^a \partial_\beta X^b G_{ab}(X) + \text{fermions}$$

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Step 2: The classical solution [Uvarov, 2009; LB, M. Bianchi, A. Bres, V. Forini, E. Vescovi, 2014]

- Poincaré coordinates on $AdS_4 \times \mathbb{CP}^3$

$$ds^2 = \frac{dx^\mu dx_\mu + dw^2}{w^2} + ds^2_{\mathbb{CP}^3} \qquad \mu = 0, 1, 2$$

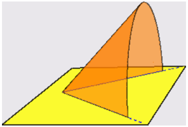
- Minimal surface

$$x^+ = \tau \qquad x^- = -\frac{1}{2\sigma} \qquad x^+ x^- = -\frac{1}{2} w^2$$

- AdS light-cone gauge [Metsaev, Thorn, Tseytlin,2000]

$$x^+ = x^0 + x^2 = \tau$$

$$\langle W_{cusp} \rangle = Z_{string} = e^{-\frac{1}{2} f(\lambda) V}$$



Cusp anomaly at two loops [LB, M. Bianchi, A. Bres, V. Forini, E. Vescovi, 2014]

Step 3: Fluctuations around the classical solution - Asymptotic spectrum

- Bosons: 1 mode $m^2 = 1$; 1 mode $m^2 = 1/2$; 6 modes $m^2 = 0$.
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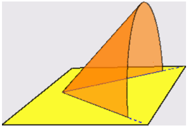

Expand the action up to quartic order in fluctuations and compute all connected 2-loop vacuum Feynman diagrams.

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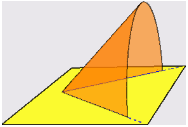

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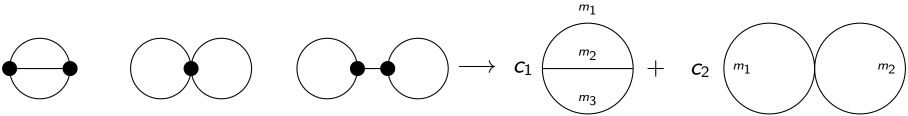
The two-loop result gives strong support to a recent conjecture for $h(\lambda)$ [Gromov, Sizov, 2014].



Quantum dispersion relation [LB, M. Bianchi, 2015]

Asymptotic spectrum

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Non-relativistic theory $\Rightarrow$ Corrections to the dispersion relation

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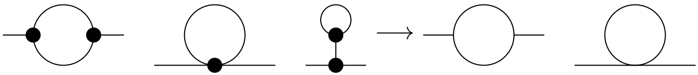
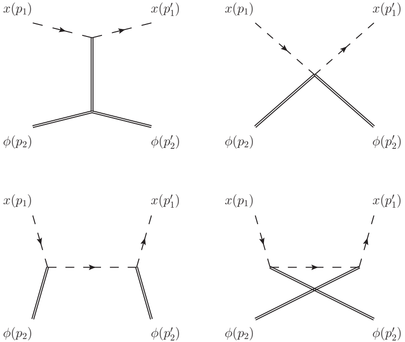


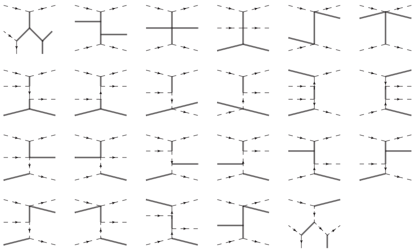
We found agreement with the integrability predictions up to some subtleties.

S-matrix [LB, M.S. Bianchi 2015]

$$\mathbb{S}\ |\Phi_A(p)\Phi_B(p')\rangle = |\Phi_C(p)\Phi_D(p')\rangle\ S_{AB}^{CD}(p,p')$$

Tree-level

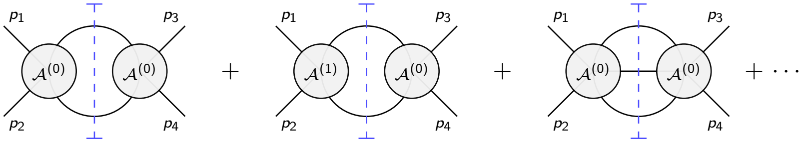




One-loop S-matrix by unitarity [LB, Hoare, Forini, 2013; Engelund, McKeown, Roiban, 2013.]

Standard unitarity in 4d [Bern, Dixon, Dunbar, Kosower, 1994]

$\mathcal{A}|_{\text{cut}} =$



Glue together the two amplitudes and uplift the integral with

$$i\pi\delta^{+}(p^2 - m^2) \to \frac{1}{p^2 - m^2 - i\epsilon}$$

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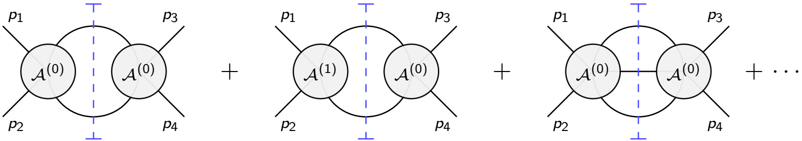

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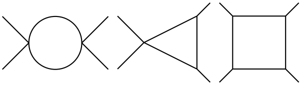
Generalized unitarity in 4d [Bern, Dixon, Kosower, 1998; Britto, Cachazo, Feng, 2004]

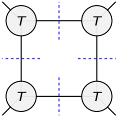
$$\mathcal{A}^L = \sum_i c_i \, \mathcal{I}_i^{(L)}$$

→ Known basis of L-loop scalar integrals

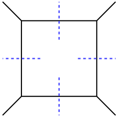
For L=1



$\Rightarrow$



$= c_{\text{box}}$



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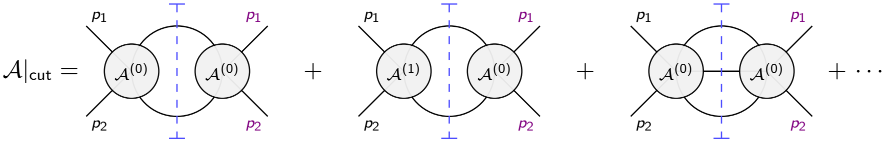

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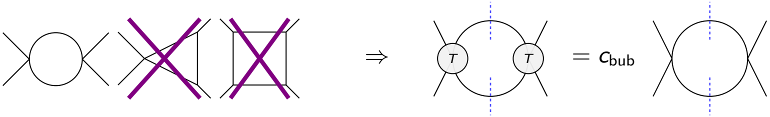

Conclusion

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- Perturbative calculations are essential to give a solid foundation and inspiration to any integrability-based construction, and thus to guarantee its predictivity.
- It is often possible to improve current perturbative techniques.
- We developed a technique to perform perturbative computations in 1+1 dimensions at one loop and applied it also to off-shell quantities.
- Higher loop computations often reveal subtleties regarding regularization, with impact on the quantum integrability of the theory.