Transverse Momentum Dependent Parton Distribution Functions

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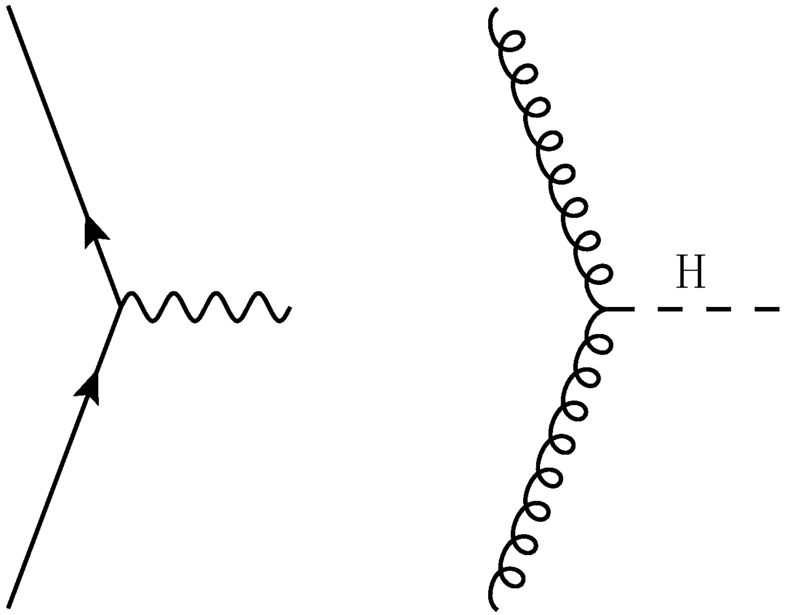
Fellow's meeting
DESY, Hamburg
November 10, 2015

Part I: motivation

Drell-Yan as an example

Motivation: Drell-Yan

- $q\overline{q} \rightarrow \gamma^*$: virtual photon production (or $gg \rightarrow H$)

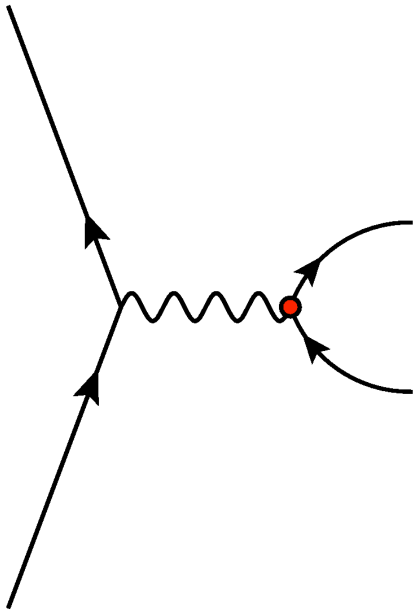


Color Factors!

Drell and Yan, Phys. Rev. Lett. **25**, 316 (1970).

Motivation: Drell-Yan

- $q\overline{q} \rightarrow \gamma^*$: virtual photon production
- Easily identifiable final state
  - Such as $\mu^+\mu^-$



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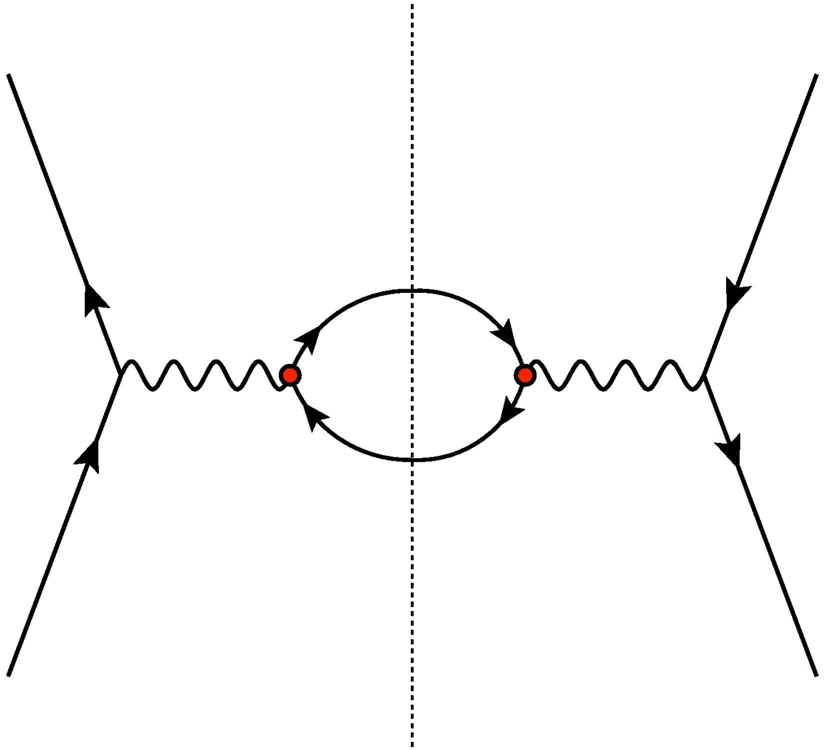
Motivation: Drell-Yan

- $q\overline{q} \rightarrow \gamma^*$: virtual photon production
- Easily identifiable final state
- Complex conjugate



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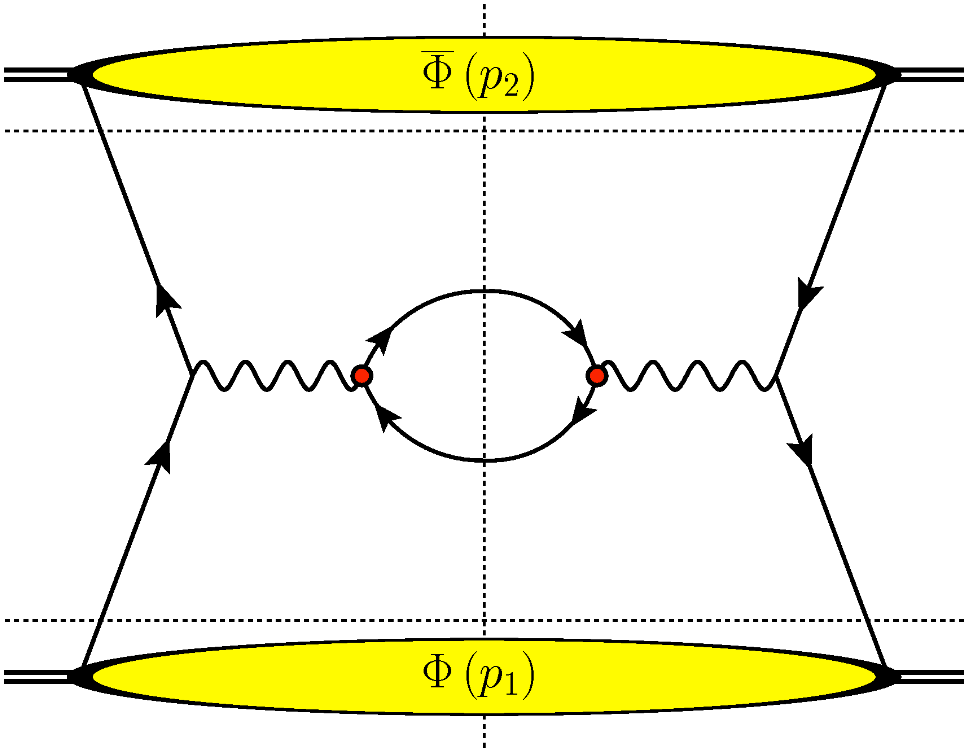
Motivation: Drell-Yan

- $q\overline{q} \to \gamma^*$: virtual photon production
- Easily identifiable final state
- Complex conjugate
- Proton is composite particle



$$\Phi^q \sim \mathrm{F.T.}\langle \mathrm{Proton}|\overline{\psi}(x)\psi(y)|\mathrm{Proton}\rangle$$

For the expert: $u(p_1)\overline{u}(p_1) \to \Phi(p_1, P_1)$

Motivation: Drell-Yan

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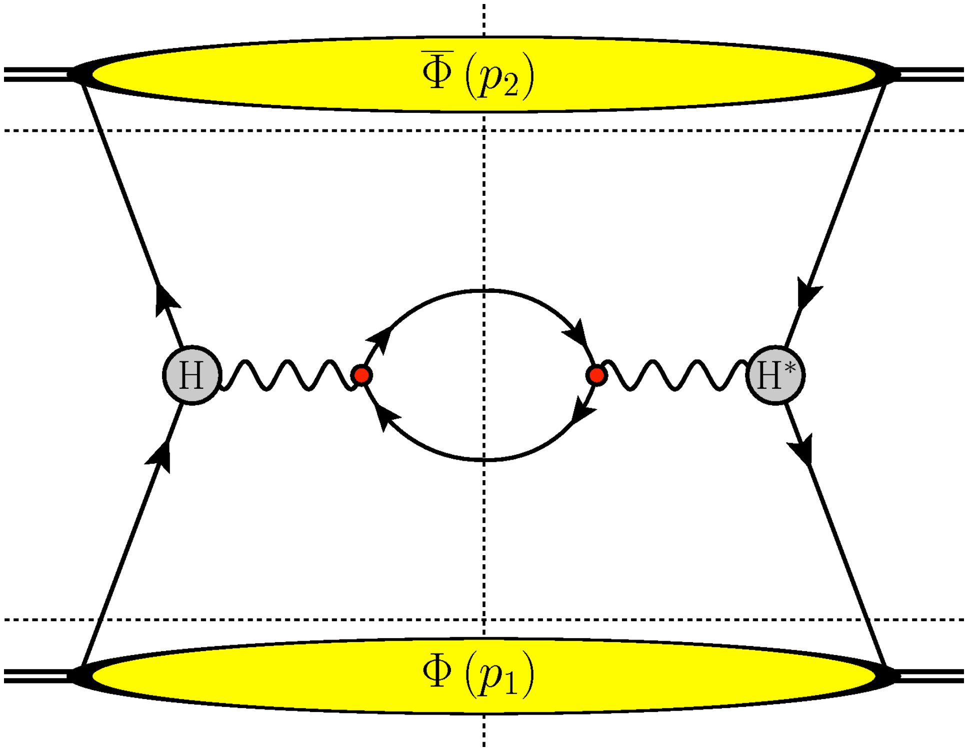
- Proton is composite particle
- Hard scattering contribution



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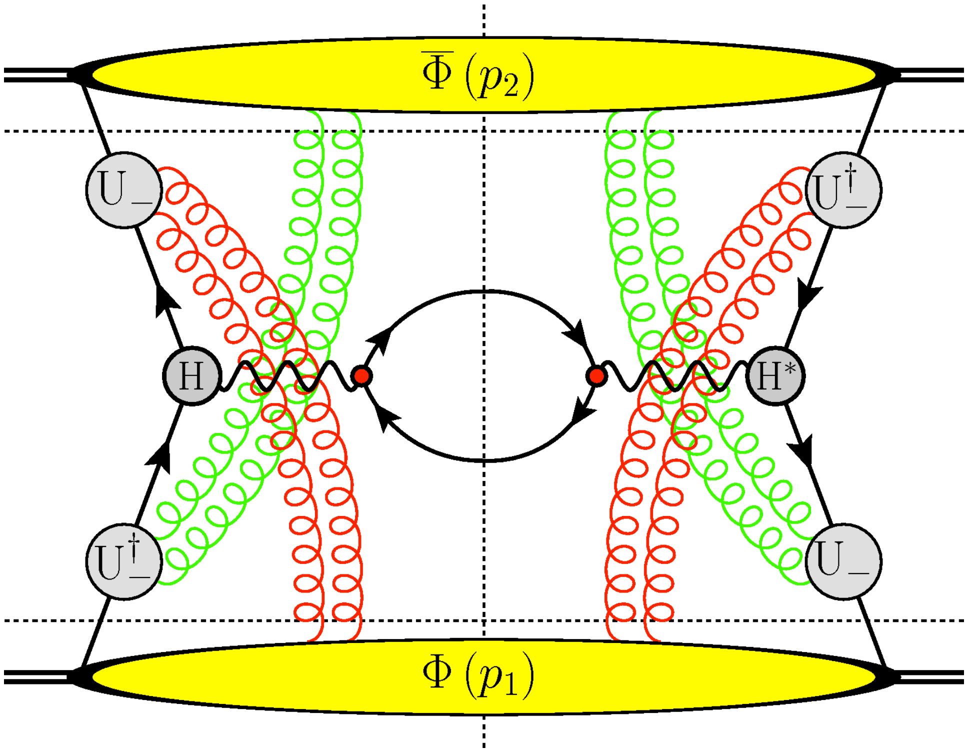
- Gluons give gauge links:

$$U^{[n]}_{[0,\xi]} = \mathcal{P}\exp\left(-i\int_0^{\xi} d\,\eta \cdot P\;n\cdot A(\eta)\right)$$

  - These gluons carry color!



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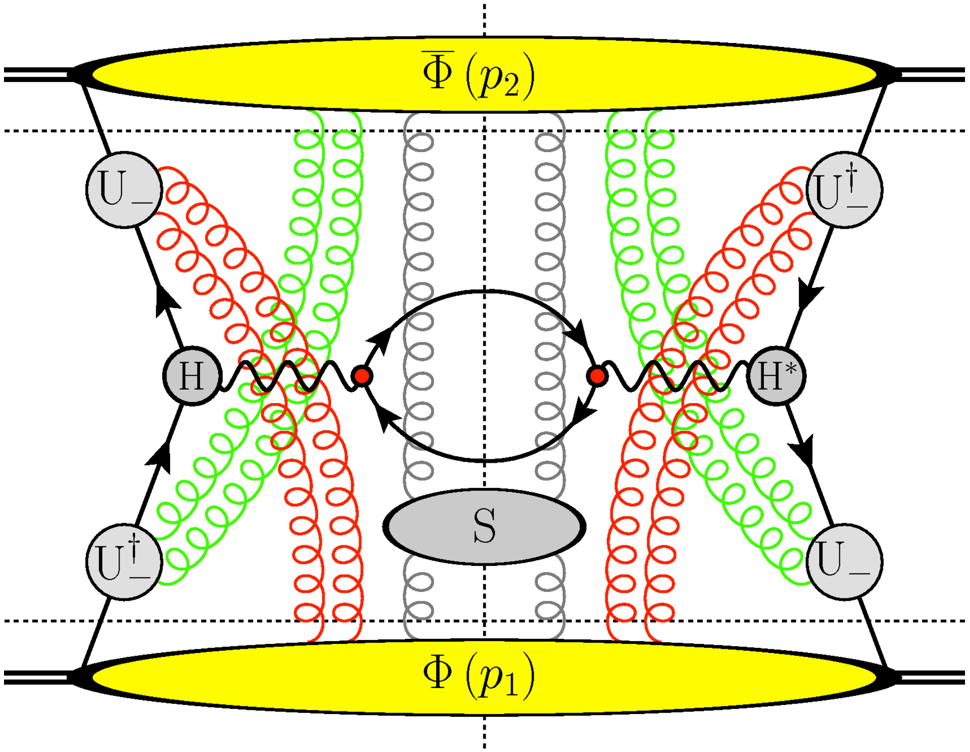

See e.g. Collins, *Foundations of perturbative QCD*, Cambridge University Press, (2011)

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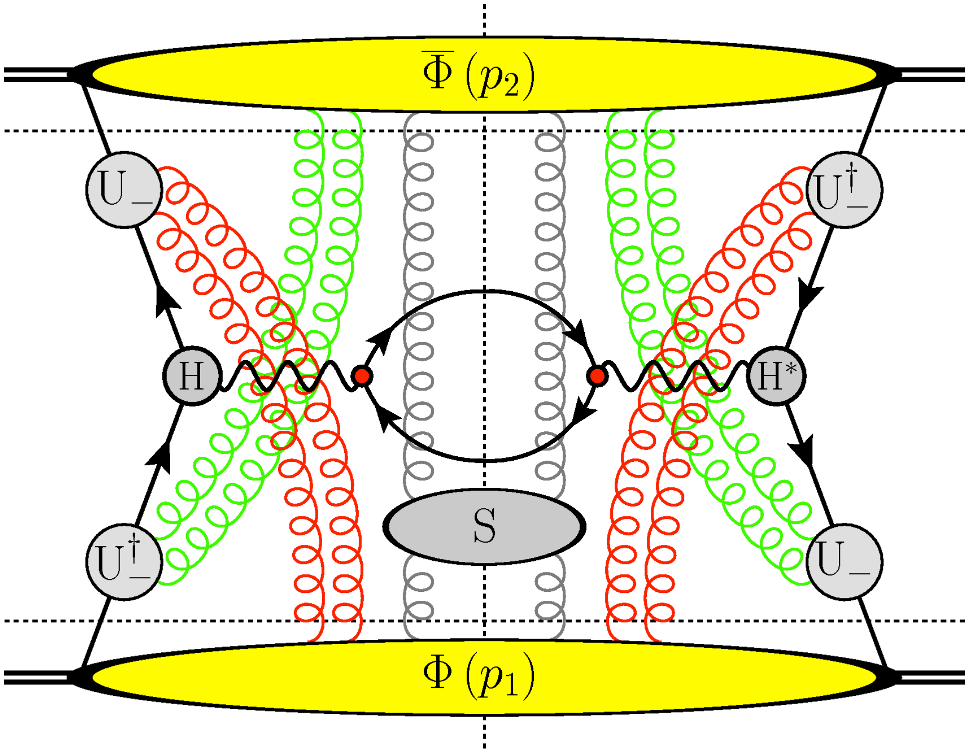
- Both the protons and the partons could be polarized



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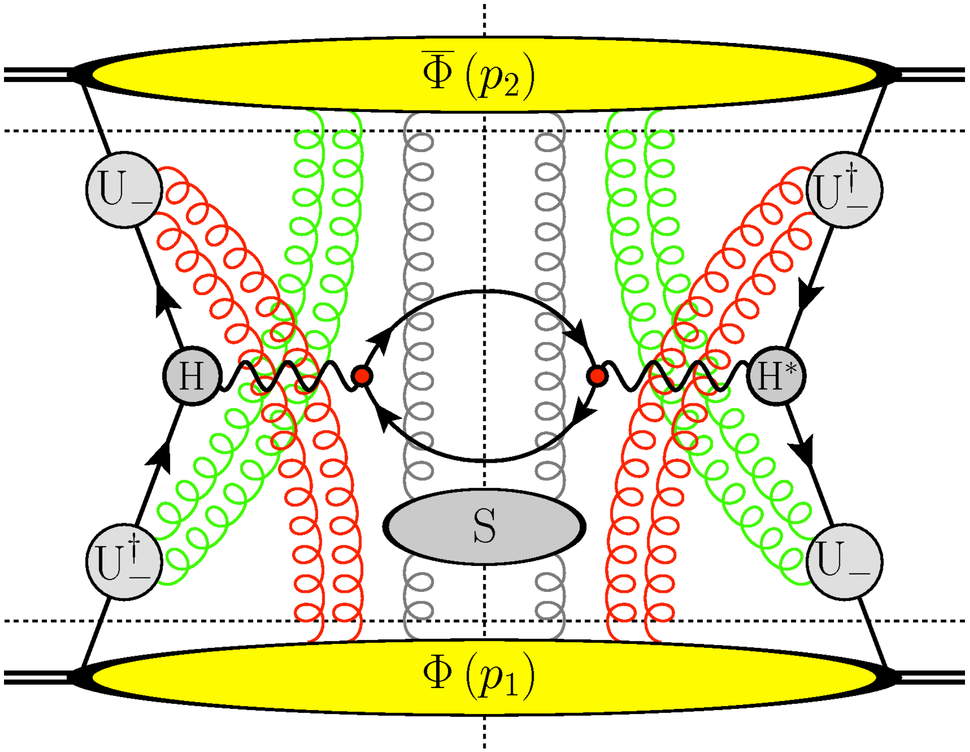

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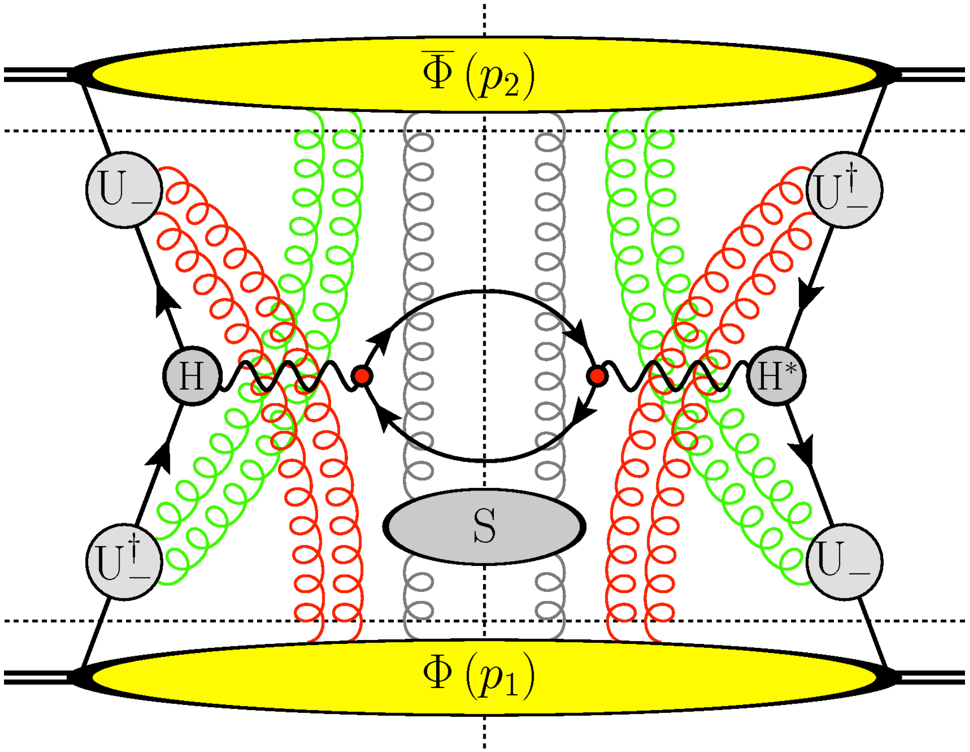
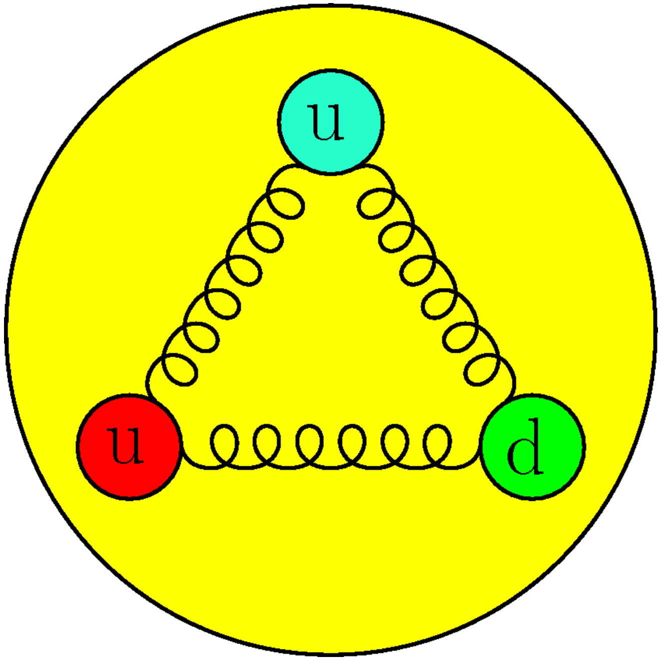


- My work: interplay between color, spin and transverse momentum

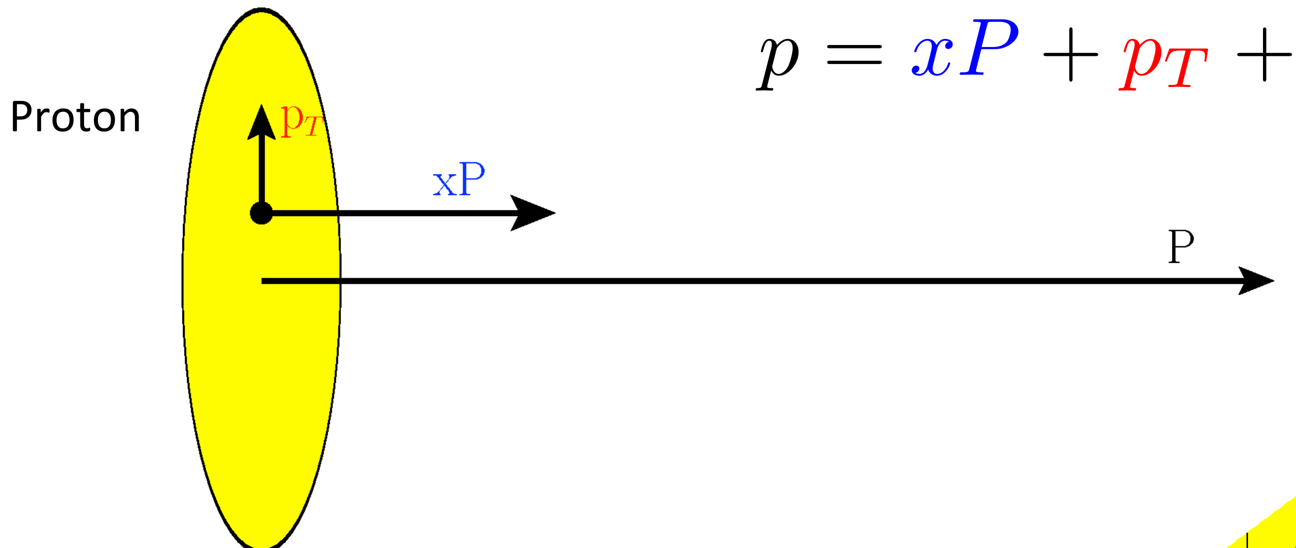
Part II: the proton as composite particle



- One has to take care of the composite nature of the proton!

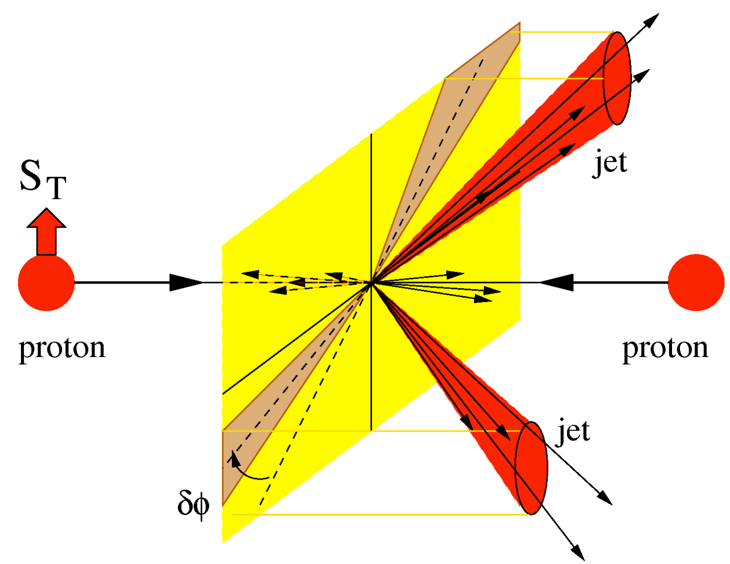
Decomposition of momentum

- Decomposition of quark momentum into collinear and transverse parts:



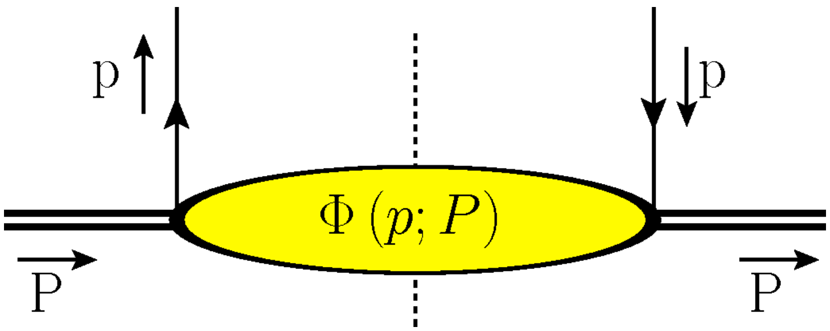
$$p = xP + p_T + \sigma\, n$$

- Correlators Φ depend on p_T as well → this shows up as azimuthal asymmetries.



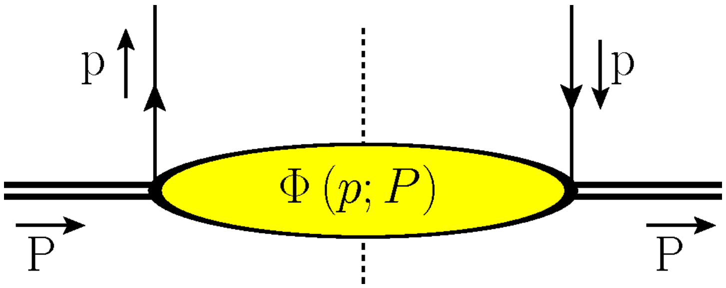
Quarks and gluons in a proton

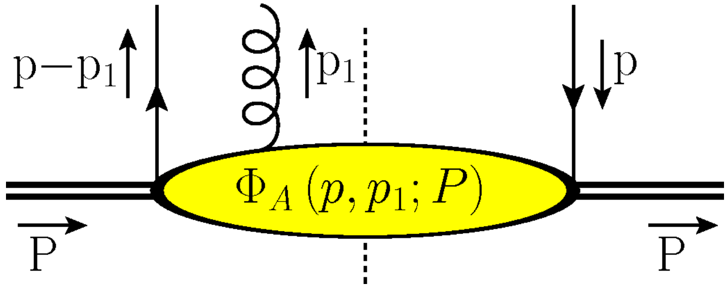
- The proton is described by a correlator Φ.



Matrix elements

- Quark correlators can be written as matrix elements





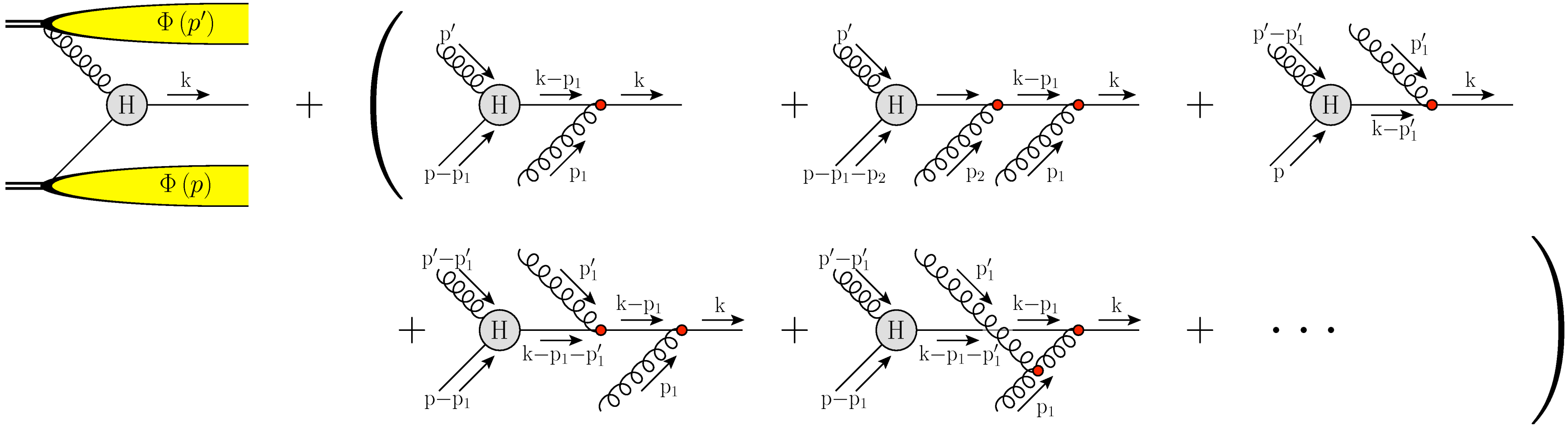
$$\Phi_{ij}(p;P) = \Phi_{ij}(p|p) = \text{F.T.}\,\langle P|\overline{\psi}_j(0)\psi_i(\xi)|P\rangle$$

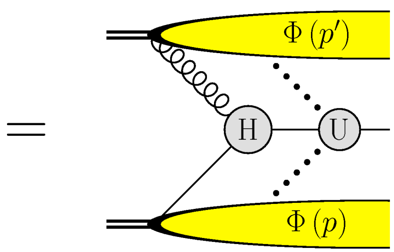
$$\Phi^{\alpha}_{A;ij}(p-p_1|p) = \text{F.T.}\,\langle P|\overline{\psi}_j(0)A^{\alpha}(\eta)\psi_i(\xi)|P\rangle$$

- The field combination is non-local → what about gauge invariance?

Feynman diagrams

Summation of collinear gluons emitted by the colliding protons nicely yields objects that are color gauge invariant.





$$U_{[0,\xi]} = \mathcal{P}\exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\Phi^q \sim \mathrm{F.T.}\langle\mathrm{proton}|\overline{\psi}(0)U_{[0,\xi]}\psi(\xi)|\mathrm{proton}\rangle$$

Color gauge invariance

- Fields transform under local gauge transformations

$$\overline{\psi}(x) \rightarrow \overline{\psi}(x)\, e^{-i\alpha(x)}$$
$$\psi(y) \rightarrow e^{i\alpha(y)}\, \psi(y)$$
$$U(x,y) \rightarrow e^{i\alpha(x)}\, U(x,y)\, e^{-i\alpha(y)}$$

- The gauge link ensures gauge invariance for a combination of fields.

$$\overline{\psi}(x)\, U(x,y)\, \psi(y) \rightarrow \overline{\psi}(x)\, U(x,y)\, \psi(y)$$

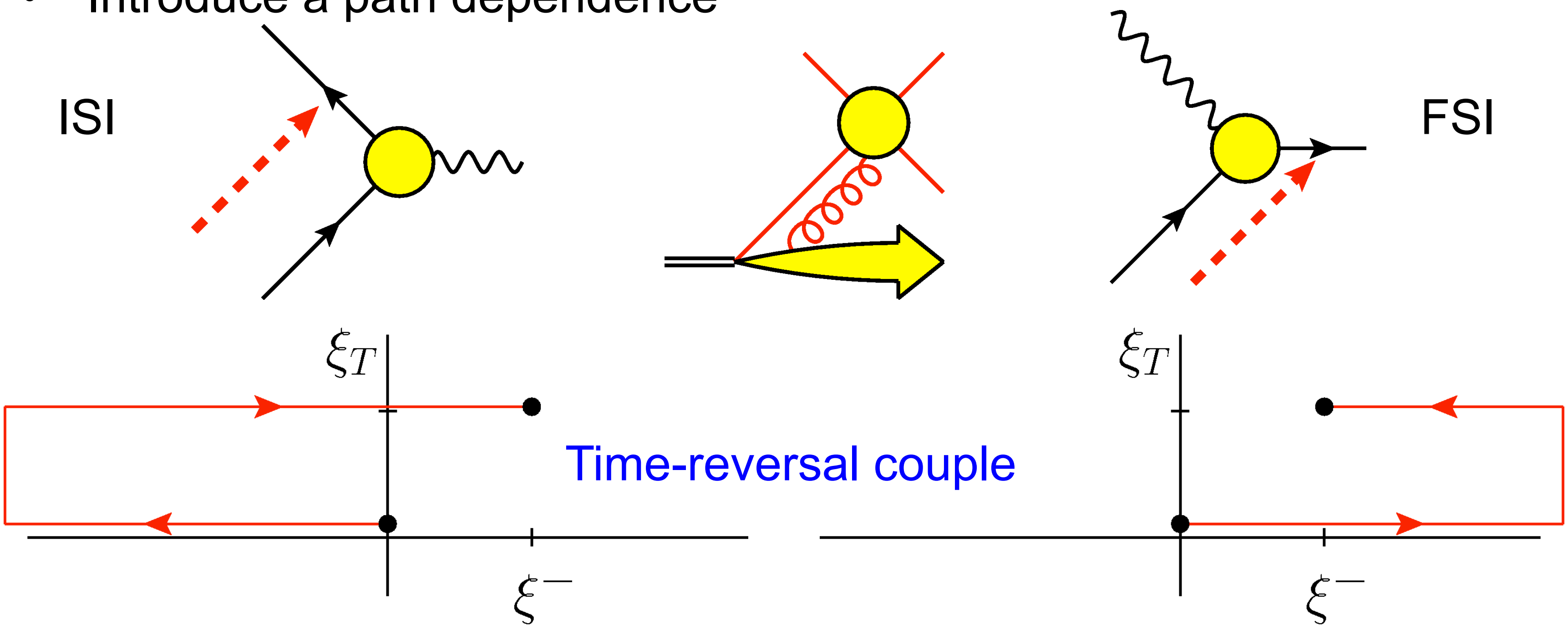
- The correlator Φ has gauge link dependence
  $\rightarrow$ the U(x,y) depends on the color flow in the hard process (in case of transverse momentum)

Gauge invariance for quark TMDs

- Gauge links make the nonlocal combinations of fields gauge invariant

$$U_{[0,\xi]} = \mathcal{P} \exp\left(-ig \int_0^{\xi} ds^{\mu} A_{\mu}\right)$$

- Introduce a path dependence



A.V. Belitsky, X. Ji, F. Yuan, NP B 656, (2003) 165
D. Boer, P.J. Mulders & F. Pijlman, NP B 667 (2003) 2012

TMD PDFs

- Correlator Φ cannot be calculated from first principles: use parton distribution functions (TMDs)

- Depending on polarization(s), differerent contributions are required

$$\Phi^{[U]}(x, p_T) = \left(f_1^{[U]}(x, p_T^2) + i h_1^{\perp [U]}(x, p_T^2) \frac{\not{p}_T}{M} + \dots \right) \frac{\not{P}}{2}$$

quark polarization

nucleon pol.	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T}, h_{1T}^{\perp}$

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Observable at LHC
(unpolarized protons)

TMD PDFs

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$$\Gamma^{[U]}(x,p_T) = \frac{1}{2x}\left(-g_T^{\mu\nu} f_1^g(x,p_T^2) + \left(\frac{p_T^\mu p_T^\nu - \frac{1}{2}p_T^2 g_T^{\mu\nu}}{M^2}\right) h_1^{\perp g[U]}(x,p_T^2) + \ldots\right)$$

gluon polarization (columns) / nucleon pol. (rows)

	U	L	Lin.
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

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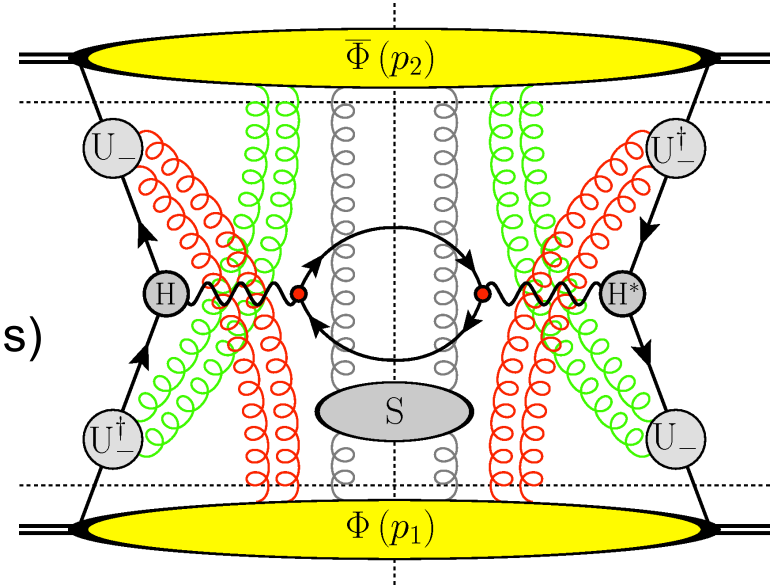
Relevant for Higgs physics

See e.g. Boer, den Dunnen, Pisano, Schlegel and Vogelsang, Phys. Rev. Lett. 108 (2012) 032002

Summary and outlook

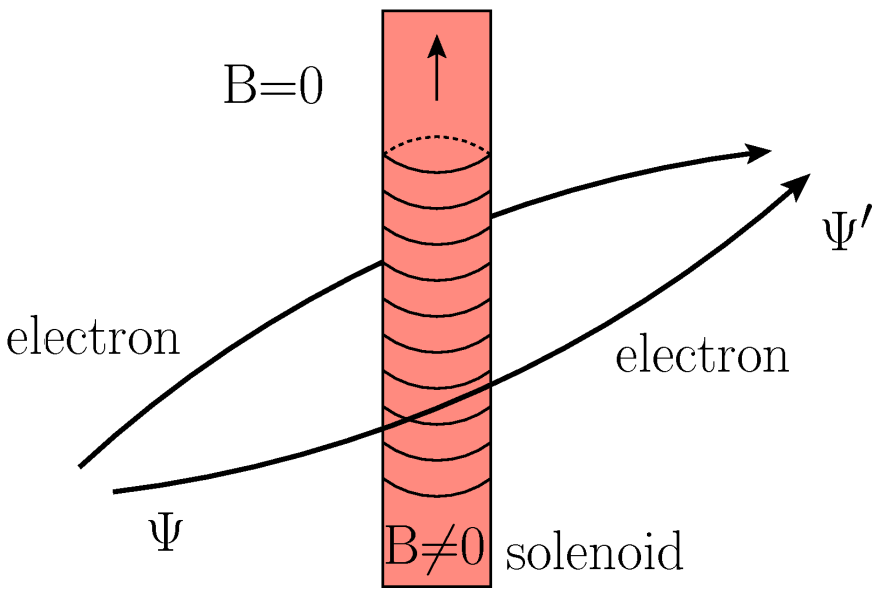
- Transverse directions are required
  - TMDs
  - Polarizations
- Gauge links are required
  - Process dependence for TMDs (in a calculable way → see e.g. my thesis)
  - Universality
- Outlook: double parton distributions
  - Multiple hard scattering exchanges simultaneously

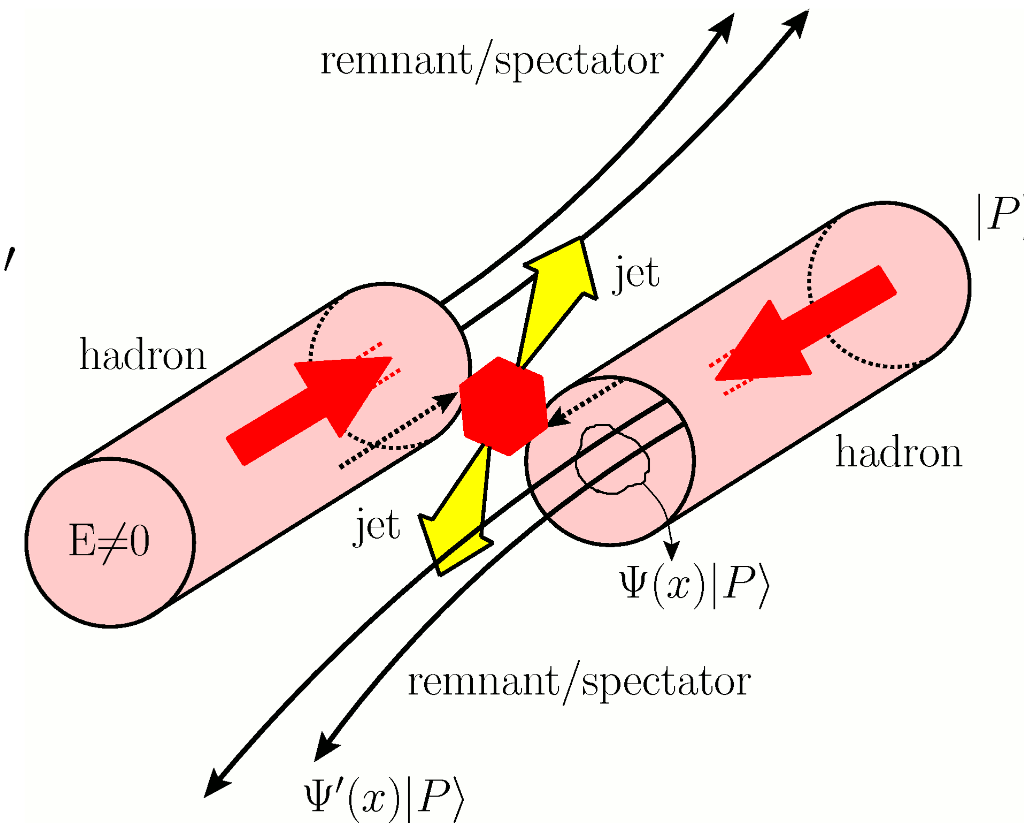


Backup slides

Theory: phases

- Gauge links are phases





$$\Psi = \mathcal{P} e^{ie\int ds\cdot A}\Psi$$

$$\Psi(x)|P\rangle = \mathcal{P} e^{-ig\int_x^{x'} ds_\mu A^\mu}\Phi(x')|P\rangle$$

- Depending on the process, multiple paths are possible

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T-odd

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Process dependent

Pretzelocity is T-even and process dependent