Neutrino Mixing from SUSY breaking

and how the instable vacuum comes into the game

Wolfgang Gregor Hollik

DESY Hamburg

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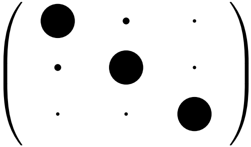
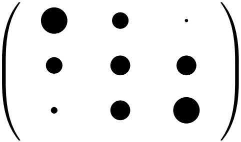
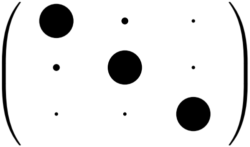
Quark and Neutrino Mixing

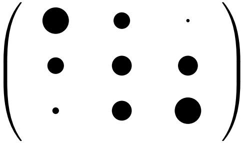
$|V_{\mathrm{CKM}}| =$ 

$|U_{\mathrm{PMNS}}| =$ 

- small mixing angles
- close to unit matrix
- remnants of new Physics?
- get mixings from loops?

- large mixing angles
- no hierarchy
- not close to trivial mixing?
- tree-level symmetries?

$|V_{\text{CKM}}| =$ 

$|U_{\text{PMNS}}| =$ 

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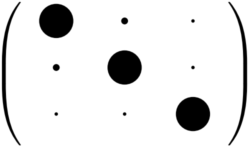
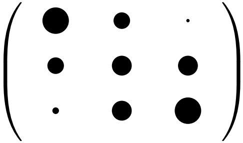
different or similar?

Let's see. . .

$|V_{\mathrm{CKM}}| =$ 

$|U_{\mathrm{PMNS}}| =$ 

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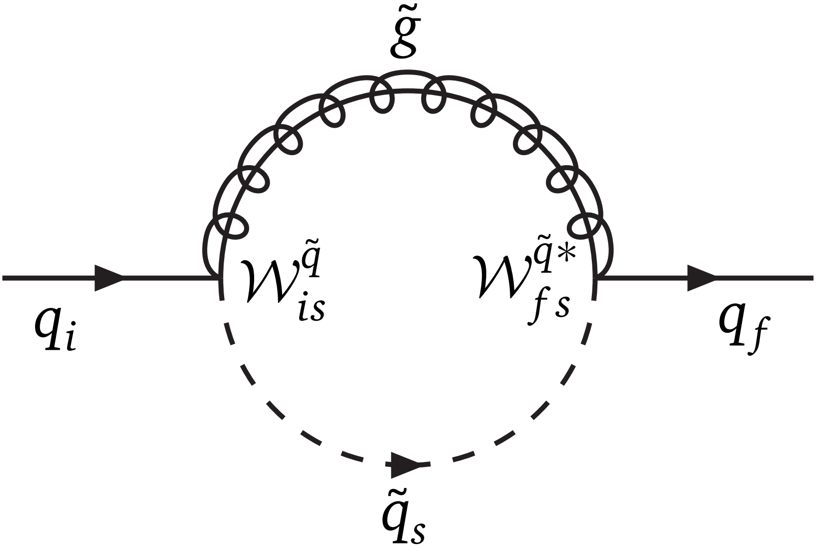
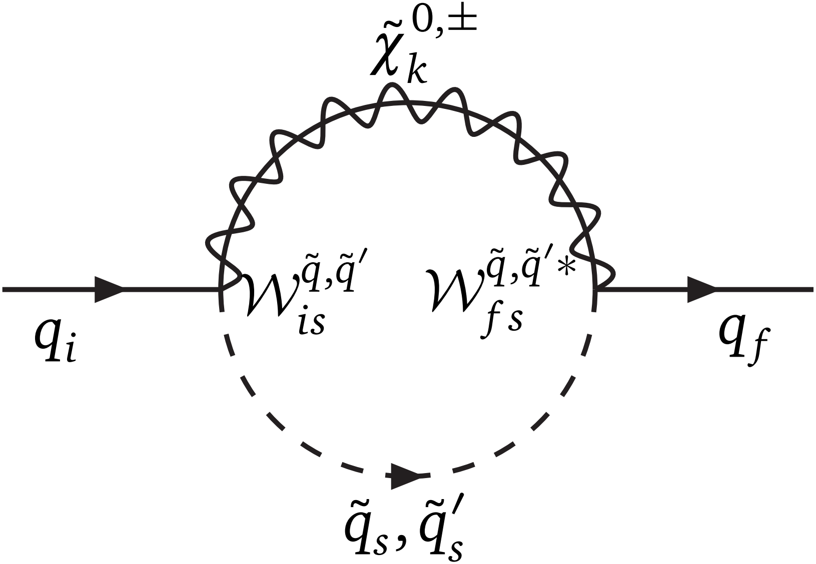
Task: Find out whether radiative corrections can change any tree-level mixing pattern in the PMNS case.

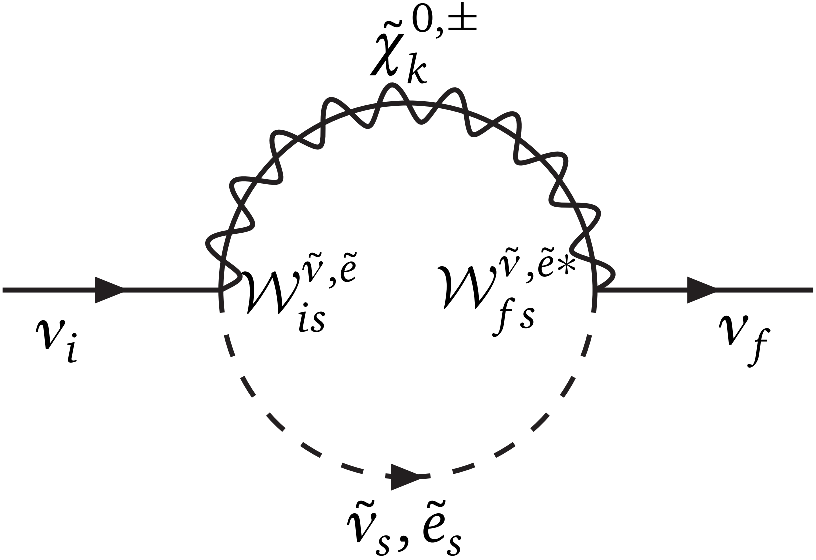
alternatively: mixing angles from mass ratios [WGH, UJSS 2015]



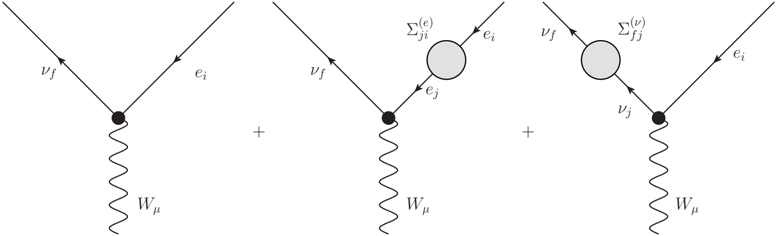
$\tilde{g}$
$\mathcal{W}^{\tilde{q}}_{is}$
$\mathcal{W}^{\tilde{q}*}_{fs}$
q_i
q_f
$\tilde{q}_s$



$\tilde{\chi}_k^{0,\pm}$
$\mathcal{W}_{is}^{\tilde{q},\tilde{q}'}$
$\mathcal{W}_{fs}^{\tilde{q},\tilde{q}'*}$
q_i
q_f
$\tilde{q}_s, \tilde{q}'_s$


$\tilde{\chi}_k^{0,\pm}$
$\mathcal{W}_{is}^{\tilde{\nu},\tilde{e}}$
$\mathcal{W}_{fs}^{\tilde{\nu},\tilde{e}*}$
ν_i
ν_f
$\tilde{\nu}_s, \tilde{e}_s$

[CKM renormalization: Crivellin, Nierste 2008; Leptons: Girrbach et al. 2010]



[Denner & Sack 1990]

mixing matrix renormalization

$$i\frac{g}{\sqrt{2}}\gamma^\mu P_L U^\dagger_{\mathsf{PMNS}} \to i\frac{g}{\sqrt{2}}\gamma^\mu P_L \left(U^{(0)\dagger} + \Delta U^e U^{(0)\dagger} + \Delta U^\nu U^{(0)\dagger}\right),$$

sensitivity to neutrino mass spectrum

$$\Delta U^\nu_{fi} \sim \frac{m_{\nu_f}\Sigma_{fi}}{\Delta m^2_{fi}}$$

Enhanced corrections for quasi-degenerate neutrinos

[WGH, arxiv: 1411.2946]

enhancement by degeneracy of neutrino mass spectrum

$$\Delta U^{\nu}_{fi} \sim \frac{m_{\nu_f} \Sigma_{fi}}{\Delta m^2_{\nu}} \sim \frac{m_{\nu_f} m_{\nu_i}}{\Delta m^2_{fi}} \leq 5 \times 10^3$$

for $m^{(0)}_{\nu} \sim 0.35$ eV and $f, i = 1, 2$

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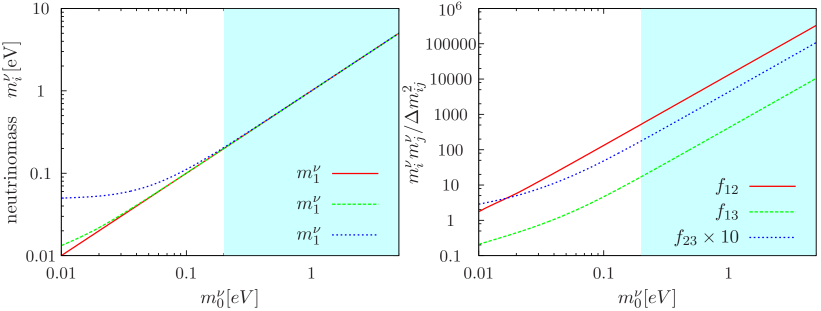

$$f_{ij} = m_{\nu_i} m_{\nu_j} / \Delta m^2_{ij}$$

[e.g. Chun and Pokorski '99; Chankowski et al. '00; WGH '15]

Generic treatment

$$m^{\nu}_{AB} = m^{(0)}_{AB} + m^{(0)}_{AC} I_{CB} + I_{AC} m^{(0)}_{CB}$$

I: threshold correction $I \sim \frac{y^2}{16\pi^2} f\left(\ln(M^2/Q^2)\right)$ (in the SM diagonal)

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Mass basis

$$m^{\nu}_{ab} = m^{(0)}_{a} \delta_{ab} + \left(m^{(0)}_{a} + m^{(0)}_{b}\right) I_{ab}$$

$I_{ab} = \sum_{AB} I_{AB} U^{(0)}_{Aa} U^{(0)}_{Bb}$

Degenerate neutrinos and threshold Corrections

[e.g. Chun and Pokorski '99; Chankowski et al. '00; WGH '15]

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- assumption: degenerate tree-level masses, $|m^{(0)}_1| = |m^{(0)}_2| = |m^{(0)}_3|$

MSSM with righthanded neutrinos

$$\mathcal{W} \supset \mu H_\mathrm{d} \cdot H_\mathrm{u} + Y^\nu_{ij} \, H_\mathrm{u} \cdot L_{L,i} N_{R,j} - Y^\ell_{ij} \, H_\mathrm{d} \cdot L_{L,i} E_{R,j} + \frac{1}{2} M^R_{ij} N_{R,i} N_{R,j}$$

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New soft SUSY breaking terms

$$V^{\tilde{\nu}}_\mathrm{soft} = \left(\boldsymbol{m}^2_{\tilde{L}}\right)_{ij} \tilde{\nu}^*_{L,i}\tilde{\nu}_{L,j} + \left(\boldsymbol{m}^2_{\tilde{R}}\right)_{ij} \tilde{\nu}_{R,i}\tilde{\nu}^*_{R,j} + \left(A^\nu_{ij}\, h^0_\mathrm{u}\, \tilde{\nu}_{L,i}\tilde{\nu}^*_{R,j} + \left(\boldsymbol{B}^2\right)_{ij} \tilde{\nu}^*_{R,i}\tilde{\nu}^*_{R,j} + \text{h.c.}\right)$$

SUSY non-diagonal threshold corrections

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- seesaw type I:

$$\boldsymbol{m}^{(0)}_{\nu} = -v_{\rm u}^2 \boldsymbol{Y}^T_{\nu}\, \boldsymbol{M}^{-1}_{\rm R} \boldsymbol{Y}_{\nu} + \mathcal{O}(v_{\rm u}^4/M_{\rm R}^3)$$

- adding SUSY 1-loop [Dedes, Haber, Rosiek 2007]

$$\left(\boldsymbol{m}^{1-\rm loop}_{\nu}\right)_{ij} = (\boldsymbol{m}_{\nu})_{ij} + {\rm Re}\left[\Sigma^{(\nu),S}_{ij} + \frac{m_{\nu_i}}{2}\Sigma^{(\nu),V}_{ij} + \frac{m_{\nu_j}}{2}\Sigma^{(\nu),V}_{ji}\right]$$

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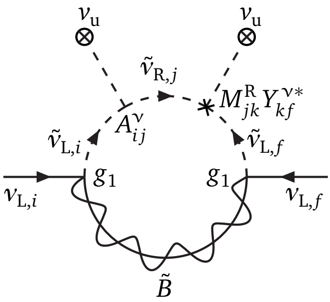
$$\Sigma^{(\nu)}_{ij}(p) = \Sigma^{(\nu),S}_{ij}(p^2) P_\mathrm{L} + \Sigma^{(\nu),S^*}_{ij}(p^2) P_\mathrm{R} + \not{p}\left[\Sigma^{(\nu),V}_{ij}(p^2) P_\mathrm{L} + \Sigma^{(\nu),V^*}_{ij}(p^2) P_\mathrm{R}\right].$$

The seesaw mechanism

[WGH PhD thesis, arXiv:1505.07764]



$$\boldsymbol{\Sigma} \sim y_\nu \boldsymbol{A}^\nu / M_R$$

$\boldsymbol{Y}^\nu = y_\nu \mathbf{1}$, $\boldsymbol{M}_R = M_R \mathbf{1}$

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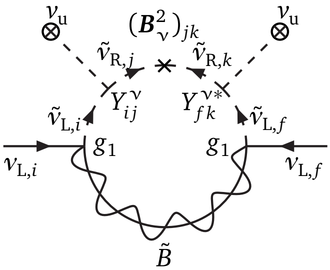
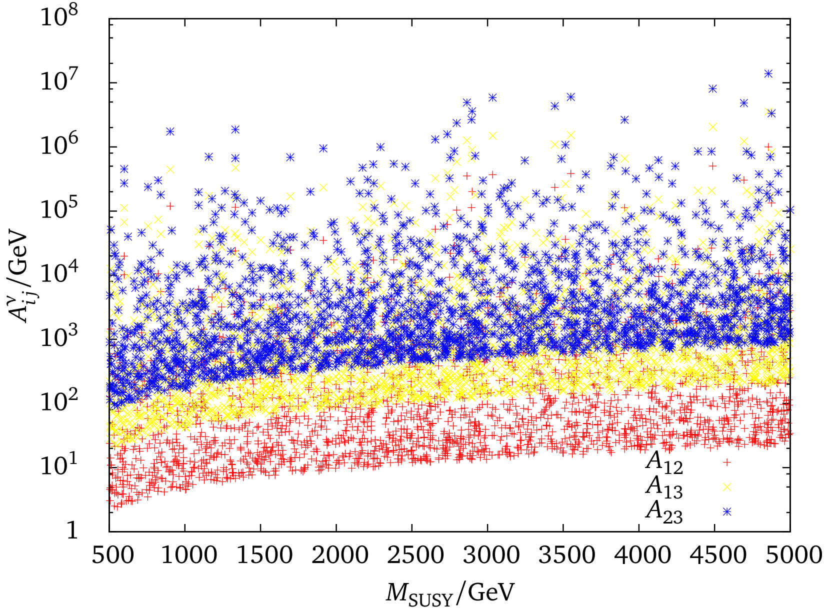


$$\boldsymbol{\Sigma} \sim y_\nu^2 \boldsymbol{b}_\nu / M_\mathrm{R}$$

$\boldsymbol{Y}^\nu = y_\nu \mathbf{1}$, $\boldsymbol{M}_\mathrm{R} = M_\mathrm{R}\mathbf{1}$, $\boldsymbol{B}_\nu^2 = \boldsymbol{b}_\nu M_\mathrm{R}$

A non-decoupling contribution



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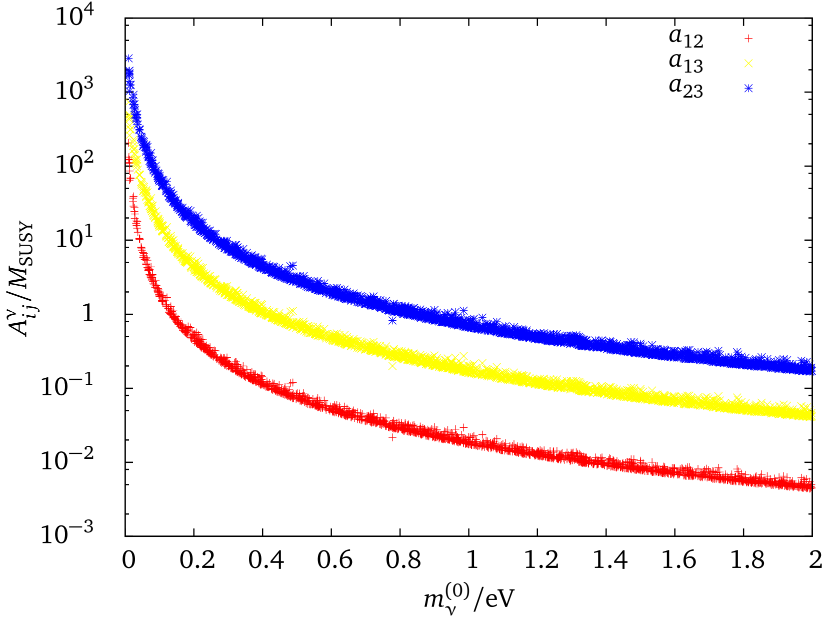

What about the (in)stable vacuum?

[mascot of the 1997 World Championships of Athletics, Athens]

Constraints on flavor-changing terms

[Casas, Dimopoulos '96]

$$|A^{\mathrm{u}}_{ij}|^2 \leq Y^2_{\mathrm{u}_k} \left(\tilde{m}^2_{Q_i} + \tilde{m}^2_{u_j} + m^2_{H_\mathrm{u}} + |\mu|^2\right)$$

Similar bounds on diagonal A-terms

[Frére, Jones, Raby '83; Gunion, Haber, Sher '88]

$$|A_\mathrm{t}|^2 \leq 3\left(\tilde{m}^2_{Q} + \tilde{m}^2_{t} + m^2_{H_\mathrm{u}} + |\mu|^2\right)$$

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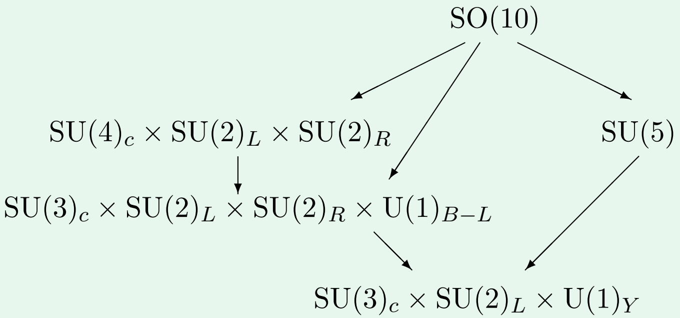
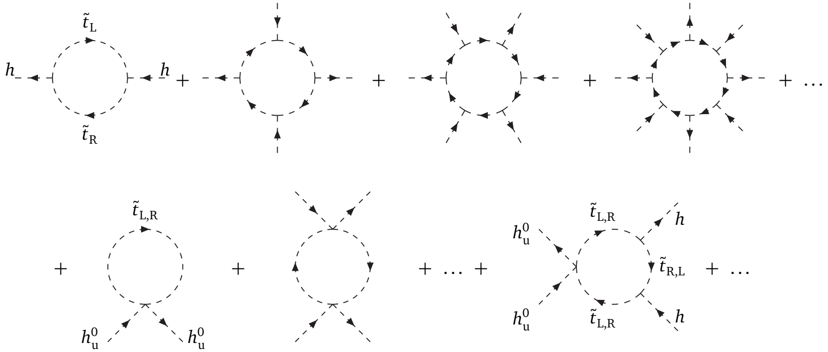
The unified picture

Relating up and down, left and right, quarks and leptons

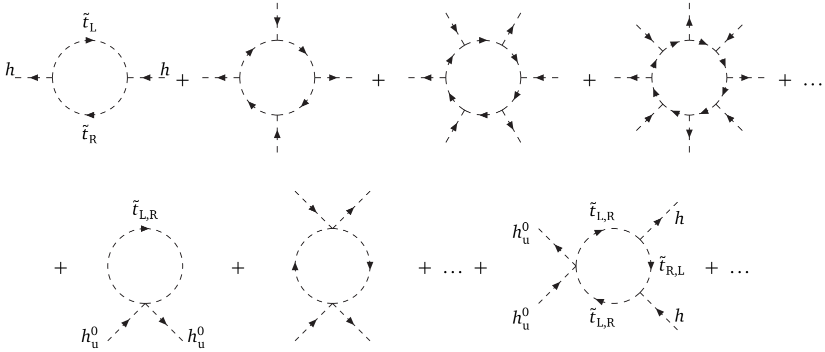


- RFV in both quark and lepton sector
- Pati–Salam unification: $\mathrm{SU}(4)_c$ adds lilac as fourth color
  $\hookrightarrow$ squark and slepton soft breaking terms unify
- large A_ν means also large A_t: check with quantum corrections

One-loop effective Higgs potential (in the MSSM)



- dominant contribution from third generation squarks
- quadrilinear couplings ($\sim |Y_t|^2$)
- trilinear coupling to a linear combination ($\mu^* Y_t h_d^\dagger - A_t h_u^0$)
- series summable to an infinite number of external legs



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- **Do not stop after renormalizable / dim 4 terms!**

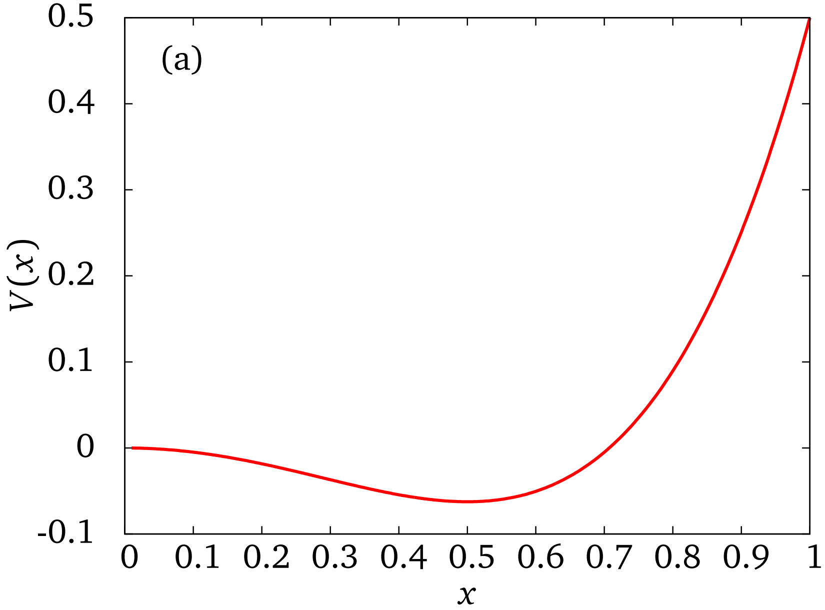
Features of the resummed series

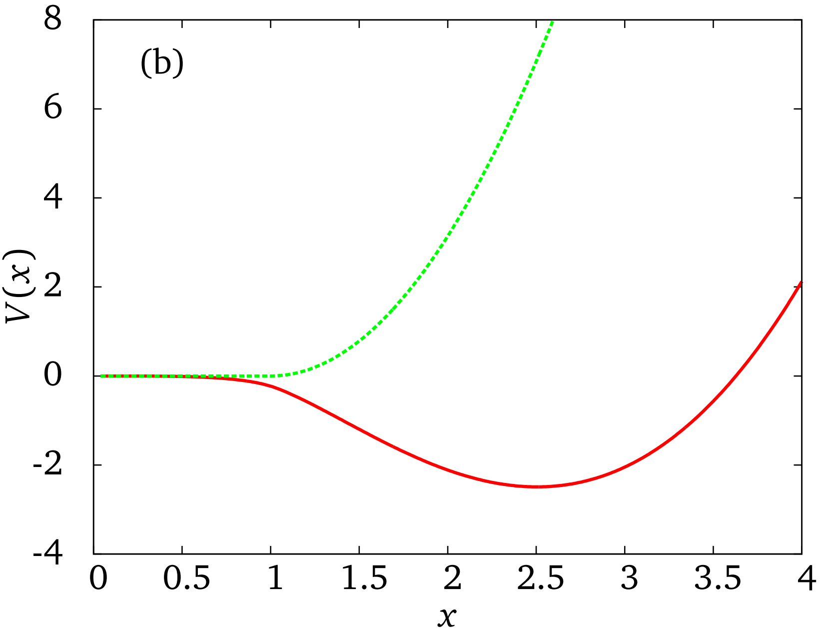
[Bobrowski, Chalons, WGH, Nierste 2014]

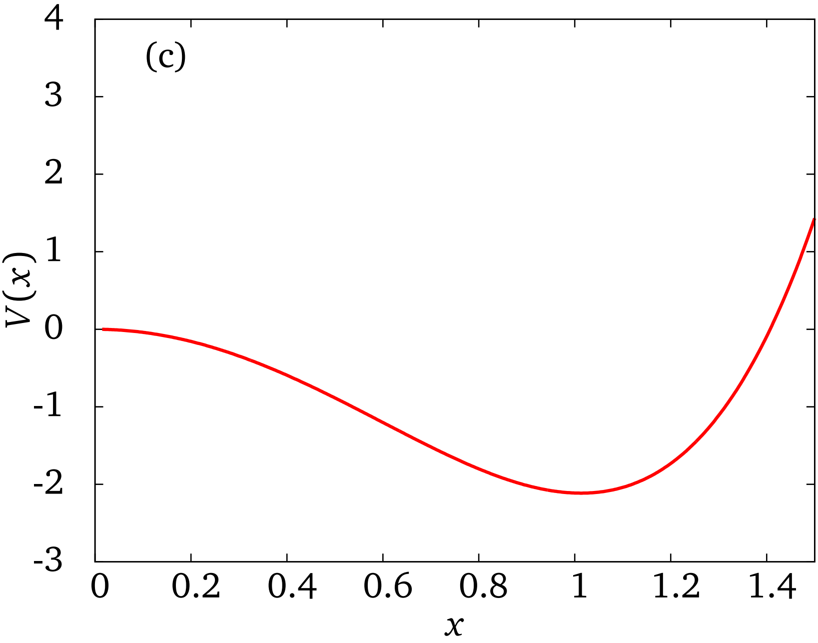
$$V_1(h_u^0, h_d^0) = \frac{N_c M^4}{32\pi^2} \Big[(1+y+x)^2 \log(1+y+x) + (1+y-x)^2 \log(1+y-x) - 3(x^2+y^2+2y) \Big]$$

$$x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4},\ h = h_d^{0*} - \frac{A_t}{\mu^* Y_t} h_u^0, \qquad y = \frac{|Y_t h_u^0|^2}{M^2}$$

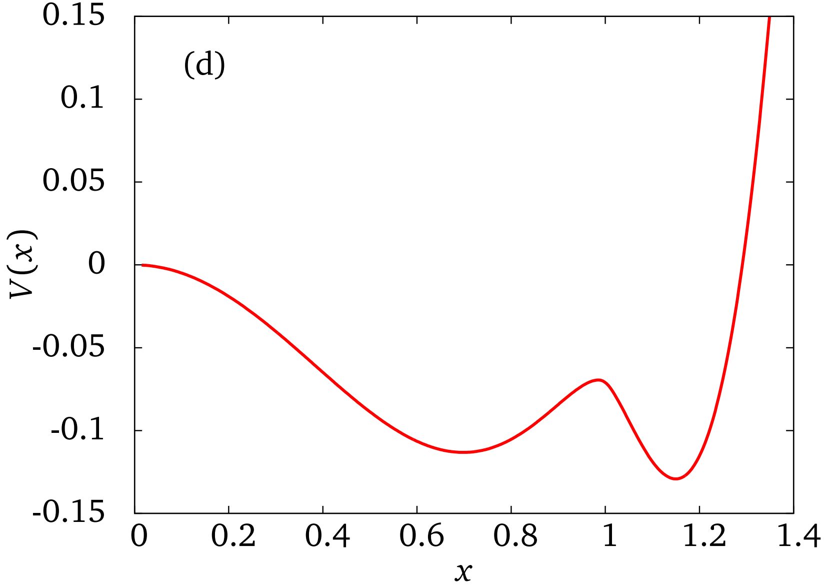
- branch cut at $x - y = 1$: take real part (analytic continuation)
- ignore imaginary part: $\log(1+y-x) = \frac{1}{2}\log\left((1+y-x)^2\right)$
- always bounded from below
- minimum independent of Higgs parameters from tree potential
- minimum determined by SUSY scale parameters


(a)
V(x)
x
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

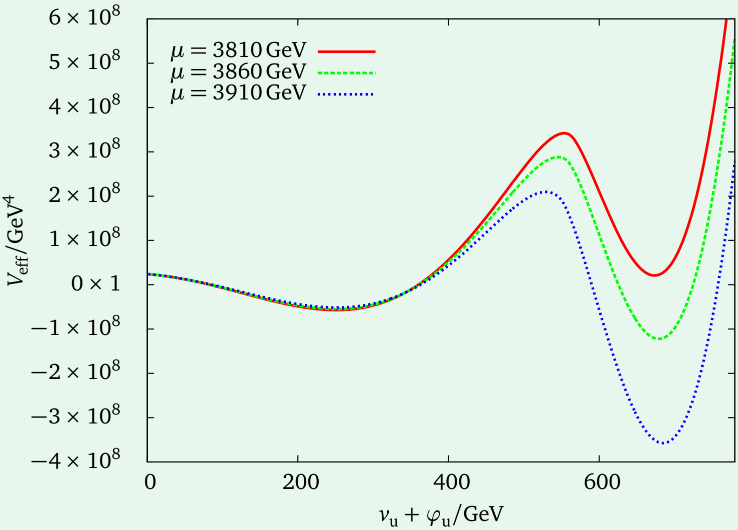

(b)
8
6
4
2
0
-2
-4
V(x)
0
0.5
1
1.5
2
2.5
3
3.5
4
x


(c)
V(x)
x
4
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1
1.2
1.4

Tree, loop and tree + loop



Access to Charge and Color breaking minima



Access to Charge and Color breaking minima

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & \tilde{m}_t^2 + |Y_t h_u^0|^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_b h_d^0|^2 & A_b h_d^0 - \mu^* Y_b h_u^{0*} \\ A_b^* h_d^{0*} - \mu Y_b^* h_u^0 & \tilde{m}_b^2 + |Y_b h_d^0|^2 \end{pmatrix}$$

- non-trivial behaviour of sfermions masses with Higgs vev

New interpretation

Access to Charge and Color breaking minima

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- non-trivial behaviour of sfermions masses with Higgs vev:

$$m^2_{\tilde{b}_{1,2}}(h_u^0, h_d^0) = \frac{\tilde{m}_Q^2 + \tilde{m}_b^2}{2} + |Y_b h_d^0|^2 \pm \frac{1}{2}\sqrt{(\tilde{m}_Q^2 - \tilde{m}_b^2)^2 + 4|A_b h_d^0 - \mu^* Y_b h_u^{0*}|^2}$$

- expand theory around new minimum: $m^2_{\tilde{b}_2} < 0$

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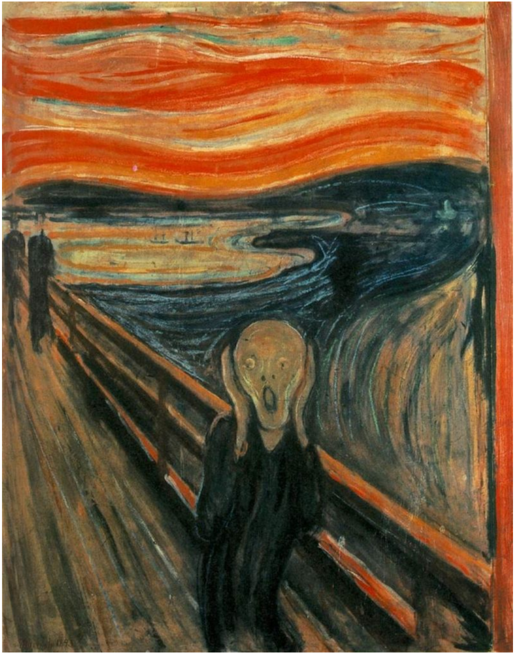
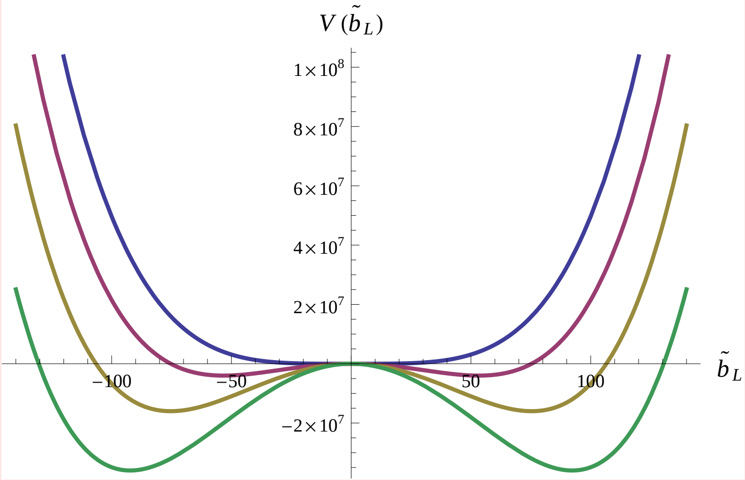
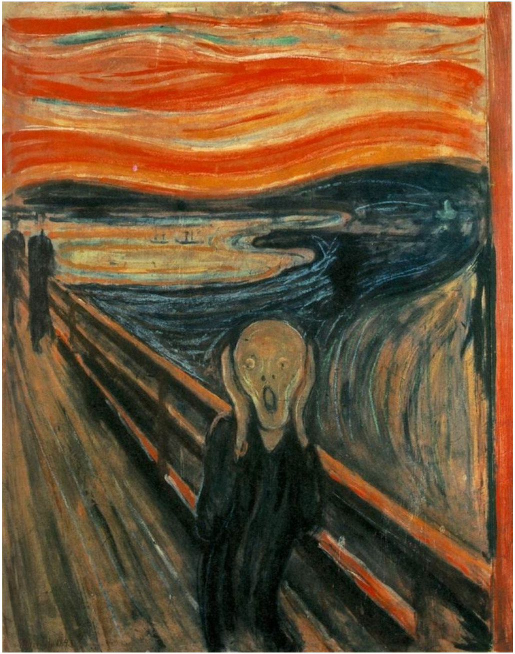
- **tachyonic squark mass!**



[commons.wikimedia.org]

Developing sbottom vev





[commons.wikimedia.org]

Keep us from charge and color breaking...

- choose appropriate direction $\hookrightarrow$ one-field problem

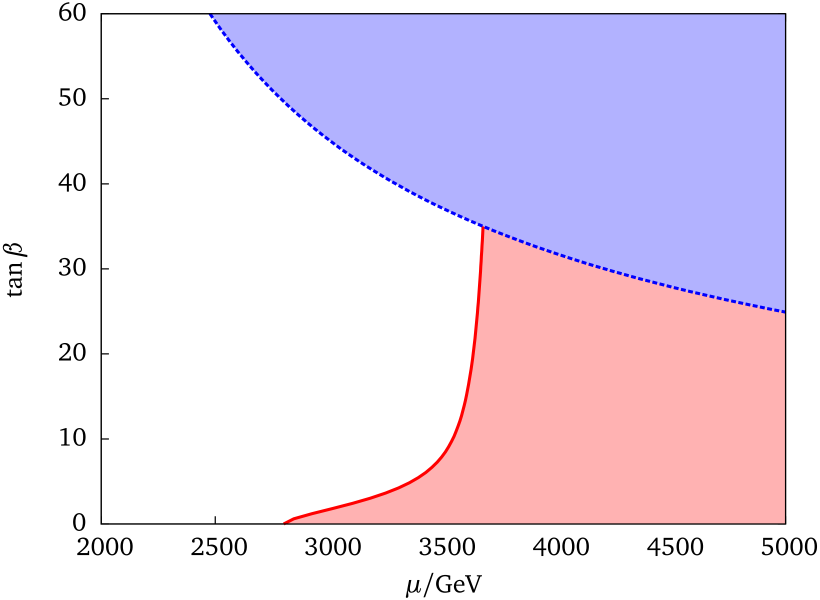
$$V_\phi^{\text{tree}} = \bar{m}^2\phi^2 - A\phi^3 + \lambda\phi^4$$

$h_{\mathrm{u}}^0 = \tilde{b}$, $h_{\mathrm{d}}^0 = 0$

$$m_{H_{\mathrm{u}}}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu Y_{\mathrm{b}})^2}{Y_{\mathrm{b}}^2 + (g_1^2 + g_2^2)/2}$$

$|h_{\mathrm{d}}^0|^2 = |h_{\mathrm{u}}^0|^2 + |\tilde{b}|^2$, $\tilde{b} = \alpha h_{\mathrm{u}}^0$

$$m_{11}^2(1+\alpha^2) + m_{22}^2 \pm 2m_{12}^2\sqrt{1+\alpha^2} + \alpha^2(\tilde{m}_Q^2 + \tilde{m}_b^2) > \frac{4\mu^2\alpha^2}{2+3\alpha^2}$$


60
50
40
30
20
10
0
2000
2500
3000
3500
4000
4500
5000
$\tan\beta$
μ/GeV

- Vacuum stability MSSM [PRD 90 (2014) 3, 035025]
- Radiative neutrino mixing [PRD 91 (2015) 3, 033001]
- ν-mix from ~~SUSY~~ [PoS CORFU2014 (2015) 077]
- new CCB constraints [arXiv:1508.07201 (PLB 75X)]
- unrelated, but interesting [NPB 892 (2015) 364]

Neutrino mixing from SUSY breaking

- projecting observed flavour on soft SUSY breaking terms (A-terms)
- large neutrino mixing from neutrino mass degeneracy
- SUSY loop corrections may change any tree-level pattern

Large A-terms destabilize the electroweak vacuum

- avoid death from vacuum-to-vacuum transition
- as we observe no charge and color breaking true vacuum
- constraints on parameters