Factorization and Resummation for Massive Quark Effects in Exclusive Drell-Yan

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Outline

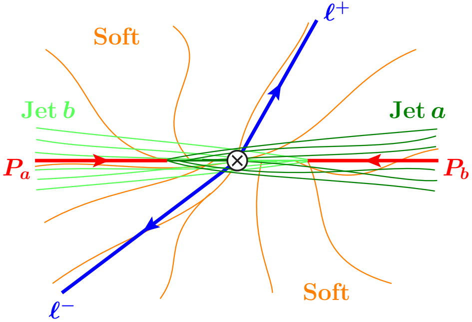
Motivation

- Drell-Yan + 0 Jets
  (different jet vetoes: p_T, beam thrust)

- p_T spectrum of Z-boson measured
  with high precision

- NNLL′ analyses available
  (see Thomas' talk)

- no systematic theoretical description of
  b-quark mass effects yet



from: I.W.Stewart,F.J.Tackmann,W.J.Waalewijn, *Phys. Rev.* **D81** (2010) 094035

- discrepancies between MC and
  experiment in low p_T region
  (splitting into massive quarks not understood)

our project: systematic treatment of quark mass effects at NNLL′ accuracy
for transverse momentum ($SCET_{II}$) and beam thrust ($SCET_{I}$) spectrum

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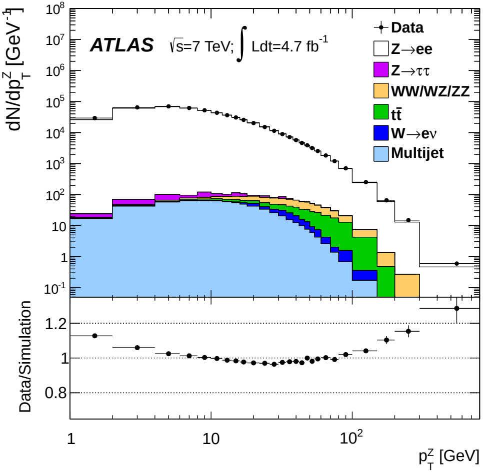

[ATLAS Collaboration (2014)]

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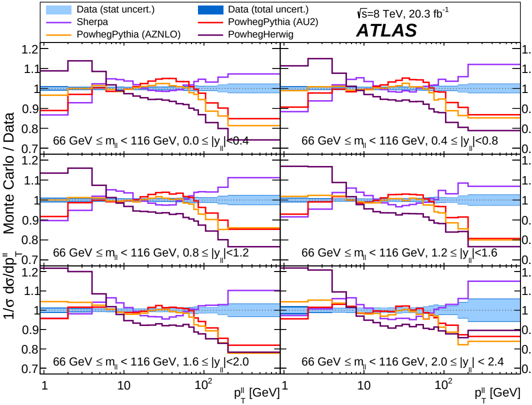

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our project: systematic treatment of quark mass effects at NNLL$'$ accuracy
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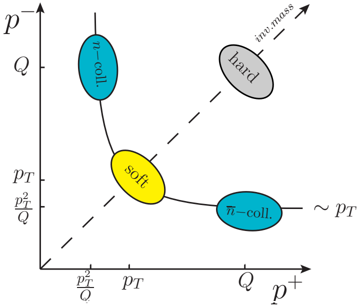
Massless Factorization for p_T



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

hard $\mu \sim Q$:

hard function $H^{(n_l)}(Q)$

n−coll. $\mu \sim p_T$:

TMD beam function

$$B_i^{(n_l)}(\vec{p}_T, x) = \sum_{k\in\{q,\bar{q},g\}} \mathcal{I}_{ik}^{(n_l)}(\vec{p}_T, x) \otimes f_k^{(n_l)}(x)$$

soft $\mu \sim p_T$:

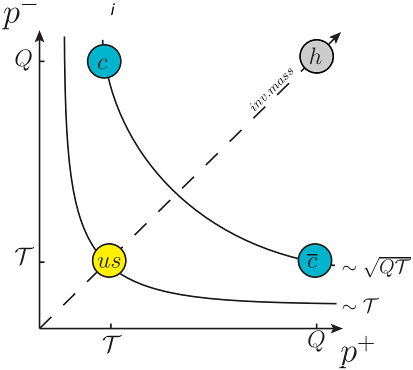
TMD soft function $S^{(n_l)}(\vec{p}_T)$

SCET$_{\text{II}}$ - rapidity divergences cancel between soft and beam functions.

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l)} \times \left[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{p_T}{Q}\right)$$

Massless Factorization for $\mathcal{T}$

$$\mathcal{T} = \sum_i \min\{\mathrm{n_a} \cdot \mathrm{p_i},\, \mathrm{n_b} \cdot \mathrm{p_i}\}$$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

SCET$_\mathrm{I}$ - no rapidity divergences.

h $\mu \sim Q$:

hard function $H^{(n_l)}(Q)$

c $\mu \sim \sqrt{Q\mathcal{T}}$:

SCET$_\mathrm{I}$ beam function

$$B_i^{(n_l)}(t,x) = \sum_{k \in \{q,\bar{q},g\}} \mathcal{I}_{ik}^{(n_l)}(t,x) \otimes f_k^{(n_l)}(x)$$

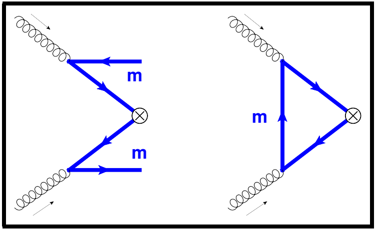
us $\mu \sim \mathcal{T}$:

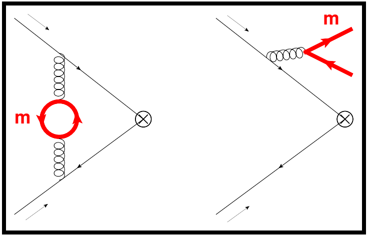
hemisphere soft function $S^{(n_l)}(\ell)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l)} \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\mathcal{T}}{Q}\right)$$

Massive Quarks

primary and secondary massive quarks.



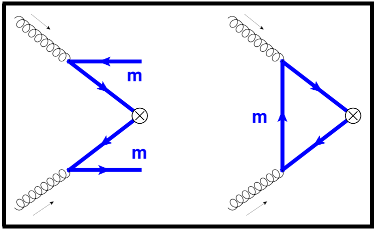


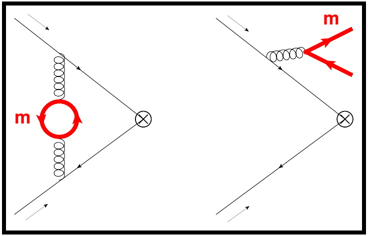
- $\mathcal{O}\left(\alpha_s^2\right)$ contributions relevant for NNLL′ resummation
- rapidity logarithms due to (secondary) massive quarks
- secondary massive quarks can contribute to all components: H, B_q, S
- heavy flavor beam function (PDF) for primary massive quarks for $m \lesssim \mu_B$: $B_Q\,(f_Q)$

$$\mu_B \sim p_T \qquad \mu_B \sim \sqrt{Q\mathcal{T}}$$

Massive Quarks

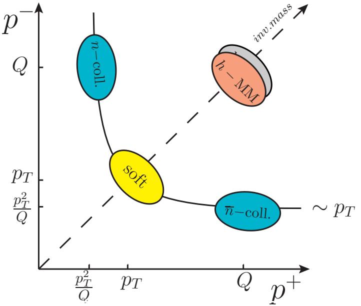
primary and secondary massive quarks.





- introduce new mass-modes: fluctuations around the the mass shell
- integrate out mass-modes at their natural scale $\mu \sim m$
  $\Rightarrow$ additional mass dependent structures in the factorization theorem
- different hierarchies between the mass and the other scales possible
- first assume large hierarchies to derive factorization theorem
- include power corrections between the different theories if necessary

$m \sim Q$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

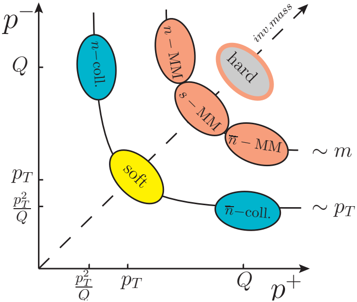
$\mu \sim Q \sim m$:

$$\left(J^\mu_{\rm QCD}\right)^{(n_l+1)} = C(Q, m) \times \left(J^\mu_{\rm SCET}\right)^{(n_l)}$$

hard function with contributions from primary and secondary massive quarks

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}(m) \times \left[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{p_T^2}{m^2}\right)$$

$Q \gg m \gg p_T$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

$\mu \sim Q$:

$C^{(n_l+1)}$
$\Rightarrow$ hard function with $(n_l + 1)$ massless flavors

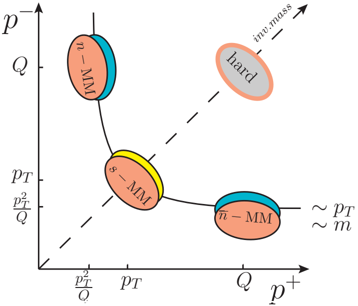
$\mu \sim m$:

$$(J^\mu_{\text{SCET}})^{(n_l+1)} = C_n(m) \times C_{\bar{n}}(m) \times C_s(m) \times (J^\mu_{\text{SCET}})^{(n_l)}$$

[S. Gritschacher, A. Hoang, I. Jemos, V. Mateu, P. Pietrulewicz (2014)]
[A. Hoang, P. Pietrulewicz, D.S. (2016)]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_n(m) \times H_{\bar{n}}(m) \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{p_T^2}{m^2}\right)$$

$m \sim p_T$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

$n-\mathrm{MM}$ $\mu \sim p_T \sim m$:

$$B_i^{(n_l+1)}(\vec{p}_T, x, m) = \sum_{k \in \{q,\bar{q},g\}} \mathcal{I}_{ik}(\vec{p}_T, x, m) \otimes f_k^{(n_l)}(x)$$

$i \in \{q, \bar{q}, Q, \bar{Q}\}$

one-loop primary massive: $\mathcal{I}_{Qg}(\vec{p}_T, x, m)$ new

two-loop secondary massive: $\mathcal{I}_{qq}(\vec{p}_T, x, m)$ new

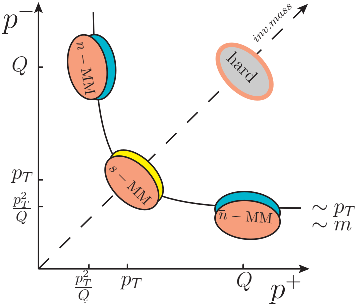
$s-\mathrm{MM}$ $\mu \sim p_T \sim m$:

$$S^{(n_l+1)}(\vec{p}_T, m)$$

two-loop secondary massive new

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2} \right)$$

$m \sim p_T$



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$\mu \sim p_T \sim m$:

$$B_i^{(n_l+1)}(\vec{p}_T, x, m) = \sum_{k\in\{q,\bar{q},g\}} \mathcal{I}_{ik}(\vec{p}_T, x, m) \otimes f_k^{(n_l)}(x)$$

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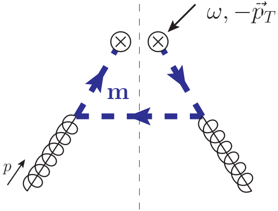

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\right]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

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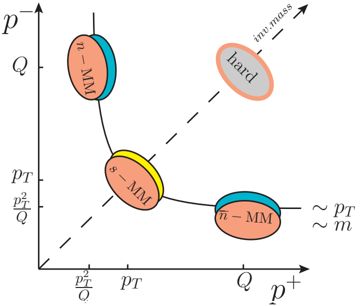

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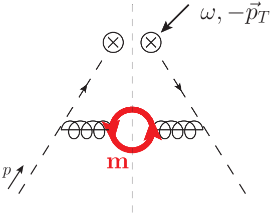

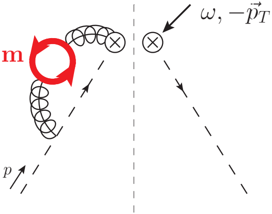

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\right]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

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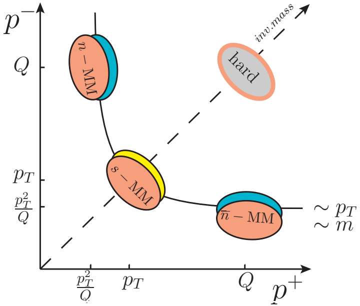

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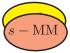

$\mu \sim p_T \sim m$:

$$S^{(n_l+1)}(\vec{p}_T, m)$$

two-loop secondary massive new

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\right]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

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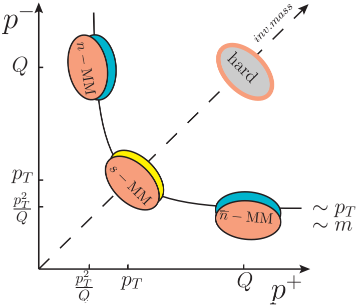

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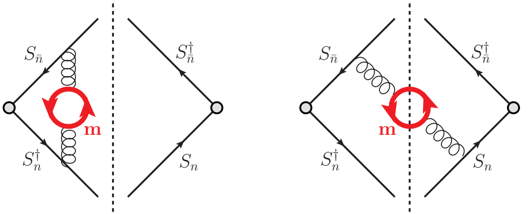
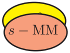




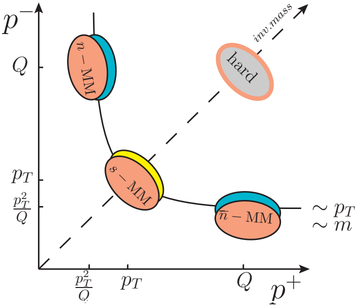
$\mu \sim p_T \sim m$:

$S^{(n_l+1)}(\vec{p}_T, m)$

two-loop secondary massive

new

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\right]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$



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$\mu \sim p_T \sim m$:

$$B_i^{(n_l+1)}(\vec{p}_T, x, m) = \sum_{k \in \{q,\bar{q},g\}} \mathcal{I}_{ik}(\vec{p}_T, x, m) \otimes f_k^{(n_l)}(x)$$

$i \in \{q, \bar{q}, Q, \bar{Q}\}$

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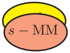
two-loop secondary massive: $\mathcal{I}_{qq}(\vec{p}_T, x, m)$ new



$\mu \sim p_T \sim m$:

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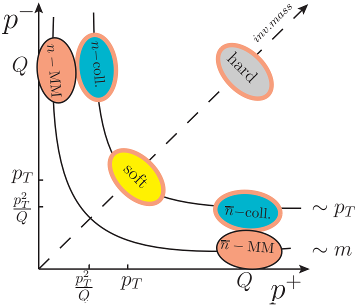
two-loop secondary massive new

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2} \right)$$

$m \ll p_T$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

$n-$coll. $\mu \sim p_T$:

$$B_i^{(n_l+1)}(\vec{p}_T, x) = \sum_{k\in\{q,\bar{q},Q,\bar{Q},g\}} \mathcal{I}_{ik}^{(n_l+1)}(\vec{p}_T, x) \otimes f_k^{(n_l+1)}(x)$$

beam function with $(n_l + 1)$ massless flavors

soft $\mu \sim p_T$:

$$S^{(n_l+1)}(\vec{p}_T)$$

TMD soft function with $(n_l + 1)$ massless flavors

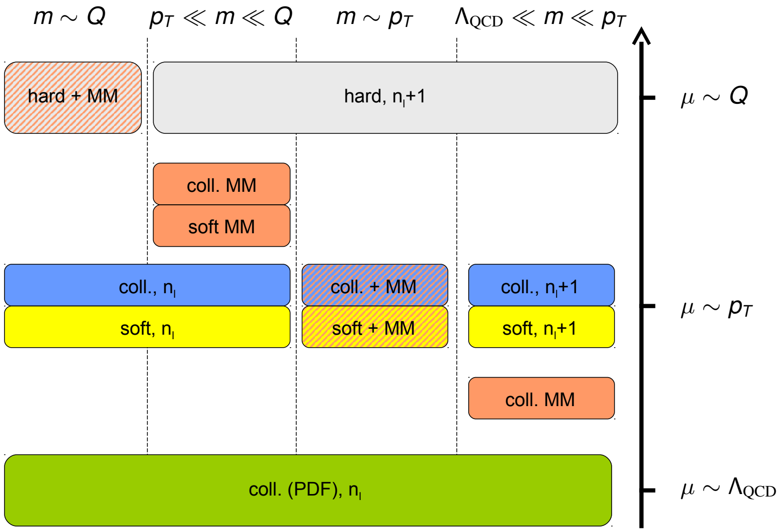
$n-$MM $\mu \sim m$:

$$\underset{i\,\in\,\{q,\,\bar{q},\,Q,\,\bar{Q}\}}{f_i^{(n_l+1)}(x, m)} = \sum_{k\in\{q,\bar{q},g\}} \mathcal{M}_{ik}(x, m) \otimes f_k^{(n_l)}(x)$$

one-loop primary massive: $\mathcal{M}_{Qg}(x, m)$
two-loop secondary massive: $\mathcal{M}_{qq}(x, m)$
[M. Buza, Y. Matiounine, J. Smith, W. van Neerven (1998)]

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{p_T^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

Summary of all Modes



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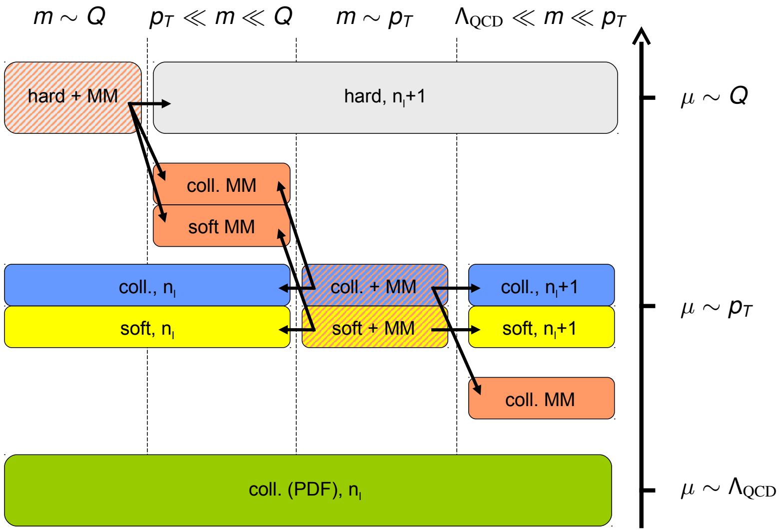


Relations between Hierarchies

components for the different hierarchies are related.

$$\mathcal{I}_{ik}(m) = \mathcal{I}_{ik}^{(n_l)} \times H_n(m) \times \left[1 + \mathcal{O}\left(\frac{|\vec{p}_T|^2}{m^2}\right)\right]$$

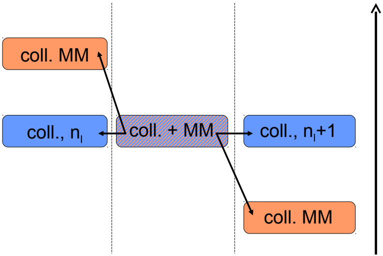
$$\mathcal{I}_{ik}(m) = \sum_{j\in\{q,\bar{q},Q,\bar{Q},g\}} \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \times \left[1 + \mathcal{O}\left(\frac{m^2}{|\vec{p}_T|^2}\right)\right]$$

$$S(m) = S^{(n_l)} \times H_s(m) \times \left[1 + \mathcal{O}\left(\frac{|\vec{p}_T|^2}{m^2}\right)\right]$$

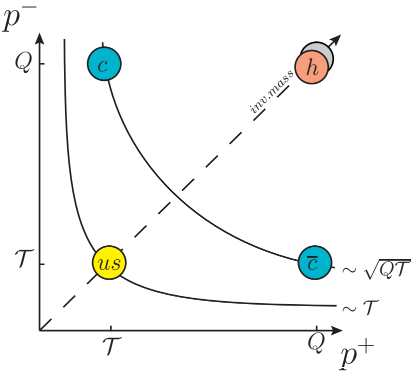
$$S(m) = S^{(n_l+1)} \times \left[1 + \mathcal{O}\left(\frac{m^2}{|\vec{p}_T|^2}\right)\right]$$

checked explicitely at two-loops.

can be used to systematically include all power corrections between the different theories.



$m \sim Q$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

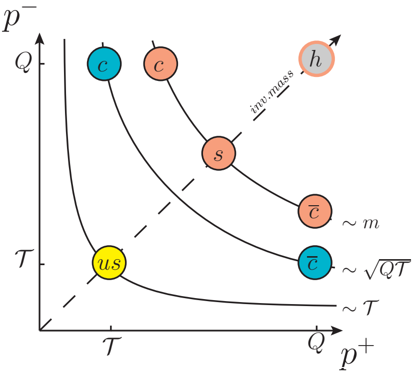
$\mu \sim Q \sim m$:

$$\left(J^\mu_{\mathrm{QCD}}\right)^{(n_l+1)} = C(Q,m) \times \left(J^\mu_{\mathrm{SCET}}\right)^{(n_l)}$$

hard function with contributions from primary and secondary massive quarks

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} \sim \sum_{i,j} H_{ij}(m) \times \left[\sum_k \mathcal{I}^{(n_l)}_{ik} \otimes f^{(n_l)}_k\right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{Q\mathcal{T}}{m^2}\right)$$

$Q \gg m \gg \sqrt{Q\mathcal{T}}$



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h $\mu \sim Q$:

$C^{(n_l+1)}$

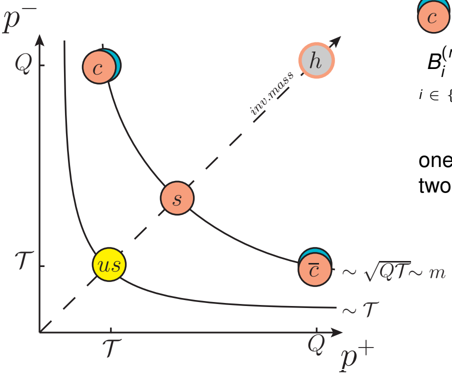
$\Rightarrow$ hard function with $(n_l + 1)$ massless flavors

c s $\mu \sim m$:

$$(J^{\mu}_{\mathrm{SCET}})^{(n_l+1)} = C_n(m) \times C_{\bar{n}}(m) \times C_s(m) \times (J^{\mu}_{\mathrm{SCET}})^{(n_l)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_n(m) \times H_{\bar{n}}(m) \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{Q\mathcal{T}}{m^2}\right)$$

$m \sim \sqrt{Q\mathcal{T}}$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

c $\mu \sim \sqrt{Q\mathcal{T}} \sim m$:

$$B_i^{(n_l+1)}(t,x,m) = \sum_{k \in \{q,\bar{q},g\}} \mathcal{I}_{ik}(t,x,m) \otimes f_k^{(n_l)}(x)$$

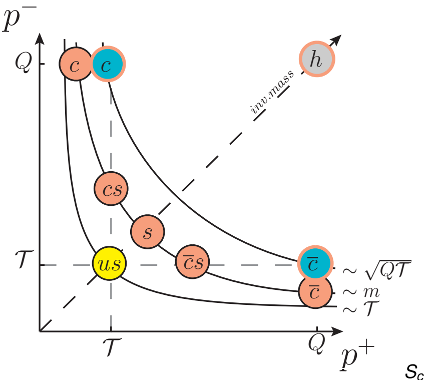
$i \in \{q, \bar{q}, Q, \bar{Q}\}$

one-loop primary massive: $\mathcal{I}_{Qg}(t,x,m)$ new

two-loop secondary massive: $\mathcal{I}_{qq}(t,x,m)$ new

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\mathcal{T}^2}{m^2}\right)$$

$\sqrt{Q\mathcal{T}} \gg m \gg \mathcal{T}$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

cs $\quad \mu \sim m$:

collinear-soft mode:

[C.W. Bauer, F.J. Tackmann, J.R. Walsh, S. Zuberi (2012)]
[M. Procura, W.J. Waalewijn, L. Zeune (2015)]

- SCET$_+$-type mode
- $(p^+, p^-, p^\perp) \sim (\mathcal{T}, \frac{m^2}{\mathcal{T}}, m)$

$\Rightarrow$ csoft function $S_c(\ell, m)$:

- matrix element of boosted Wilson lines
- two-loop secondary massive $\qquad$ new

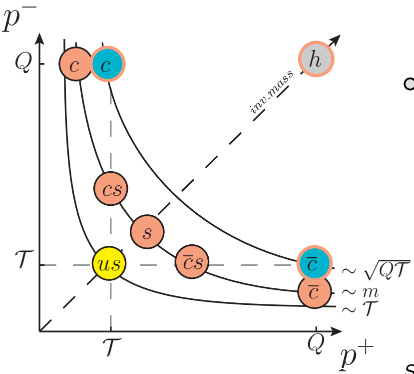
$$S_c(\ell, m) = \frac{1}{N_c} \langle 0 | \mathrm{Tr} \left[\overline{\mathbf{T}} \left(X_n^\dagger V_n(0) \right) \delta(\ell - \hat{p}^+) \mathbf{T} \left(V_n^\dagger X_n(0) \right) \right] |0\rangle$$

results with csoft function equivalent to threshold corrections in previous works

[S. Gritschacher, A. Hoang, I. Jemos, V. Mateu, P. Pietrulewicz (2014)], [A. Hoang, P. Pietrulewicz, D.S. (2016)]

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S_c(m) \otimes S^{(n_l)} \otimes S_{\bar{c}}(m)$$

$\sqrt{Q\mathcal{T}} \gg m \gg \mathcal{T}$



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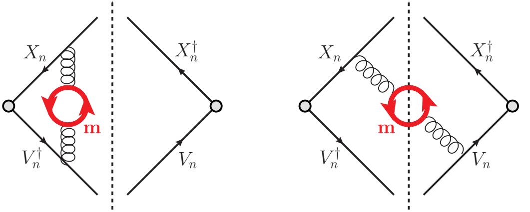

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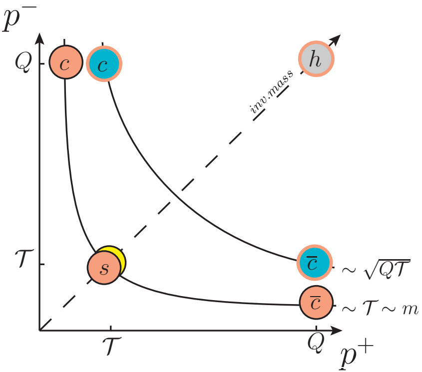
$m \sim \mathcal{T}$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

$\mu \sim \mathcal{T} \sim m$:

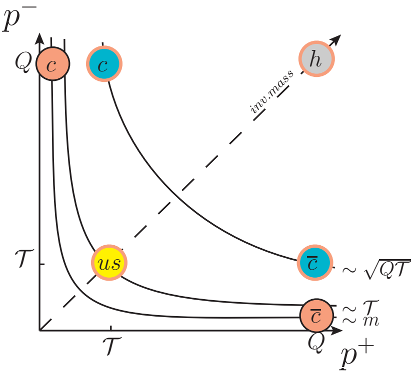
$S^{(n_l+1)}(\ell, m)$

two-loop secondary massive

[S. Gritschacher, A. Hoang, I. Jemos, P. Pietrulewicz (2014)]

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)}\right]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q\mathcal{T}}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

$m \ll \mathcal{T}$



(non-pert. n, $\bar{n}$-modes (PDF) not shown)

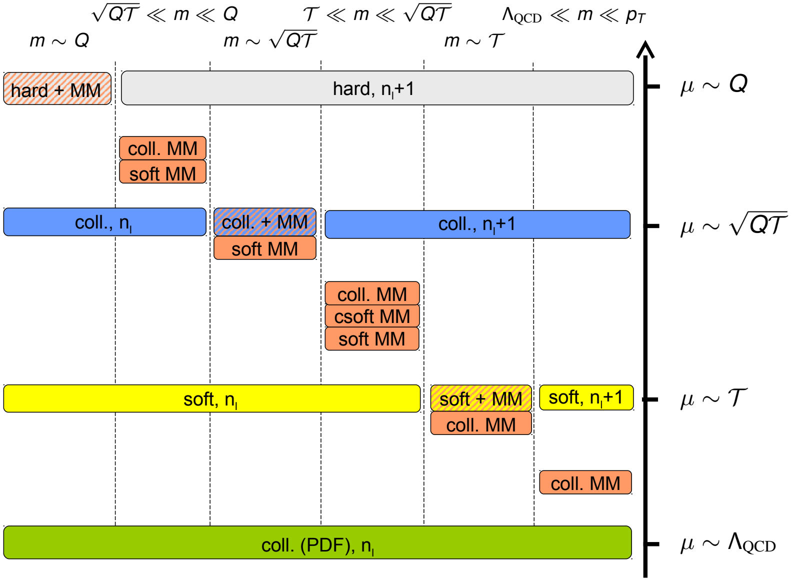
us $\mu \sim \mathcal{T}$:

$S^{(n_l+1)}(\ell)$

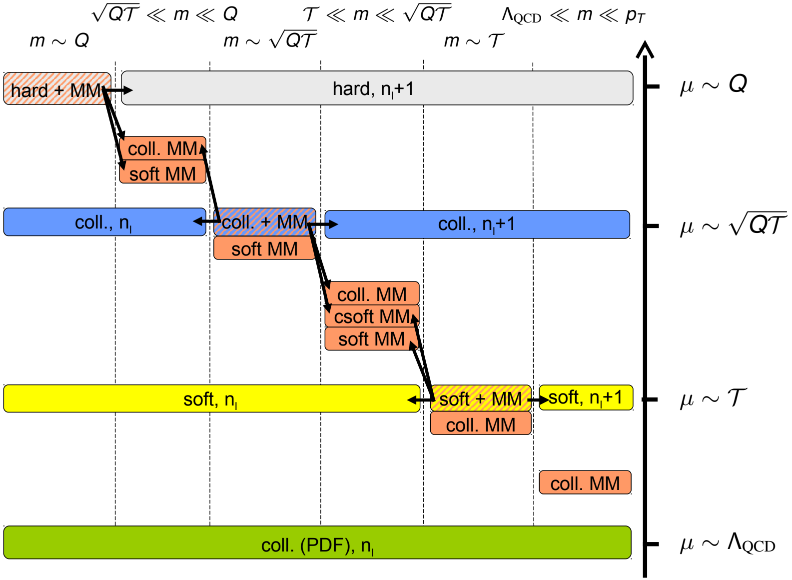
hemisphere soft function with $(n_l + 1)$ massless flavors

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)} \right]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{\mathcal{T}^2}, \frac{\Lambda_{\mathrm{QCD}}}{m^2} \right)$$

Summary of all Modes



Summary of all Modes



Relations between Hierarchies

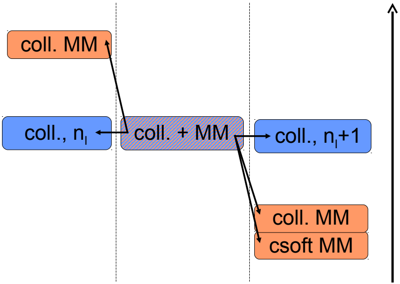
components for the different hierarchies are related.

$$\mathcal{I}_{ik}(m) = \mathcal{I}_{ik}^{(n_l)} \times H_n(m) \times \left[1 + \mathcal{O}\left(\frac{Q\mathcal{T}}{m^2}\right)\right]$$

$$\mathcal{I}_{ik}(m) = \sum_{j\in\{q,\bar{q},Q,\bar{Q},g\}} \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \otimes S_c(m) \times \left[1 + \mathcal{O}\left(\frac{m^2}{Q\mathcal{T}}\right)\right]$$

$$S(m) = S^{(n_l)} \otimes S_c(m) \times H_s(m) \times \left[1 + \mathcal{O}\left(\frac{\mathcal{T}^2}{m^2}\right)\right]$$

$$S(m) = S^{(n_l+1)} \times \left[1 + \mathcal{O}\left(\frac{m^2}{\mathcal{T}^2}\right)\right]$$



mass singularities absorbed also into the csoft function S_c.

checked explicitely at two-loops.

can be used to systematically include all power corrections between the different theories

Resummation of Logs from Massive Quarks

- logs of the form $\ln \frac{\mu_m}{\mu_i}$ with $i = H, B, S$ are resummed in the evolution of the matching factors

- e.g. hard function for $\mu_H \gg \mu_m \gg \mu_B, \quad \mu < \mu_m$:

$$U_H^{n_l+1}(\mu_H,\mu) \times \underbrace{U_{H_n}(\mu_m,\mu) \times U_{H_{\bar{n}}}(\mu_m,\mu) \times U_{H_S}(\mu_m,\mu)}_{U_H^{n_l}(\mu_m,\mu)\times\left(U_H^{n_l+1}(\mu_m,\mu)\right)^{-1}}$$

$$= U_H^{n_l+1}(\mu_H,\mu_m) \times U_H^{n_l}(\mu_m,\mu)$$

additional flavor in the running above μ_m resums $\ln \frac{\mu_m}{\mu_H}$.

- secondary massive quarks introduce new rapidity logarithms

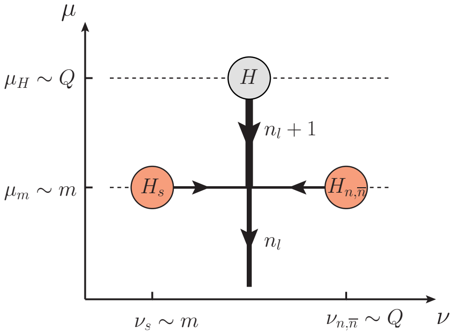
- rapidity logarithms resummed via rapidity RGE

[J.-Y. Chiu, A. Jain, D. Neill, I. Rothstein (2012)]

- e.g. hard matching functions H_i, $i = n, \bar{n}, s$:

$$H_i(m,\nu) = V_{H_i}(\nu,\nu_i) \times H_i(m,\nu_i)$$

[A. Hoang, A. Pathak, P. Pietrulewicz, I. Stewart (2015)]



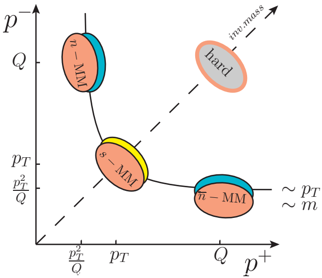
Resummation of Rapidity Logarithms

- solution of rapidity RGE straight forward for local matching functions and $\mathcal{T}$ dependent functions.

- problems for TMD beam and soft function (mass effects if $m \sim p_T$) without proper choice of scales.

- solve rapidity RGE in impact parameter space:

$$V(\vec{p}_T, \mu, \nu, \nu_i) = \int \mathrm{d}^2\vec{b}\, \mathrm{e}^{i\vec{b}\cdot\vec{p}_T} \left(\frac{\nu}{\nu_i}\right)^{\tilde{\gamma}_\nu(b,m,\mu)}$$



- logs of μ in γ_ν need to be resummed. (see Markus' talk)

$$\tilde{\gamma}_\nu(b, m, \mu) = \int_{\ln\mu_0}^{\ln\mu} \mathrm{d}\ln\mu' \,\frac{\mathrm{d}\tilde{\gamma}_\mu(b,\mu',\nu)}{\mathrm{d}\ln\nu} + \tilde{\gamma}_\nu^{\mathrm{FO}}(b, m, \mu_0)$$

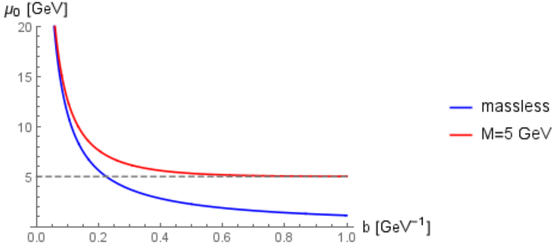
- choose μ_0 such that logs in $\tilde{\gamma}_\nu^{\mathrm{FO}}$ are minimized.

Resummation of γ_ν

soft function with massive gauge boson:

$$\text{massless: } \tilde{\gamma}_\nu^{\text{FO}}(b,\mu_0) = -\frac{\alpha_s(\mu_0)C_F}{2\pi^3}\ln\left(\frac{b^2\mu_0^2 e^{2\gamma_E}}{4}\right) \qquad \Rightarrow \mu_0 \sim \frac{2e^{-\gamma_E}}{b}$$

$$\text{massive: } \tilde{\gamma}_\nu^{\text{FO}}(b,M,\mu_0) = \frac{\alpha_s(\mu_0)C_F}{2\pi^3}\left(\ln\frac{M^2}{\mu_0^2} + 2K_0(b\,M)\right) \qquad \Rightarrow \mu_0 \sim M e^{K_0(b\,M)}$$



$$\mu_0(M, b\to 0) \to \frac{2e^{-\gamma_E}}{b}$$
$$\mu_0(M, b\to \infty) \to M$$

mass introduces IR cutoff

no Landau pole for $b \to \infty$

similar behavior for effects of secondary massive quarks.

correct scheme choice for α_s required in the two limits $b \to 0$, $b \to \infty$.

Outlook and Conclusions

- included massive quarks into the factorization theorem for Drell-Yan + 0 jets for a $\mathrm{SCET_I}$ and a $\mathrm{SCET_{II}}$ jet veto

- showed how structure of rapidity logarithms changes for massive quarks

- resummation of all mass related logarithms at NNLL$'$ accuracy (one and two loop matrix elements with massive quarks)

- parts of this also relevant for other processes, e.g. $b\bar{b}H$ production

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- parts of this also relevant for other processes, e.g. $b\bar{b}H$ production

Thank you for your attention!

Rapidity Anomalous Dimension

TMD soft function rapidity anomalous dimensions at 1-loop:

massless:

$$\gamma_\nu(\vec{p}_T, \mu) = \frac{\alpha_s(\mu) C_F}{4\pi} \times 16\, \mathcal{L}_0(\vec{p}_T, \mu) \qquad \mathcal{L}_0(\vec{p}_T, \mu) = \frac{1}{2\pi\mu^2} \left[\frac{\mu^2}{p_T^2}\right]_+$$

massive gluon:

$$\gamma_\nu(\vec{p}_T, M, \mu) = \frac{\alpha_s(\mu) C_F}{4\pi} \times 8 \left[\delta^{(2)}(\vec{p}_T) \ln\frac{M^2}{\mu^2} + \frac{1}{\pi\left(p_T^2 + M^2\right)}\right]$$