Structure of transverse momentum dependent (TMD) distributions at NNLO

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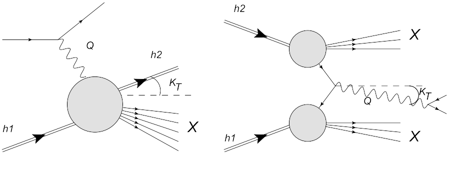
There are three main processes for TMD factorization: DY,SIDIS and $e^+e^- \to hadrons$

- Simultaneous description of all these processes involves both TMD PDF and TMD FF.
- Modern phenomenology fails to describe all processes with needed accuracy using same TMDs.
- TMD PDFs are known up to NNLO [Catani et al,12][Gehrmann et al,14]
- TMD FF known up NLO only (gluon part unknown even at this level)



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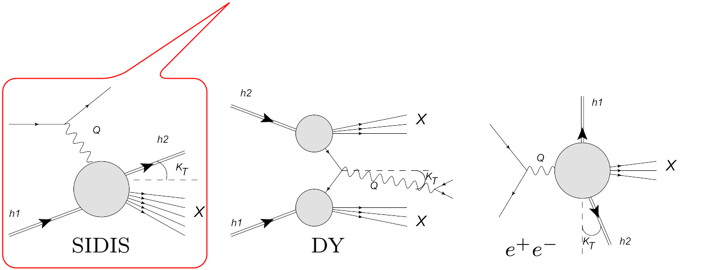
TMD factorization

Hadronic tensor is alike for all processes. We consider SIDIS

$$d\sigma \sim \int d^4x\, e^{iqx} \sum_X \langle h_1|J^\mu(x)|X;h_2\rangle\langle X;h_2|J^\nu(0)|h_1\rangle$$



TMD factorization

Applying TMD factorization ($Q^2 \gg q_T^2$) we factorize the cross-section

$$d\sigma \sim \int d^4x \, e^{iqx} \sum_X \langle h_1 | J^\mu(x) | X; h_2 \rangle \langle X; h_2 | J^\nu(0) | h_1 \rangle$$

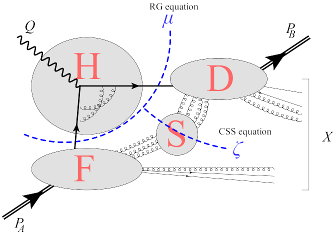
TMD factorization

TMD soft factor (very singular)

$$d\sigma \sim \int d^2 b_T \, e^{-i(qb)_T} H(Q^2) \;\; \Phi_{h1}(z_1, b_T) S(b_T) \Delta_{h_2}(z_2, b_T) + Y$$

TMD FF (singular)

TMD PDF (singular)



It is not complete factorization
due to Soft Factor
that mixes singularities
of PDF and FF

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TMD factorization

At $Q^2 \gg q_T^2 \gg \Lambda_{QCD}^2$, collinear factorization allows to recombine singularities

$$d\sigma \sim \int d^2 b_T\, e^{-i(qb)_T} H(Q^2)\;\; \Phi_{h1}(z_1, b_T) S(b_T) \Delta_{h_2}(z_2, b_T) + Y$$

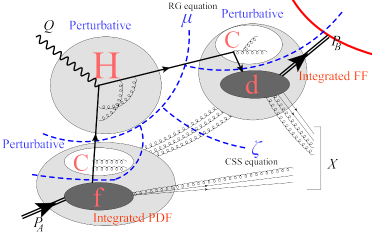
collinear factorization

integrated FF

$$d\sigma \sim \int d^2 b_T\, e^{-i(qb)_T} H(Q^2) \Big[q_{h1} \otimes C(b_T)\Big](z_1) \Big[d_{h2} \otimes \mathbb{C}(b_T)\Big](z_2)$$

TMD matching coef.
(regular)

integrated PDF



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TMD factorization

Splitting rapidity singularities individual TMD can be defined

$$d\sigma \sim \int d^2 b_T\, e^{-i(qb)_T} H(Q^2)\quad \Phi_{h1}(z_1, b_T) S(b_T) \Delta_{h_2}(z_2, b_T) + Y$$

spliting rapidity singularities
$S(b_T) \to \sqrt{S(b_T;\zeta)}\sqrt{S(b_T;\zeta^{-1})}$

$$d\sigma \sim \int d^2 b_T\, e^{-i(qb)_T} H(Q^2)\ F(z_1, b_T)\ D(z_2, b_T)$$

TMD PDF (regular)

TMD FF (regular)

TMD distributions are non-perturbative objects defined on the whole range of b_T.
At $b_T \to 0$ they can be expressed via integrated distributions

$$F(z, b_T) = \Big[C(b_T) \otimes q\Big](z)$$

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TMD factorization

In this way we come to the previous result

$$\int H(Q^2)\ \Phi_{h1}(z_1, b_T) S(b_T) \Delta_{h_2}(z_2, b_T) + Y$$

collinear factorization

spliting rapidity singularities

$$\int H(Q^2) \big[q \otimes C\big](z_1) \big[d \otimes \mathbb{C}\big](z_2)$$

$$\int H(Q^2)\ F(z_1, b_T)\ D(z_2, b_T)$$

small-b_T OPE

$$D(z, b_T) = \Big[\mathbb{C}(b_T) \otimes d\Big](z)$$

$$F(x, b_T) = \Big[C(b_T) \otimes q\Big](x)$$

TMD factorization

In this way we come to the previous result

$$\int H(Q^2)\ \Phi_{h1}(z_1,b_T)S(b_T)\Delta_{h_2}(z_2,b_T)+Y$$

collinear factorization

spliting rapidity singularities

$$\int H(Q^2)\big[q\otimes C\big](z_1)\big[d\otimes \mathbb{C}\big](z_2)$$

$$\int H(Q^2)\ F(z_1,b_T)\ D(z_2,b_T)$$

small-b_T OPE

$$D(z,b_T)=\Big[\mathbb{C}(b_T)\otimes d\Big](z)$$

$$F(x,b_T)=\Big[C(b_T)\otimes q\Big](x)$$

Formal definition of TMD operator

Operator for (unpolarized) TMD PDF

$$O_q^{bare}(x, b_T) = \frac{1}{2}\sum_X \int \frac{d\xi^-}{2\pi} e^{-ixp^+\xi^-} \left\{ T\left[\bar{q}_i\, \tilde{W}_n^T\right]_a \left(\frac{\xi}{2}\right) \, |X\rangle \gamma^+_{ij} \langle X| \, \bar{T}\left[\tilde{W}_n^{T\dagger} q_j\right]_a \left(-\frac{\xi}{2}\right)\right\},$$

Operator for (unpolarized) TMD FF

$$\mathbb{O}_q^{bare}(z, b_T) = \frac{1}{4zN_c}\int \frac{d\xi^-}{2\pi} e^{-ip^+\xi^-/z}$$

$$\langle 0|T\left[\tilde{W}_n^{T\dagger} q_j\right]_a \left(\frac{\xi}{2}\right) \sum_X |X, \frac{\delta}{\delta J}\rangle \gamma^+_{ij} \langle X, \frac{\delta}{\delta J}| \bar{T}\left[\bar{q}_i\, \tilde{W}_n^T\right]_a \left(-\frac{\xi}{2}\right)|0\rangle,$$

$$\xi = [0, \xi^-, \xi_T]$$

- W_n is Wilson line along n ($n^2 = 0$).
- Gluon operators are similar $O_g \sim T[F_{+\mu}W](\xi/2)\bar{T}[W^\dagger F_{+\mu}](-\xi/2)$.

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Formal definition of TMD operator

Applying these operators to the hadron states we obtain unsubtracted TMDs

$$\begin{aligned}\Phi_{q\leftarrow h}(x,b_T) &= \langle h|O_q^{bare}(x,b_T)|h\rangle \\ \Delta_{q\to h}(z,b_T) &= \langle h|\mathbb{O}_q^{bare}(z,b_T)|h\rangle\end{aligned}$$

To define individual TMD we have to take into account rapidity divergences, UV divergences and overlap regions

$$\begin{aligned}F_{q\leftarrow h}(x,b_T;\zeta,\mu) &= \sqrt{S(b_T;\zeta)}\langle h|Z_q(\mu)O_q^{bare}(x,b_T)|h\rangle\Big|_{zero-bin} \\ D_{q\to h}(x,b_T;\zeta,\mu) &= \sqrt{S(b_T;\zeta)}\langle h|Z_q(\mu)\mathbb{O}_q^{bare}(x,b_T)|h\rangle\Big|_{zero-bin}\end{aligned}$$

- μ is scale of UV renormalization.
- ζ is scale of rapidity-divergences separation.

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Formal definition of the TMD operator

In this way we come to the definition of TMD operator

$$O_q(x,b_T,\mu,\zeta)=Z_q(\zeta,\mu)R_q(\zeta,\mu)O_q^{bare}(x,b_T)$$
$$\mathbb{O}_q(z,b_T,\mu,\zeta)=Z_q(\zeta,\mu)R_q(\zeta,\mu)\mathbb{O}_q^{bare}(z,b_T),$$

Universal UV and rapidity renormalization constants

$R_q(\zeta,\mu)=\frac{\sqrt{S(b_T)}}{\text{zero-bin}}$
contains all IR divergences of operator

Z_q is UV renormalization const.

Similarly, one defines the gluon TMD operators

$$O_g(x,b_T,\mu,\zeta)=Z_g(\zeta,\mu)R_g(\zeta,\mu)O_g^{bare}(x,b_T),$$
$$\mathbb{O}_g(z,b_T,\mu,\zeta)=Z_g(\zeta,\mu)R_g(\zeta,\mu)\mathbb{O}_g^{bare}(z,b_T).$$

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Unlike usual operators, TMD operator has IR divergences, that cured by the multiplier R

$$R(\zeta,\mu)=\frac{\sqrt{S(b_T)}}{\text{zero-bin}}$$

Form of R is dependent on the rapidity regularization

Tilted WL's [Collins]

$$R_q(\zeta,\mu)=\frac{\sqrt{S(b_T;+\infty,y_s)}}{\sqrt{S(b_T;+\infty,-\infty)S(b_T;y_s,-\infty)}}$$

$$\zeta\sim m^2e^{-2y_s}$$

δ-regularization [EIS]

zero-bin coincides with soft-factor

$$R_q(\zeta,\mu)=\frac{1}{\sqrt{S(b_T;\alpha\delta^+,\delta^+)}}$$

$$\zeta\sim\alpha Q^2$$

Universality of R

- R is universal for different processes (thus, definition of TMD operator is process independent)
- R obeys Casimir scaling $\frac{R_q}{R_g}=\sqrt{\frac{C_A}{C_F}}$

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Modified δ-regularization scheme

"Old-fashion" δ-regularization

$$\delta-\text{regularization} \qquad \frac{1}{k^+ + i0} \to \frac{1}{k^+ + i\delta}$$

Such regularization does not suite the demands at higher pert.orders:

- Violates non-Abelian exponentiation
- Zero-bin $\neq$ soft factor

Both occur at NNLO.

Modified δ-regularization

Collinear WL's

$$TMDPDF: \quad P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)e^{-\delta\sigma x}\right)$$

$$TMDFF: \quad P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)e^{-\delta\sigma/z}\right)$$

Soft WL's

$$SF: \quad P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)e^{-\delta\sigma}\right)$$

Modified δ-regularization

$$\frac{1}{(k_1^+ + i0)(k_1^+ + k_2^+ + +i0)...(k_1^+ + ... + k_n^+ + i0)} \rightarrow$$
$$\frac{1}{(k_1^+ + i\delta)(k_1^+ + k_2^+ + 2i\delta)...(k_1^+ + ... + k_n^+ + ni\delta)}$$

Proc.

- Non-Abelian exponentiation is satisfied at all orders [AV,1501.03316].
- Factors x, z makes zero-bin be equal to soft-factor (explicitly checked at NNLO)
- At NLO there is no difference between usual and modified δ-regularization.

Cons.

- δ-regularization violates gauge properties of WL by power-suppressed in δ terms.

Only calculation at $\delta \to 0$ is legitimate.
Note: Be aware of power divergent integrals!

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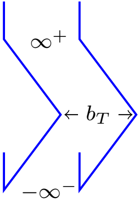
Soft factor

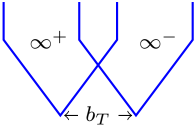


SIDIS

$$S(b_T) = \langle 0|T\Big[W_{\bar n}(-\infty, b_T)W_n(b_T,\infty)\Big]\bar T\Big[W_n^\dagger(\infty,0)W_{\bar n}^\dagger(0,-\infty)\Big]|0\rangle$$

Soft factor is function of $\delta^+\delta^- = \boldsymbol{\delta}$ and $b_T^2 = \boldsymbol{B}$:

$$S(b_T) = \exp\Big(C_K\Big(a_s S^{[1]} + a_s^2 S^{[2]} + ...\Big)\Big)$$



DY

Singularities are presented in SF as

- $\frac{1}{\epsilon}$ from UV singularities and UV part of rapidity singularities
- $(\boldsymbol{\delta})^{-\epsilon}$ from collinear and rapidity singularities
- $\ln(\boldsymbol{\delta B})$ from IR part of rapidity singularities

The most important property of SF is that its logarithm is linear in $\ln(\delta^+\delta^-)$

$$S(b_T) = \exp\left(A(b_T,\epsilon)\ln(\delta^+\delta^-) + B(b_T,\epsilon)\right)$$

It allows to split rapidity divergences and define individual TMDs.

Linearity in $\ln(\boldsymbol{\delta})$

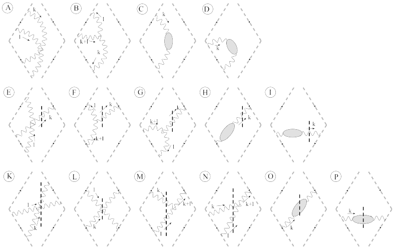
Generally (say at NNLO) one expects the following form (finite ϵ)

$$S^{[2]} = \underbrace{A_1\boldsymbol{\delta}^{-2\epsilon} + A_2\boldsymbol{\delta}^{-\epsilon}\boldsymbol{B}^{\epsilon} + \boldsymbol{B}^{2\epsilon}\Big(A_3\ln^2(\boldsymbol{\delta}B)}_{\text{cancel in sum of diagram}} + A_4\ln(\boldsymbol{\delta}B) + A_5\Big)$$

Proof

- A_1 should cancel since $\lim_{b_T\to 0} S^{[2]} = 0$ (modified δ-regularization supports!)
- A_2 should cancel since $\lim_{b_T\to 0} S^{[2]} = 0$ at $\delta = \delta b_T$ (mod. δ-regularization supports!)
- A_3 cancels due to Ward identity (alike leading UV pole for cusp)

These arguments work at all orders.



Result for Soft Factor [Echevarria,Scimemi,AV,1511.05590]

- Soft factor has been evaluated at NNLO at fixed(positive) ϵ
- All cancellations shown explicitly
- Depends only on $|\boldsymbol{\delta}|$, process independent.

$$
\begin{aligned}
S^{[2]} \quad = \quad & \Big[d^{(2,2)} \left(\frac{3}{\epsilon^3} + \frac{2\mathbf{l}_\delta}{\epsilon^2} + \frac{\pi^2}{6\epsilon} + \frac{4}{3}\mathbf{L}_\mu^3 - 2\mathbf{L}_\mu^2 \mathbf{l}_\delta + \frac{2\pi^2}{3}\mathbf{L}_\mu + \frac{14}{3}\zeta_3 \right) - \\
& d^{(2,1)} \left(\frac{1}{2\epsilon^2} + \frac{\mathbf{l}_\delta}{\epsilon} - \mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{l}_\delta - \frac{\pi^2}{4} \right) - d^{(2,0)} \left(\frac{1}{\epsilon} + 2\mathbf{l}_\delta \right) + \\
& C_A \left(\frac{\pi^2}{3} + 4\ \ln 2 \right) \left(\frac{1}{\epsilon^2} + \frac{2\mathbf{L}_\mu}{\epsilon} + 2\mathbf{L}_\mu^2 + \frac{\pi^2}{6} \right) + C_A \left(8\ \ln 2 - 9\zeta_3 \right) \left(\frac{1}{\epsilon} + 2\mathbf{L}_\mu \right) + \frac{656}{81} T_R N_f + \\
& C_A \left(-\frac{2428}{81} + 16\ \ln 2 - \frac{7\pi^4}{18} - 28\ \ln 2\, \zeta_3 + \frac{4}{3}\pi^2 \ln^2 2 - \frac{4}{3}\ \ln^4 2 - 32\mathrm{Li}_4\left(\frac{1}{2}\right) \right) + \mathcal{O}(\epsilon) \Big], \qquad (1)
\end{aligned}
$$

Small-b_T OPE

One can consider "transverse"-twist expansion of TMD at small-b_T

Coef. function (matching coef.)

$$O_q(x,b_T)=\sum_{n=0}^{\infty}\left(\frac{b_T^2}{B^2}\right)^n C^n_{q\to f}(x,\mathbf{L}_\mu;\mu,\zeta)\otimes O_{n,f}(x)$$

$$\boxed{\int e^{ixp\xi}T[\bar q W^\dagger](\xi,b_T)\bar T[Wq](0)}$$

$$\boxed{\int e^{ixp\xi}T[\bar q W^\dagger](\xi)(\overleftrightarrow{\partial}_T B)^n\bar T[Wq](0)}$$

Some unknown parameter
(character size)

Small-b_T OPE

One can consider "transverse"-twist expansion of TMD at small-b_T

Coef. function (matching coef.)

$$O_q(x,b_T) = \sum_{n=0}^{\infty}\left(\frac{b_T^2}{B^2}\right)^n C^n_{q\to f}(x,\mathbf{L}_\mu;\mu,\zeta)\otimes O_{n,f}(x)$$

Matrix elemnent over hadron states

TMD PDF

PDF (of higher twists)

$$F_{q\leftarrow h}(x,b_T;\mu,\zeta) = \sum_{n=0}^{\infty}\left(\frac{b_T^2}{B^2}\right)^n C^n_{q\leftarrow f}(x,\mathbf{L}_\mu;\mu,\zeta)\otimes f^n_{f\leftarrow h}(x)$$

- At $n=0$ f^0 is usual integrated PDF
- FF kinematics is analogous, but with overall factor $z^{-2+2\epsilon}$ (Collins normalization)

Evaluation of partonic TMD

- Simplest way to find (leading) coefficient function is to calculate partonic matrix element.
- For $n = 0$ we can set parton on-mass-schell $p^2 = 0$.

$$D^{[0]} = \Delta^{[0]},$$
$$D^{[1]} = \Delta^{[1]} - \frac{S^{[1]}\Delta^{[0]}}{2} + \left(Z_q^{[1]} - Z_2^{[1]}\right)\Delta^{[0]},$$
$$D^{[2]} = \Delta^{[2]} - \frac{S^{[1]}\Delta^{[1]}}{2} + \frac{3S^{[1]}S^{[1]}\Delta^{[0]}}{8} - \frac{S^{[2]}\Delta^{[0]}}{2} + \left(Z_D^{[1]} - Z_2^{[1]}\right)\left(\Delta^{[1]} - \frac{S^{[1]}\Delta^{[0]}}{2}\right)$$
$$+ \left(Z_D^{[2]} - Z_2^{[2]} - Z_2^{[1]}Z_D^{[1]} + Z_2^{[1]}Z_2^{[1]}\right)\Delta^{[0]}.$$

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Evaluation of partonic TMD

- Simplest way to find (leading) coefficient function is to calculate partonic matrix element.
- For $n = 0$ we can set parton on-mass-schell $p^2 = 0$.

UV renormalization

$$D^{[0]} = \Delta^{[0]},$$

$$D^{[1]} = \Delta^{[1]} - \frac{S^{[1]}\Delta^{[0]}}{2} + \left(Z_q^{[1]} - Z_2^{[1]}\right)\Delta^{[0]},$$

$$D^{[2]} = \Delta^{[2]} - \frac{S^{[1]}\Delta^{[1]}}{2} + \frac{3S^{[1]}S^{[1]}\Delta^{[0]}}{8} - \frac{S^{[2]}\Delta^{[0]}}{2} + \left(Z_D^{[1]} - Z_2^{[1]}\right)\left(\Delta^{[1]} - \frac{S^{[1]}\Delta^{[0]}}{2}\right)$$
$$+ \left(Z_D^{[2]} - Z_2^{[2]} - Z_2^{[1]}Z_D^{[1]} + Z_2^{[1]}Z_2^{[1]}\right)\Delta^{[0]}.$$

Evaluation of partonic TMD

- Simplest way to find (leading) coefficient function is to calculate partonic matrix element.
- For $n = 0$ we can set parton on-mass-schell $p^2 = 0$.

UV renormalization

$$
\begin{aligned}
D^{[0]} &= \Delta^{[0]},\\
D^{[1]} &= \Delta^{[1]} - \frac{S^{[1]}\Delta^{[0]}}{2} + \left(Z_q^{[1]} - Z_2^{[1]}\right)\Delta^{[0]},\\
D^{[2]} &= \Delta^{[2]} - \frac{S^{[1]}\Delta^{[1]}}{2} + \frac{3S^{[1]}S^{[1]}\Delta^{[0]}}{8} - \frac{S^{[2]}\Delta^{[0]}}{2} + \left(Z_D^{[1]} - Z_2^{[1]}\right)\left(\Delta^{[1]} - \frac{S^{[1]}\Delta^{[0]}}{2}\right)\\
&\quad + \left(Z_D^{[2]} - Z_2^{[2]} - Z_2^{[1]}Z_D^{[1]} + Z_2^{[1]}Z_2^{[1]}\right)\Delta^{[0]}.
\end{aligned}
$$

rapidity renormalization

Evaluation of partonic TMD

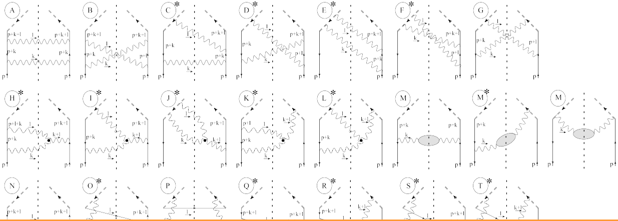
- Simplest way to find (leading) coefficient function is to calculate partonic matrix element.
- For $n = 0$ we can set parton on-mass-schell $p^2 = 0$.

UV renormalization

$$D^{[0]} = \Delta^{[0]},$$
$$D^{[1]} = \Delta^{[1]} - \frac{S^{[1]}\Delta^{[0]}}{2} + \left(Z_q^{[1]} - Z_2^{[1]}\right)\Delta^{[0]},$$
$$D^{[2]} = \Delta^{[2]} - \boxed{\frac{S^{[1]}\Delta^{[1]}}{2}\ \text{MIX}} + \frac{3S^{[1]}S^{[1]}\Delta^{[0]}}{8} - \frac{S^{[2]}\Delta^{[0]}}{2} + \left(Z_D^{[1]} - Z_2^{[1]}\right)\left(\Delta^{[1]} - \frac{S^{[1]}\Delta^{[0]}}{2}\right)$$
$$+ \left(Z_D^{[2]} - Z_2^{[2]} - Z_2^{[1]}Z_D^{[1]} + Z_2^{[1]}Z_2^{[1]}\right)\Delta^{[0]}.$$

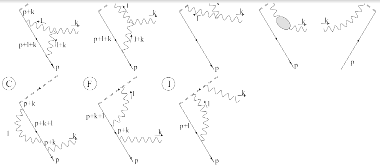
rapidity renormalization

All rapidity and UV divergences cancel
only collinear remains
(checked at NNLO)



We have evaluated all flavor-channels TMD PDF and TMD FF at NLO and NNLO.

- $\gtrsim 100$ non-zero diagrams
- ~ 20 basic integrals (all taken at finite ϵ)
- Algebra done by *Mathematica*
- Multiple checks performed (cancellation of IR divergences by topologies, Ward identities, RGEs)
- Anomalous dimensions, operator renormalization constants found



$$\left[\frac{F^{(a0)}_{0101}}{\bar\eta + i\delta}\right]_+ = \frac{\Gamma(-\epsilon)\Gamma(1-\epsilon)}{\Gamma(-2\epsilon)}\frac{x^a}{1-x}\left(\ln(-i\delta) - \ln\bar x + \psi_{-2\epsilon} + \gamma_E\right) \qquad (8.19)$$

$$\left[\frac{F^{(a0)}_{0101}}{\bar\eta + i\delta}\right]_\delta = \frac{\Gamma(-\epsilon)\Gamma(1-\epsilon)}{\Gamma(-2\epsilon)}\frac{1}{6}\Big[-3(\psi(1+a)+\psi(-2\epsilon))(\psi(1+a)+\psi(-2\epsilon)+2\ln(-i\delta)+4\gamma_E)$$
$$-3\ln(-i\delta)(4\gamma_E+\ln(i\delta))+3\psi'(1+a)+3\psi'(-2\epsilon)-12\gamma_E^2-2\pi^2\Big]$$

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Evaluation of coefficient coefficient

- Leading order are δ-function $\Longrightarrow$ coefficient functions from straightforward matching.
- **LO:** $C^{[0]}_{f\leftarrow f'} = \delta_{ff'}\delta(1-x)$, $\qquad \mathbb{C}^{[0]}_{f'\to f} = \delta_{ff'}\delta(1-z)$.
- **NLO:** $C^{[1]}_{f\leftarrow f'} = F^{[1]}_{f\leftarrow f'} - f^{[1]}_{f\leftarrow f'}$, $\qquad \mathbb{C}^{[1]}_{f\to f'} = D^{[1]}_{f'\to f} - \frac{d^{[1]}_{f'\to f}}{z^{2-2\epsilon}}$.
- **NNLO:**

$$
\begin{aligned}
C^{[2]}_{f\leftarrow f'} &= F^{[2]}_{f\leftarrow f'} - \sum_r C^{[1]}_{f\leftarrow r} \otimes f^{[1]}_{r\leftarrow f'} - f^{[2]}_{f\leftarrow f'},\\
\mathbb{C}^{[2]}_{f'\to f} &= D^{[2]}_{f'\to f} - \sum_r \mathbb{C}^{[1]}_{f\to r} \otimes \frac{d^{[1]}_{r\to f'}}{z^{2-2\epsilon}} - \frac{d^{[2]}_{f'\to f}}{z^{2-2\epsilon}}.
\end{aligned}
$$

Note: f and d are zero in our scheme. Thus, only UV counter remains

$$
f^{[1]}_{f\leftarrow f'} = \frac{-1}{\epsilon} P^{(1)}_{f\leftarrow f'}(x), \qquad f^{[2]}_{f\leftarrow f'} = \frac{-1}{2\epsilon}\left(\frac{P^{(1)}_{f\leftarrow r}\otimes P^{(1)}_{r\leftarrow f'}(x) + \beta_0 P^{(1)}_{f\leftarrow f'}(x)}{\epsilon} + P^{(1)}_{f\leftarrow f'}(x)\right)
$$

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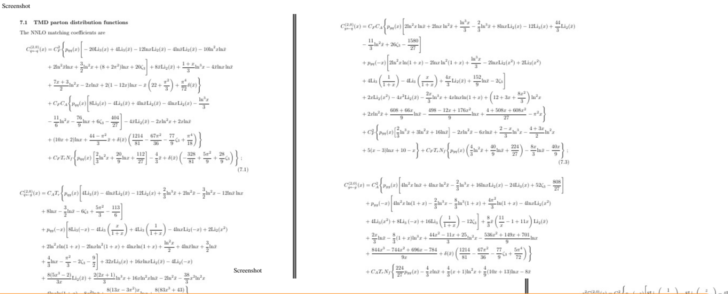
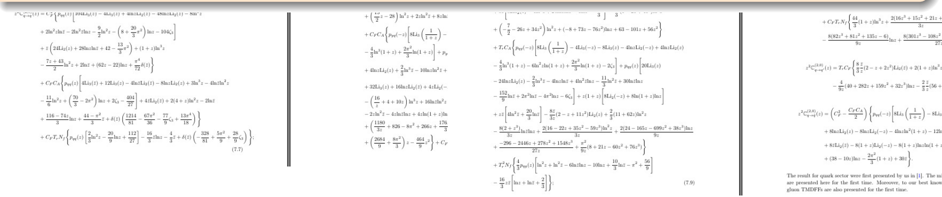


We have evaluated all flavor-channels TMD PDF and TMD FF and the their small-b_T matching coefficients at NLO and NNLO.

The expression are ready for phenomenology and will be published in the shortest time

see also [Echevarria,Scimemi,AV,1509.06392]



Crossing symmetry for TMD

TMD PDF vs. TMD FF operators

On level of unsubtracted TMDs the exact relation holds (at any order of pert.theory)

$$D_{f\to f'}(z) \quad = \quad -\frac{\mathcal{N}_{f,f'}}{z}\, F_{f\leftarrow f'}(z^{-1})$$

$$\mathcal{N}_{f,f'} = \frac{\#\text{physical states}_f}{\#\text{physical states}_{f'}}$$

This nice relation is significantly violated for matching coefficients

- ϵ-expansion and renormalization (choice of brunch for logs, factors of ζ_n)
- Extra factor from integrated FF normalization $\mathbb{O}(z, b_T) = z^{2-2\epsilon}\mathbb{O}(z)$ (while for TMD PDF $O(x, b_T) = O(x)$)

Finally: There are very little traces of crossing between FF and PDF

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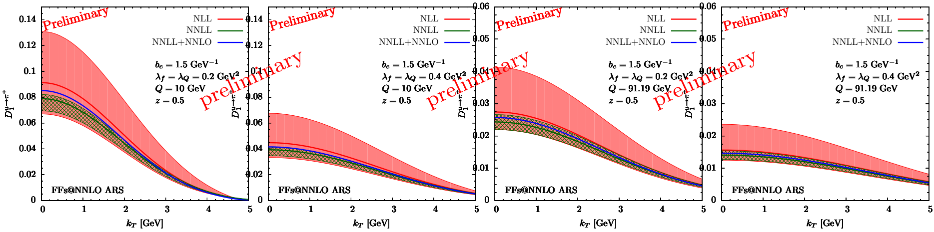
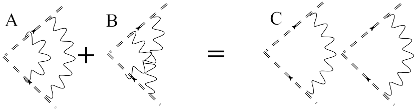
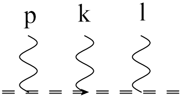
Conclusion



- Definition of TMD operators elaborated for PDF and FF kinematics
- UV and rapidity renormalization constants evaluated at NNLO (in modified δ-reg.scheme)
- Partonic TMD PDF and FF are evaluated at NNLO
- All matching coefficients are found at NNLO (for PDF coincide with [Catani at al,Gehrmann at al], for q/q TMD FF [Echevarria,Scememi,AV;1509.06392])
- Gluon TMD FF is considered for the first time
- Various properties and relations are discussed

Violation of exponentiation in δ-regularization



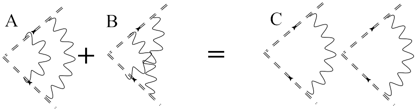


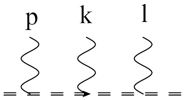
$$= \frac{1}{(p^+ + i\delta)(p^+ + k^+ + i\delta)(p^+ + k^+ + l^+ + i\delta)}$$

Within original δ-regularization, the exponentiation is broken

$$\mathrm{Diag}_A + \mathrm{Diag}_B = \frac{\mathrm{Diag}_C^2}{2} + \delta^+ \underbrace{\int \frac{d^dk}{k^2}\frac{d^dl}{l^2}\frac{1}{(k^+ + l^+)k^+ l^+ (k^- + l^-)k^-}}_{\frac{1}{\delta^+}\ \text{divergent}}$$

- That can result to artificial singularities in δ
- To incomplete cancellation of $\ln\delta$, that will cause problems at higher loops.





$$= \frac{1}{(p^+ + i\delta)(p^+ + k^+ + 2i\delta)(p^+ + k^+ + l^+ + 3i\delta)}$$

δ-regularization preserving exponentiation

The regularization should be implemented on the level of operator

$$P\exp\left[-ig\int_0^\infty d\sigma A_\pm(\sigma n)\right] \longrightarrow P\exp\left[-ig\int_0^\infty d\sigma A_\pm(\sigma n)e^{-\delta^\pm|\sigma|}\right]$$

Then exponentiation is exact

$$\text{Diag}_A + \text{Diag}_B = \frac{\text{Diag}_C^2}{2}$$

In any form, δ-regularization violate gauge-invariance linearly, beware of linearly divergent integrals.

- Is there any regularization with scale for light-like half-infinite Wilson lines without any problem?

Structure of anomalous dimensions

$$O_f(x,b_T) = \underbrace{Z_f(\mu,\zeta;\epsilon)}_{\to\gamma_V}\underbrace{R_f(\zeta;\epsilon,\delta)}_{\to\mathcal{D}}O^{bare}(x,b_T)$$

Anomalous dimension for CSS evolution

$$\boxed{\mathcal{D}^f = \frac{1}{2}\frac{dS}{d\mathbf{l}_\zeta} - \underbrace{\frac{dZ_f}{d\ln\zeta}}_{\sim\frac{1}{\epsilon}} = \frac{1}{2}\frac{dS}{d\mathbf{l}_\zeta}\Big|_{finite}}$$

$$\begin{aligned} S^{[2]} &= \Big[d^{(2,2)}\left(\frac{3}{\epsilon^3} + \frac{2\mathbf{l}_\delta}{\epsilon^2} + \frac{\pi^2}{6\epsilon} + \frac{4}{3}\mathbf{L}_\mu^3 - 2\mathbf{L}_\mu^2\mathbf{l}_\delta + \frac{2\pi^2}{3}\mathbf{L}_\mu + \frac{14}{3}\zeta_3\right) - \\ &\quad d^{(2,1)}\left(\frac{1}{2\epsilon^2} + \frac{\mathbf{l}_\delta}{\epsilon} - \mathbf{L}_\mu^2 + 2\mathbf{L}_\mu\mathbf{l}_\delta - \frac{\pi^2}{4}\right) - d^{(2,0)}\left(\frac{1}{\epsilon} + 2\mathbf{l}_\delta\right) + \dots. \end{aligned}$$

$$\Longrightarrow \mathcal{D}^{[2]} = d^{(2,2)}\ln^2\left(\frac{b_T^2\mu^2}{4e^{-2\gamma_E}}\right) + d^{(2,1)}\ln\left(\frac{b_T^2\mu^2}{4e^{-2\gamma_E}}\right) + d^{(2,0)}$$

$$d^{(2,2)} = \frac{\Gamma^{(0)}\beta_0}{4}, \qquad d^{(2,1)} = \frac{\Gamma^{(1)}}{2}, \qquad d^{(2,0)} = C_K\left(\left(\frac{404}{27} - 14\zeta_3\right)C + A - \frac{112}{27}T_rN_f\right)$$

$$O_f(x,b_T)=\underbrace{Z_f(\mu,\zeta;\epsilon)}_{\to\gamma_V}\underbrace{R_f(\zeta;\epsilon,\delta)}_{\to\mathcal{D}}O^{bare}(x,b_T)$$

TMD anomalous dimension

$$Z_f^{[1]}=\frac{-\Gamma^{[1]}}{2\epsilon^2}\left(1+\epsilon\mathbf{l}_\zeta\right)+Z_f^{[1]}+\frac{\gamma_V^{[1]f}}{2\epsilon}$$

$$Z_f^{[2]}=\frac{\Gamma^{[2]^2}}{8\epsilon^4}\left(1+2\epsilon\mathbf{l}_\zeta+\epsilon^2\mathbf{l}_\zeta^2\right)+...+Z_f^{[2]}+\frac{\gamma_V^{[2]f}}{4\epsilon}$$

$$Z_q^{[2]}=\frac{2C_F^2}{\epsilon^4}+.....+\frac{C_F}{\epsilon}\left[C_F\left(\pi^2-12\zeta_3\right)+C_A\left(-\frac{355}{27}-\frac{11\pi^2}{12}+13\zeta_3+\left(-\frac{67}{9}+\frac{\pi^2}{3}\right)\mathbf{l}_\zeta\right)+T_rN_f\left(\frac{92}{27}+\frac{\pi^2}{3}+\frac{20}{9}\mathbf{l}_\zeta\right)\right],$$

$$Z_g^{[2]}=\frac{2C_A^2}{\epsilon^4}+.....+\frac{C_A}{\epsilon}\left[C_A\left(-\frac{2147}{216}+\frac{11\pi^2}{36}+\zeta_3+\left(-\frac{67}{9}+\frac{\pi^2}{3}\right)\mathbf{l}_\zeta\right)+T_rN_f\left(\frac{121}{54}-\frac{\pi^2}{9}+\frac{20}{9}\mathbf{l}_\zeta\right)\right].$$

$$\Longrightarrow\gamma_V^{q(2)}=C_F^2\left(-3+4\pi^2-48\zeta_3\right)+C_FC_A\left(-\frac{961}{27}-\frac{11\pi^2}{3}+52\zeta_3\right)+C_FT_rN_f\left(\frac{260}{27}+\frac{4\pi^2}{3}\right),$$

$$\Longrightarrow\gamma_V^{g(2)}=C_A^2\left(-\frac{1384}{27}+\frac{11\pi^2}{9}+4\zeta_3\right)+C_AT_rN_f\left(\frac{512}{27}-\frac{4\pi^2}{9}\right)+8C_FT_rN_f.$$

RGE for TMD
and
coefficient functions

RGE for operators

$$\mu^2 \frac{d}{d\mu^2} O_f(x, b_T) = \frac{1}{2}\gamma_D^f(\mu,\zeta) O_f(x, b_T), \qquad \mu^2 \frac{d}{d\mu^2} \mathbb{O}_f(z, b_T) = \frac{1}{2}\gamma_D^f(\mu,\zeta) \mathbb{O}_f(z, b_T).$$

$$\zeta \frac{d}{d\zeta} O_f(x, b_T) = -\mathcal{D}^f(\mu, b_T) O_f(x, b_T), \qquad \zeta \frac{d}{d\zeta} \mathbb{O}_f(z, b_T) = -\mathcal{D}^f(\mu, b_T) \mathbb{O}_f(z, b_T).$$

RGE for coefficient functions

The ζ-dependance can be solved out from the functions

$$\begin{aligned} C_{f\leftarrow f'}(x, b_T; \mu, \zeta) &= \exp\left(-\mathcal{D}^f(\mu, b_T)\mathbf{L}_{\sqrt{\zeta}}\right) \hat{C}_{f\leftarrow f'}(x, \mathbf{L}_\mu) \\ \mathbb{C}_{f\rightarrow f'}(x, b_T; \mu, \zeta) &= \exp\left(-\mathcal{D}^f(\mu, b_T)\mathbf{L}_{\sqrt{\zeta}}\right) \hat{\mathbb{C}}_{f\rightarrow f'}(z, \mathbf{L}_\mu). \end{aligned}$$

RGE for operators

$$\mu^2 \frac{d}{d\mu^2} O_f(x, b_T) = \frac{1}{2} \gamma_D^f(\mu, \zeta) O_f(x, b_T), \qquad \mu^2 \frac{d}{d\mu^2} \mathbb{O}_f(z, b_T) = \frac{1}{2} \gamma_D^f(\mu, \zeta) \mathbb{O}_f(z, b_T).$$

$$\zeta \frac{d}{d\zeta} O_f(x, b_T) = -\mathcal{D}^f(\mu, b_T) O_f(x, b_T), \qquad \zeta \frac{d}{d\zeta} \mathbb{O}_f(z, b_T) = -\mathcal{D}^f(\mu, b_T) \mathbb{O}_f(z, b_T).$$

RGE for coefficient functions

The μ-dependence is given by equation

$$\mu^2 \frac{d}{d\mu^2} \hat{C}_{f \leftarrow f'}(x, \mathbf{L}_\mu) = \sum_r \hat{C}_{f \to r}(x, \mathbf{L}_\mu) \otimes K^f_{r \leftarrow f'}(x, \mathbf{L}_\mu),$$
$$\mu^2 \frac{d}{d\mu^2} \hat{\mathbb{C}}_{f \to f'}(z, \mathbf{L}_\mu) = \sum_r \hat{\mathbb{C}}_{f \to r}(z, \mathbf{L}_\mu) \otimes \mathbb{K}^f_{r \to f'}(z, \mathbf{L}_\mu).$$

The kernels $\mathbf{K}$ and $\mathbb{K}$ are

$$\begin{aligned} K^f_{r \leftarrow f'}(x, \mathbf{L}_\mu) &= \frac{\delta_{rf'}}{2} \left(\Gamma^f_{cusp} \mathbf{L}_\mu - \gamma^f_V \right) - P_{r \leftarrow f'}(x), \\ \mathbb{K}^f_{r \to f'}(z, \mathbf{L}_\mu) &= \frac{\delta_{rf'}}{2} \left(\Gamma^f_{cusp} \mathbf{L}_\mu - \gamma^f_V \right) - \frac{\mathbb{P}_{r \to f'}(z)}{z^2}. \end{aligned}$$