Transverse momentum spectra at NNLL'+NNLO with CuTe

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Outline

1. Framework
   - Resummation
   - Some technical points
2. Phenomenology
   - Confront with data
   - Conclusions

Observable

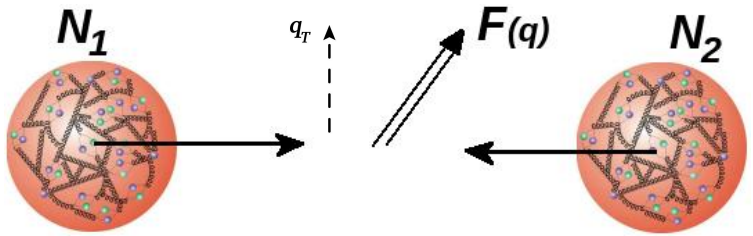
Consider:
$N_1 + N_2 \to F(q) + X$

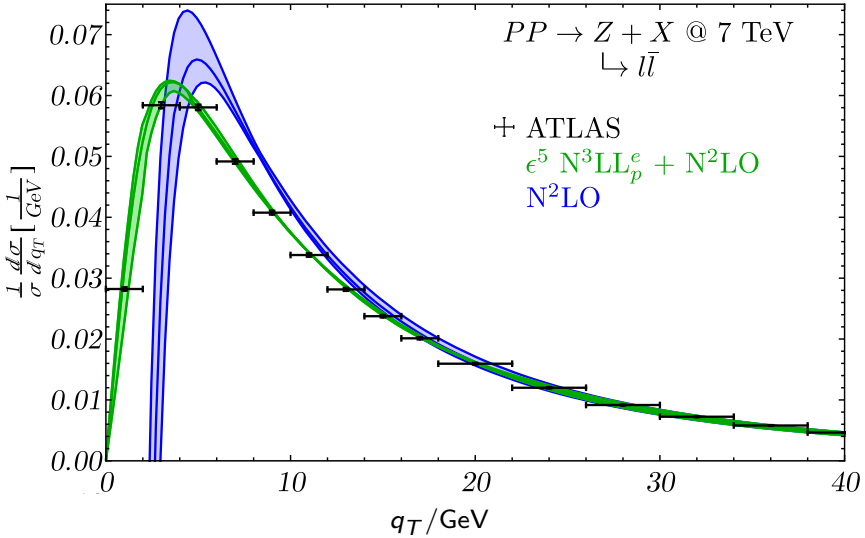
- $F = \gamma^*, Z, W, H, Z', \ldots$
- Test (B)SM to high precision.
- $d\sigma/dq_T$ in region $q_T^2 \ll M^2$.
  Large $\log q_T^2/M^2$.

$\Rightarrow$ Need to **resum** these.
Soft/collinear origin.

$\Rightarrow$ Transverse PDFs
(Beam functions).

$\Rightarrow$ F recoils against initial state radiation.





[Becher, TL, Neubert, Wilhelm]$_{\text{prg.}}$

$\frac{d\sigma}{dq_T dy}$ - counting a_s and L

$\lambda = q_T/M$, $L = \log\lambda$, $a_s = \frac{\alpha_s}{4\pi}$, PC= power correction.

$$\frac{d\sigma}{dq_T dy} = C(a_s)\exp\left[L g_1(a_s L) + g_2(a_s L) + a_s g_3(a_s L) + a_s^2 g_4(a_s L) + \dots\right] + \mathcal{O}(\lambda)\,.$$

When expand:

FO\RES		LL	NLL	NLL'	N^{2}LL	N^{2}LL'	N^{3}LL	...	PC	
LO	$a_s^0[$	1								]
NLO	$a_s^1[$	L^2	L^1	1					$\mathcal{O}(\lambda)$	]
N^{2}LO	$a_s^2[$	L^4	L^3	L^2	L^1	1			$\mathcal{O}(\lambda)$	]
N^{3}LO	$a_s^3[$	L^6	L^5	L^4	L^3	L^2	L^1	1	$\mathcal{O}(\lambda)$	]
		$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		

In this talk will discuss **CuTe**'s

- N^{3}LL$_p$ * resummation and
- N^{2}LO matching ($\Rightarrow$ recover PC).

* p=partial: lacking values of Γ_3^i and $F_i^{(3,0)}$.

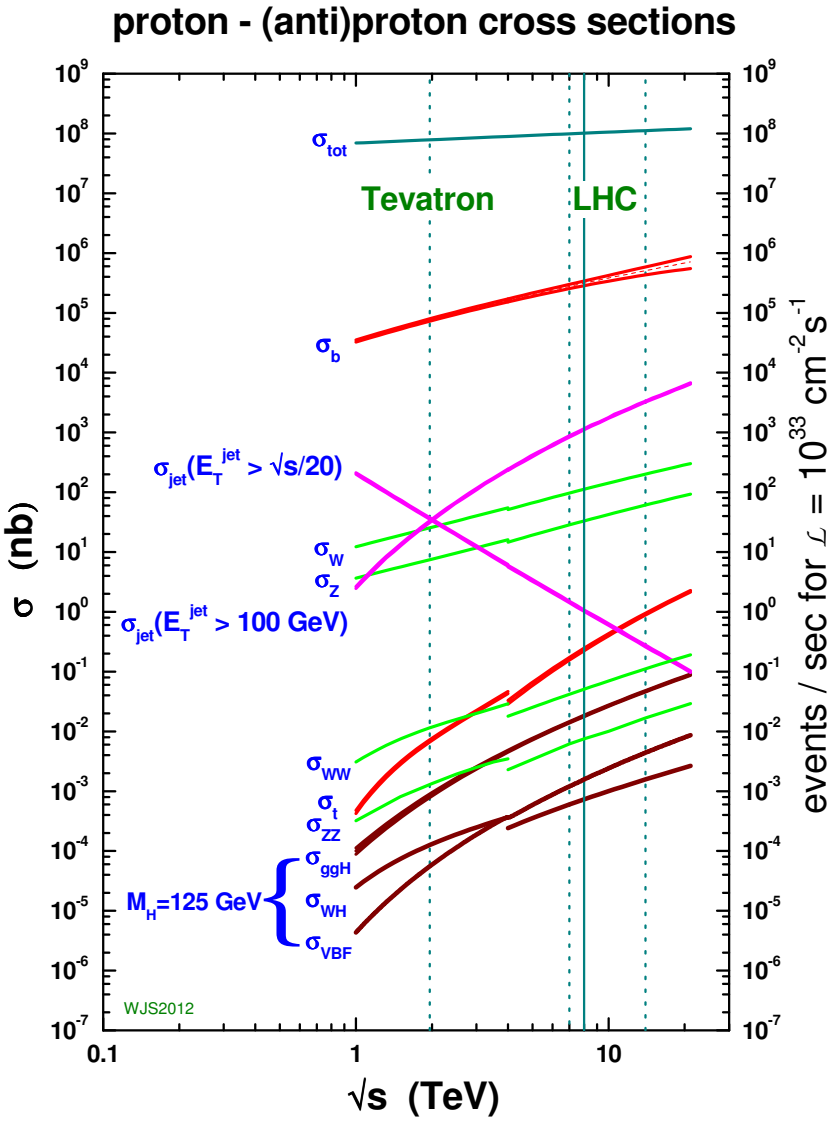
Selected tools for q_T resummation

Monte Carlo event generators: Sherpa, Herwig, Pythia, ...
Matched: Geneva [Alioli, Bauer, Berggren, Tackmann, Walsh].

Analytic resummation:

- CSS type: DYqT, DYRes, HRes [Catani, Cieri, de Florian, Ferrera, Grazzini], $\gamma\gamma$: [Cieri, Coradeschi, de Florian], ZZ, W^+W^-: [Grazzini, Kallweit, Rathlev, Wiesemann], ResBos [Balazs, Yuan] ...
- SCET: CuTe 1.0 [Becher, Neubert, Wilhelm], [Neill, Rothstein, Vaidya] and [Echevarria, Kasemets, Mulders, Pisano].
- Here: CuTe 2.0 [Becher, TL, Neubert, Wilhelm]$_{\text{prog.}}$. Currently available: γ^*, Z, W, H.
- Method can be applied to **all** other processes with color neutral final states: V, H, VV', HH, Z', ...
- For others to include F.O. part. VV' at fixed order see e.g. Mainz/Zürich/Karlsruhe-groups [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, v.Manteuffel, Pozzorini, Rathlev, Tancredi, Torre, Weihs], [Caola, Henn, Melnikov, Smirnov, Smirnov].

Cross sections



- Popular: H.
- σ much larger for V production.
- For Z/γ^* very clean final state.
- Test SM to high precision.

Factorization formula

- [Collins, Soper, Sterman], ..., [Becher, Neubert] for $\Lambda^2_{\rm QCD} \ll q_T^2 \ll M^2$:

$$\frac{d^2\sigma_c}{dq_T^2 dy} = \sigma_c^{(0)} \sum_{k,j} \int d^2x_T \, e^{iq_T\cdot x_T} \, \widetilde{C}_{c\leftarrow kj}(z_1,z_2,x_T^2,M^2,\mu) \otimes f_{k/N_1}(z_1,\mu) \otimes f_{j/N_2}(z_2,\mu) \, ,$$

- in impact parameter (x_T) space
- with **perturbative** ↙hard collinear&soft↘

$$\widetilde{C}_{c\leftarrow ij}(z_1,z_2,x_T^2,M^2,\mu) = |C_c(-M^2,\mu)|^2 \bar{I}_{i\leftarrow k}(z_1,L_\perp,a_s)\bar{I}_{\bar{i}\leftarrow j}(z_2,L_\perp,a_s) e^{g_i(M,x_T,\mu)} \, ,$$

- where $i(c) = q,\, g$; $L_\perp = \log \frac{x_T^2\mu^2}{b_0^2}$, $b_0 = 2e^{-\gamma_E}$, $a_s = \alpha_s(\mu)/4\pi$ and

$$g_i(M,x_T,\mu) = 2h_i(L_\perp,a_s) - F_i(L_\perp,a_s)\log\frac{x_T^2 M^2}{b_0^2} \, .$$

  Note: $F_i(L_\perp,a_s) = \gamma^i_{B,\nu}$ in RRG framework of [Chiu, Jain, Neill, Rothstein] → Joel Oredsson's talk.
- Each function depends on **single** physical scale ⇒ Safely determine pert..
- Solving **RGEs** (μ) ⇒ **resummation** of $L = \log(x_T^2 M^2/b_0^2)$.

In $\widetilde{C}$ suppressed sum over tensor components ($i(c) = g$) or quark charges ($i(c) = q$).

x_T integral and scale choice

- CuTe: Combine everything and numerically evaluate $\otimes f$, x_T, y integrals.

Perform $\int d^2x_T\, e^{iq_T\cdot x_T}\, \widetilde{C}_{c\leftarrow ij} = \int_0^\infty dx_T\, x_T J_0(x_T q_T)\widetilde{C}_c(z_1,z_2,x_T^2,M^2,\mu)\,.$

- Essentially two kind of logs: $L_M = \log M^2/\mu^2$ and $L_\perp = \log {x_T}^2\mu^2/b_0^2$.
- Aim: small $L_\perp$. $b_0 = 2e^{-\gamma_E}$
- $\mu_x = b_0/x_T$: Run into Landau pole.
- Moreover, prefer physical choice $\mu(q_T, M)$.
- $\boldsymbol{\mu = \mu_* = q_T + q_* \exp(-q_T/q_*)}$, $q_* = M_i/\exp\left(1/(2\Gamma_0^i a_s)\right)$
  $\Rightarrow \langle L_\perp\rangle$ small. Set $\mu_{c,s} = \mu$, $\mu_h = M$.
- Thanks to $x_T e^{g_i}$ $\rightarrow$Gaussian peak: determines q_*, width $\sim 1/\sqrt{a_s}$.
- **Power counting** for $\int dx_T$: $\mathbf{a_s \sim \epsilon^2}$, $\mathbf{L_M \sim \epsilon^{-2}}$, $\mathbf{L_\perp \sim \epsilon^{-1}}$
  Do up to ϵ^5.

Matching to fixed order

Restoring power (qT/M) suppressed contributions.

$$\frac{d\sigma^{\text{matched}}}{dq_T} = \frac{d\sigma^{\text{res}}}{dq_T} + \left.\frac{d\sigma^{\text{MC}}}{dq_T}\right|_{\text{MS}},$$

$$\left.\frac{d\sigma^{\text{MC}}}{dq_T}\right|_{\text{MS}} = R_{\text{ms}}\left(\frac{d\sigma^{\text{FO}}}{dq_T} - \left.\frac{d\sigma^{\text{res}}}{dq_T}\right|_{\text{expanded to FO}}\right)$$

with $H(-M^2,\mu_h,\mu) = U_H(\mu,\mu_h)\cdot|C_i(-M^2,\mu_h)|^2$ and

$$R_{\text{add}} = R_{\text{ms}}(\mu_h) = 1\,,$$

$$R_{\text{ms}}(\mu_*) = H(-M^2,\mu_h,\mu_*)\cdot H^{-1}(-M^2,\mu_*,\mu_*)\,,$$

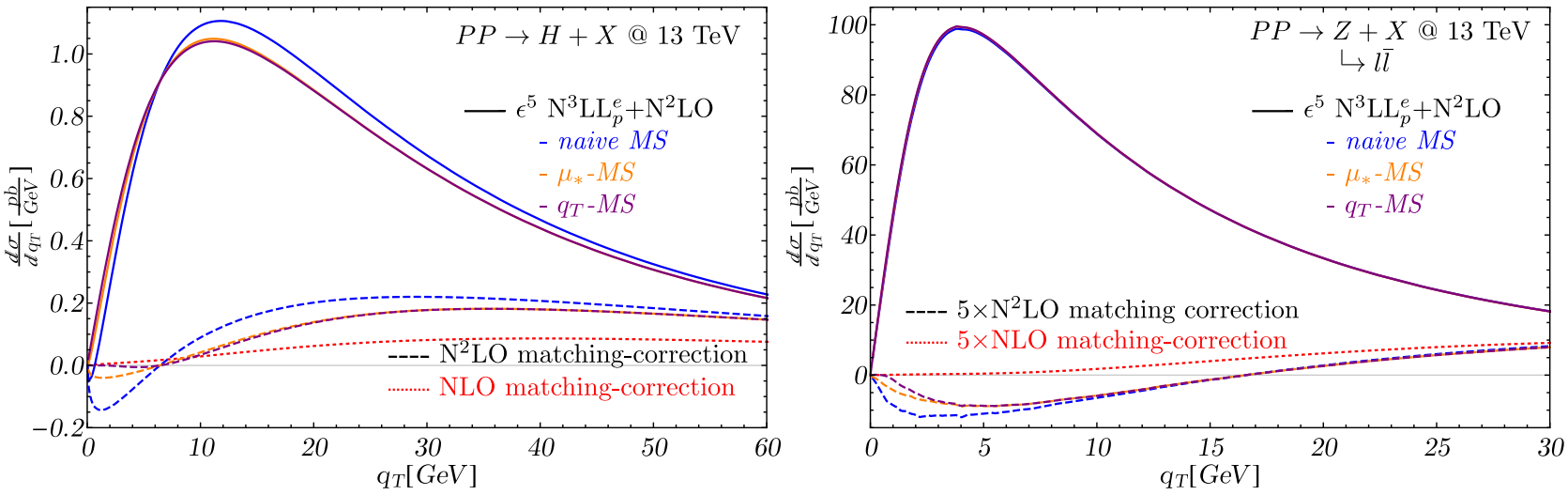
$$R_{\text{ms}}(q_T) = H(-M^2,\mu_h,q_T)\cdot H^{-1}(-M^2,q_T,q_T)\,.$$

$H(-M^2,\mu,\mu)$ corresponds to the FO expansion of resummed $H(-M^2,\mu_h,\mu)$.
$R_{\text{ms}} = 1 + \mathcal{O}(a_s^3)$ but **can supply Sudakov suppression** to $d\sigma^{\text{MC}}$.

Model our ignorance of power suppressed but log enhanced terms beyond NNLO.

FO results: V→[Gonsalves,Pawlowski,Wai], H→[Glosser,Schmid].

Matching schemes - numerical effects



- For Drell-Yan:
  - Small matching corrections.
  - Negligible matching scheme dependence.
- For H larger uncertainties
- Will use q_T-MS.

Non-perturbative effects

- Modeling leading NPE beyond $B_{i/N} = \sum_k I^{\rm pert}_{i/k} \otimes f_{k/N}$.
- TPDFs must vanish rapidly at $x_T > r_{\rm proton}$. Ansatz:

$$B_{i/N}(z, x_T^2, \mu) = f_{\rm hadr}(x_T \Lambda_{\rm NP})\, B^{\rm pert}_{i/N}(z, x_T^2, \mu) \ ,$$

- with

$$f^{\rm enhanced}_{\rm hadr}(x_T \Lambda_{\rm NP}) = \exp\left[-\Lambda_{\rm NP}^2\, x_T^2 \log(x_T^2 M^2/b_0^2)\right] \quad \text{[Becher, Bell]}\,.$$

- $\Lambda_{\rm NP} = 0$GeV: no correction.
- Effect: Suppression at very small q_T. Peak shifted.

CuTe: old vs new version

CuTe: handle all of this for γ^*, Z, W and H.

Now up to **N^{2}LO+N^{3}LL$_p$ precision**.

CuTe 1.1

- Public C^{++}-code.
- LHAPDF 5.
- N^{2}LL resummation.
- NLO matching.
- Power counting for very small q_T: $\mathcal{O}(\epsilon^2)$.

CuTe 2.0

- Not yet public.
- Linked to LHAPDF 6, Cuba.
- Full N^{3}LL implementation:
  - 2-loop beam-functions
  - Unknown Γ_3^i and $F_i^{(3,0)}$ $\Rightarrow$ N^{3}LL$_p$ (partial)
- N^{2}LO matching.
- $\mathcal{O}(\epsilon^5)$.
- Phase space cuts y, p_T^l, η^l . narrow width approx.

[Becher,Neubert,TL,Wilhelm]$_{\text{pgr.}}$

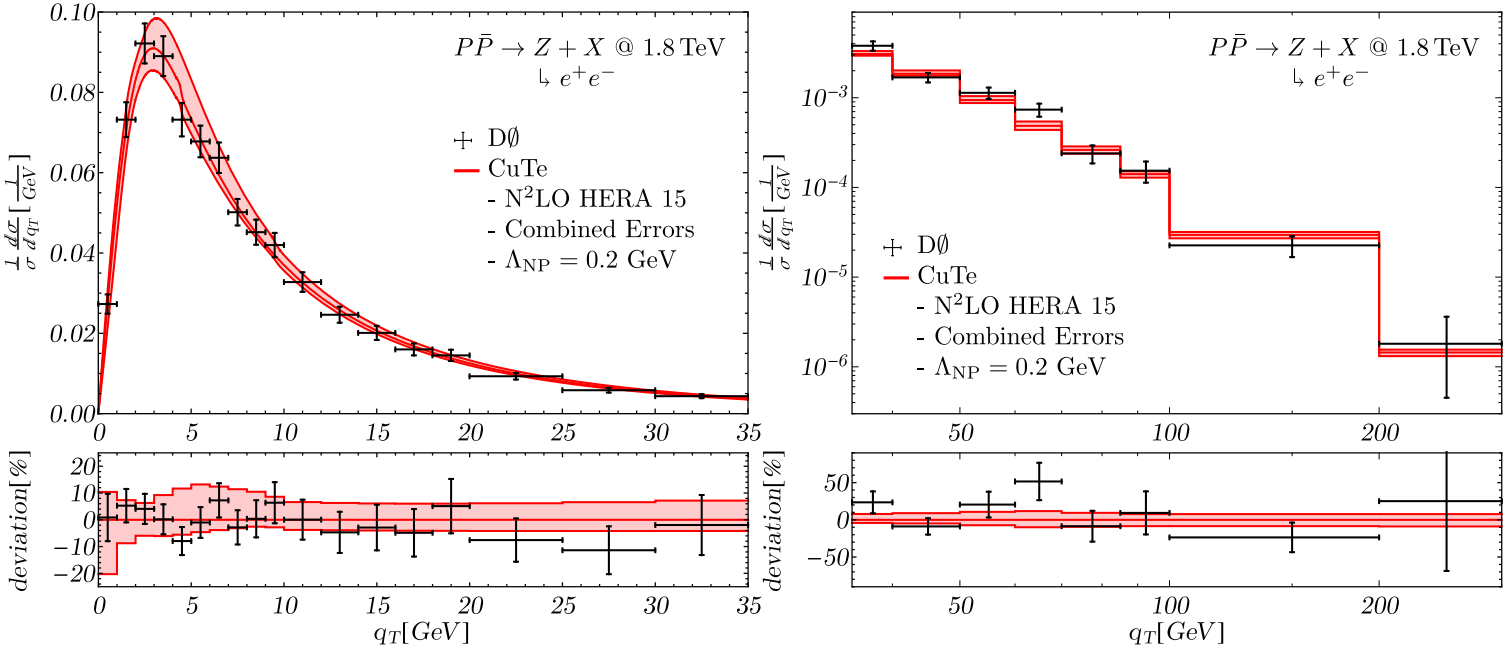
Phenomenology

Apply CuTe

Settings

- ϵ^5 N^{3}LL$_p$+N^{2}LO precision.
- $\mu = q_T + q_* \exp(-q_T/q_*)$, $q_* = M_i/\exp\left(1/2\Gamma_0^i a_s\right)$.
  $q_*^Z \sim 2GeV$, $q_*^H \sim 8GeV$.
- q_T matching scheme.
- Smooth transit to pure fixed order for q_T between $\frac{M_i}{2}$ and $\frac{3M_i}{4}$.
- HERA PDF 15 NNLO.
- π^2 resummation.
- Γ_3 Pade approximation.
- Uncertainty bands: vary μ by factor 2, $F^{(3,0)}$ between $\pm 2(4\pi)F^{(2,0)}$.

VS Z D0 data

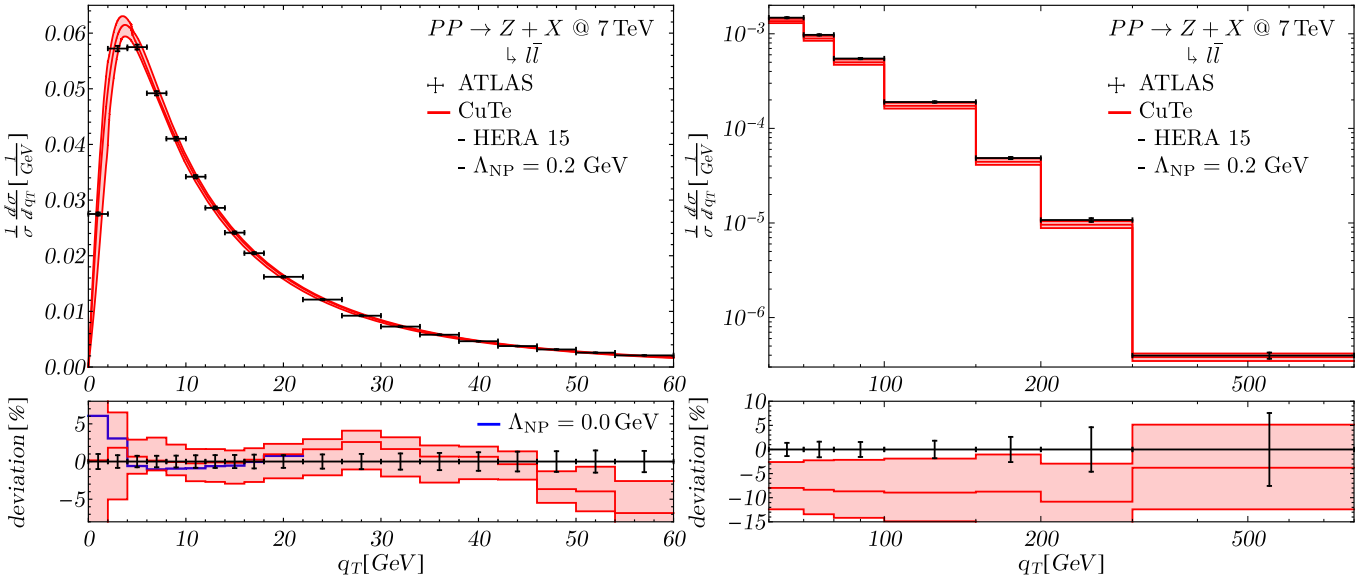


- Upper plots: left linear; right log-log.
  Lower plots: ratio. Experimental bins.

[Becher,TL,Neubert,Wilhelm]$_{\text{pgr.}}$

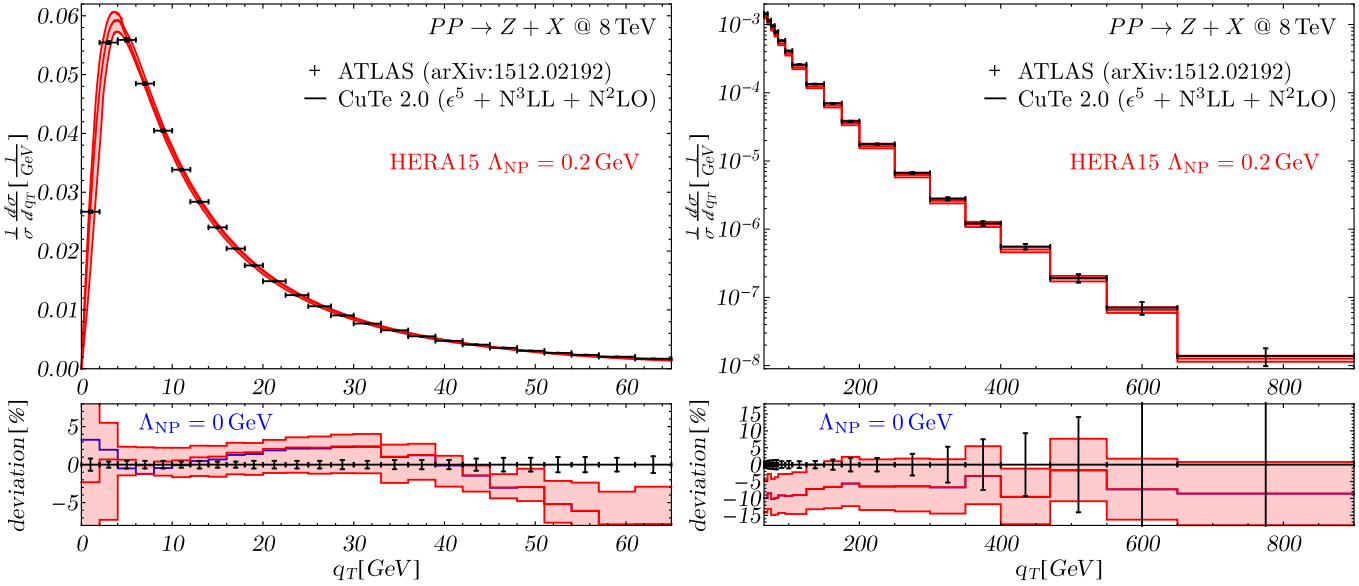
- Very good agreement with data:
  - D0, hep-ex/9909020, Z at 1.8TeV.
  - Cuts for $d\sigma_{\text{fiducial}}/dq_T$: $60 < M_{ll}/GeV < 120$,
    $p_{T,l} > 25\text{GeV}$, $|\eta_{l,1}| < 1.1$, $|\eta_{l,2}| < 2.4$, excluding $1.11 < |\eta_{l,2}| < 1.5$
- Effect of cuts: Normalization and shift peak to the right.
- In theory prediction, symmetrized lepton cuts.

VS Z ATLAS 7 TeV data



- Good agreement with data: [Becher,TL,Neubert,Wilhelm]$_{\text{pgr.}}$
  - ATLAS hep-ex/1406.3660 Z/γ^* 4.7 fb^{-1} at 7TeV.
  - Cuts for $d\sigma_{\text{fiducial}}/dq_T$: $66 < M_{ll}/GeV < 116$,
  - $p_{T,l} > 20\text{GeV}$, $|\eta_l| < 2.4$, excluding $1.37 < |\eta_l| < 1.52$
- Uncertainty band: μ & $F^{(3,0)}$ variation.
- Tail FO. On-shell approx. less accurate.

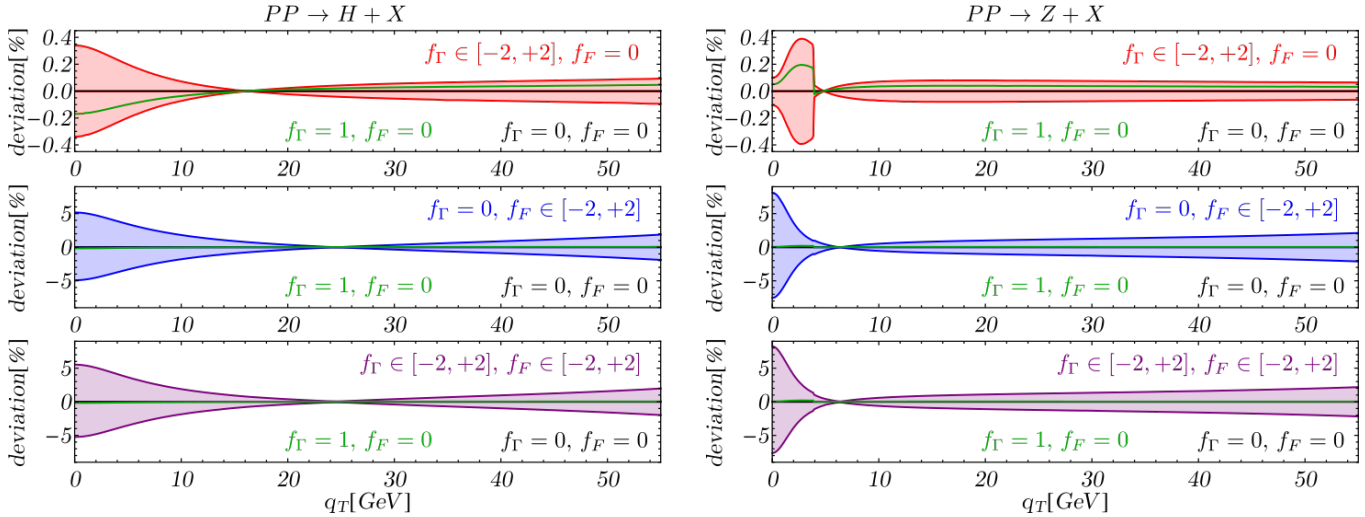
VS Z ATLAS 8 TeV



- Good agreement with data: [Becher,TL,Neubert,Wilhelm]$_{\text{pgr.}}$
  - ATLAS hep-ex/1512.0219 Z/γ^* 20.3 fb^{-1} at 8TeV.
  - Cuts for $d\sigma_{\text{fiducial}}/dq_T$: $66 < M_{ll}/GeV < 116$,
  - $p_{T,l} > 20\text{GeV}$, $|\eta_l| < 2.4$, excluding $1.37 < |\eta_l| < 1.52$
- Suppressed tail and overshoot $\sigma^{\text{exp}}_{Z/\gamma^* \to l^+l^-} = 537.10$pb by $\sim +6\%$.

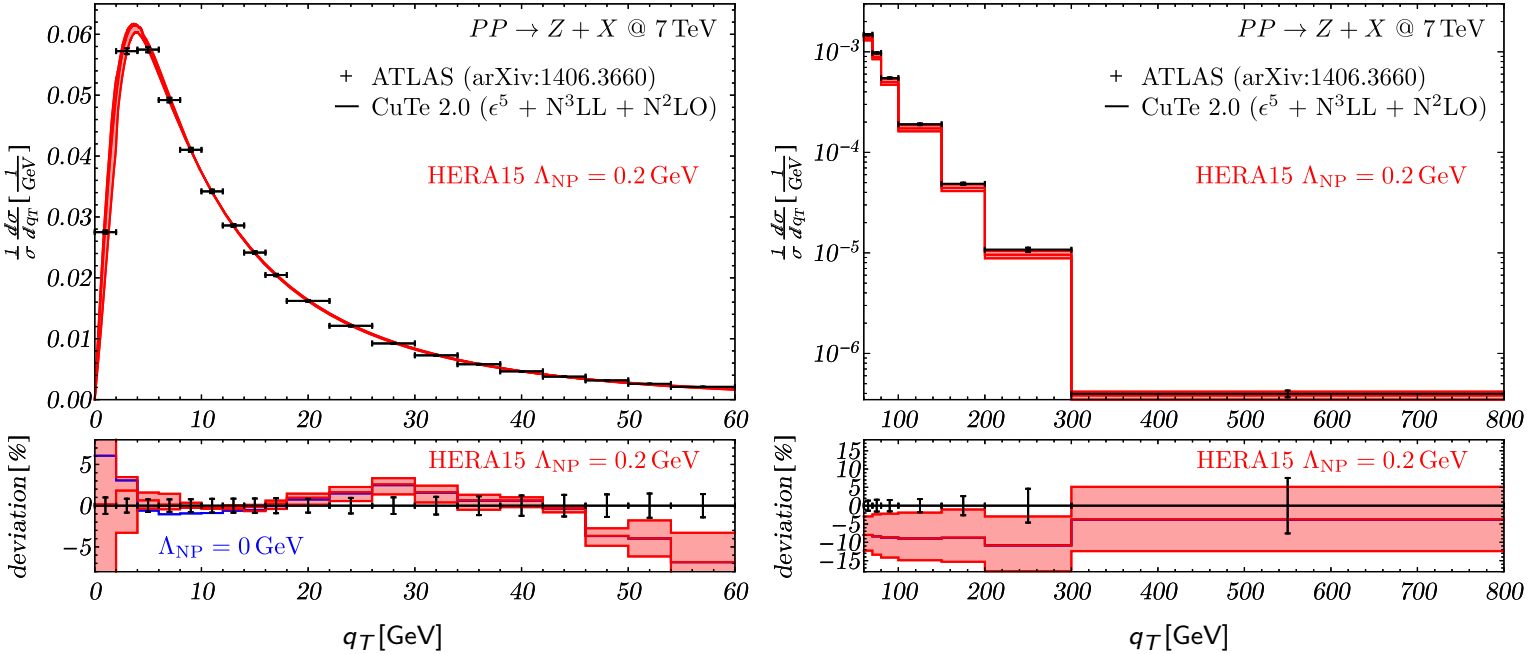
  At this precision, potentially relevant λ^n and $\lambda^0 \log^0 \lambda\ \alpha_s^3$ contributions:
  $K_{qT/GeV \in [10,20]} \sim 0.9$, $K_{qT/GeV \in [20,40]} \sim 0.95$
  [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan].
- All three experiments well described. No specific tuning.

Estimated impact of unknown N^{3}LL ingredients



- Vary unknowns Γ_3, $F^{(3,0)}$:
  - Deviation plots w.r.t. $f_\Gamma, f_F = 0$.
  - $\Gamma_3 = f_\Gamma \Gamma_3^{\text{Pade}}$, $\to$ tiny.
  - $F^{(3,0)} = f_F(4\pi)F^{(2,0)}$, $\to$ few percent.
  - Largest impact at small q_T.

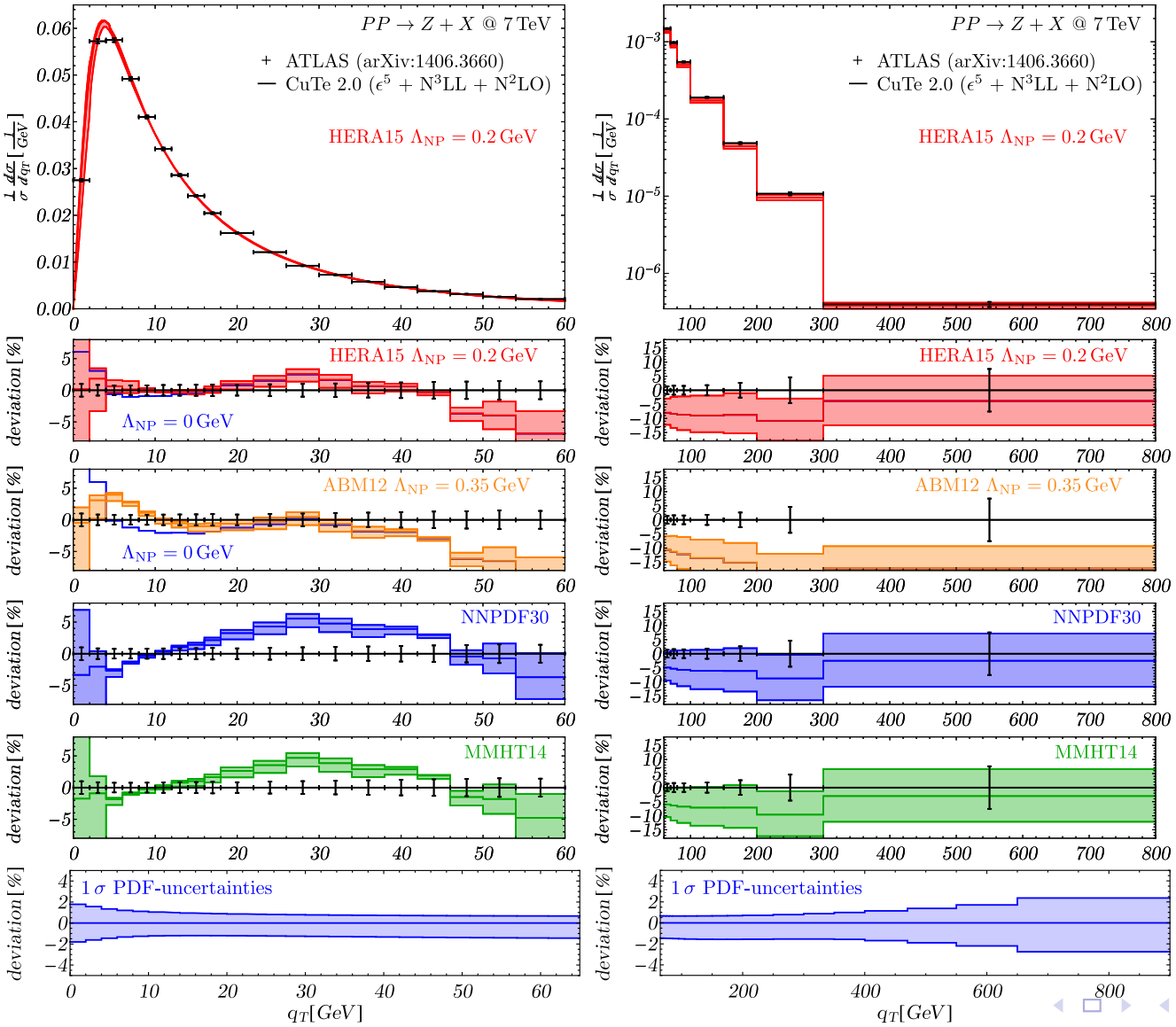
μ variation



μ uncertainties:

- In tail: $\sim 8\%$, pure FO.
- Towards peak: $< 1\%$ (ϵ^5 N^{3}LL$_p$+N^{2}LO ...)
- At very small q_T: increases again (huge logs).

PDF uncertainties



Various NNLO pdf sets.

μ variation for each.

Sizeable pdf uncertainty.

Not fully reflected by 1-σ pdf uncertainty.
Here for HERA 15.

Uncertainties

- CuTe accurately describes precise data sets over large q_T range.
- Uncertainties $\sim 7\%$ for Z/γ^*.
- Discussed above:
  - Higher order in QCD:
    $F^{(3,0)}$ (& Γ_3) ($\sim 2\%$, medium q_T);
    μ-variations ($\sim 5\%$, tail).
  - Pdf uncertainty ($\sim 2\%^{++}$).
  - Non-perturbative effects ($\sim 2\%$, $q_T < 3\text{GeV}$, $M_{ll} > 66\,GeV$).
- Not discussed here:
  - Off-shell effects.
  - Electro-weak effects.
  - Quark mass effects.
  - New physics.
  - ...

Conclusions

- **Resummation** essential for small q_T/M.
- **Generic** framework to obtain precise $d\sigma/dq_T(/dy)$ for large class of processes at hadron colliders.
- Implemented to $\mathbf{N^2LO{+}N^3LL_p}$ precision for γ^*, Z, W, H in **CuTe** 2.0.
- Obtain very accurate description of q_T spectrum.
- Code will become public.

Appendix

Towards N^{3}LL, required elements

Numbers refer to power n in expansion $X = \sum_n a_s^n X^{(n)}$.

expression	needed to	known to	
Γ^i	4	3	for RGEs
γ_i	3	3	
$P_{i/j}(z)$	3	3	
β	4	4	
$C_c(M^2)$	2	2	at appr. μ
$F_i(L_\perp)$	3	2	
$h_i(L_\perp)$	2	2	
$\bar{I}_{i/j}(z, L_\perp)$	2	2	
$\bar{I}'_{g/j}(z, L_\perp)$	1 (2)*	1	

*: I' starts at α_s^1.

$\Rightarrow$ $n = 1$ sufficient if C_{gg} does not mix I' & I. (E.g. for Higgs.)

Resummation

- RGEs $\Rightarrow$ Dependence on μ $\qquad i = i(c)\,,$

$$\frac{d}{d\log\mu}\log C_c(-M^2,\mu) = \Gamma^i(a_s)\log\frac{-M^2-i0^+}{\mu^2} + 2\gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}F_i(L_\perp,a_s) = 2\,\Gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}h_i(L_\perp,a_s) = \Gamma^i(a_s)L_\perp - 2\gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}\bar{I}_{i/j}(z,L_\perp,\mu) = -2\sum_k \bar{I}_{i/k}(z,L_\perp,a_s)\otimes P_{k/j}(z,a_s)\,,$$
$$\frac{d}{d\log\mu}f_{i/j}(z,\mu) = 2\sum_k P_{ik}(z,\mu)\otimes f_{k/j}(z,\mu)\,.$$

$\Rightarrow$ Resum logarithms. E.g.:

$$|C_c(-M^2,\mu)|^2 = |C_c(-M^2,\mu_h)|^2\exp\left\{2\mathrm{Re}[\mathrm{E_{C_c}}(-\mathrm{M}^2,\mu,\mu_\mathrm{h})]\right\}\,,$$
$$E_{C_c}(-M^2,\mu,\mu_h) = \int_{\mu_h}^{\mu}\frac{d\mu'}{\mu'}\left(\Gamma^i\log\frac{-M^2-i0^+}{\mu'^2} + 2\gamma^i\right)\,.$$

- Log indep. parts and anom. dims. from pert. calc..

Relation to framework by Collins, Soper, Sterman

$$\widetilde{C}_{q\bar{q}\leftarrow ij}\left(z_1,z_2,x_T^2,q^2,\mu\right)$$

$$= \left|C_V\left(-q^2,\mu\right)\right|^2 \left(\frac{x_T^2 q^2}{b_0^2}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}\left(z_1,x_T^2,\mu\right) I_{\bar{q}\leftarrow j}\left(z_2,x_T^2,\mu\right),$$

compare this to [Collins, Soper, Sterman]:

$$= \exp\left\{ -\int_{\mu_b}^{q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\log\frac{q^2}{\bar{\mu}^2} A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu})) \right] \right\}$$
$$\times\, C_{qi}(z_1, \alpha_s(\mu_b)) C_{\bar{q}j}(z_2, \alpha_s(\mu_b)),$$

x_T dependence via $\mu_b = b_0 {x_T}^{-1}$.

Relations:

- $C_{qi}(z, \alpha_s(\mu_b)) = \left|C_V(-\mu_b^2, \mu_b)\right| I_{q/i}(z, x_T^2, \mu_b)$,
- A & B related to F and anomalous dimensions.

Dictionary CSS vs BN

Using $b_0 = 2e^{-\gamma_e}$, $\mu_b = b_0 x_T^{-1}$ and $\bar{x}_T = b_0 \bar{\mu}^{-1}$:

$$C_{qi}(z, \alpha_s(\mu_b)) = \left| C_V(-\mu_b^2, \mu_b) \right| I_{q/i}(z, \bar{x}_T^2, \mu_b) \,,$$

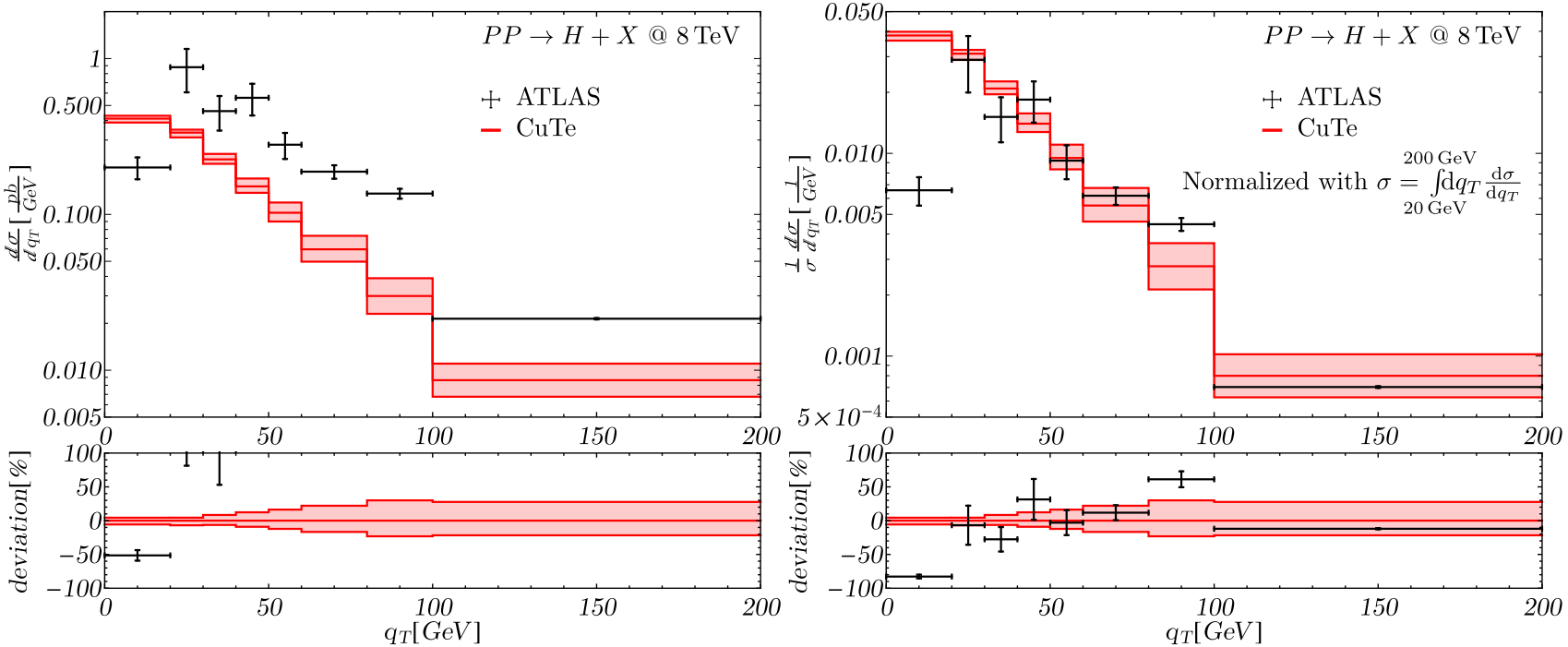
$$A(\alpha_s(\bar{\mu})) = \Gamma^q(\alpha_s) - \bar{\mu}^2 \frac{dF_{q\bar{q}}(\bar{x}_T, \bar{\mu})}{d\bar{\mu}^2} \,,$$

$$B(\alpha_s(\bar{\mu})) = 2\gamma_q(\alpha_s) + F_{q\bar{q}}(\bar{x}_T, \bar{\mu}) - \bar{\mu}^2 \frac{d \log |C_V(-\bar{\mu}^2, \bar{\mu})|^2}{d\bar{\mu}^2} \,.$$

$\Rightarrow$ Apply results in preferred resummation framework.

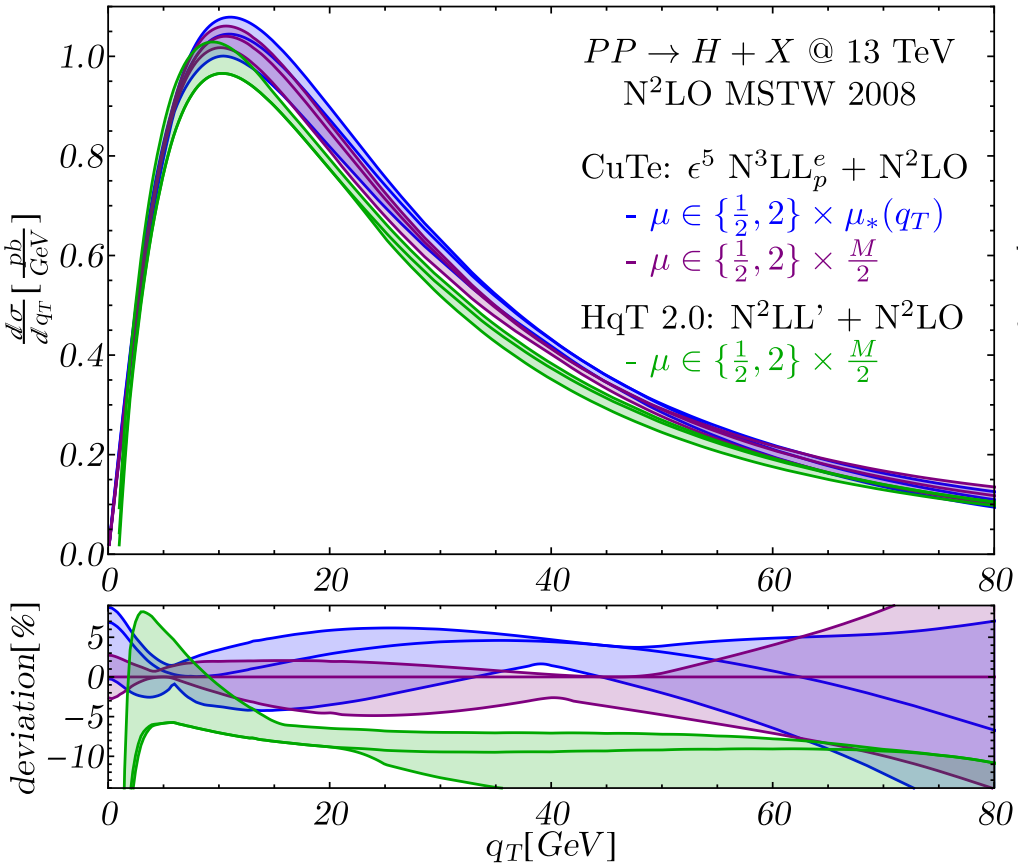
Can reconstruct $\mathcal{H}^{DY}_{q\bar{q}\leftarrow i\bar{j}}$, $\mathcal{H}^{H}_{gg\leftarrow i\bar{j}}$, ... in [Catani, Cieri, de Florian, Ferrera, Grazzini].

Vs H ATLAS 8 TeV



- Sizeable uncertainties. Mainly from FO part.
- Low statistics for data:
  - ATLAS, hep-ex/1504.05833, H 20.3 fb^{-1} at 8TeV .
  - Cuts unfolded.
- Can we trust fist experimental bin?

H 13 TeV vs HqT



- CuTe (μ as μ_* and $\frac{M}{2}$) vs HqT 2.0.
- Very similar shape.
- Integrated σ

$$\sigma_{\text{CuTe}} = 44\text{pb} \pm 8\%,$$
$$\sigma_{\text{HqT}} = 40\text{pb} \pm 6\%,$$
$$\sigma_{\text{nnlo}} = 43\text{pb} \pm 9\%,$$
$$\sigma_{\text{nnnlo}} = 44.31\text{pb}^{+0.31\%}_{-2.64\%}.$$

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger].
π^2 resummation in CuTe.