

p_T -Resummation in momentum space.

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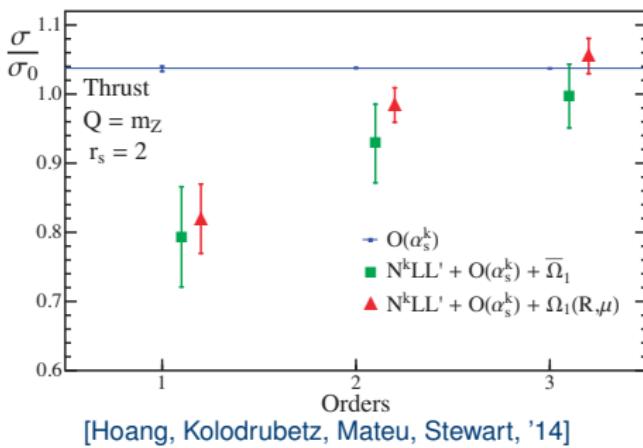
Outline

- 1 Introduction
- 2 Transverse momentum distribution in SCET
- 3 Resummation of γ_ν
- 4 Solution to Rapidity RGE
- 5 Conclusion

Introduction.

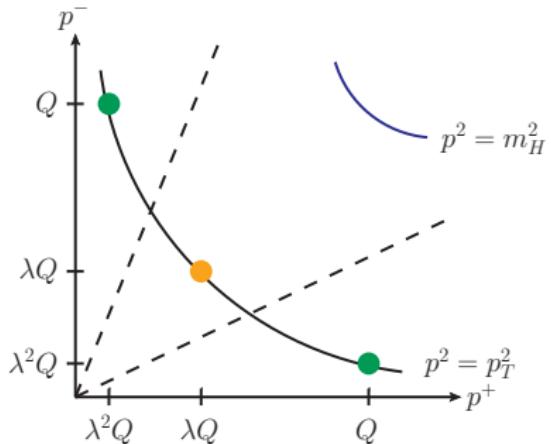
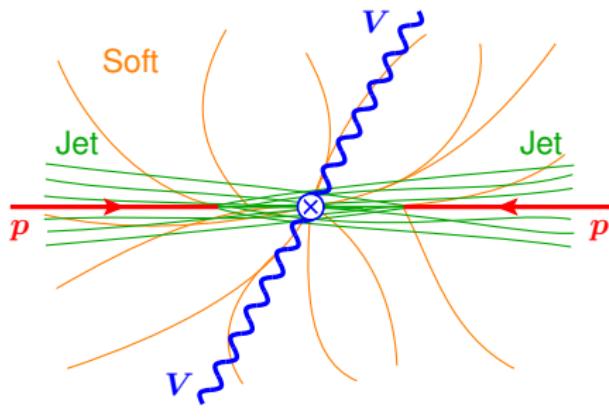
Resummation in b -space vs p_T -space.

- Spectrum $\frac{d\sigma}{dp_T}$ contains large logarithms $\ln \frac{p_T}{Q}$
- Resummation performed in impact parameter space $b \sim p_T^{-1}$
 - ▶ Resums logarithms $\ln(b Q)$ rather than $\ln \frac{p_T}{Q}$
- Possible impact on [Ellis, Veseli '98; Frixione, Nason, Ridolfi '99; Kulesza, Stirling '99]
 - ▶ momentum space spectrum
 - ▶ uncertainties
 - ▶ matching to fixed order
- Analogy:
 - ▶ Scale setting in spectrum vs scale setting in cumulant
 - ▶ Example: Thrust distribution
- Goal: Perform resummation directly in momentum space



Transverse momentum distribution in SCET.

Factorization in SCET.



- Factorized cross section: $\sigma = \mathbf{H}(\mathbf{B}_1 \otimes \mathbf{B}_2 \otimes \mathbf{S})(\vec{p}_T)$
- Rapidity divergences require additional regulator
 - We use the η -regulator [Chiu, Jain, Neill, Rothstein '11] (\rightarrow talk by J. Oredsson)
- Other regulators:
 - Non-light like axial gauge [Collins, Soper '81]
 - Wilson lines off light-cone [Collins '11]
 - α -regulator [Becher, Bell '11] (\rightarrow talk by T. Lübbert)
 - δ -regulator [Echevarria, Idilbi, Scimemi '11] (\rightarrow talk by A. Vladimirov)

RG structure of the cross section.

$$\sigma = H(\mu) [B_1(\mu, \nu) \otimes B_2(\mu, \nu) \otimes S(\mu, \nu)](\vec{p}_T)$$

- μ -RGE:

$$\mu \frac{dH(Q, \mu)}{d\mu} = \gamma_H H(Q, \mu)$$

$$\mu \frac{dB(\vec{p}_T, \omega, \mu, \nu)}{d\mu} = \gamma_B B(\vec{p}_T, \omega, \mu, \nu)$$

$$\mu \frac{dS(\vec{p}_T, \mu, \nu)}{d\mu} = \gamma_S S(\vec{p}_T, \mu, \nu)$$

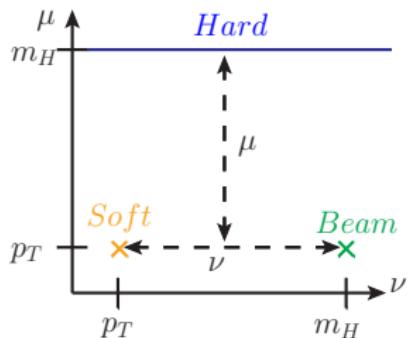
- ν -RGE (RRGE):

$$\nu \frac{dB(\vec{p}_T, \omega, \mu, \nu)}{d\nu} = -\frac{1}{2} \gamma_\nu(\vec{p}_T, \mu) \otimes B(\vec{p}_T, \omega, \mu, \nu)$$

$$\nu \frac{dS(\vec{p}_T, \mu, \nu)}{d\nu} = \gamma_\nu(\vec{p}_T, \mu) \otimes S(\vec{p}_T, \mu, \nu)$$

- Commutativity of μ and ν RGE:

$$\mu \frac{d\gamma_\nu(\vec{p}_T, \mu)}{d\mu} = -4\Gamma_C[\alpha_s(\mu)]\delta(\vec{p}_T)$$



Toy example: Resummation of large logs.

- Toy function $F(q, \mu)$ containing large logs $\ln(\mu/q)$

- Toy RGE:

$$\mu \frac{dF(q, \mu)}{d\mu} = F(q, \mu)$$

- Solution:

$$F(q, \mu) = F(q, \mu_0)U(\mu_0, \mu)$$

- ▶ Shifts logs from $\ln(q/\mu_0)$ to $\ln(q/\mu)$
- ▶ Sufficient only if $F(q, \mu_0)$ known “exactly” (Example: PDFs)

- Otherwise choose μ_0 such that the boundary $F(q, \mu_0)$ can be calculated

- ▶ Simplest case: multiplicative running (e.g. hard function: $q = Q$)

$$F(q, \mu_0) = 1 + \alpha_s \ln(\mu_0/q) + \alpha_s^2 \ln^2(\mu_0/q) + \dots$$

- ▶ Pick $\mu_0 = q$

- “All-order” $F(q, \mu_0 = q) = 1 + 0 + 0 + \dots = 1 \quad \checkmark$

- $F(q, \mu) = 1 \cdot U(\mu_0 = q, \mu)$ correctly resums $\ln(\mu/q) \quad \checkmark$

Toy example: Resummation of large logs.

- Toy function $F(q, \mu)$ containing large logs $\ln(\mu/q)$
- Toy RGE:
$$\mu \frac{dF(q, \mu)}{d\mu} = F(q, \mu)$$
- Solution:
$$F(q, \mu) = F(q, \mu_0)U(\mu_0, \mu)$$
 - ▶ Shifts logs from $\ln(q/\mu_0)$ to $\ln(q/\mu)$
 - ▶ Sufficient only if $F(q, \mu_0)$ known “exactly” (Example: PDFs)
- Otherwise choose μ_0 such that the boundary $F(q, \mu_0)$ can be calculated
 - ▶ More complicated case: multiplicative running in conjugate space (e.g. $\vec{p}_T \rightarrow \vec{b}_T$)
$$\tilde{F}(x, \mu_0) = 1 + \alpha_s \ln(\mu_0 x) + \alpha_s^2 \ln^2(\mu_0 x) + \dots$$
 - ▶ Pick $\mu_0 = q \sim 1/x$
 - $\tilde{F}(x, \mu_0 = q) = 1 + \alpha_s \ln(qx) + \alpha_s^2 \ln^2(qx) + \dots = 1$?
 - Need to *assume* that $\ln(qx)$ can be neglected!

Resummation in b -space.

- Differential equations easily solved in b -space:

$$\frac{d\sigma}{db} \sim \textcolor{blue}{H}(\mu_H) \tilde{B}(\vec{b}, \mu_L, \nu_B)^2 \tilde{S}(\vec{b}, \mu_L, \nu_S) \\ \times \underbrace{\exp \left[\int_{\mu_L}^{\mu_H} \frac{d\mu'}{\mu'} \gamma_H[\mu'] \right]}_{\mu\text{-evolution}} \underbrace{\exp \left[\ln \frac{\nu_S}{\nu_B} \tilde{\gamma}_\nu(b, \mu_L) \right]}_{\nu\text{-evolution}}$$

- Fixed order expansions:

$$H(Q, \mu_H) \sim 1 - \frac{\alpha_s}{4\pi} 2\Gamma_0 \ln^2 \frac{Q}{\mu_H} + \dots$$

$$B(\vec{b}_T, \mu_L, \nu_B) \sim 1 - \frac{\alpha_s}{4\pi} \Gamma_0 \ln \frac{Q}{\nu_B} \ln(\mu_L^2 b^2) + \dots$$

$$S(\vec{b}_T, \mu_L, \nu_S) \sim 1 + \frac{\alpha_s}{4\pi} 2\Gamma_0 \ln \frac{\mu_L}{\nu_S} \ln(\mu_L^2 b^2) + \dots$$

$$\tilde{\gamma}_\nu(b, \mu_L) \sim -2\Gamma_C \ln(b^2 \mu_L^2) + \dots$$

- Minimize logs in b -space:

$$\mu_H \sim Q, \mu_L \sim 1/b, \nu_B \sim Q, \nu_S \sim 1/b$$

Resummation in p_T space.

Naive attempt:

- Solve RGEs in Fourier space

$$\sigma \sim \int d^2\vec{b} e^{i\vec{p}_T \cdot \vec{b}} \mathbf{H}(\mu_H) \tilde{B}(\vec{b}, \mu_L, \nu_B)^2 \tilde{S}(\vec{b}, \mu_L, \nu_S)$$
$$\times \underbrace{\exp \left[\int_{\mu_L}^{\mu_H} \frac{d\mu'}{\mu'} \gamma_H[\mu'] \right]}_{\mu\text{-evolution}} \underbrace{\exp \left[\ln \frac{\nu_S}{\nu_B} \tilde{\gamma}_\nu(b, \mu) \right]}_{\nu\text{-evolution}}$$

- with scales in momentum space:

$$\mu_H \sim Q, \mu_L \sim p_T, \nu_B \sim Q, \nu_S \sim p_T$$

- Idea: Singular cross section dominated by $k_1 \sim k_2 \sim k_3 \sim p_T$

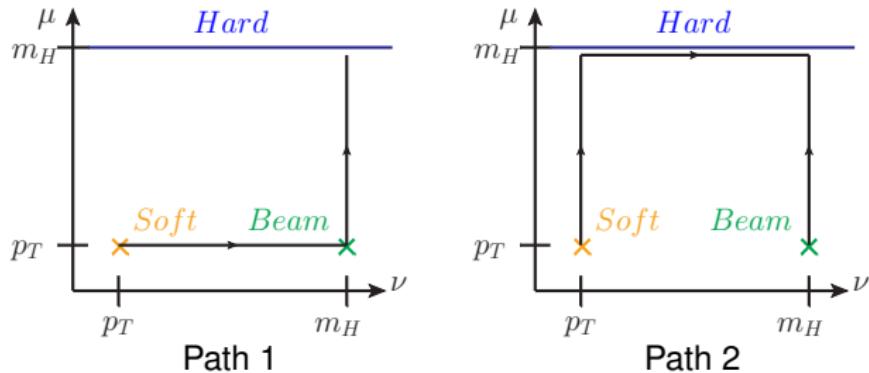
$$\sigma \sim \mathbf{H} \int d^2\vec{k}_1 d^2\vec{k}_2 d^2\vec{k}_3 B(\vec{k}_1) B(\vec{k}_2) S(\vec{k}_3) \delta(\vec{p}_T - \vec{k}_1 - \vec{k}_2 - \vec{k}_3)$$

- Fixed order logs are $\sim \ln \frac{p_T}{\mu_L}, \ln \frac{p_T}{\nu_S}$
- Similar argument: Fourier transform dominated by $b \sim p_T^{-1}$

Resummation in p_T space.

$$\sigma \sim \int d^2\vec{b} e^{i\vec{p}_T \cdot \vec{b}} H(\mu_H) \tilde{B}(\vec{b}, \mu_L, \nu_B)^2 \tilde{S}(\vec{b}, \mu_L, \nu_S)$$
$$\times \exp \left[\int_{\mu_L}^{\mu_H} \frac{d\mu'}{\mu'} \gamma_H[\mu'] \right] \exp \left[\ln \frac{\nu_S}{\nu_B} \tilde{\gamma}_\nu(b, \mu) \right]$$

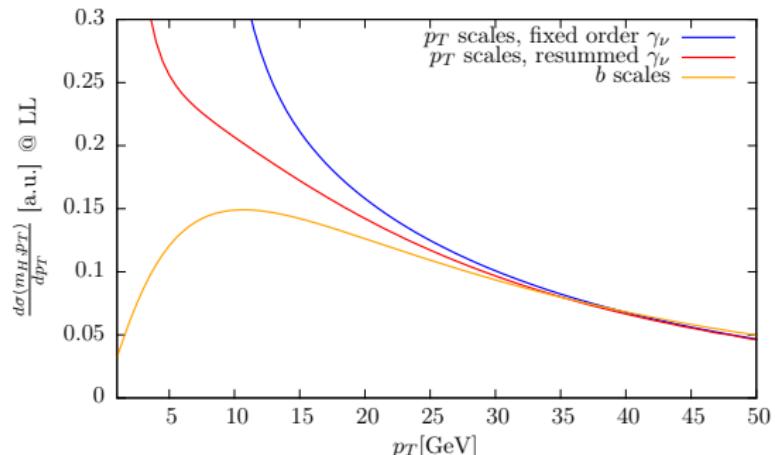
- μ -scale for $\tilde{\gamma}_\nu(b, \mu)$ depends on ordering of μ and ν -evolution:



- Evolve along path 1
 - ▶ $\mu \sim p_T$
 - ▶ Can use fixed order expansion for γ_ν
 - ▶ Typical choice in the literature

Resummation in p_T space.

Result:



- No resummation of γ_ν : Well known divergence at $\Gamma_C[\mu_L] \ln \frac{\nu_B^2}{\nu_S^2} = 1$
[Frixione, Nason, Ridolfi '99; Becher, Neubert, Wilhelm '12; Chiu, Jain, Neill, Rothstein '12]
- With resummation of γ_ν : spectrum finite, but diverges for $p_T \rightarrow 0$
- Scale setting in b -space is well behaved

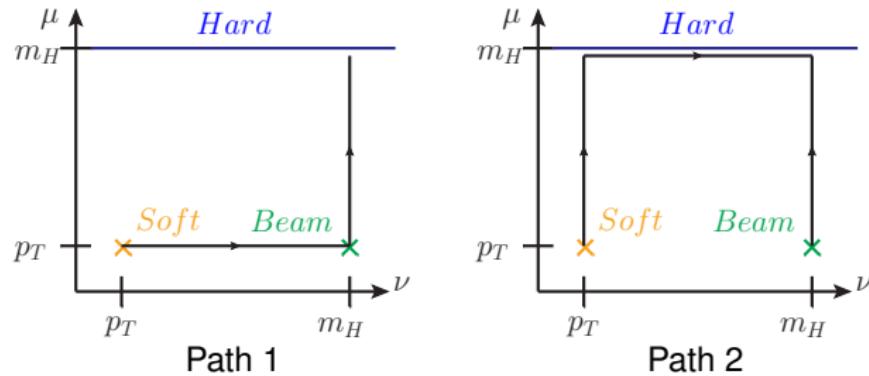
Conclusion:

Naive p_T scale choice does not correctly resum momentum space logs.

Resummation of γ_ν .

The RGE for γ_ν .

- Freedom to evolve in (μ, ν) -space: [Chiu, Jain, Neill, Rothstein '12]



- Expressed through commutativity $[d/d\mu, d/d\nu] = 0$
- Induced RGE for γ_ν :

$$\mu \frac{d\gamma_\nu(\vec{k}_T, \mu)}{d\mu} = -(4\pi)^2 \Gamma_C[\alpha_s(\mu)] \delta(\vec{k}_T)$$

- Formal solution:

$$\gamma_\nu(\vec{k}_T, \mu) = \gamma_\nu(\vec{k}_T, \mu_0) - (4\pi)^2 \delta(\vec{k}_T) \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \Gamma_C[\alpha_s(\mu')]$$

- Crucial: Distributional way to set $\mu_0 \sim k_T$

γ_ν -Resummation in p_T -space.

- Recall the toy example:

$$\begin{aligned} F(k, \mu) &= F(k, \mu_0) U(\mu_0, \mu) \\ \leftrightarrow \ln(\mu/k) &= \ln(\mu_0/k) + \ln(\mu/\mu_0) \\ &= \ln(\mu_0/k) + \ln(\mu/k) - \ln(\mu_0/k) \end{aligned}$$

- Distributional analogue:

$$\begin{aligned} \left[\frac{1}{2\pi k_T^2} \right]_+^\mu &= \left[\frac{1}{2\pi k_T^2} \right]_+^{\mu_0} + \ln \frac{\mu}{\mu_0} \delta(\vec{k}_T) \\ &= \left[\frac{1}{2\pi k_T^2} \right]_+^{\mu_0} + \left[\frac{1}{2\pi k_T^2} \right]_+^\mu - \left[\frac{1}{2\pi k_T^2} \right]_+^{\mu_0} \end{aligned}$$

► Plus distribution: $\int d^2 \vec{k}_T [f(\vec{k}_T)]_+^\mu = 0$
 $| \vec{k}_T | < \mu$

- Distributional way to implement $\mu_0 = k_T$:

$$\ln \frac{\mu}{\mu_0} \delta(\vec{k}_T) \Big|_{\mu_0=k_T} = \left[\frac{1}{2\pi k_T^2} \right]_+^\mu , \quad \left[\frac{1}{2\pi k_T^2} \right]_+^{\mu_0} \Big|_{\mu_0=k_T} = 0$$

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γ_ν -Resummation in p_T -space.

- RGE:

$$\mu \frac{d\gamma_\nu(\vec{k}_T, \mu)}{d\mu} = -(4\pi)^2 \Gamma_C[\alpha_s(\mu)] \delta(\vec{k}_T)$$

- All-order solution:

$$\begin{aligned}\gamma_\nu(\vec{k}_T, \mu) = & \left[\frac{1}{k_T^2} \left(8\pi \Gamma_C[\alpha_s(k_T)] + \frac{1}{2\pi} \frac{d\gamma_\nu[\alpha_s(k_T)]}{d \ln k_T} \right) \right]_+^\mu \\ & + \delta(\vec{k}_T) \gamma_\nu[\alpha_s(\mu)]\end{aligned}$$

- Physical interpretation:

- ▶ Propagator of a single soft emission
- ▶ Distribution regulates IR divergences
→ Boundary condition ensures RGE is fulfilled ✓
- ▶ Virtual corrections to emission resummed in $\alpha_s(k_T)$
- ▶ Fixed order boundary ensured by $\delta(\vec{k}_T) \gamma_\nu[\alpha_s(\mu)]$
→ All large logs are resummed ✓

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Comparison to b -space resummation.

- RGE solved in momentum space:

$$\gamma_\nu(\vec{k}_T, \mu) = \left[\frac{1}{k_T^2} \left(8\pi \Gamma_C[\alpha_s(k_T)] + \frac{1}{2\pi} \frac{d\gamma_\nu[\alpha_s(k_T)]}{d \ln k_T} \right) \right]_+^\mu + \delta(\vec{q}_T) \gamma_\nu[\alpha_s(\mu)]$$

- RGE solved in b space:

$$\gamma_\nu^{(b)}(\vec{k}_T, \mu) = \int d^2 \vec{b}_T e^{i \vec{k}_T \cdot \vec{b}_T} \left(-4 \int_{1/b}^\mu \frac{d\mu'}{\mu'} \Gamma_C[\alpha_s(\mu')] + \tilde{\gamma}_\nu[\alpha_s(1/b)] \right)$$

- Compare fixed order expansions in $\alpha_s(\mu)$:

$$\gamma_\nu^{(2)} \supset \gamma_{\nu 2} \delta(\vec{k}_T)$$

$$\gamma_\nu^{(b)(2)} \supset \left(\tilde{\gamma}_{\nu 2} + \frac{8}{3} \zeta_3 \beta_0^2 \Gamma_0 \right) (2\pi)^2 \delta(\vec{k}_T)$$

- ▶ Different definition of non-cusp $\gamma_{\nu n}$ in b -space / p_T -space
- ▶ Differences always appear at subleading order



Outlook: Non-perturbative effects in γ_ν .

- For simplicity: take only Γ_C -term

$$\gamma_\nu(\vec{k}_T, \mu) = \left[\frac{8\pi \Gamma_C[\alpha_s(k_T)]}{k_T^2} \right]_+^\mu$$

- $\gamma_\nu(\vec{k}_T, \mu)$ enters convolutions (e.g. $\nu \frac{dS}{d\nu} = \gamma_\nu \otimes S$)
 - Require non-perturbative modeling for $k_T \lesssim \Lambda_{\text{QCD}}$
- Split using a profile function μ_0 :

$$\begin{aligned}\gamma_\nu(\vec{k}_T, \mu) &= \left[\frac{8\pi}{k_T^2} \Gamma_C[\alpha_s(\mu_0(k_T))] \right]_+^\mu \\ &\quad + \left[\frac{8\pi}{k_T^2} \left(\Gamma_C[\alpha_s(k_T)] - \Gamma_C[\alpha_s(\mu_0(k_T))] \right) \right]_+^\mu\end{aligned}$$

- For suitable profiles μ_0 , we can expand in moments:

$$\gamma_\nu(\vec{k}_T, \mu) = \left[\frac{8\pi}{k_T^2} \Gamma_C[\alpha_s(\mu_0(k_T))] \right]_+^\mu + \sum_{n=1}^{\infty} \Omega_n \Delta^n \delta(\vec{k}_T)$$

Outlook: Non-perturbative effects in γ_ν .

- Moments become more intuitive in b -space:

$$\int d^2\vec{k}_T e^{i\vec{k}_T \cdot \vec{b}_T} \sum_{n=1}^{\infty} \Omega_n \Delta^n \delta(\vec{k}_T) = \sum_{n=1}^{\infty} \Omega_n (-b_T^2)^n$$

- Implication on cross section (b -space):

$$\begin{aligned}\sigma &\sim \int d^2\vec{b}_T e^{i\vec{k}_T \cdot \vec{b}_T} H(Q) B(b_T)^2 S(b_T) e^{\ln \frac{\nu_S}{\nu_B} \tilde{\gamma}_\nu(b, \mu)} \\ &\rightarrow \int d^2\vec{b}_T e^{i\vec{k}_T \cdot \vec{b}_T} H(Q) B(b_T)^2 S(b_T) e^{\ln \frac{\nu_S}{\nu_B} \tilde{\gamma}_\nu^{(p)}(b, \mu)} e^{-\ln \frac{\nu_S}{\nu_B} \Omega_1 b_T^2 + \dots}\end{aligned}$$

- γ_ν is intrinsically non-perturbative in b -space [Collins, Soper, Sterman '85]
→ Confirmed in p_T -space
- Agrees with non-perturbative Gaussian modeling
[Collins, Soper, Sterman '85]
- Non-perturbative effects scale with rapidity logarithm
[Collins, Soper, Sterman '85; Becher, Bell '13]

Resummation of γ_ν .

Summary:

- RGE:

$$\mu \frac{d\gamma_\nu(\vec{k}_T, \mu)}{d\mu} = -(4\pi)^2 \Gamma_C[\alpha_s(\mu)] \delta(\vec{k}_T)$$

- Momentum space solution:

$$\begin{aligned}\gamma_\nu(\vec{k}_T, \mu) = & \left[\frac{1}{k_T^2} \left(8\pi \Gamma_C[\alpha_s(k_T)] + \frac{1}{2\pi} \frac{d\gamma_\nu[\alpha_s(k_T)]}{dk_T} \right) \right]_+^\mu \\ & + \delta(\vec{k}_T) \gamma_\nu[\alpha_s(\mu)]\end{aligned}$$

- ▶ Requires careful distributional scale setting
- Formally subleading differences to b -space solution
- Non-perturbative modeling of γ_ν with momenta expansion
 - ▶ Leading non-perturbative contribution: Gaussian in b -space
 - ▶ Scales with rapidity $\log \ln \frac{\nu_S}{\nu_B}$

Solution to Rapidity RGE.

Solution to Rapidity RGE.

- Focus on soft function (beam functions similar)
- μ -RGE:

$$\mu \frac{dS(\vec{k}_T, \mu, \nu)}{d\mu} = \gamma_S S(\vec{k}_T, \mu, \nu)$$

- ν -RGE (RRGE):

$$\begin{aligned}\nu \frac{dS(\vec{k}_T, \mu, \nu)}{d\nu} &= \gamma_\nu(\vec{k}_T, \mu) \otimes S(\vec{k}_T, \mu, \nu) \\ &= \int \frac{d^2 \vec{k}_1}{(2\pi)^2} \gamma_\nu(\vec{k}_T - \vec{k}_1, \mu) S(\vec{k}_1, \mu, \nu)\end{aligned}$$

- Complicated due to convolution structure
- Main reason for b -space formalism
- Focus on solution of RRGE in momentum space

Formal solution to RRGE.

- RRGE: $\nu \frac{dS(\vec{k}_T, \mu, \nu)}{d\nu} = \gamma_\nu(\vec{k}_T, \mu) \otimes S(\vec{k}_T, \mu, \nu)$

- Formal solution (integrate RRGE iteratively):

$$S(\vec{k}_T, \mu, \nu_B) = S(\vec{k}_T, \mu, \nu_S) + \sum_{n=1}^{\infty} \frac{1}{n!} \ln^n \left(\frac{\nu_B}{\nu_S} \right) (\gamma_\nu \otimes^n) \otimes S(\vec{k}_T, \mu, \nu_S)$$

- Easy to see in Fourier space:

$$\tilde{S}(\vec{b}_T, \mu, \nu_B) = \exp \left[\ln \left(\frac{\nu_B}{\nu_S} \right) \tilde{\gamma}_\nu(\vec{b}_T, \mu) \right] \tilde{S}(\vec{b}_T, \mu, \nu_S)$$

- Formal solution correctly shifts logarithms from ν_S to ν_B ✓
- ... but is $\nu_S \sim p_T$ the correct boundary condition ?

Formal solution to RRGE.

- Formal solution

$$S(\vec{k}_T, \mu, \nu_B) = S(\vec{k}_T, \mu, \nu_S) + \sum_{n=1}^{\infty} \frac{1}{n!} \ln^n \left(\frac{\nu_B}{\nu_S} \right) (\gamma_\nu \otimes^n) \otimes S(\vec{k}_T, \mu, \nu_S)$$

assumes $S(\vec{k}_T, \mu, \nu_S)$ at $\nu_S \sim p_T$ permits perturbative truncation, i.e.

$$S(\vec{k}_T, \mu, \nu_S) = \delta(\vec{k}_T) + \dots$$

- Recall cross section:

$$\sigma \sim H \int d^2 \vec{k}_1 d^2 \vec{k}_2 d^2 \vec{k}_3 B(\vec{k}_1) B(\vec{k}_2) S(\vec{k}_3) \delta(\vec{p}_T - \vec{k}_1 - \vec{k}_2 - \vec{k}_3)$$

- ▶ Expect σ to be dominated by $k_1 \sim k_2 \sim k_3 \sim p_T$
- ▶ This is crucial for above assumption!

- Need to investigate effects from $k_i \gg p_T$

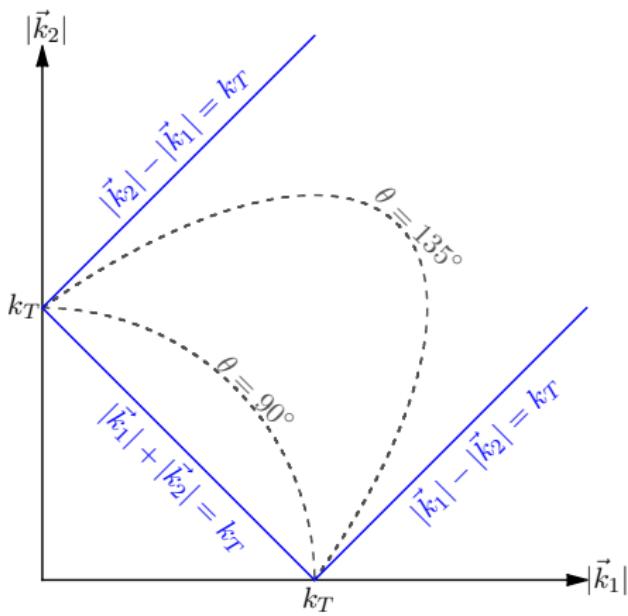
Formal solution to RRGE.

$$n = 1 : \ln \left(\frac{\nu_B}{\nu_S} \right) \int d^2 \vec{k}_1 \int d^2 \vec{k}_2 \gamma_\nu(\vec{k}_2, \mu) S(\vec{k}_1, \mu, \nu_S) \delta(\vec{k}_T - \vec{k}_1 - \vec{k}_2)$$

Pictorially:



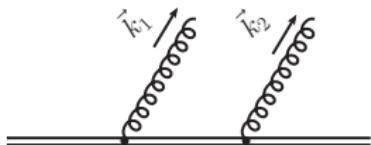
Relevant momenta:



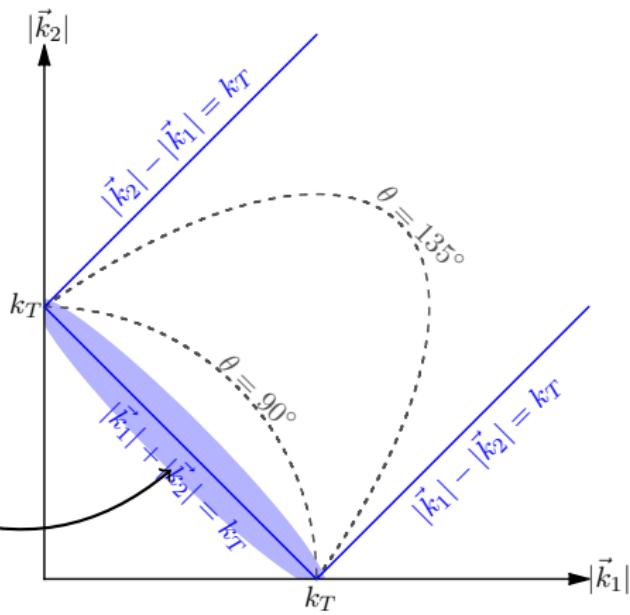
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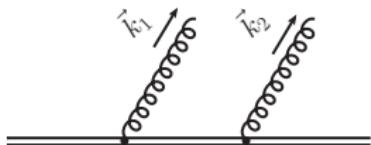


Soft momenta $k_1, k_2 \sim k_T$
Well described by $\nu_S \sim k_T$
(Corresponds to 1D case)

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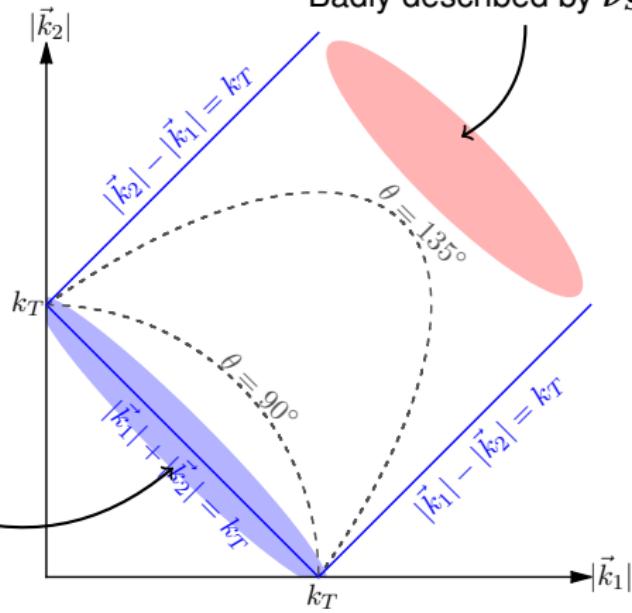
Pictorially:



Relevant momenta:

Large momenta $k_1, k_2 \sim k_T$
Badly described by $\nu_S \sim k_T$

Soft momenta $k_1, k_2 \sim k_T$
Well described by $\nu_S \sim k_T$
(Corresponds to 1D case)



Correct solution to RRGE.

Illustration: Distributional scale setting

- Soft function at NLO fulfills

$$\nu \frac{dS^{(1)}(\vec{k}_T, \mu, \nu)}{d\nu} = \gamma_\nu^{(0)} \otimes S^{(0)} = 2\Gamma_0 \left[\frac{1}{2\pi k_T^2} \right]_+^\mu$$

- Integrate over ν :

$$\begin{aligned} S^{(1)}(\vec{k}_T, \mu, \nu) &= S^{(1)}(\vec{k}_T, \mu, \nu_0) + 2\Gamma_0 \left[\frac{1}{2\pi k_T^2} \ln \frac{\nu}{\nu_0} \right]_+^\mu \\ &= \left(S^{(1)}(\vec{k}_T, \mu, \nu_0) - 2\Gamma_0 \left[\frac{1}{2\pi k_T^2} \ln \frac{\nu_0}{k_T} \right]_+^\mu \right) + 2\Gamma_0 \left[\frac{1}{2\pi k_T^2} \ln \frac{\nu}{k_T} \right]_+^\mu \end{aligned}$$

- ▶ ν_0 -dependence must cancel
- ▶ Logs are distributionally minimized by $\nu_0 = k_T$

- Inserting correct boundary condition:

$$S^{(1)}(\vec{k}_T, \mu, \nu) = S^{(1)}(\vec{k}_T, \mu, \nu_0 = k_T) + 2\Gamma_0 \left[\frac{1}{2\pi k_T^2} \ln \frac{\nu}{k_T} \right]_+^\mu$$



Correct solution to RRGE.

- All-order solution:

$$S(\vec{k}, \mu, \nu) = \sum_{n=0}^{\infty} \left(\prod_{i=1}^n \int_{\nu_0=k_{i-1}}^{\nu_{i-1}} \frac{d\nu_i}{\nu_i} \int d^2 \vec{k}_i \gamma_\nu(\vec{k}_{i-1} - \vec{k}_i, \mu) \right) S(\vec{k}_n, \mu, \nu_0 = k_n)$$

- Fulfils RRGE: $\nu \frac{dS}{d\nu} = \gamma_\nu \otimes S$ ✓
- Distributional scale setting ensures resummation of all logarithms
 - ▶ Setting $\nu_0 = k_i$ under convolution minimizes logs in boundary term
 - ▶ Explicitly checked at two loops ✓
- Comparison to b -space resummation:
 - ▶ Find similar subleading terms as for γ_ν
- Formal proof that resummation in p_T space *is* possible

Conclusion.

Conclusion.

- Large logarithms $\ln \frac{p_T}{Q}$ require resummation for $p_T \ll Q$
- Standard approach: scale setting in impact parameter space
 - ▶ Resums logarithms $\ln(b Q)$ instead of $\ln \frac{p_T}{Q}$
- Solution in momentum space is possible
 - ▶ Requires distributional scale setting rather than scale setting $\mu_S, \nu_S = p_T$ in Fourier space
 - ▶ Compatible with b -space approach up to subleading effects
- New insight into non-perturbative effects from γ_ν in momentum space
 - ▶ γ_ν is intrinsically non-perturbative
 - ▶ Gaussian model motivated by moment expansion

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Thank you for your attention!

Backup slides.

Non-perturbative effects in γ_ν .

Another motivation for the moment expansion:

- Fourier transform to b -space:

$$\begin{aligned}\tilde{\gamma}_\nu(b, \mu) &= \int d^2 \vec{k}_T e^{i \vec{k}_T \cdot \vec{b}_T} \left[\frac{8\pi \Gamma_C[\alpha_s(k_T)]}{k_T^2} \right]_+^\mu \\ &= 4 \int_0^\mu dk_T \frac{J_0(b_T k_T) - 1}{k_T} \Gamma_C[\alpha_s(k_T)] + \int_\mu^\infty \dots\end{aligned}$$

- Expand Bessel function:

$$\tilde{\gamma}_\nu(b, \mu) = 4 \sum_{n=1}^{\infty} \frac{1}{n!^2} (-b^2/4)^n \underbrace{\int_0^\mu dk_T k_T^{2n-1} \Gamma_C[\alpha_s(k_T)]}_{\sim \Omega_n} + \dots$$

- We naturally obtain an expansion in $(-b^2)^n$

Differences in γ_ν -resummation (1).

More details on subleading differences:

- Formal comparison possible through fixed order expansion
- At each subleading order in $\alpha_s(\mu)$, differences can arise
- Formal all-order difference:

$$\begin{aligned}\gamma_\nu(\vec{p}_T, \mu) &= \int d^2 \vec{b}_T e^{i \vec{p}_T \cdot \vec{b}_T} \tilde{\gamma}_\nu(b, \mu) \\ &\quad - \left[\frac{1}{2\pi p_T^2} \frac{d\Delta\gamma_\nu[\alpha_s(p_T)]}{d \ln p_T} \right]_+^\mu - \delta^2(\vec{p}_T) \Delta\gamma_\nu[\alpha_s(\mu)]\end{aligned}$$

$$\Delta\gamma_\nu[\alpha_s(\mu)] = (2\pi)^2 \int_0^\infty d(b\mu) J_1(b\mu) \tilde{\gamma}_\nu(b, \mu) - \gamma_\nu[\alpha_s(\mu)]$$

- Cancels differences at known fixed order

Differences in γ_ν -resummation (2).

More details on subleading differences:

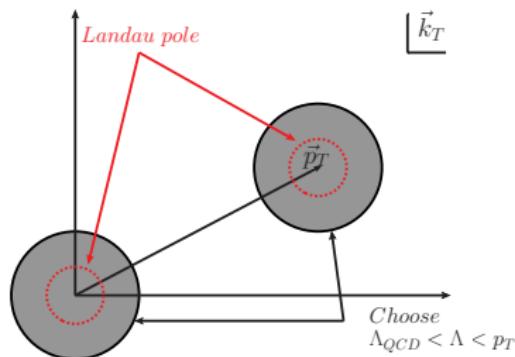
- The subleading terms arise from the Fourier transform of logs $\ln(b\mu)$
- Example:

$$\int d^2 \vec{b}_T e^{i \vec{p}_T \cdot \vec{b}_T} \ln \frac{b^2 \mu^2 e^{2\gamma_E}}{4} = -\frac{4\pi}{\mu^2} \left[\frac{p_T^2}{\mu^2} \right]_+^\mu$$
$$\int d^2 \vec{b}_T e^{i \vec{p}_T \cdot \vec{b}_T} \ln^2 \frac{b^2 \mu^2 e^{2\gamma_E}}{4} = +\frac{8\pi}{\mu^2} \left[\frac{p_T^2}{\mu^2} \ln \frac{p_T^2}{\mu^2} \right]_+^\mu$$
$$\int d^2 \vec{b}_T e^{i \vec{p}_T \cdot \vec{b}_T} \ln^3 \frac{b^2 \mu^2 e^{2\gamma_E}}{4} = -\frac{12\pi}{\mu^2} \left[\frac{p_T^2}{\mu^2} \ln^2 \frac{p_T^2}{\mu^2} \right]_+^\mu - (4\pi)^2 \zeta_3 \delta(\vec{p}_T)$$
$$\int d^2 \vec{b}_T e^{i \vec{p}_T \cdot \vec{b}_T} \ln^4 \frac{b^2 \mu^2 e^{2\gamma_E}}{4} = +\frac{16\pi}{\mu^2} \left[\frac{p_T^2}{\mu^2} \ln^3 \frac{p_T^2}{\mu^2} \right]_+^\mu + \frac{64\pi}{\mu^2} \zeta_3 \left[\frac{p_T^2}{\mu^2} \right]_+^\mu$$

Perturbativity of ν -kernel.

- The ν -kernel involves convolutions $\gamma_\nu \otimes^n$
- Problematic due to Landau pole in: $\gamma_\nu(\vec{k}_T, \mu) \sim \left[\frac{8\pi\Gamma_C[\alpha_s(k_T)]}{k_T^2} \right]_+^\mu$
 - ▶ Expect $(\gamma_\nu \otimes \gamma_\nu)(\vec{p}_T, \mu) = \int d^2\vec{k}_T \gamma_\nu(\vec{p}_T - \vec{k}_T, \mu) \gamma_\nu(\vec{k}_T, \mu)$ to be non-perturbative
- $p_T, \mu \gg \Lambda_{QCD}$: Landau pole effectively regulated by plus prescription
- Illustration:

$$\begin{aligned} & \int d^2\vec{k}_T \gamma_\nu(\vec{p}_T - \vec{k}_T, \mu) \gamma_\nu(\vec{k}_T, \mu) \\ & \sim -2 \left[\frac{\Gamma_C[\vec{p}_T]}{p_T^2} + \mathcal{O}\left(\frac{\Lambda}{p_T}\right) \right]_+^\mu \int_\Lambda^\mu \frac{dq_T}{q_T} \Gamma_C[\alpha_s(q_T)] \\ & + (\gamma_\nu \otimes \gamma_\nu) \Big|_{\mathbb{R} \setminus (B_\Lambda(\vec{0}) \cup B_\Lambda(\vec{p}_T))} \end{aligned}$$



Soft function at two loops.

- Expand soft function as

$$\begin{aligned} S(\vec{k}_T, \mu, \nu) &= S^{(0)}(\vec{k}_T, \mu, \nu) + a_s S^{(1)}(\vec{k}_T, \mu, \nu) + a_s^2 S^{(2)}(\vec{k}_T, \mu, \nu), \\ S^{(0)}(\vec{k}_T, \mu, \nu) &= \delta(\vec{k}_T), \\ S^{(1)}(\vec{k}_T, \mu, \nu) &= S_1 \delta(\vec{k}_T) + \frac{\Gamma_0}{\pi \mu^2} \left[\frac{\mu^2}{k_T^2} \ln \frac{\nu^2}{k_T^2} \right]_+^\mu, \\ S^{(2)}(\vec{k}_T, \mu, \nu) &= \delta(\vec{k}_T) \left\{ S_2 - (\gamma_{S1} + 2\beta_0 S_1) \frac{1}{2\pi\mu^2} \left[\frac{\mu^2}{k_T^2} \right]_+^\mu \right\} \\ &\quad - \frac{\gamma_{\nu 1}}{2\pi\nu^2} \left[\frac{\nu^2}{k_T^2} \right]_+^\mu - \frac{\Gamma_1 + \Gamma_0 S_1}{\pi\mu^2} \left[\frac{\mu^2}{k_T^2} \ln \frac{k_T^2}{\nu^2} \right]_+^\mu \\ &\quad + \frac{\beta_0 \Gamma_0}{\pi\mu^2} \left[\frac{\mu^2}{k_T^2} \ln \frac{k_T^2}{\mu^2} \ln \frac{k_T^2}{\nu^2} \right]_+^\mu \\ &\quad + \frac{\Gamma_0^2}{2\pi\mu^2} \left[\frac{\mu^2}{k_T^2} \ln \frac{k_T^2}{\mu^2} \ln \frac{k_T^2}{\nu^2} \ln \frac{k_T^2 \mu^2}{\nu^4} \right]_+^\mu \end{aligned}$$

- Logs are minimized by $\nu = k_T$