

# Resummation for top pair production at the LHC

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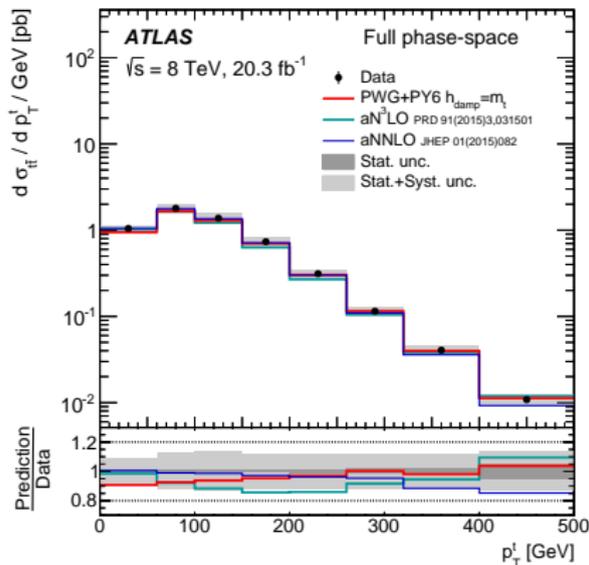
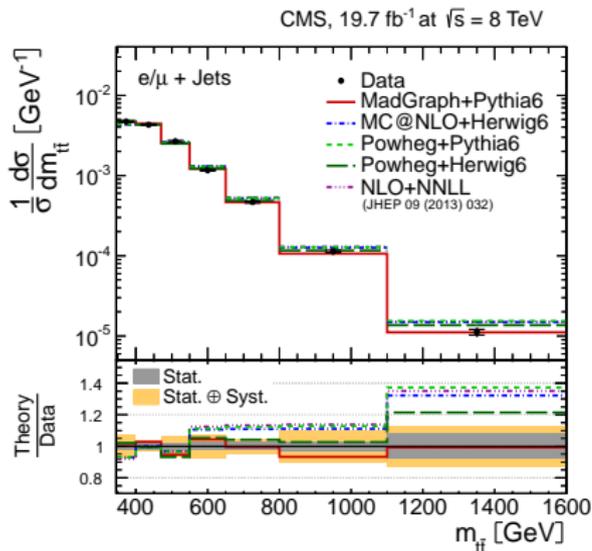


# Outline

- 1 Motivation
- 2 Threshold Resummation
- 3 Threshold + Small mass Resummation
- 4 Comparing Mellin and Momentum space
- 5 Results for Distributions
- 6 Conclusions

# Motivation

- Want to address the issue of boosted top production
- LHC 8 TeV results beginning to probe the “boosted” regime  
[\[1505.04480, 1511.04716\]](#)



- Already some 13 TeV results from boosted regime

# Framework

Consider  $t\bar{t}$  production at hadron colliders.

$$i(p_1) + j(p_2) \rightarrow t(p_3) + \bar{t}(p_4) + X(p_X)$$

With  $ij \in \{q\bar{q}, gg\}$  at leading order. QCD factorisation allows us to write the cross section for such processes as

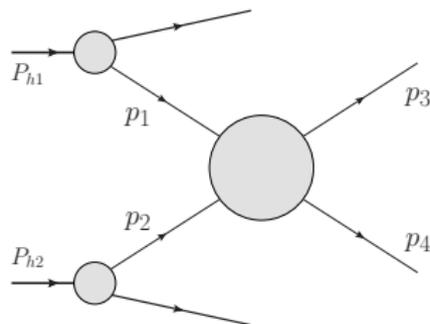
$$\frac{d^2\sigma(\tau)}{dM d\cos\theta} = \frac{8\pi\beta_t}{3sM} \sum_{ij} \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z, \mu_f) C_{ij}(z, M, m_t, \cos\theta, \mu_f)$$

$$\mathcal{L}_{ij}(y) = \int_0^1 \frac{dx}{x} \phi_i(x) \phi_j(y/x)$$

$$s = (P_{h1} + P_{h2})^2 \quad \hat{s} = (p_1 + p_2)^2$$

$$M_{t\bar{t}}^2 = (p_3 + p_4)^2$$

$$\tau = \frac{M_{t\bar{t}}^2}{s} \quad z = \frac{M_{t\bar{t}}^2}{\hat{s}}$$



True threshold:  $\tau \rightarrow 1$

Partonic threshold:  $z \rightarrow 1$

# Framework

Considering the invariant mass distribution we have

$$\frac{d\sigma(\tau)}{dM_{t\bar{t}}} = \int_{\tau}^1 \frac{dz}{z} \mathcal{L}(\tau/z, \mu_f) \frac{d\hat{\sigma}}{dM_{t\bar{t}}}(z, M_{t\bar{t}}, m_t, \mu_f)$$

Partonic cross section contains two types of logs

- Soft logs:  $\left[ \frac{\ln^m(1-z)}{1-z} \right]_+$
- Small mass logs:  $\ln(m_t/M_{t\bar{t}})$

Can consider two limits where these logs become important but also resumable

- Soft limit:  $\hat{s}, t_1, m_t^2 \gg \hat{s}(1-z)^2$ 
  - Realised as:  $z \rightarrow 1$
- Boosted Soft limit  $\hat{s}, t_1 \gg m_t^2 \gg \hat{s}(1-z)^2 \gg m_t^2(1-z)^2$ 
  - Realised as  $z \rightarrow 1$  and  $M_{t\bar{t}} \gg m_t$

# Factorisation

The partonic cross section factorises in Mellin space

[Kidonakis, Sterman, 97]

Using SCET it is possible to factorise the cross-section in the  $z \rightarrow 1$  limit. [Ahrens, Ferroglia, Neubert, Pecjak, Yang, '10]

Factorisation allows Resummation

$$C_{ij} = \text{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, \dots) \mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, \dots)] + \mathcal{O}(1-z)$$

Matrices in colour space

$\mathbf{H}_{ij}$  - Hard Function. Related to virtual corrections

$\mathbf{S}_{ij}$  - Soft Function. Related to real emission of soft gluons.

Contains distributions singular in  $(1-z)$ .

*Key to the resummation of soft logs*

# Factorisation

$$C_{ij} = \text{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, \dots) \mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, \dots)] + \mathcal{O}(1-z)$$

- Two one-scale problems, no ideal  $\mu_f$
- Derive and solve RG equations; soft function in Laplace (or Mellin) space
- We have a choice how to pick the soft scale
  - Pick numerically in momentum space (minimize higher order corrections)
  - Pick in Laplace space
- In SCET this is traditionally done in momentum space
- Here we will work in Mellin space.
  - Equivalent to Laplace space implementation up to power corrections

# Mellin v Momentum

We could have chosen to stay in momentum space

## Momentum (z) space

- Avoid issues with the Landau pole
- Soft scale has to be chosen numerically
- Resum logs containing this dynamically generated scale  $\mu_s$

$$\left[ \frac{\ln \left( \frac{M(1-z)}{\mu_s} \right)}{1-z} \right]_+$$

## Mellin (N) space

- Resum (almost) the correct logs

$$\left[ \frac{\ln^n(-\ln z)}{-\ln z} \right]_+ \vee \left[ \frac{\ln^m(1-z)}{1-z} \right]_+$$

- Need to choose a prescription to deal with the Landau pole
- Need to numerically invert the transform

# Mellin Space

Convolutions become products in Mellin space, we have

$$d\tilde{\sigma}(N) = \tilde{\mathcal{L}}(N)\tilde{\mathcal{C}}(N)$$

where  $\tilde{f}(N) = \mathcal{M}[f](N) = \int_0^1 dx x^{N-1} f(x)$

In Mellin space, the  $z \rightarrow 1$  limit corresponds to  $N \rightarrow \infty$

$$P_n(z) = \left[ \frac{\ln^n(1-z)}{1-z} \right]_+ \quad \bar{N} = Ne^{\gamma_E}$$

$$\mathcal{M}[P_0] = -\ln \bar{N} + \mathcal{O}(1/N)$$

$$\mathcal{M}[P_1] = \frac{1}{2} \left( \ln^2 \bar{N} + \frac{\pi^2}{6} \right) + \mathcal{O}(1/N)$$

$$\mathcal{M}[P_2] = -\frac{1}{3} \left( \ln^3 \bar{N} + \frac{\pi^2}{2} \ln \bar{N} + 2\zeta(3) \right) + \mathcal{O}(1/N)$$

The transform out of Mellin space requires a prescription to avoid Landau pole problems. We will return to this later.

# Mellin Space

$$C(N) = \text{Tr} \left[ \mathbf{H}(M_{t\bar{t}}, m_t, \mu_f, \dots) \tilde{\mathbf{S}} \left( \ln \frac{M_{t\bar{t}}^2}{\bar{N}^2 \mu_f^2}, m_t, \mu_f, \dots \right) \right] + \mathcal{O}(1/N)$$

Our RG equations are

$$\frac{d}{d \ln \mu} \mathbf{H} = \Gamma_H \mathbf{H} + \mathbf{H} \Gamma_H^\dagger$$

In particular, the Mellin space soft function becomes

$$\frac{d}{d \ln \mu} \tilde{\mathbf{S}} = - \left[ \Gamma_{\text{cusp}} \ln \frac{M^2}{\bar{N}^2 \mu^2} + \gamma^{s\dagger} \right] \tilde{\mathbf{S}} - \tilde{\mathbf{S}} \left[ \Gamma_{\text{cusp}} \ln \frac{M^2}{\bar{N}^2 \mu^2} + \gamma^s \right]$$

$$\gamma^s = \gamma^h + 2\gamma^\phi \mathbb{1}$$

$\gamma^\phi$  - PDF anomalous dimension.

# Threshold Resummation in Mellin Space

Solving the RG equations, our result for the hard scattering kernel becomes (suppressing some arguments)

$$C(N) = \text{Tr} \left[ \mathbf{U}(\mu_h, \mu_s) \mathbf{H}^m(M, m_t, \cos \theta, \mu_h) \mathbf{U}^\dagger(\mu_h, \mu_s) \tilde{\mathbf{S}}^m \left( \ln \frac{M}{\bar{N} \mu_s}, m_t, \cos \theta, \mu_s \right) \right]$$

$$\mathbf{U}(\mu_h, \mu_s) = \exp \left\{ 2S(\mu_h, \mu_s) - 2a_\Gamma(\mu_h, \mu_s) \ln \left( \frac{M}{\mu_h} \right) + 2a_\Gamma(\mu_f, \mu_s) \ln \bar{N} + 2a_{\gamma_\phi}(\mu_s, \mu_f) \right\} \\ \times \mathbf{u}^m(\mu_h, \mu_s)$$

Where:

$$S(\mu_h, \mu_s) = - \int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_s)} d\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\mu_h)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} \\ a_\gamma(\mu_h, \mu_s) = - \int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_s)} d\alpha \frac{\gamma(\alpha)}{\beta(\alpha)} \\ \mathbf{u}^m(\mu_h, \mu_s) = \mathcal{P} \exp \left\{ \int_{\mu_h}^{\mu_s} \frac{d\mu'}{\mu'} \gamma^{h,m}(M, m_t, \cos \theta, \alpha_s(\mu')) \right\}$$

# Threshold Resummation in Mellin Space

We can re-write this as

$$C(N) = \exp \left\{ \underbrace{\frac{4\pi}{\alpha_s(\mu_h)} g_1^m(\lambda, \lambda_f) + g_2^m(\lambda, \lambda_f)}_{\text{LL}} + \frac{\alpha_s(\mu_h)}{4\pi} g_3^m(\lambda, \lambda_f) + \dots \right\}^{\text{NLL}}$$
$$\times \text{Tr} \left[ \mathbf{u}^m(\mu_h, \mu_s) \mathbf{H}^m(M, m_t, \cos \theta, \mu_h) \mathbf{u}^{m\dagger}(\mu_h, \mu_s) \tilde{\mathbf{S}}^m \left( \ln \frac{M}{\mu_s \bar{N}}, M, m_t, \cos \theta, \mu_s \right) \right]$$

where

$$\lambda = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \left( \frac{\mu_h}{\mu_s} \right) \quad \lambda_f = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \left( \frac{\mu_h}{\mu_f} \right)$$

Choosing  $\mu_h \sim M$  and  $\mu_s \sim M/\bar{N}$  removes the presence of large logs in

the hard and soft functions respectively and resums (to a given accuracy) in the exponential

# Boosted Soft Resummation

In the boosted soft limit  $\hat{s}, t_1 \gg m_t^2 \gg \hat{s}(1-z)^2 \gg m_t^2(1-z)^2$   
( $z \rightarrow 1$  and  $M \gg m_t$ ) logs of the form  $\ln(M/m_t)$  become important.

Our previous cross section factorises in this limit

$$C(N) = \text{Tr} \left[ \mathbf{H}^m(M, m_t, \mu_f, \dots) \tilde{\mathbf{S}}^m \left( \ln \frac{M^2}{\bar{N}^2 \mu_f^2}, m_t, \mu_f, \dots \right) \right] + \mathcal{O}(1/N)$$



$$M^2 \gg m_t^2$$

$$C(N) = \text{Tr} \left[ \mathbf{H}(M, \mu_f, \dots) C_D^2(m_t, \mu_f) \tilde{\mathbf{S}} \left( \ln \frac{M^2}{\bar{N}^2 \mu_f^2}, \mu_f, \dots \right) \tilde{\mathbf{s}}_D^2 \left( \ln \frac{m_t}{\bar{N} \mu_f}, \mu_f \right) \right] \\ + \mathcal{O}(1/N) + \mathcal{O}(m_t/M)$$

Each of these functions  
known to two-loops

**H:** [Ferrogli et. al '15]

**S:** [Ferrogli, Pecjak, Yang '12]

$s_D, C_D$ : [Melnikov, Mitov '04] & [Becher, Neubert '05]

# Boosted Soft Resummation

We can repeat the same procedure as before, deriving and solving RG equations for each of the scales.

$$\begin{aligned} C(N) = & \exp \left\{ \frac{4\pi}{\alpha_s(\mu_h)} (g_1(\lambda, \lambda_f) + g_1^D(\lambda_{dh}, \lambda_{ds}, \lambda_f)) \right. \\ & \left. + (g_2(\lambda, \lambda_f) + g_2^D(\lambda_{dh}, \lambda_{ds}, \lambda_f)) + \dots \right\} \\ & \times \text{Tr} \left[ \mathbf{u}(M, \cos \theta, \mu_h, \mu_s) \mathbf{H}(M, \cos \theta, \mu_h) \mathbf{u}^\dagger(M, \cos \theta, \mu_h, \mu_s) \right. \\ & \left. \times \tilde{\mathbf{S}} \left( \ln \frac{M^2}{\bar{N}^2 \mu_s^2}, M, \cos \theta, \mu_s \right) \right] C_D^2(m_t, \mu_{dh}) \tilde{s}_D^2 \left( \ln \frac{m_t^2}{\bar{N}^2 \mu_{ds}^2}, \mu_{ds} \right) \end{aligned}$$

With two-loop matching functions NNLL  $\rightarrow$  NNLL'

Where, similarly

$$\lambda_i = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \left( \frac{\mu_h}{\mu_i} \right) \quad \mathbf{u}(M, \cos \theta, \mu_h, \mu_s) = \mathcal{P} \exp \left\{ \int_{\mu_h}^{\mu_s} \frac{d\mu'}{\mu'} \gamma^h(M, \cos \theta, \alpha(\mu')) \right\}$$

Again, we can pick the scale for each function to free it of large logs.

$$\mu_h \sim M, \mu_s \sim M/\bar{N}, \mu_{dh} \sim m_t \text{ and } \mu_{ds} \sim m_t/\bar{N}$$

# Mellin Inversion

To obtain results in momentum space, we need to invert the Mellin transform

$$\frac{d\sigma(\tau)}{dM d\cos\theta} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \frac{d\tilde{\sigma}(N)}{dM d\cos\theta}$$

With  $c$  to the right of all singularities. But our resummed coefficient function contains (exponentiated)

$$g_1(\lambda, \lambda_f) = \frac{\Gamma_0}{4\beta_0^2} [\lambda + (1 - \lambda \ln(1 - \lambda) + \lambda \ln(1 - \lambda_f))] \quad \lambda = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln\left(\frac{\mu_h}{\mu_s}\right)$$

Since we pick  $\mu_s \sim M/N$ , pole at  $\lambda = 1$

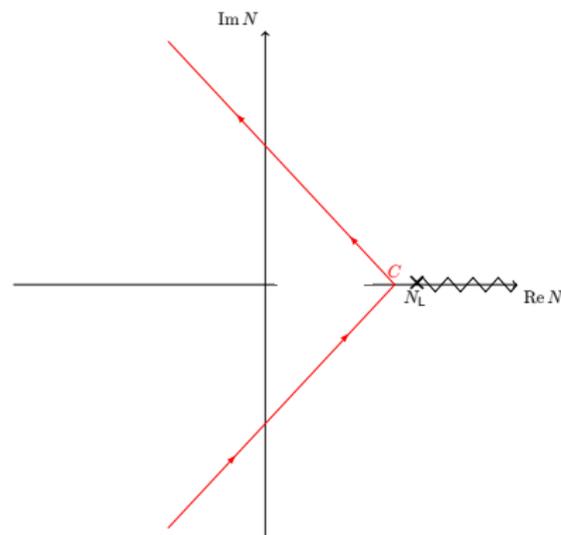
$$N_L = \exp\left(\frac{2\pi}{\alpha_s \beta_0}\right)$$

# Minimal Prescription

- We need to select a method to deal with the Landau pole.
- We use the *Minimal Prescription*:  
Select our point on the real axis to be to the *left* of the Landau pole, but to the right of all other singularities in the integrand.

[Catani, Mangano, Nason, Trentadue '96]

$$\frac{d\sigma(\tau)}{dM d\cos\theta} = \frac{1}{2\pi i} \int_{\text{MP}_C} dN \tau^{-N} \frac{d\tilde{\sigma}(N)}{dM d\cos\theta}$$



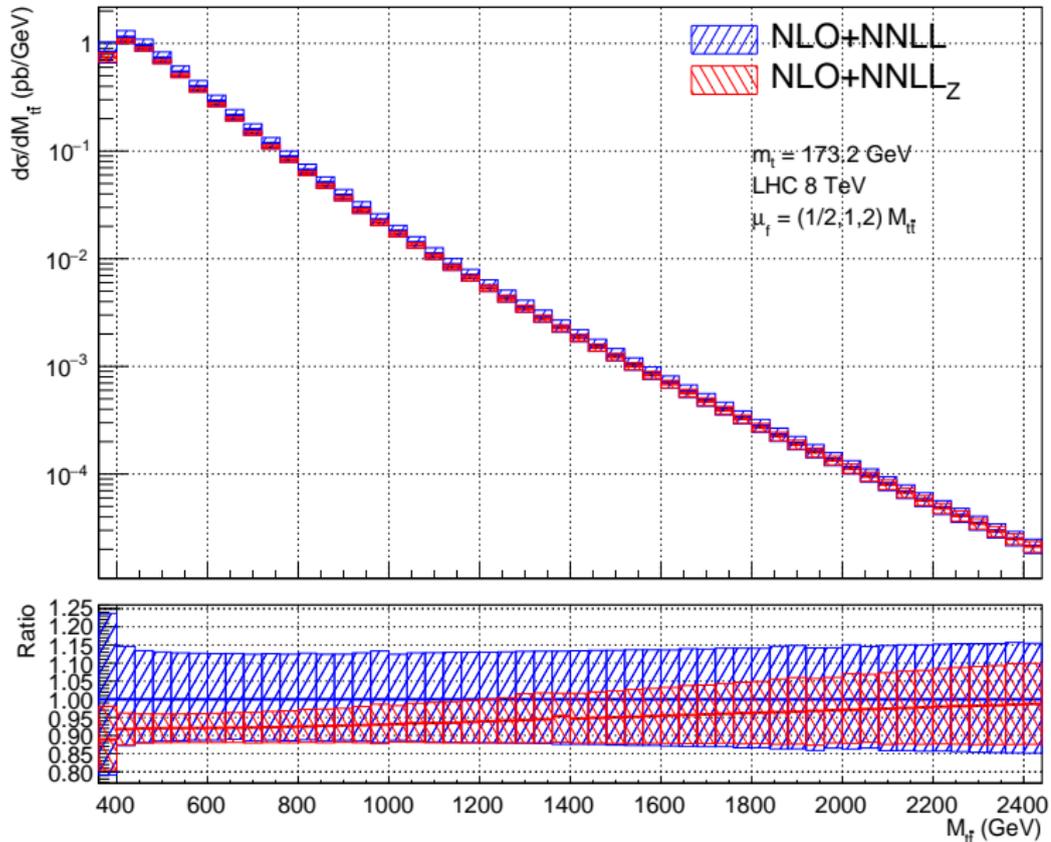
# Matching with NLO

We want to combine our results for boosted soft resummation with standard soft resummation and exact NLO (eventually NNLO).

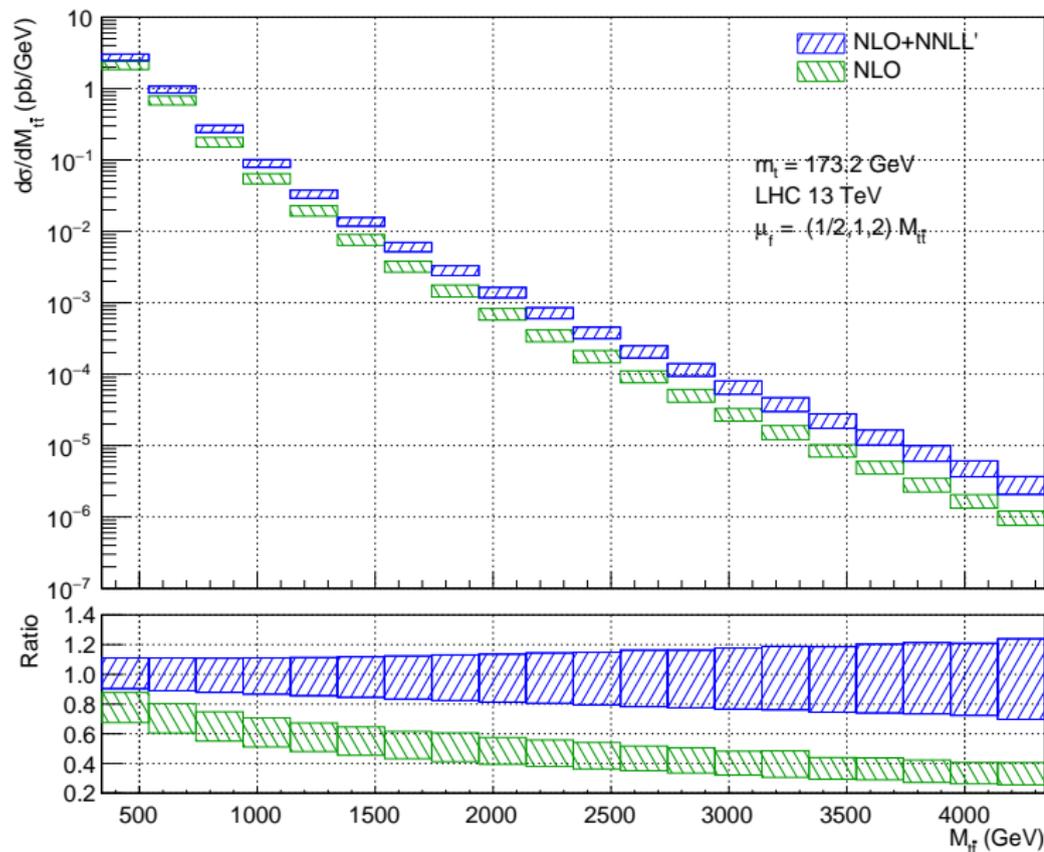
## Matching:

$$d\sigma^{\text{NLO+NNLL}'} = \underbrace{d\sigma^{\text{NNLL}'}}_{\text{Missing parts subleading in } m_t/M \text{ and } 1/N} + \underbrace{\left( \underbrace{d\sigma_m^{\text{NNLL}}}_{\text{Missing parts subleading in } 1/N} - \underbrace{d\sigma^{\text{NNLL}} \Big|_{\substack{\mu_{dh}=\mu_h \\ \mu_{ds}=\mu_s}}}_{\text{Removes double counting}} \right)}_{\text{Adds in parts subleading in } m_t/M \text{ but enhanced by } \ln N} + \underbrace{\left( d\sigma^{\text{NLO}} - d\sigma_m^{\text{NNLL}} \Big|_{\substack{\mu_h=\mu_f \\ \mu_s=\mu_f}} \right)}_{\text{Adds exact NLO results, avoiding double counting}}$$

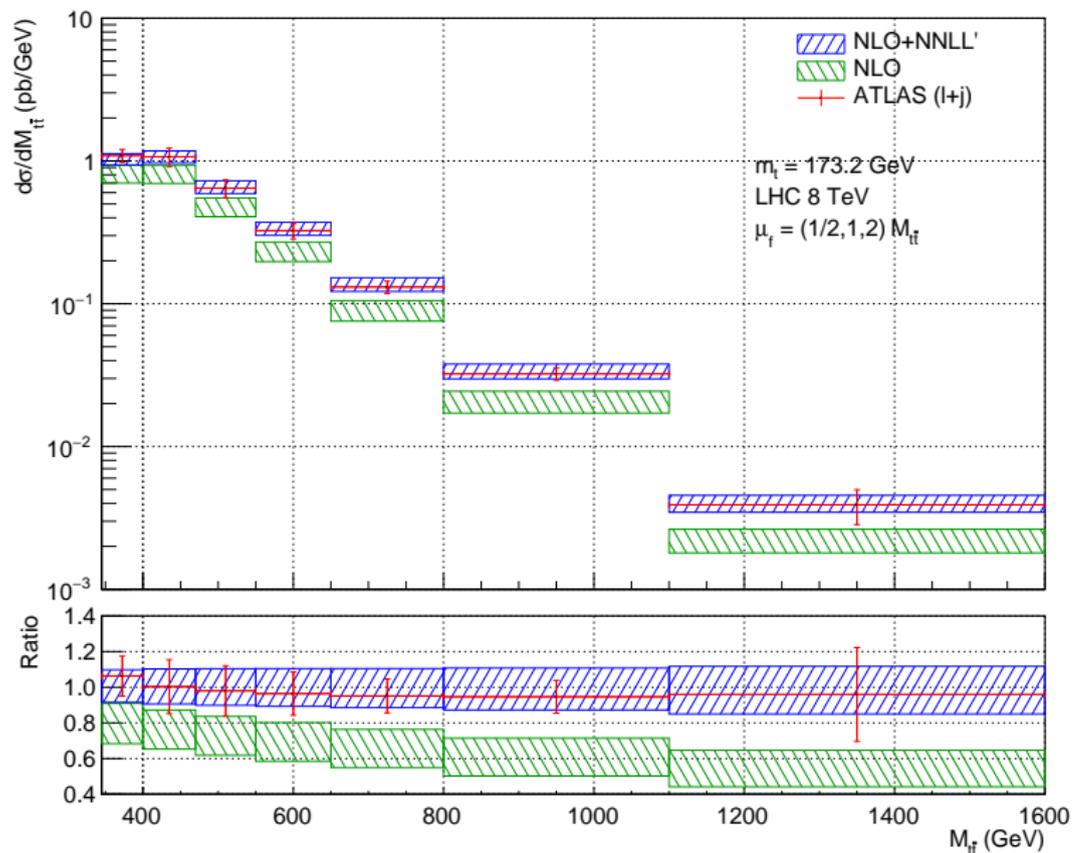
# Mellin v Momentum



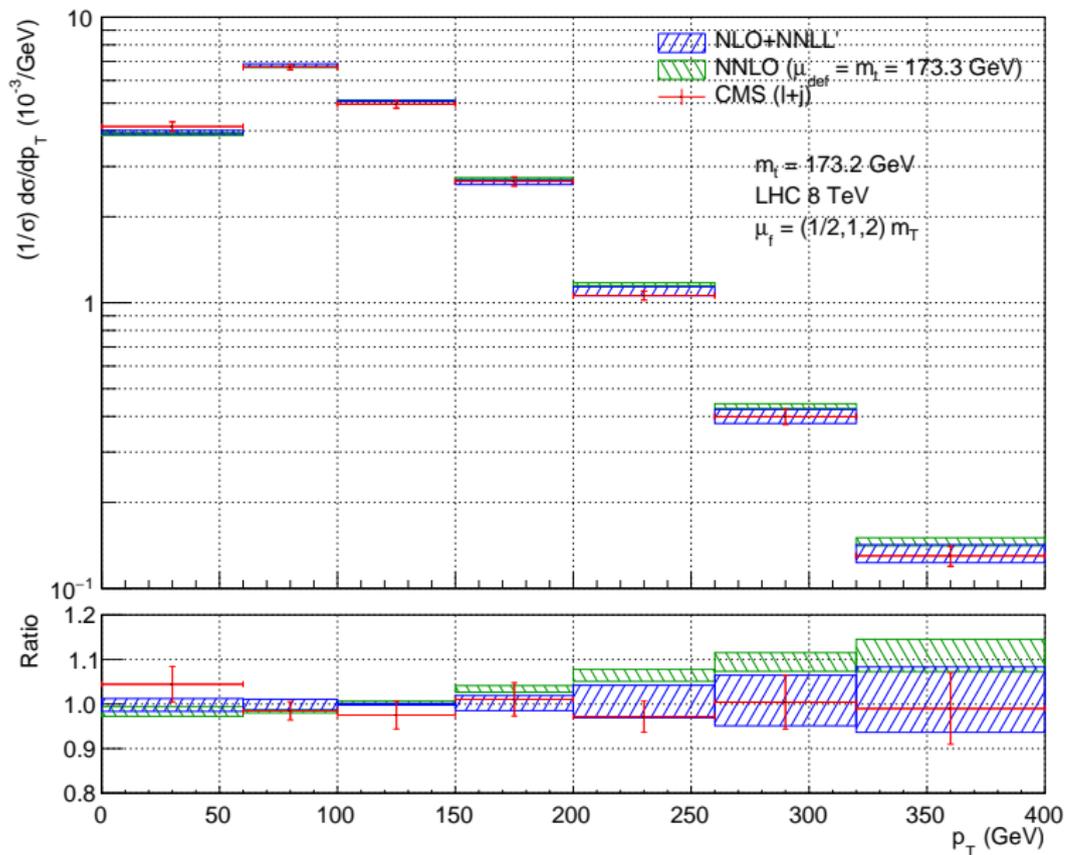
# Applications to the LHC: Effects at 13 TeV



# Application to the LHC: 8 TeV Data



# Applications to the LHC: 8 TeV $p_T$ Data



# Matching with NNLO

Eventually we will wish to match with the exact NNLO result. Only available for fixed scales at present. [Czakon, Fiedler, Heymes, Mitov]

$$\begin{aligned}
 d\sigma^{\text{NNLO}+\text{NNLL}'} &= \overbrace{d\sigma^{\text{NLO}+\text{NNLL}'-\text{NNLO}}}^{\text{Our result with only terms beyond NNLO}} + d\sigma^{\text{NNLO}} \\
 d\sigma^{\text{NLO}+\text{NNLL}'-\text{NNLO}} &= d\sigma^{\text{NLO}+\text{NNLL}'} - d\sigma^{\text{NLO}+\text{NNLL}'} \Big|_{\text{NNLO expansion}}
 \end{aligned}$$

Recall that:

$$d\sigma^{\text{NLO}+\text{NNLL}'} = d\sigma^{\text{NNLL}'} + \left( d\sigma_m^{\text{NNLL}} - d\sigma^{\text{NNLL}} \Big|_{\substack{\mu_{dh}=\mu_h \\ \mu_{ds}=\mu_s}} \right) + \text{standard NLO matching}$$

When performing the NNLO expansion, since the matching functions in  $d\sigma_m^{\text{NNLL}}$  and  $d\sigma^{\text{NNLL}} \Big|_{\substack{\mu_{dh}=\mu_h \\ \mu_{ds}=\mu_s}}$  are known only to NLO, the NNLO terms keep dependence on  $\mu_h$  and  $\mu_s$ .

Given by:

$$d\sigma_m^{\text{NNLL}} \Big|_{\text{NNLO expansion}} = d\sigma_m^{\text{NNLL}} \Big|_{\substack{\text{NNLO Expansion} \\ \text{with } \mu_i=\mu_f}} - d\sigma_m^{\text{NNLL}} \Big|_{\substack{\text{NNLO part only} \\ \text{with default scales} \\ \mu_h, \mu_s}}$$

# Beyond NNLO

It is instructive to quantify the contributions from resummation beyond NNLO.

Since NNLO is not yet available with dynamic scale choices we use approximate NNLO

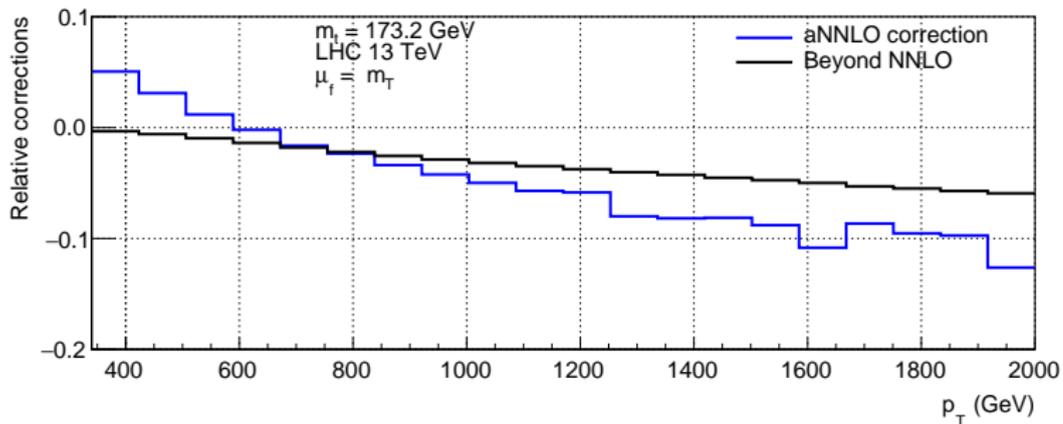
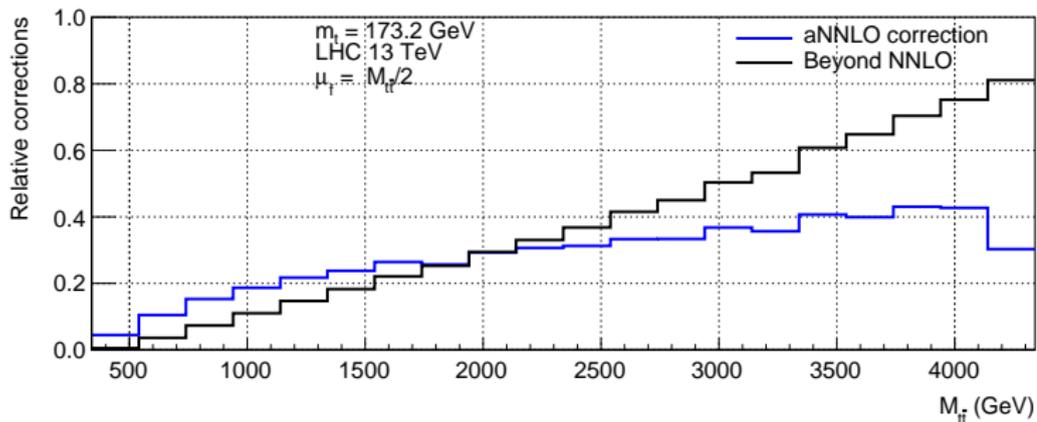
- Construct approx NNLO from the expansion of our resummed formula, truncated at NNLO.

$$\text{aNNLO correction} = \frac{d\sigma^{\text{aNNLO}} - d\sigma^{\text{NLO}}}{d\sigma^{\text{NLO}}}$$

- Effects beyond aNNLO

$$\text{beyond NNLO} = \frac{d\sigma^{\text{NLO+NNLL}'} - d\sigma^{\text{aNNLO}}}{d\sigma^{\text{NLO}}}$$

# Beyond NNLO



# Conclusions

- Likely to see many more boosted top events
- Important to have accurate predictions for top physics
- SCET allows a simultaneous resummation of soft and small mass logs
- Here, we presented a methodology for resumming such logs to  $\text{NLO}+\text{NNLL}'$  accuracy
- Slight differences between Mellin and momentum space results
- Phenomenological application: differential top pair invariant mass and top quark transverse momentum distributions.

# BACKUP SLIDES

# Total Cross section

Can consider the impact of resummation on the total cross section

	Tevatron	LHC 8 TeV	LHC 13 TeV
NNLO	$7.031^{+0.260(3.7\%)}_{-0.375(5.3\%)}$	$239.9^{+9.3(3.9\%)}_{-14.9(6.2\%)}$	$790.6^{+27.2(3.4\%)}_{-43.9(5.6\%)}$
NNLO+NNLL	$7.187^{+0.110(1.5\%)}_{-0.201(2.8\%)}$	$246.6^{+6.3(2.5\%)}_{-8.5(3.4\%)}$	$808.6^{+19.4(2.4\%)}_{-28.6(3.5\%)}$
NNLO+NNLL'	$7.44^{+0.56(7.5\%)}_{-0.95(12.8\%)}$	$248.8^{+13.8(5.5\%)}_{-12.5(5.0\%)}$	$816.7^{+41.3(5.1\%)}_{-65.2(8.0\%)}$

Slight enhancement of the total Cross section.