


QCD factorization for exclusive hadronic Z decays into flavor singlet mesons

Matthias König
THEP, Johannes Gutenberg-University (Mainz)

XIIIth annual workshop on Soft-Collinear Effective Theory

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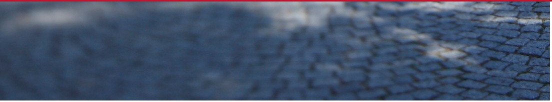


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Flashback: Hadronic Z decays in QCD factorization

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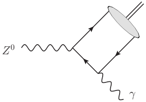

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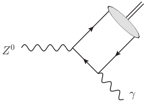

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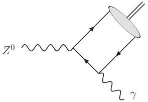

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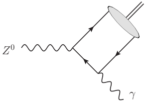

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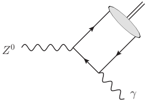

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$\Rightarrow$ Subject of today's talk!

Based on:

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Exclusive Radiative Decays of W and Z Bosons in QCD Factorization

Yuval Grossman, MK, Matthias Neubert

JHEP 1504 (2015) 101, arXiv:1501.06569

Exclusive Radiative Z-Boson Decays to Mesons with Flavor-Singlet Components

Stefan Alte, MK, Matthias Neubert

JHEP 1602 (2016) 162, arXiv:1512.09135

Outline

1. QCD factorization
   - The factorization formula for the di-quark operator
   - Renormalization of the effective operator
   - The factorization formula for the di-gluon operator
   - Renormalization of the di-guon operator
2. The full amplitude
   - RG evolution of the full amplitude
   - Mixing in the η-η' system
   - Numbers

QCD factorization

The factorization formula for the di-quark operator

History

The framework of QCD factorization was originally developed by Brodsky, Efremov, Lepage and Radyushkin in the beginning of the 1980's.

[Brodsky, Lepage (*1979*), Phys. Lett. B 87, 359]

[Brodsky, Lepage (*1980*), Phys. Rev. D 22, 2157]

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The factorization formula was **derived using light-cone perturbation theory**.

The derivation **can also be phrased in** the language of **soft-collinear effective theory**.

[Bauer et al. (*2001*), Phys. Rev. D 63, 114020]

[Bauer Pirjol, Stewart (*2002*), Phys. Rev. D 65, 054022]

[Beneke, Chapovsky, Diehl, Feldmann (*2002*), Nucl. Phys. B 643, 431]

An effective theory approach

QCD factorization: The **hadronization** happens well **after the hard scattering has taken place** $\rightarrow$ separation of scales.

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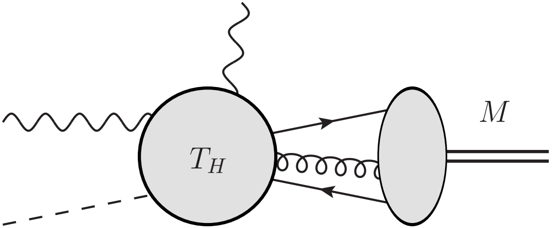

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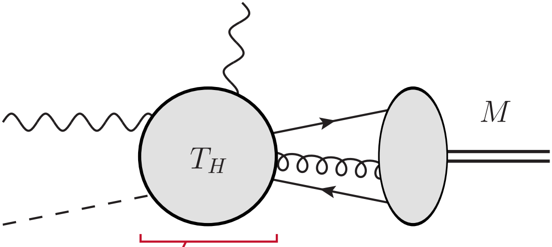

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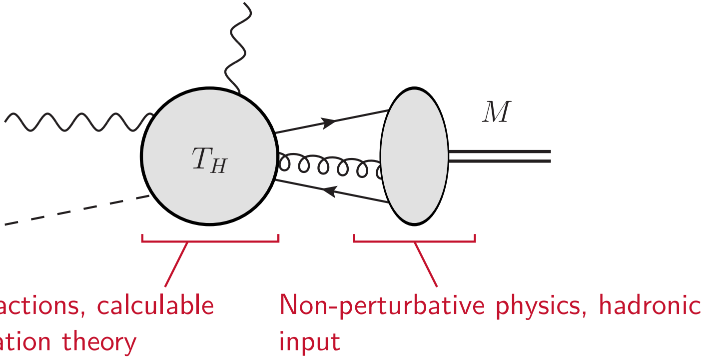

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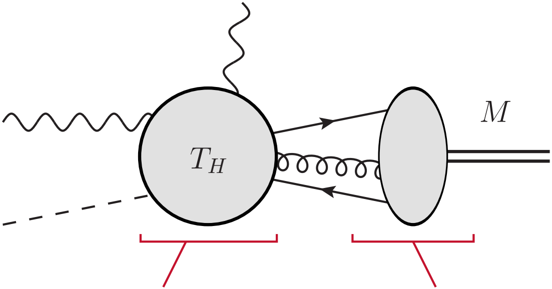


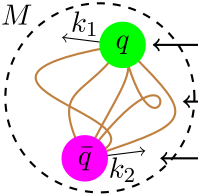
Hard interactions, calculable in perturbation theory

Non-perturbative physics, hadronic input

The **scale separation** in the case at hand **calls for an effective theory** description!

An effective theory approach

The simplest way to imagine the meson in its rest frame:



valence quark, momentum $k_1^\mu \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$

non-perturbative muck

valence anti-quark, momentum $k_2^\mu \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$

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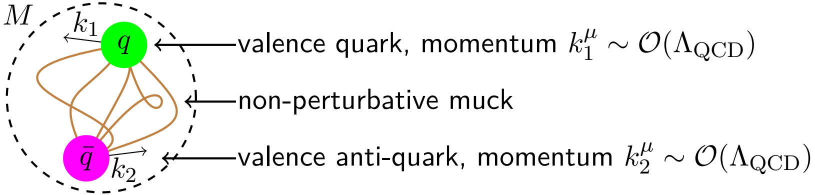
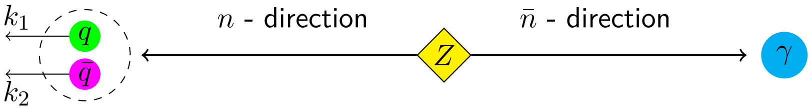


Now: Boost to the rest frame of the decaying Z boson:



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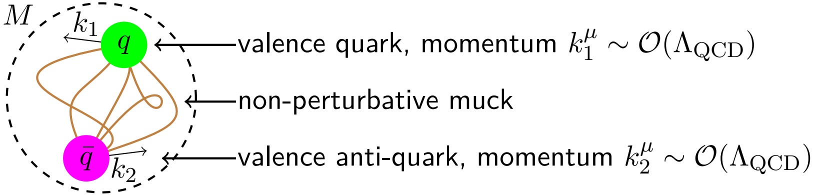

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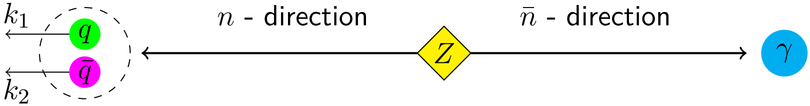

In the Z boson's rest frame, the two quarks move collinear with momenta

$$k_i^\mu = \frac{m_Z}{2}\left(x_i n^\mu + \lambda\, n_{i\perp}^\mu\right) \qquad \lambda = \frac{\Lambda_{\rm QCD}}{m_Z}$$

An effective theory approach

According to this picture we construct an effective operator from two collinear quarks:

$$J_q \quad \sim \quad \bar{q}_c \ldots q_c + \bar{q}_c \ldots (\bar{n} \cdot \partial) q_c + \quad \ldots \quad = \quad \bar{q}_c(x) \ldots q_c(x + t\bar{n})$$

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The amplitude for $Z \to M\gamma$ is then given by:

$$i\mathcal{A} = \mathcal{C}(t, \dots) \otimes \langle M(k)| \, J_q(t, \dots) \, |0\rangle$$

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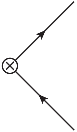
This matrix element defines a **hadronic function**, analogous to the decay constants. In fact, these are just the local case $(t = 0)$ above. The generalization to our **bi-local current operator**

$$\langle M(k) | \, J_q(t, \dots) \, | 0 \rangle \sim f_M \int e^{i(t\bar{n}) \cdot (xk)} \phi_M^q(x) dx$$

defines the light-cone distribution amplitude (LCDA).

Renormalization of the effective operator

The matrix element receives radiative corrections beginning at $\mathcal{O}(\alpha_s)$.



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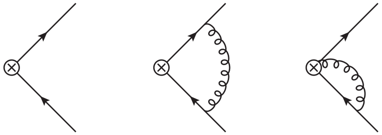

These corrections are UV-divergent. From computing the corrections at $\mathcal{O}(\alpha_s)$, the **renormalized LCDA** can be extracted.

$$\Rightarrow \quad \phi_M^{\text{ren}}(x,\mu) = \int Z(x,y,\mu)\,\phi_M^{\text{bare}}(y)dy$$

$$\text{with} \quad Z(x,y,\mu) = \delta(x,y) + \frac{\alpha_s(\mu)}{4\pi\epsilon} V(x,y)$$

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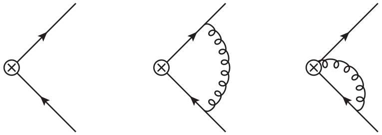

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From this, the counterterms for the Wilson coefficients are constructed:

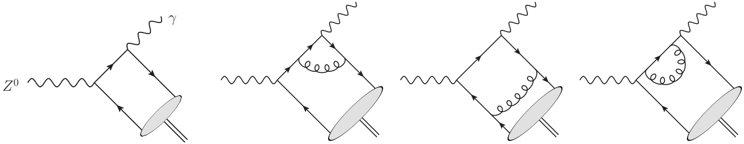
$$\mathcal{C}^{\text{ren}}(x,\mu) = \int Z^{-1}(y,x,\mu) \mathcal{C}^{\text{bare}}(y) dy$$

where the tree-level coefficient convoluted with $V(x,y)$ yields the counterterm - as usual.

The corresponding hard scattering kernel

We want to use these considerations to calculate $Z \to \eta^{(\prime)}\gamma$!

$\Rightarrow$ Match the diagrams contributing to $Z \to \bar{q}q\gamma$ onto our operator J:



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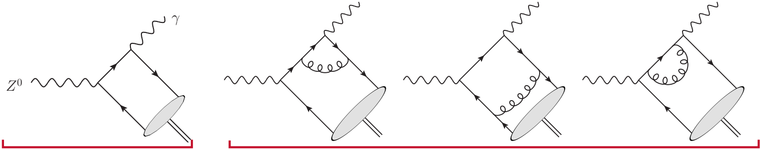

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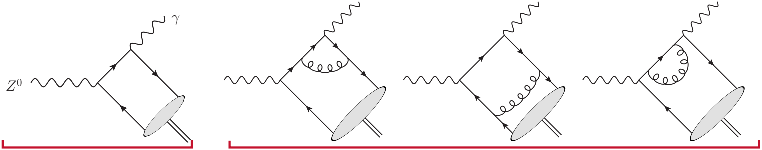

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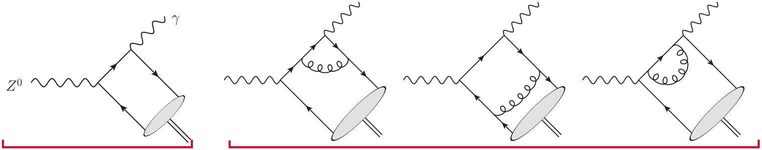

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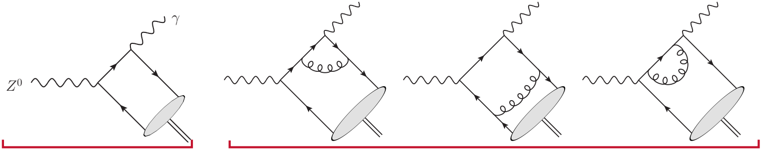

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The rest is book-keeping, but this is all old news from last year. So what exactly *is* special about the $\eta^{(\prime)}$ mesons?

QCD factorization

The factorization formula for the di-gluon operator

The answer: Gluons

So far, we have - in the parton picture - looked at the diagrams that give us $q\bar{q}$ but what about **gluons**?

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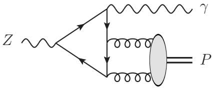
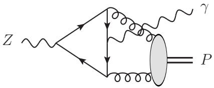
The **gluon contribution indeed exists** and is an $\mathcal{O}(\alpha_s)$-correction to the hard function. Contributing diagrams look like this:





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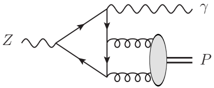

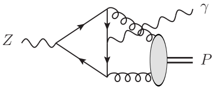

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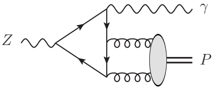

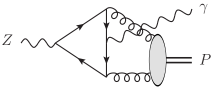

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Gluon contribution exists only for **flavor singlet mesons**!

Effective operator construction

Using the **gauge covariant building block** $\mathcal{A}_c^\mu = W^\dagger(x)\,(iD_c^\mu W(x))$, we can swiftly write down the bi-local current:

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It gives us the **distribution** amplitude of two collinear **gluons**, **separated along the light-cone** by $t\bar{n}$, to excite a meson from the vacuum.

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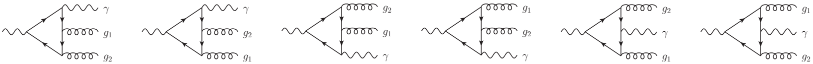
It gives us the **distribution** amplitude of two collinear **gluons**, **separated along the light-cone** by $t\bar{n}$, to excite a meson from the vacuum.

We can now go ahead and match this onto our diagrams $Z \to gg\gamma$.

The gluon amplitude

Integrating out the hard scattering is done by computing the appropriate diagrams.



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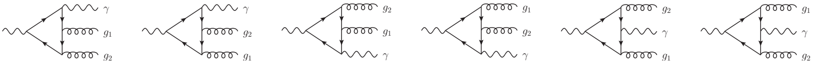

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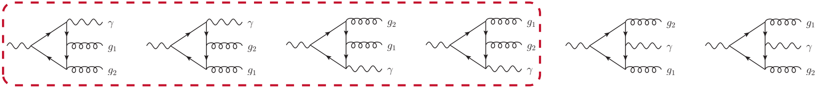

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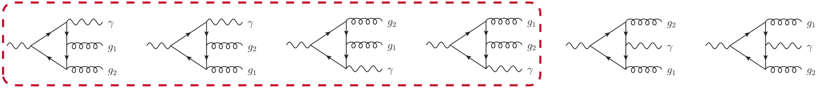

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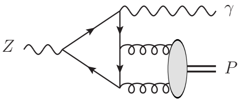

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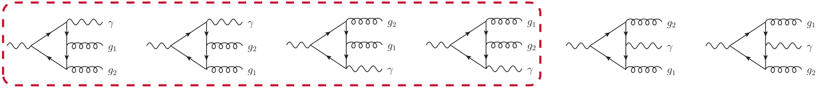

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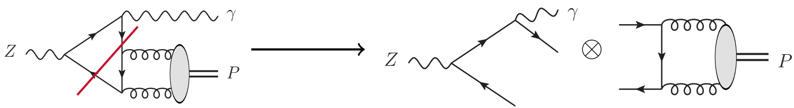
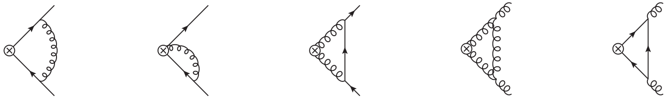


Renormalization of the di-gluon operator

Compute radiative corrections to the operators:



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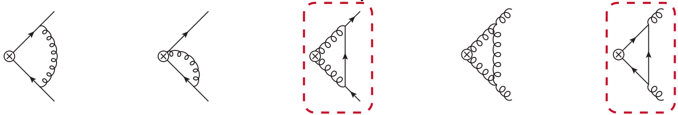

UV-divergent corrections **mix** the **quark and gluon current** operators.

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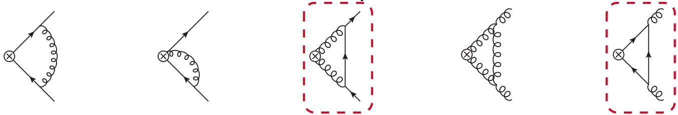
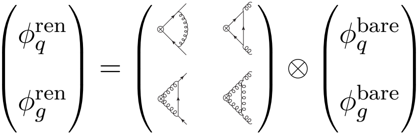


UV-divergent corrections **mix** the **quark and gluon current** operators.

Promote $\phi^{\text{ren}} = Z \otimes \phi^{\text{bare}}$ to a matrix equation:



[Terentev (*1981*), Sov. J. Nucl. Phys. 33, 911]

[Ohrndorf (*1981*), Nucl. Phys. B 186, 153]

[Shifman, Vysotsky (*1981*), Nucl. Phys. B 186, 475]

[Baier, Grozin (*1981*), Nucl.Phys. B192 476-488]

To be more specific:

$$\begin{pmatrix} \phi_q^{\text{ren}}(x,\mu) \\ \phi_g^{\text{ren}}(x,\mu) \end{pmatrix} = \int_0^1 \left[\mathbf{1} \cdot \delta(x-y) \; + \; \frac{\alpha_s(\mu)}{4\pi\epsilon} \begin{pmatrix} V_{qq}(x,y) & V_{qg}(x,y) \\ V_{gq}(x,y) & V_{gg}(x,y) \end{pmatrix} \right] \begin{pmatrix} \phi_q^{\text{bare}}(y) \\ \phi_g^{\text{bare}}(y) \end{pmatrix} dy$$

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The tree-level Wilson coefficient of the quark contribution convoluted with $V_{qg}(x,y)$ now cancels the infrared divergences in the gluon contribution:

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$\Rightarrow$ The result is **finite**!

The full amplitude

RG evolution of the full amplitude

The flavor of etas: Singlets and octets

Remember: We are trying to compute the rate for $Z \to \eta^{(\prime)}\gamma$.

The $\eta^{(\prime)}$ mesons contain **admixtures of flavor singlets and octets**:

$$|\eta^{(8)}\rangle = \frac{1}{\sqrt{6}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle - 2\,|s\bar{s}\rangle\right)$$
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$$

The singlet component scale-evolves according to what we just learned. The octet just evolves like every other non-singlet meson (no gluon contribution).

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However, at the factorization scale $\mu \approx m_Z$, a flavor singlet is

$$\frac{1}{\sqrt{5}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle + |c\bar{c}\rangle + |b\bar{b}\rangle\right).$$

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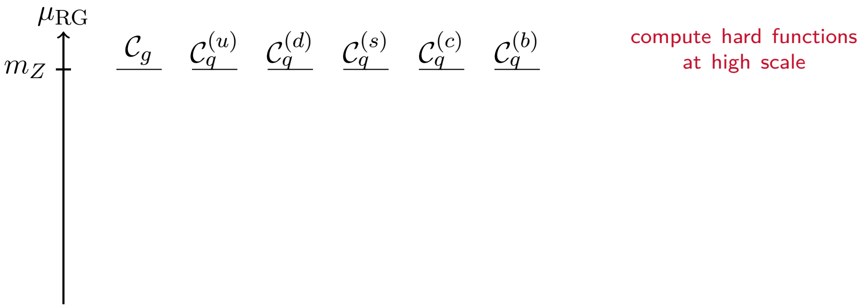

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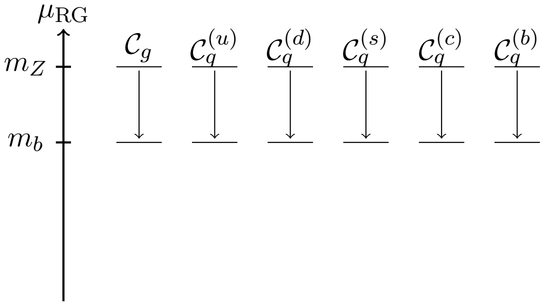

compute hard functions at high scale

RG running $m_Z \to m_b$

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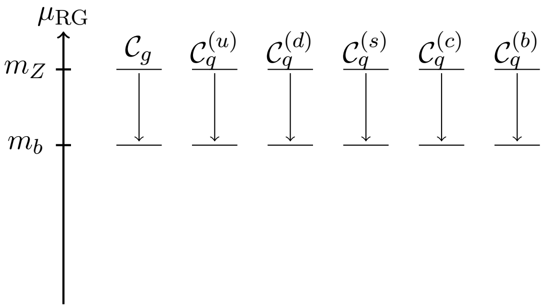

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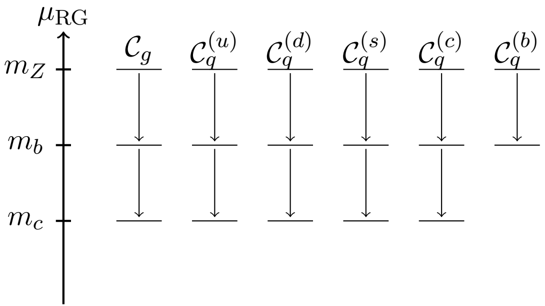

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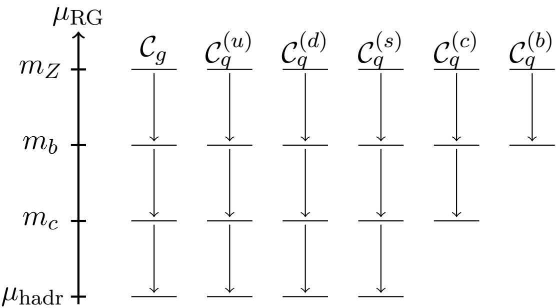

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RG running $m_c \to \mu_{\mathrm{hadr}}$

The anomalous dimensions of the Wilson coefficients are defined by:

$$\left[\frac{d}{d\log\mu} - \frac{\alpha_s}{4\pi}\begin{pmatrix}\gamma_n^{qq} & \gamma_n^{qg}\\ \gamma_n^{gq} & \gamma_n^{gg}\end{pmatrix}^T + \mathcal{O}\left(\alpha_s^2\right)\right]\begin{pmatrix}\mathcal{C}_n^{(1)}\\ \mathcal{C}_n^{g}\end{pmatrix} = 0$$

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$\Rightarrow$ **Weak sensitivity to LCDA moments** a_n^P, b_n^P thanks to **RGE**.

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The **final** factorization **formula** (after rearranging) at the **low scale** is then:

$$\mathcal{A}_{Z\to P\gamma} = \mathcal{Q}^{(1)}\left(\mathcal{C}_q^{(1)} \otimes \sum_{q=u,d,s}\left(f_P^{(q)}\phi_P^{(q)}\right) + \mathcal{C}_g \otimes \left(f_P^{uds}\phi_P^{g}\right)\right) + \mathcal{Q}^{(8)}\mathcal{C}_q^{(8)} \otimes \left(f_P^{(8)}\phi_P^{(8)}\right)$$
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The convolutions $\mathcal{C} \otimes \phi$ can be written as sums and are:

$$\mathcal{C}_q^{(i)} \otimes \phi_P^{(q)} = \sum_{n=0}^{\infty} \left(1 + \frac{C_F \alpha_s(\mu)}{4\pi} c_n(\mu) \right) a_n^P \qquad \mathcal{C}_g \otimes \phi_P^{g} = \frac{T_F \alpha_s(\mu)}{4\pi} \sum_{n=0}^{\infty} 5 d_n(\mu)\, b_n^P$$

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Hadronic shape parameters a_n^P, b_n^P from the Gegenbauer expansion (eigenfunctions of the RG evolution kernels V_{qq} and V_{gg}):

$$\phi_P^{(q)}(x) = 6x(1-x)\sum_n a_n^{P,q} C_n^{(3/2)}(2x-1)$$

$$\phi_P^{g}(x) = 30x^2(1-x)^2\sum_n b_n^P C_{n-1}^{(5/2)}(2x-1)$$

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Hard scattering coefficients:

$$c(\mu) = -\left[4H_{n+1} - 3 - \frac{2}{(n+1)(n+2)} \right] \left(\log \frac{m_Z^2}{\mu^2} - i\pi \right)$$
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The flavor of etas: Singlets and octets

The **final** factorization **formula** (after rearranging) at the **low scale** is then:

$$\mathcal{A}_{Z\to P\gamma} = \mathcal{Q}^{(1)}\left(\mathcal{C}_q^{(1)} \otimes \sum_{q=u,d,s}\left(f_P^{(q)}\phi_P^{(q)}\right) + \mathcal{C}_g \otimes \left(f_P^{uds}\phi_P^{g}\right)\right) + \mathcal{Q}^{(8)}\mathcal{C}_q^{(8)} \otimes \left(f_P^{(8)}\phi_P^{(8)}\right)$$
$$+ \mathcal{Q}^{(c)}\mathcal{C}_q^{(c)} \otimes \left(f_P^{(c)}\phi_P^{(c)}\right) + \mathcal{Q}^{(b)}\mathcal{C}_q^{(b)} \otimes \left(f_P^{(b)}\phi_P^{(b)}\right)$$

The convolutions $\mathcal{C} \otimes \phi$ can be written as sums and are:

$$\mathcal{C}_q^{(i)} \otimes \phi_P^{(q)} = \sum_{n=0}^{\infty}\left(1 + \frac{C_F\alpha_s(\mu)}{4\pi}c_n(\mu)\right)a_n^P \qquad \mathcal{C}_g \otimes \phi_P^{g} = \frac{T_F\alpha_s(\mu)}{4\pi}\sum_{n=0}^{\infty} 5\underline{d_n(\mu)}\, b_n^P$$

Hard scattering coefficients:

$$d_n(\mu) = \frac{20n(n+3)}{3(n+1)(n+2)}\left[\left(\log\frac{m_Z^2}{\mu^2} - i\pi\right) - 2H_{n+1} - 1 + \frac{1}{(n+1)(n+2)}\right]$$

The full amplitude

Mixing in the η-η' system

The flavor of etas: η and η'

The physical η and η' are linear combinations of octets and singlets.

When $SU(3)$ flavor symmetry is assumed, the η is a pure octet whereas the η' is a pure singlet.

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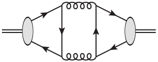

"OZI-violating effects"

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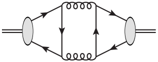

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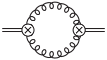
- Axial anomaly:



$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = 2im_q\bar{q}\gamma_5 q - \frac{\alpha_s}{4\pi}G^A_{\mu\nu}\tilde{G}^{A,\mu\nu}$$

(local interaction $\eta_{q/s}G\tilde{G}$)

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The **dominant** contribution to this mixing is the **anomaly** and one neglects OZI-violating effects. This allows one to describe η-η' mixing with a **single mixing angle**.

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The physical $\eta^{(\prime)}$ states are now:

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Now all we need is the hadronic input parameters f_q, f_s, φ and the LCDAs ϕ_q, ϕ_s and ϕ_g.

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The **LCDA shape parameters** a_n^P and b_n^P are determined from $\gamma\gamma^* \to \eta^{(\prime)}$ form factors, assuming a **pion-like shape for the quark-LCDA** and **fitting** the **gluon-LCDA** to the data.

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These fits rely on the FKS-scheme and thus every set of **LCDA-parameters** is **tied to** a set of **FKS-parameters**.

Branching ratios

Choosing six different sets of hadronic parameters we find varying results:

Model	(i)	(ii)	(iii)
$10^9 \cdot \mathrm{Br}(Z \to \eta\gamma)$	0.16 ± 0.05	0.17 ± 0.05	0.16 ± 0.05
$10^9 \cdot \mathrm{Br}(Z \to \eta'\gamma)$	4.70 ± 0.23	4.77 ± 0.24	4.73 ± 0.24
Model	**(iv)**	**(v)**	**(vi)**
$10^9 \cdot \mathrm{Br}(Z \to \eta\gamma)$	0.11 ± 0.03	0.10 ± 0.03	$0.010\,^{+0.014}_{-0.010}$
$10^9 \cdot \mathrm{Br}(Z \to \eta'\gamma)$	3.43 ± 0.17	3.08 ± 0.15	4.84 ± 0.23

$\Rightarrow$ With enough statistics, one could use these decays to test the hadronic parameters.

However, the branching ratios of $\mathcal{O}(10^{-9})$ for η' and even $\mathcal{O}(10^{-10})$ for η makes this very challenging, even for a future Z-factory.

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Thank you for your attention!

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