Jet shapes in dijet events at the LHC in SCET

Yiannis Makris
Duke University

In collaboration with
Andrew Hornig (LANL) and Thomas Mehen (Duke U.)

[arXiv: 1601.01319]

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Outline

Boost Invariant Angularities

Jet, Hard, Beam Functions

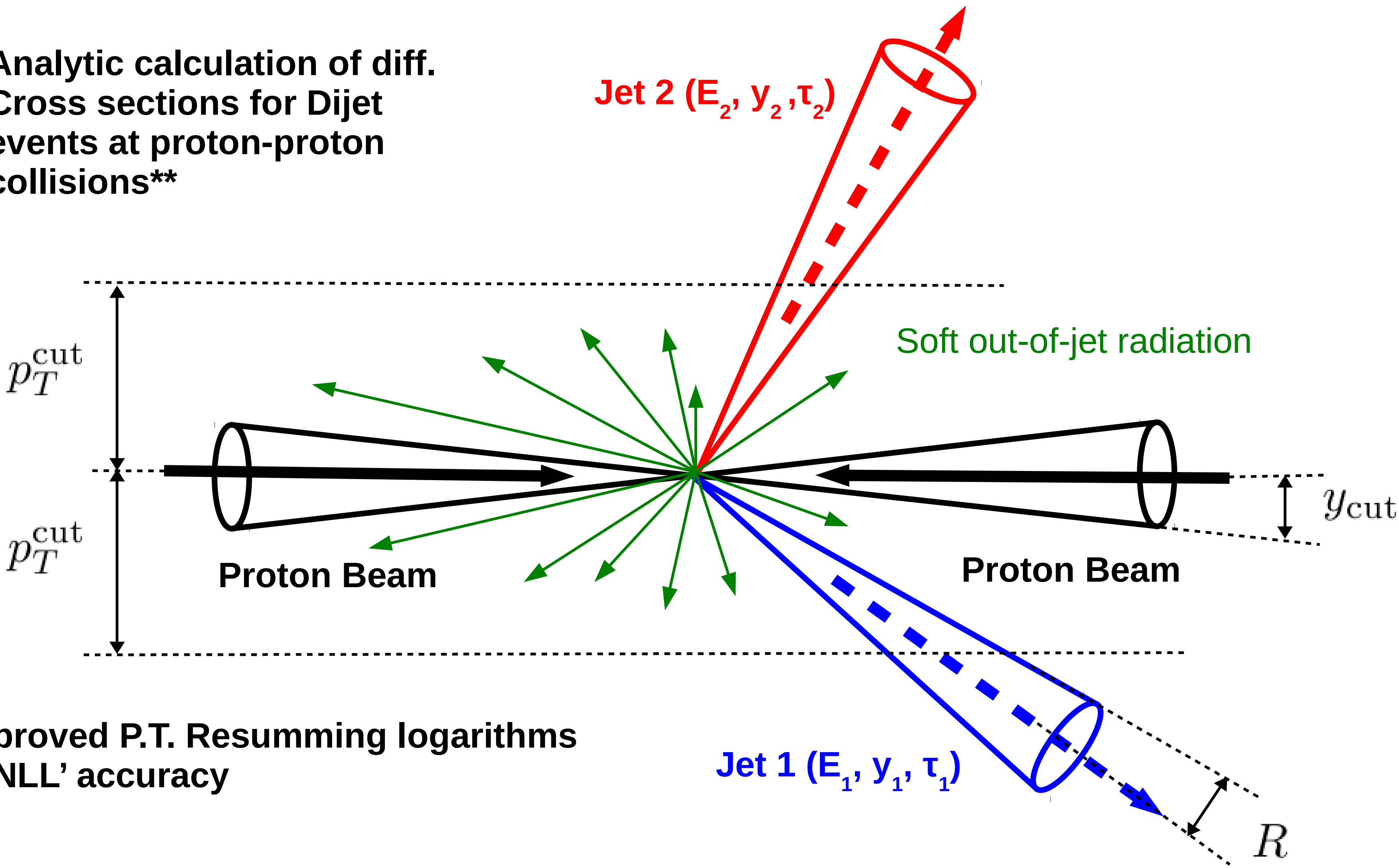
Soft Function

Scales and R.G. Evolution - Theoretical Uncertainties –
Plots for NLL'

Summary - Applications

Problem Setup

Analytic calculation of diff. Cross sections for Dijet events at proton-proton collisions**



Improved P.T. Resumming logarithms at NLL' accuracy

**Extension of the work on e+e- to N jets by Ellis, Vermilion, Walsh, Hornig and Lee, [arXiv: 1001.0014]

Angularities

Rotational invariant

Almeida et al. [arXiv: 0807.0234]
Berger, Kucs, and Sterman [hep-ph/ 0303051]

$$\tau_a^{e^+e^-} = \frac{1}{2E_J} \sum_i |p_T^{iJ}| e^{-(1-a)|y_{iJ}|}$$

$$= (2E_J)^{-(2-a)} (p_T)^{1-a} \sum_i |p_T^i| \left(\frac{\theta_{iJ}}{\sin \theta_J} \right)^{2-a} (1 + \mathcal{O}(\theta_{iJ}^2))$$

Boost invariant

$$\tau_a \equiv \tau_a^{pp} \equiv \frac{1}{p_T} \sum_i |p_T^i| (\Delta \mathcal{R}_{iJ})^{2-a}$$

$$= \left(\frac{2E_J}{p_T} \right)^{2-a} \tau_a^{e^+e^-} + \mathcal{O}(\tau_a^2)$$

where $\quad \Delta \mathcal{R}_{ij} \equiv \sqrt{(\Delta y_{ij})^2 + (\Delta \phi_{ij})^2}$

Angularities

Rotational invariant

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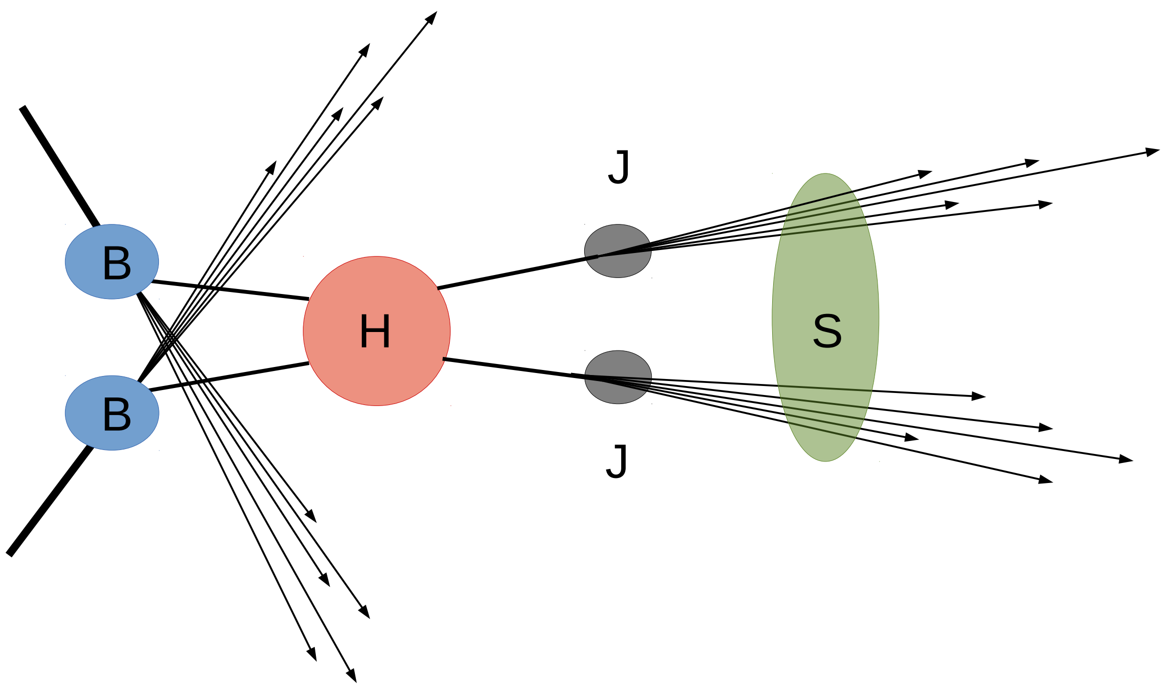
The Factorization Theorem in SCET

$$d\sigma(\tau_a^1, \tau_a^2) \equiv \frac{d\sigma}{dy_1 dy_2 dp_T d\tau_a^1 d\tau_a^2}$$

$$= \frac{p_T}{8\pi x_1 x_2 E_{\rm cm}^4} \frac{1}{N} B(x_1, \mu) \bar{B}(x_2, \mu) \, \mathrm{Tr}\{\mathbf{H}(\mu)\mathbf{S}(\tau_a^1, \tau_a^2, \mu)\} \otimes [J_1(\tau_a^1, \mu) J_2(\tau_a^2, \mu)]$$



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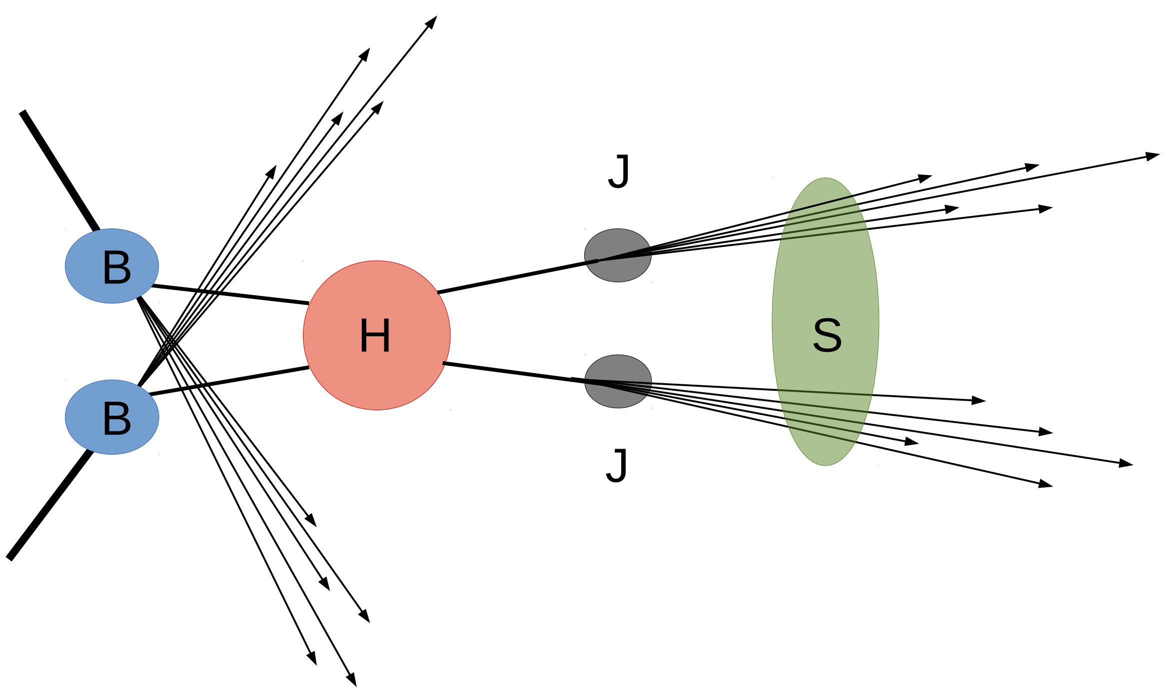
Hard Function



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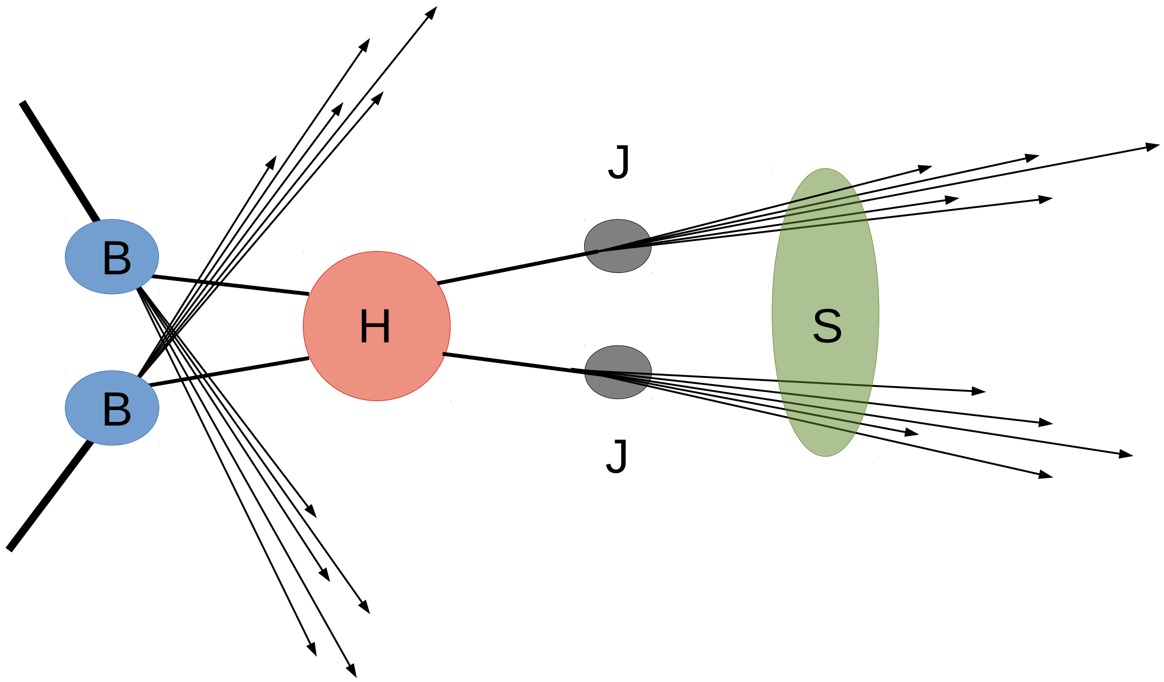
Hard Function

Soft Function



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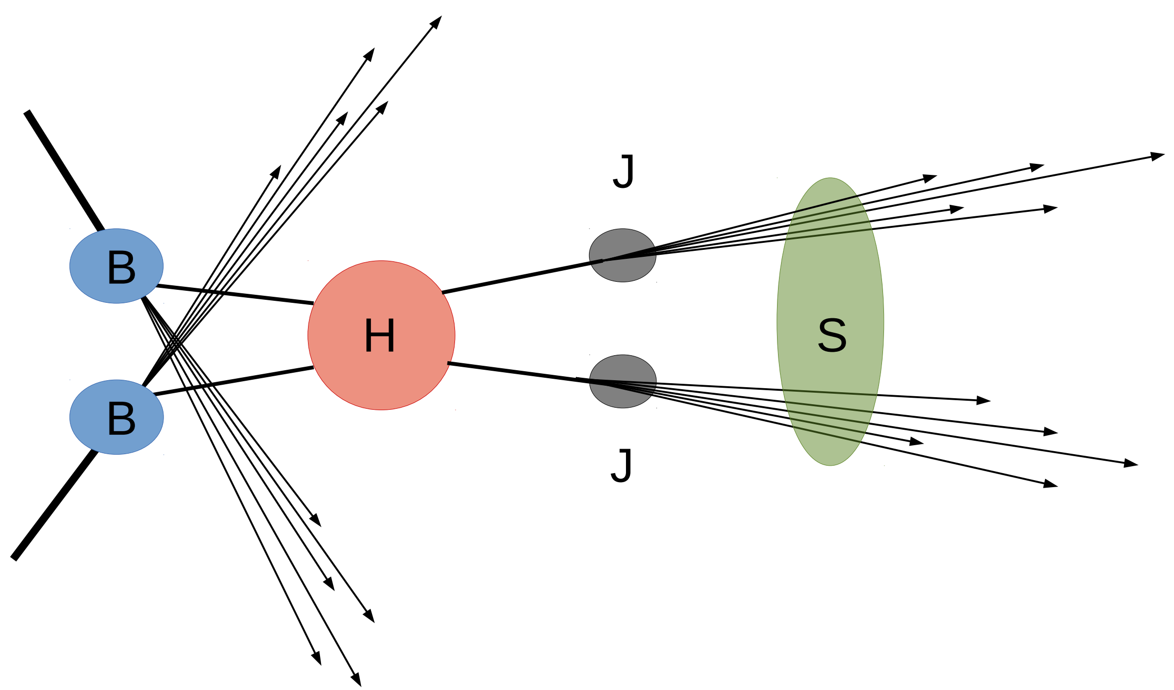
Hard Function **Soft Function** **Jet Functions**



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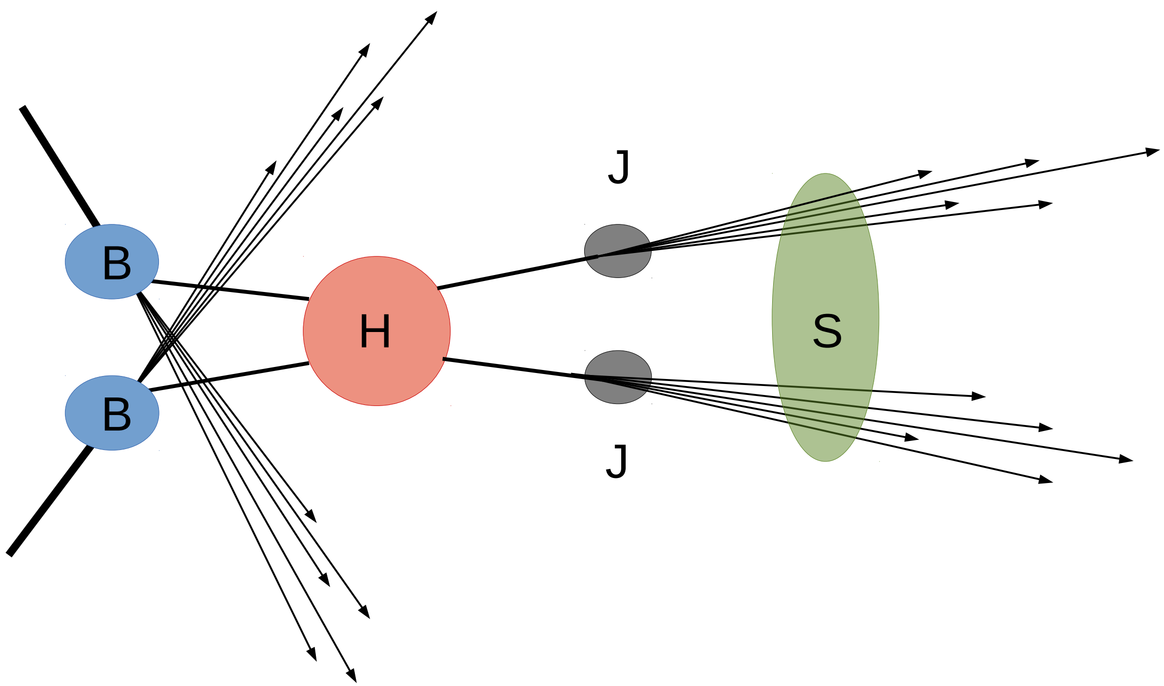
Unmeas. Beam Functions **Hard Function** **Soft Function** **Jet Functions**



Jet Functions

$$\int \frac{dk}{(2\pi)^4} \exp(-ik \cdot x) J_{n,\omega}(\tau, k^-) \left(\frac{\not{n}}{2}\right)_{\alpha\beta} = \langle \Omega | \chi^{\alpha}_{n,\omega}(x) \delta(\tau - \hat{\tau}) \bar{\chi}^{\beta}_{n,\omega}(0) | \Omega \rangle$$

Jet Functions

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Quark Jet Function

Similarly for Gloun Jets

Jet Functions

$$\int \frac{dk}{(2\pi)^4} \exp(-ik\cdot x) J_{n,\omega}(\tau, k^-) \left(\frac{\not{n}}{2}\right)_{\alpha\beta} = \langle \Omega | \chi^{\alpha}_{n,\omega}(x) \delta(\tau - \hat{\tau}) \bar{\chi}^{\beta}_{n,\omega}(0) | \Omega \rangle$$

$$A^{-1}\delta\left(A^{-1}\tau - \hat{\tau}\right) = \delta\left(\tau - A\hat{\tau}\right)$$

$$\longrightarrow \quad J_i(\tau_a) = \left(\frac{p_T}{2E_J}\right)^{2-a} J_i^{e^+e^-}\left(\left(\frac{p_T}{2E_J}\right)^{2-a} \tau_a\right)$$

Ellis, Vermilion, Walsh, Hornig and Lee, [arXiv: 1001.0014]

Hard Function

$$H_{IJ}(\mu) = C_I(\mu) C_J^*(\mu)$$

$C_I(\mu)$ **Wilson Coefficients**

Kelley and Schwartz [arXiv: 1008.2759]

$$\frac{d\,\mathbf{H}}{d\ln\mu} = \mathbf{\Gamma}_H\,\mathbf{H} + \mathbf{H}\,\mathbf{\Gamma}_H^\dagger$$

$$\mathbf{\Gamma}_H = \frac{1}{2}\Gamma_H \mathbf{1} + \Gamma_c \mathbf{M}(m_i)$$

$$\mathbf{H}(\mu, \mu_H) = \Pi_H(\mu, \mu_H)\,\mathbf{\Pi}_H(\mu, \mu_H)\mathbf{H}(\mu_H)\mathbf{\Pi}_H^\dagger(\mu, \mu_H)$$

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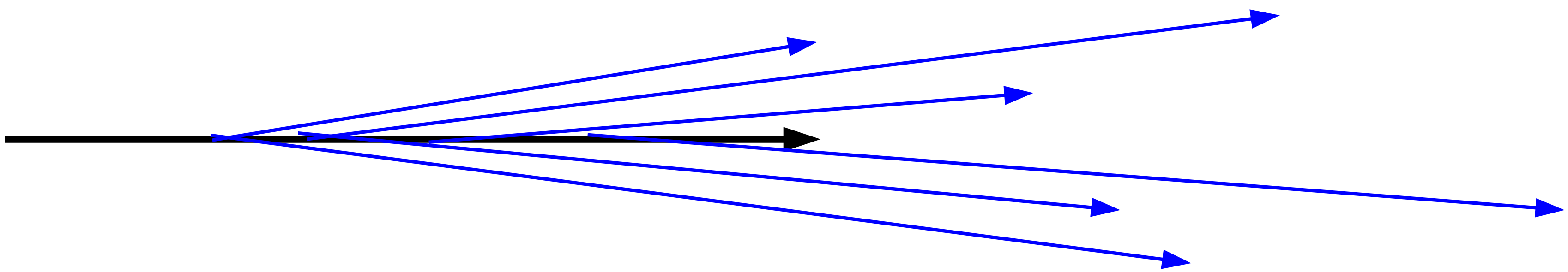
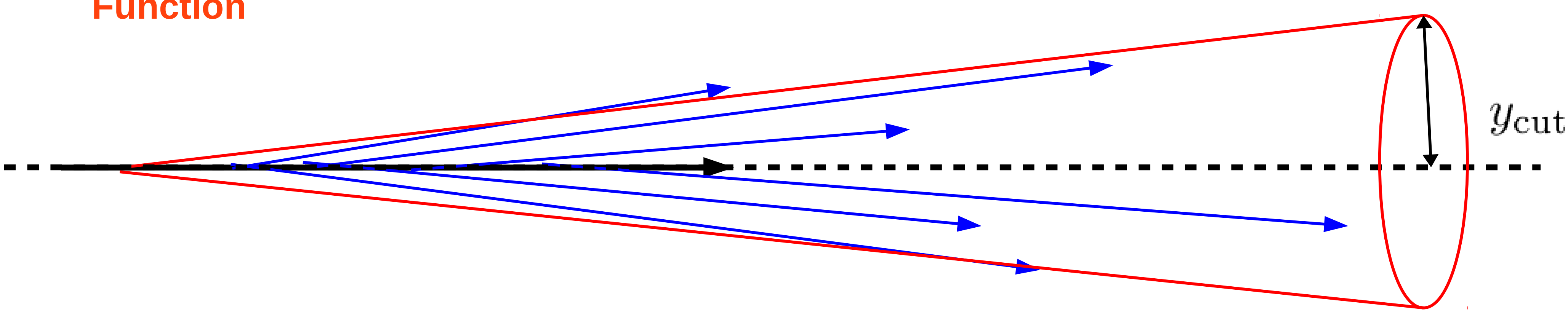
Beam Function

Ritzmann and Waalewijn [arXiv:1407.3272]
Stewart, Tackmann and Waalewijn [arXiv:0910.0467]

Measured Beam Function $B_i(s, x, \mu)$



Unmeasured Beam Function $B_i(x_i, \mu) \equiv B_i(\mathrm{E_{cm}}, y_{\mathrm{cut}}, x_i, \mu)$



Beam Function ("Unmeasured")

$$B_i(x_i,\mu) \equiv B_i(\mathrm{E}_{\mathrm{cm}}, y_{\mathrm{cut}}, x_i, \mu)$$

$$= \sum_j \int \frac{dz}{z} \mathcal{J}_{ij}(x_i \mathrm{E}_{\mathrm{cm}} e^{-y_{\mathrm{cut}}}, z, \mu) f_j(x_i/z, \mu) + \mathcal{O}(\Lambda^2_{\mathrm{QCD}}/E^2)$$

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Short Distance Matching Coefficients.

At NLO by Procura and Waalewijn, [arXiv: 1111.6605] for the (final state) FJF

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Short Distance Matching Coefficients.

Parton Distribution Functions (PDF)

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Short Distance Matching Coefficients.

Parton Distribution Functions (PDF)

At NLO by Procura and Waalewijn, [arXiv: 1111.6605] for the (final state) FJF

$$\mu_J = 2E \tan(R/2) \longrightarrow \mu_{B_i} = x_i E_{\mathrm{CM}} \exp(-y^{\mathrm{cut}})$$

$$\Rightarrow \gamma_B(\mu_B, \mu) = \gamma_J(\mu_B, \mu)$$

Soft Function

$$S_{IJ}(\tau_1,\tau_2) = \mathcal{N}\sum_X \langle\Omega|\mathcal{W}_I^\dagger|X\rangle\langle X|\mathcal{W}_J|\Omega\rangle\ \delta(\tau_1-\tau_1^X)\delta(\tau_2-\tau_2^X)$$

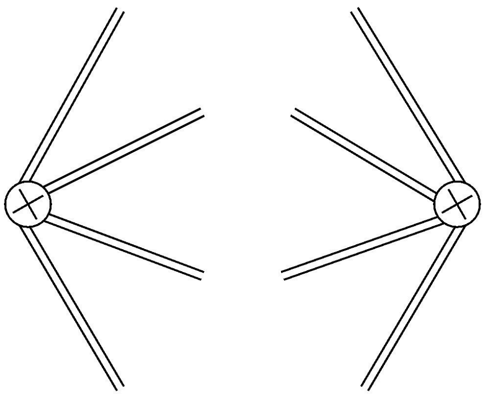
$$\text{where}\quad \tau_i^X = \begin{cases} \tau^X & : \text{inside jet}\ \ \text{i} \\ 0 & : \text{outside jet}\ \ \text{i} \end{cases}$$

Soft Function

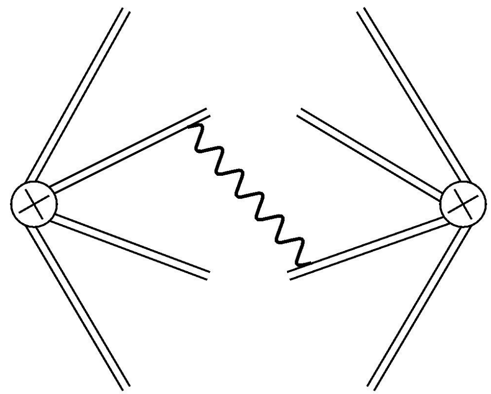
$$S_{IJ}(\tau_1, \tau_2) = \mathcal{N} \sum_X \langle \Omega | \mathcal{W}_I^\dagger | X \rangle \langle X | \mathcal{W}_J | \Omega \rangle \; \delta(\tau_1 - \tau_1^X)\delta(\tau_2 - \tau_2^X)$$

Time ordered product of Wilson lines.

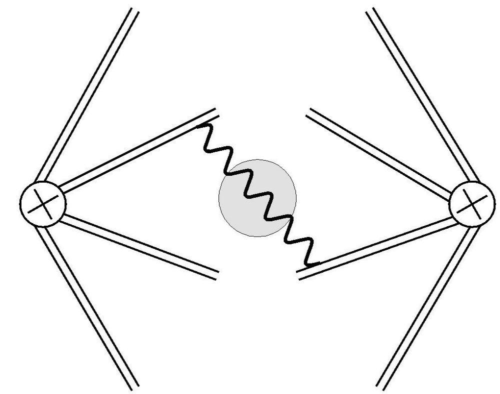
$$\text{where} \quad \tau_i^X = \begin{cases} \tau^X & : \text{inside jet } \; \text{i} \\ 0 & : \text{outside jet } \; \text{i} \end{cases}$$



Leading Order (LO) contribution



Next to Leading Order (NLO) contribution outside Jets



Next to Leading Order (NLO) contribution inside Jets

Next to Leading Order Form of the Soft Function

2-measured 0-unmeasured Jets

$$\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1)\delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1)\delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)$$

1-measured 1-unmeasured Jets

$$\mathbf{S}(\tau_a) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a) + \mathbf{S}_0 S^{\text{meas}}(\tau_a) + \mathcal{O}(\alpha_s^2)$$

0-measured 2-unmeasured Jets

$$\mathbf{S} = \mathbf{S}^{\text{unmeas}} + \mathcal{O}(\alpha_s^2)$$

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \mathcal{O}(\alpha_s)$$

Next to Leading Order Form of the Soft Function

2-measured 0-unmeasured Jets

$$\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1)\delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1)\delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)$$

1-measured 1-unmeasured Jets

$$\mathbf{S}(\tau_a) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a) + \mathbf{S}_0 S^{\text{meas}}(\tau_a) + \mathcal{O}(\alpha_s^2)$$

0-measured 2-unmeasured Jets

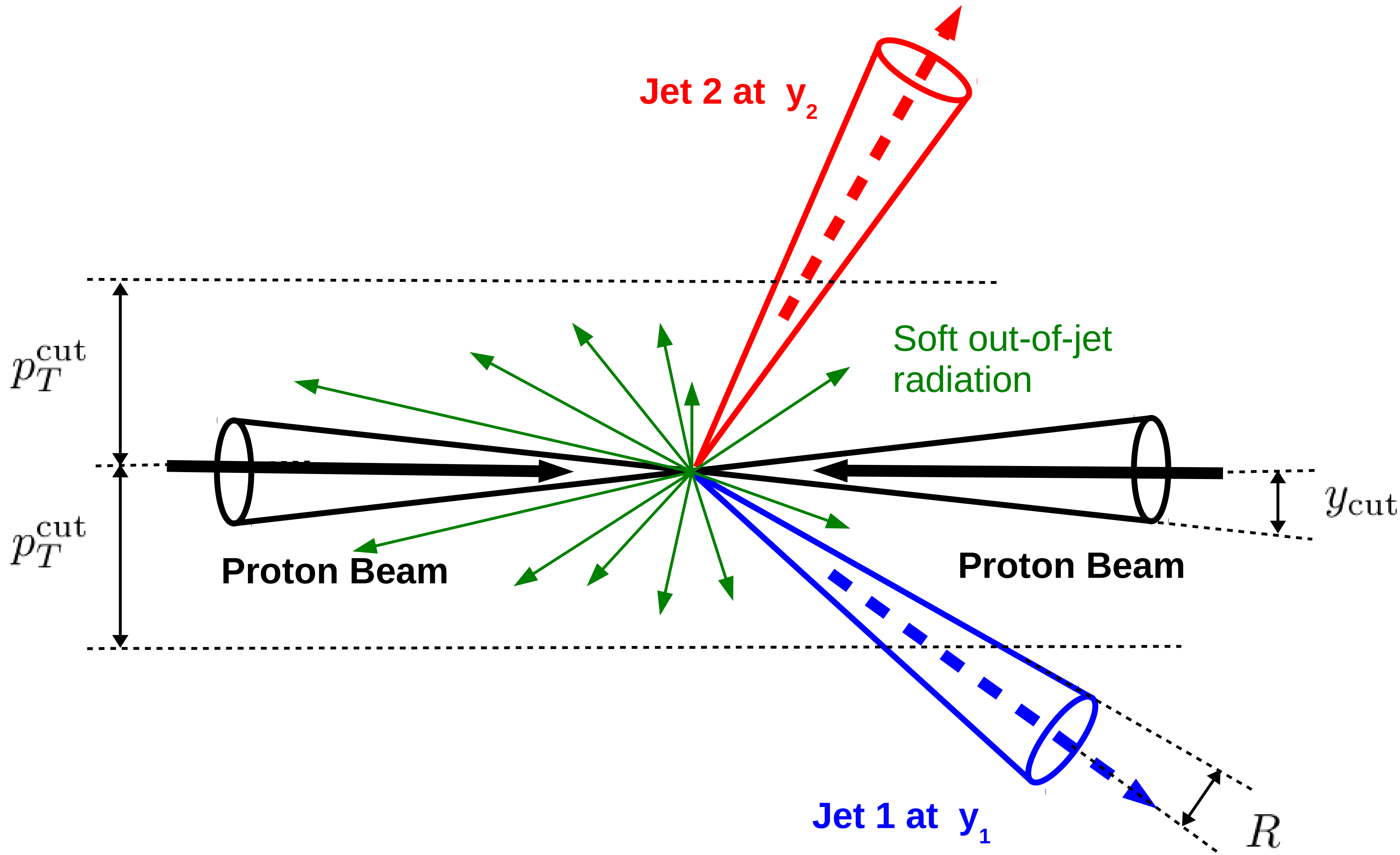
$$\mathbf{S} = \mathbf{S}^{\text{unmeas}} + \mathcal{O}(\alpha_s^2)$$

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \mathcal{O}(\alpha_s)$$

New calculation !

Phase-Space of Integration

$$\int \frac{d^d k}{(2\pi)^{d-1}}\ \delta(k^2)\Theta(E>0)\Theta(k_T<p_T^{\text{cut}})\Theta(\text{out of Jets and Beams})$$



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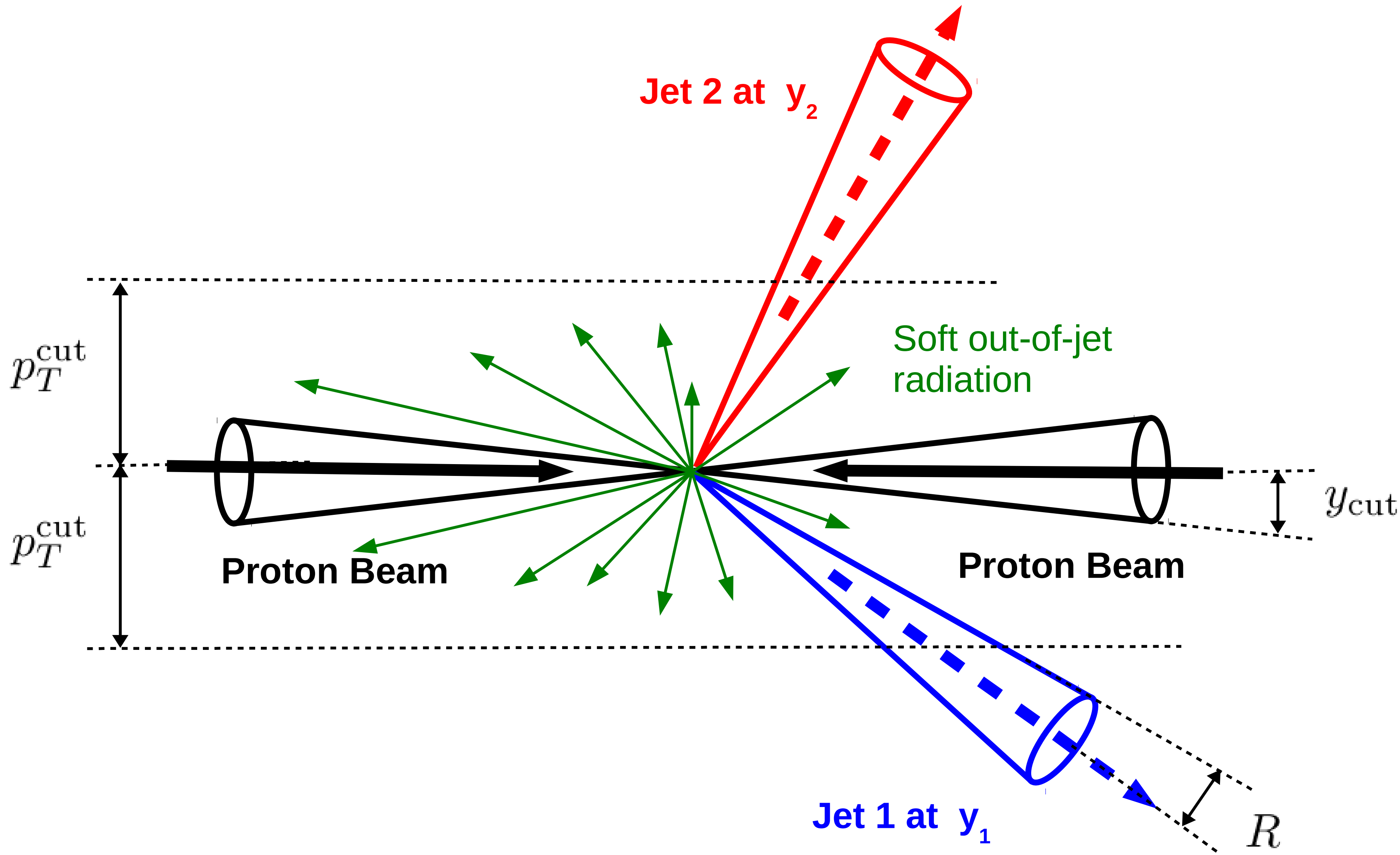

Unmeasured Evolution

$$\frac{d}{d\ln\mu}\mathbf{S}^{\text{unmeas}} = \mathbf{S}^{\text{unmeas}}\mathbf{\Gamma}_S^{\text{unmeas}} + \text{h.c.}$$

$$\mathbf{\Gamma}_S = \frac{1}{2}\Gamma_S\mathbf{1} - \Gamma_c\mathbf{M}(m_i)$$

Unmeasured Evolution

$$\frac{d}{d\ln\mu}\mathbf{S}^{\text{unmeas}} = \mathbf{S}^{\text{unmeas}}\mathbf{\Gamma}_S^{\text{unmeas}} + \text{h.c.}$$

$$\mathbf{\Gamma}_S = \frac{1}{2}\Gamma_S\mathbf{1} - \Gamma_c\mathbf{M}(m_i)$$

$$\mathbf{S}^{\text{unmeas}}(\mu,\mu_S) = \Pi_S^{\text{unmeas}}(\mu,\mu_S)\left[\mathbf{\Pi}_S^\dagger(\mu,\mu_S)\mathbf{S}^{\text{unmeas}}(\mu_S)\mathbf{\Pi}_S(\mu,\mu_S)\right]$$

Unmeasured Evolution

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Unmeasured Evolution

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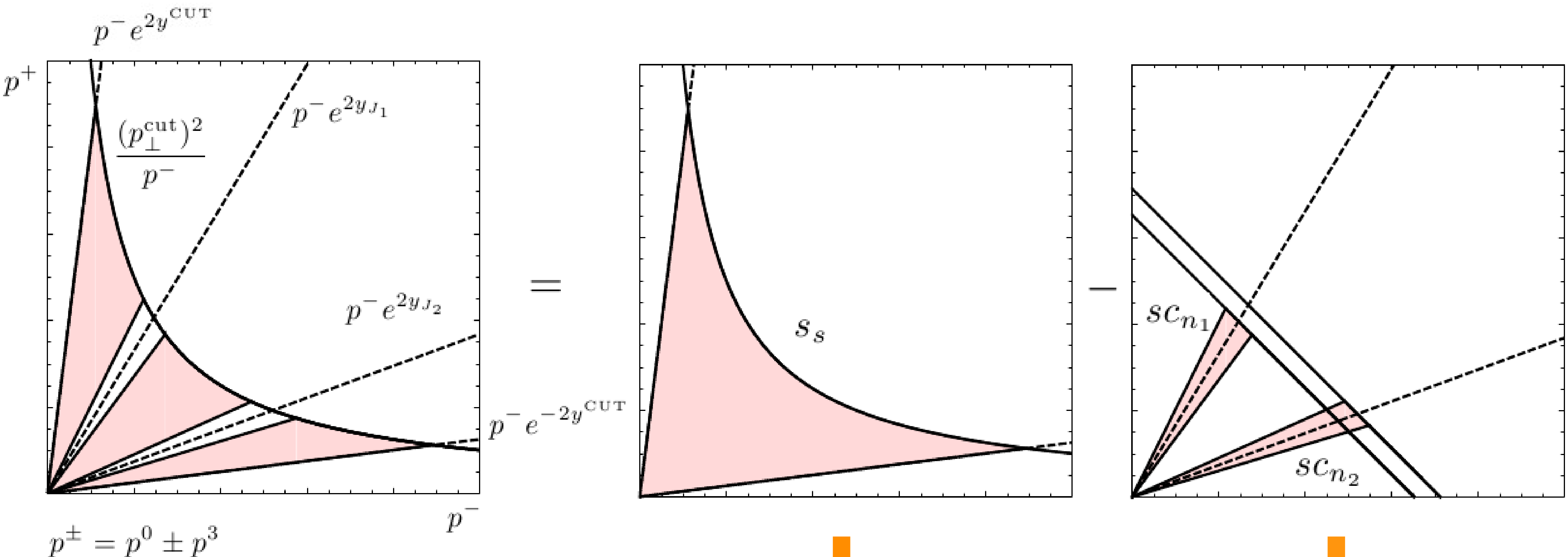
$$\mathbf{\Gamma}_S = \frac{1}{2}\Gamma_S\mathbf{1} - \Gamma_c\mathbf{M}(m_i)$$

$$\mathbf{\Gamma}_H = \frac{1}{2}\Gamma_H\mathbf{1} + \Gamma_c\mathbf{M}(m_i)$$

Necessary for the cancellation of μ dependence in the cross section

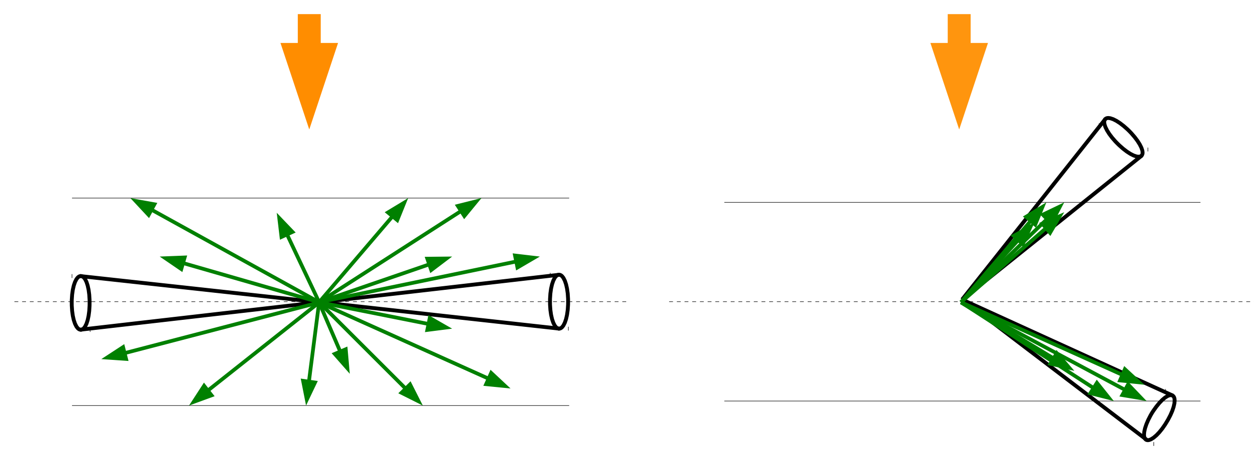
Soft-Collinear Refactorization

Chien, Hornig, and Lee, [arXiv:1509.04287]

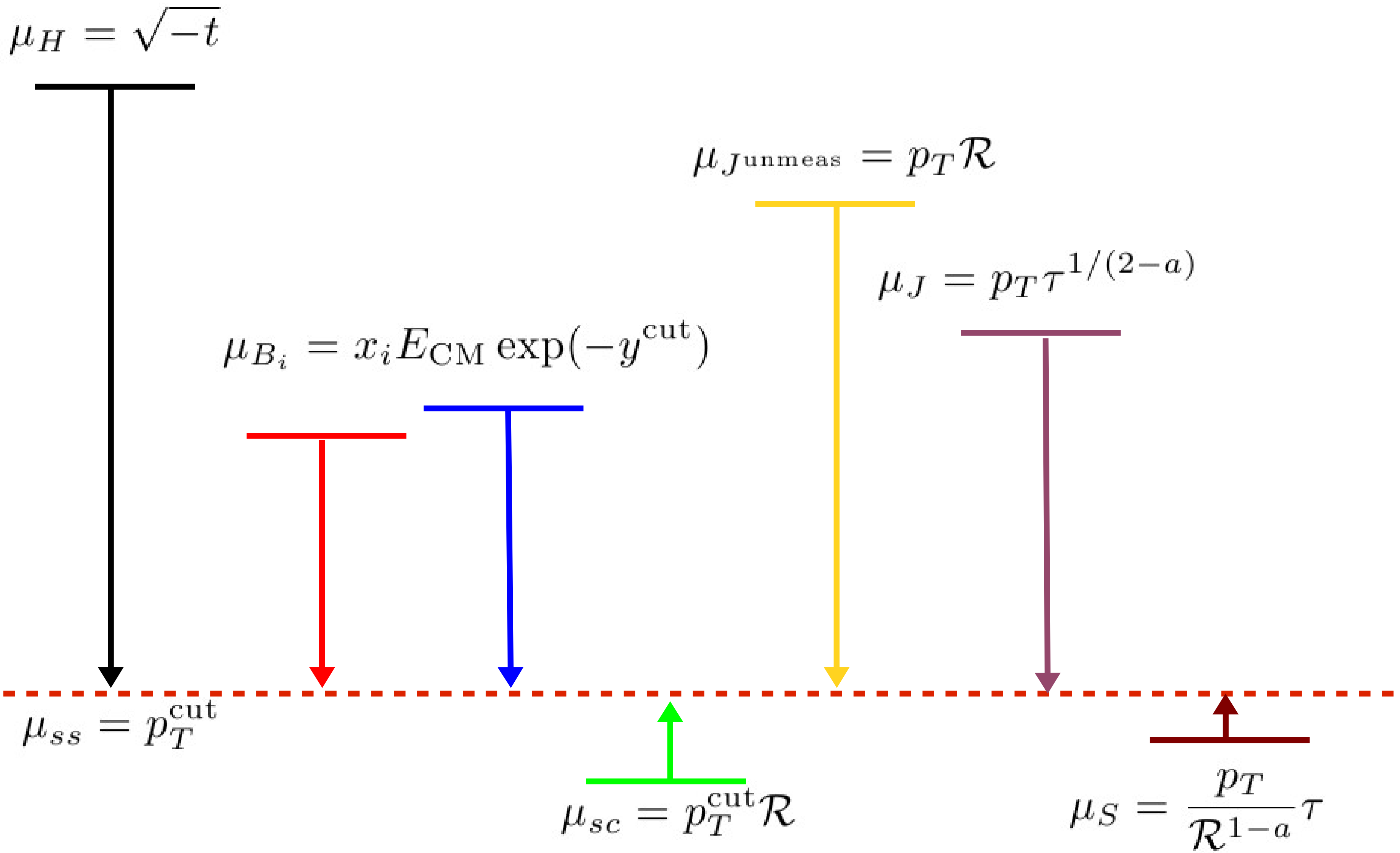


global-soft scale $\mu_{ss} = p_T^{\mathrm{cut}}$

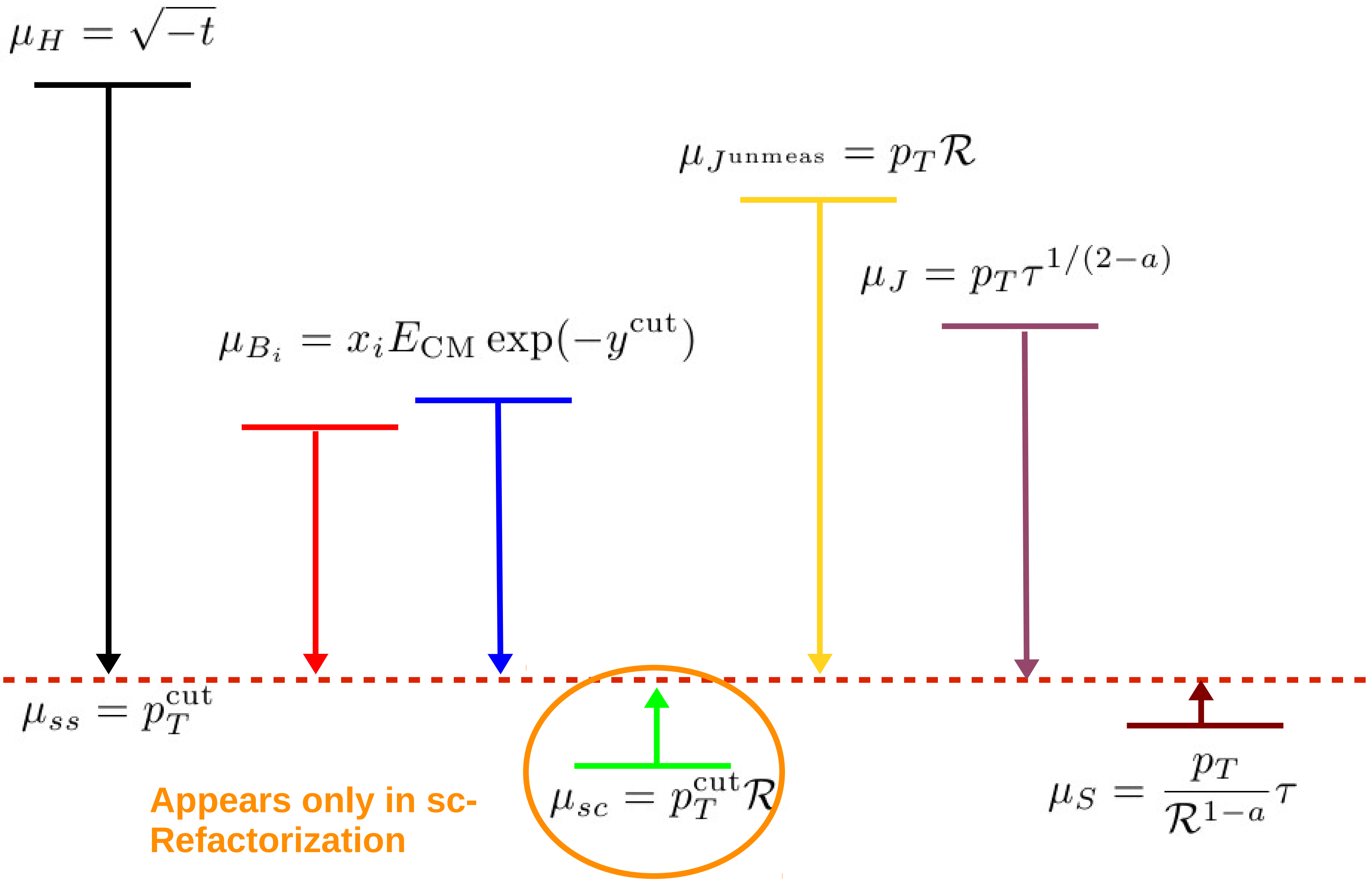
soft-collinear scale $\mu_{sc} = p_T^{\mathrm{cut}} \mathcal{R}$



Scales and R.G. Evolution



Scales and R.G. Evolution



Theoretical Uncertainties

Variation of the characteristic scales

Hard

Soft (Unmeasured)

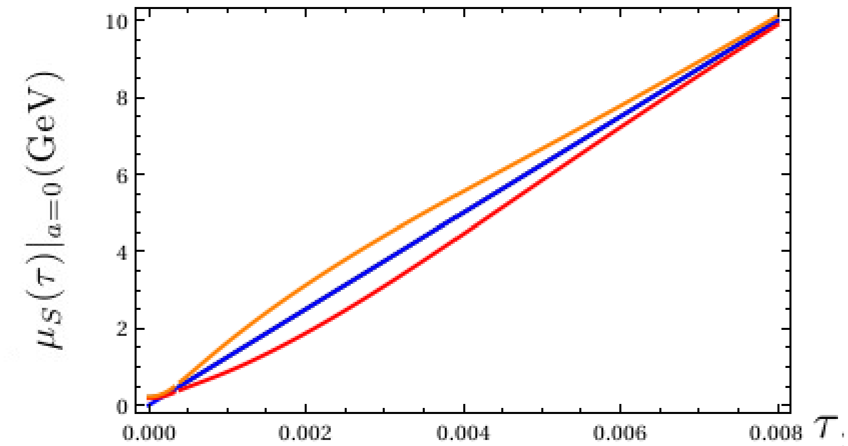
Beam

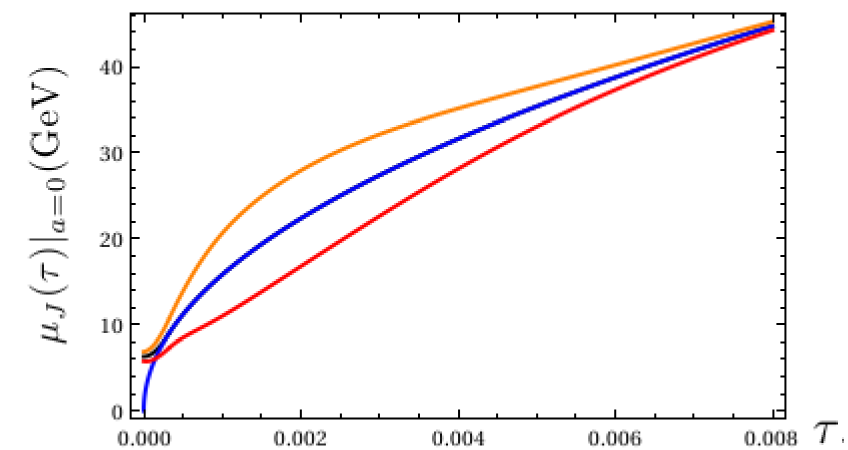
± 50 %

Jet (Measured)

Soft (Measured)

Profile Scales





Abbate, Fickinger, Hoang, Mateu and Stewart, [arxiv: 1006.3080].

Plots

$$d\tilde{\sigma}(\tau_a) \equiv \frac{B(x_1, \mu = \mu_H)\bar{B}(x_2, \mu = \mu_H)}{B(x_1, \mu = \mu_B^1)\bar{B}(x_2, \mu = \mu_B^2)} \frac{d\sigma(\tau_a^1, \tau_a^2)}{\sigma^{\mathrm{LO}}(\mu = \mu_H)}\bigg|_{\tau_a^1 = \tau_a^2 = \tau_a}$$

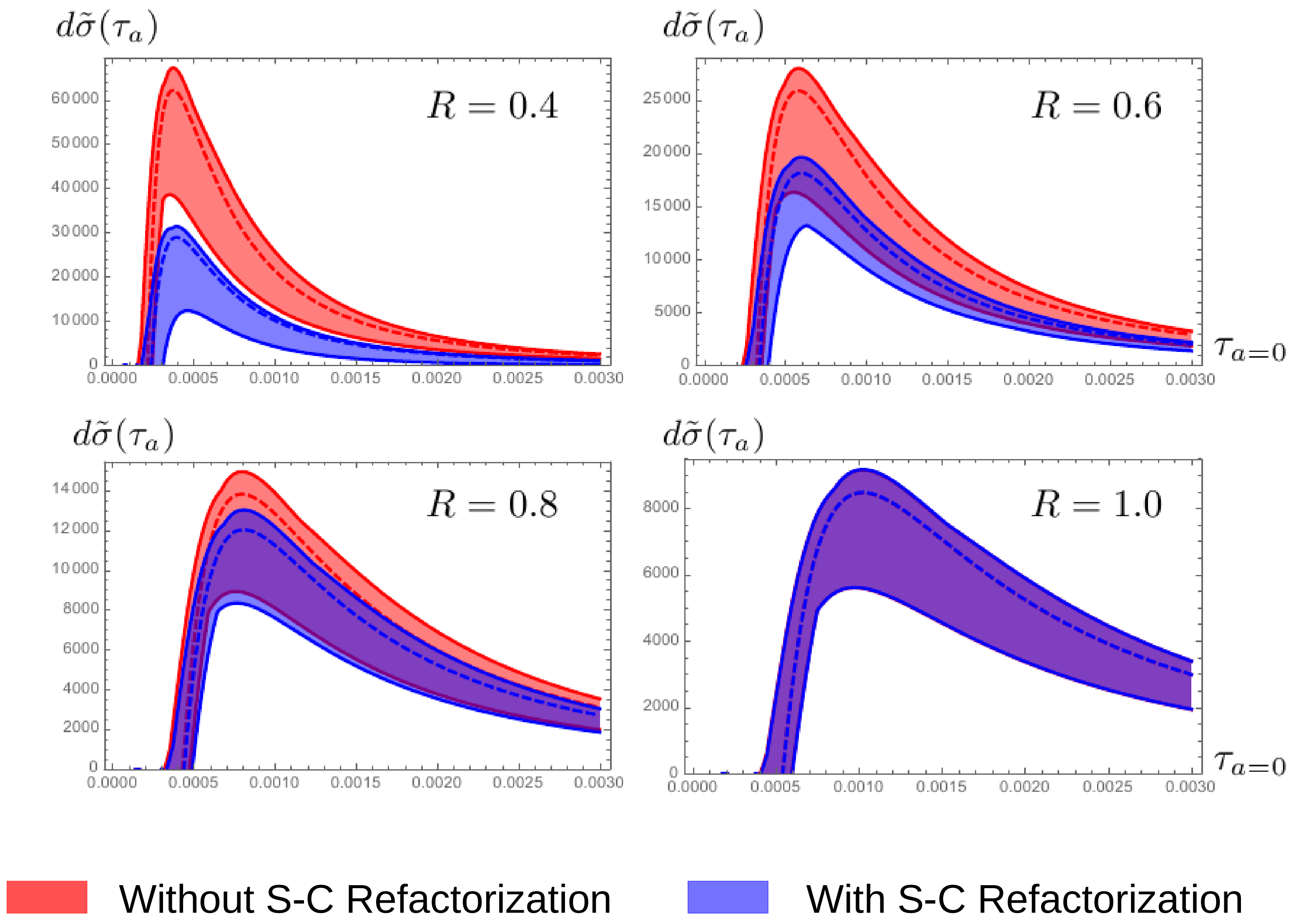
$$\boldsymbol{\tau_a^1 = \tau_a^2 = \tau_a}$$

Partonic Channel: qq' $\longrightarrow$ qq'

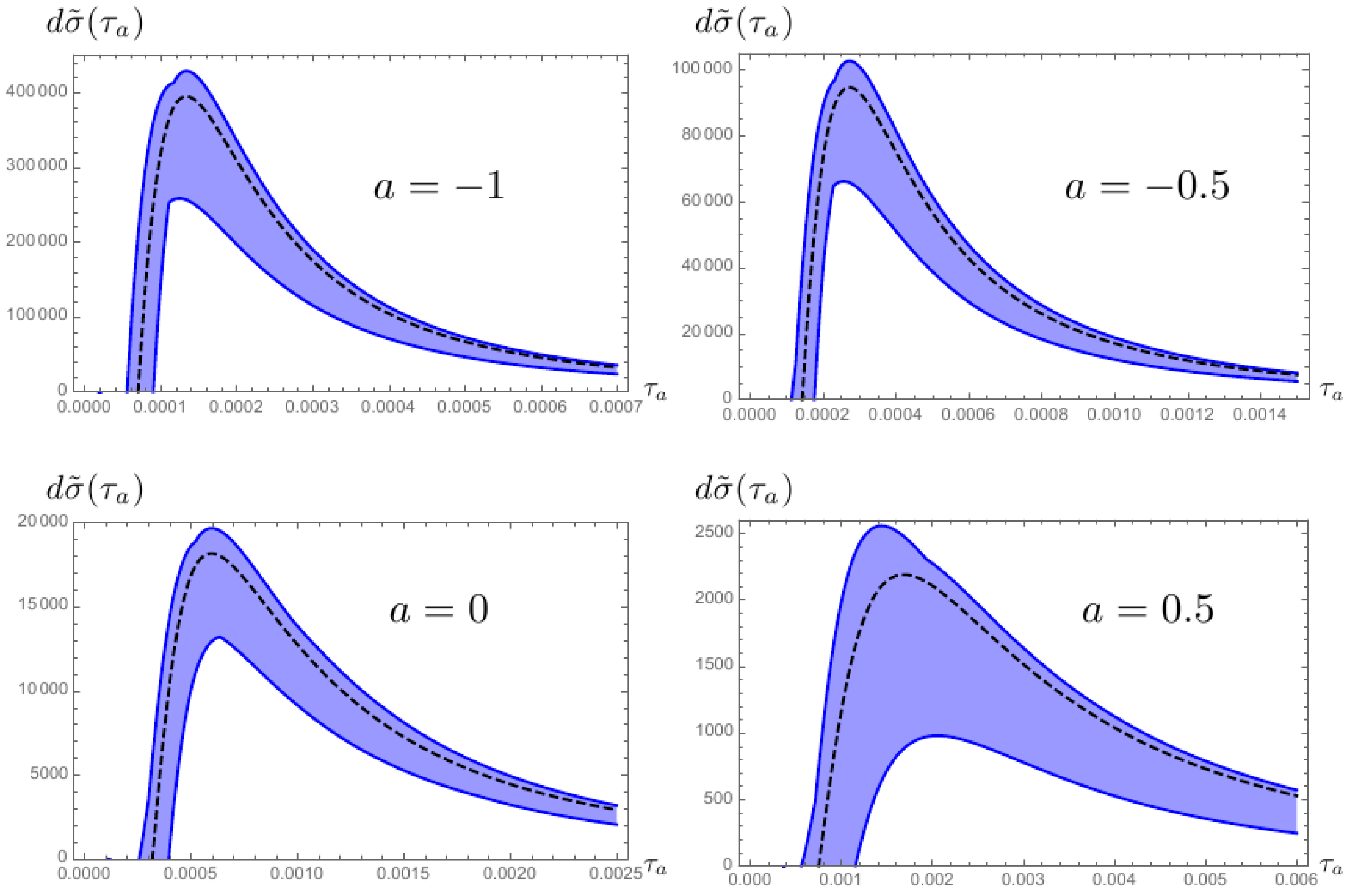
$$\mathrm{E_{cm}} = 10\ \mathrm{TeV} \qquad y_1 = 1.0 \qquad p_T = 500\ \mathrm{GeV} \qquad R = 0.6$$

$$a = 0 \qquad y_2 = 1.4 \qquad p_T^{\mathrm{cut}} = 20\ \mathrm{GeV} \qquad y_{\mathrm{cut}} = 5.0$$

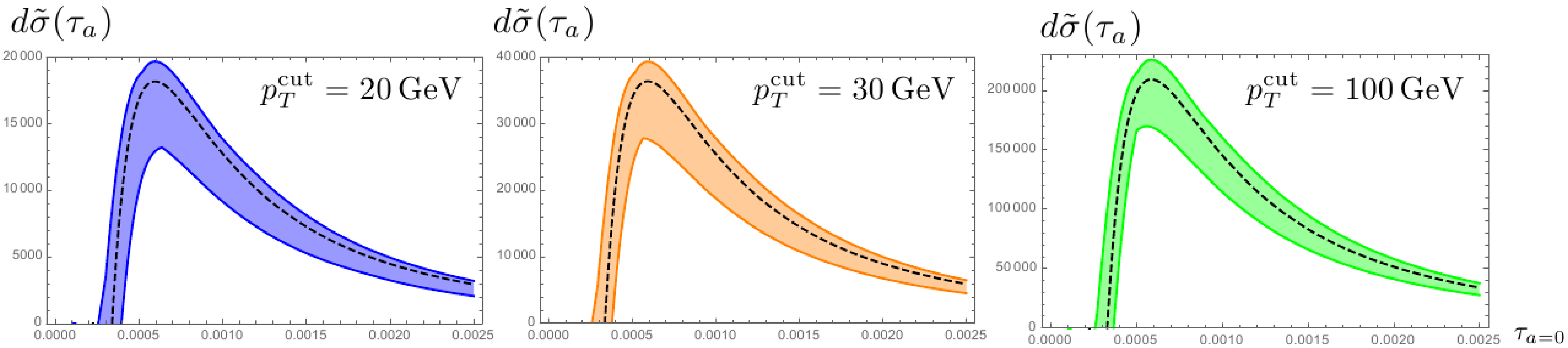
Plots - Variation of cone size R



Plots - Variation of a



Plots - Variation of p_T^{cut}



Increase of p_T^{cut} corresponds to increase of normalization

Peak location and shape independent of p_T^{cut}

Non-Global-Logarithms : $\alpha_s^n \ln^n(p_T^{\rm cut}\,\mathcal{R}^2/p_T^J\,\tau_a)$ not included

Summary

Establish framework for calculation of dijet events in proton-proton collisions

Veto on out-of-jet transverse momentum radiation and rapidity constrains

Introduce the unmeasured beam functions

Calculate differential cross section at NLL’ accuracy

Apply s-c refactorization for improved accuracy

Future Work

Apply to different partonic channels and compute physically observable cross section

NNLL calculation

Study other jet substructure observables

Exclusive cross sections for heavy meson and quarkonium production (In collaboration with Bain, Dai, Hornig, Leibovich, Mehen)

Compare to Monte Carlo simulations and experimental data

Thank you!

Scales and R.G. Evolution (1/2)

$$\frac{d}{d\ln\mu}F(\mu) = \Big(\Gamma_F[\alpha]\ln\frac{\mu^2}{m_F^2} + \gamma_F[\alpha]\Big)F(\mu)$$

$$F(\mu) = \exp[K_F(\mu,\mu_0)]\Big(\frac{\mu_0}{m_F}\Big)^{\omega_F(\mu,\mu_0)}F(\mu_0)$$

Unmeasured

$$\frac{d}{d\ln\mu}F(\tau,\mu) = \Big[\Gamma_F[\alpha]\Big(\ln\frac{\mu^2}{m_F^2}\delta(\tau) - \frac{2}{j_F}\Big[\frac{\Theta(\tau)}{\tau}\Big]_+\Big) + \gamma_F[\alpha]\delta(\tau)\Big]\otimes F(\tau,\mu)$$

$$F(\tau,\mu) = \frac{\exp[K_F(\mu,\mu_0) + \gamma_E\omega(\mu,\mu_0)]}{\Gamma(-\omega(\mu,\mu_0))}\Big(\frac{\mu_0}{m_F}\Big)^{j_F\omega_F(\mu,\mu_0)}\Big[\frac{\Theta(\tau)}{(\tau)^{1+\omega(\mu,\mu_0)}}\Big]_+\otimes F(\tau,\mu_0)$$

Measured

$$\omega_F(\mu,\mu_0) \equiv \frac{2}{j_F}\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta[\alpha]}\Gamma_F[\alpha]\,,$$

$$K_F(\mu,\mu_0) \equiv \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta[\alpha]}\gamma_F[\alpha] + 2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta[\alpha]}\Gamma_F[\alpha]\int_{\alpha_s(\mu_0)}^{\alpha}\frac{d\alpha'}{\beta[\alpha']}$$

Scales and R.G. Evolution (2/2)

	$\Gamma_F[\alpha_s]$	$\gamma_F[\alpha_s]$	j_F	m_F	μ_F
γ_H	$-\Gamma \sum_i C_i$	$-\sum_i \frac{\alpha_s}{\pi} \gamma_i$	1	$\prod_i m_i^{C_i/\sum_j C_j}$	m_i
$\gamma_{J_i}(\tau_a^i)$	$\Gamma C_i \frac{2-a}{1-a}$	$\frac{\alpha_s}{\pi} \gamma_i$	$2-a$	p_T	$p_T (\tau_a^i)^{1/(2-a)}$
$\gamma_S^{\rm meas}(\tau_a^i)$	$-\Gamma C_i \frac{1}{1-a}$	0	1	$p_T/\mathcal{R}^{1-a}$	$p_T\ \tau_a^i/\mathcal{R}^{1-a}$
γ_{J_i}	ΓC_i	$\frac{\alpha_s}{\pi} \gamma_i$	1	$p_T \mathcal{R}$	$p_T \mathcal{R}$
γ_{B_i}	ΓC_i	$\frac{\alpha_s}{\pi} \gamma_i$	1	$x_i \mathrm{E}_{\rm cm} e^{-y_{\rm cut}}$	$x_i \mathrm{E}_{\rm cm} e^{-y_{\rm cut}}$
$\gamma_S^{\rm unmeas}$	0	$\frac{2\alpha_s}{\pi} \Delta\gamma_{ss}(m_i)$ $+\frac{2\alpha_s}{\pi}(C_1+C_2)\ln\mathcal{R}$	1	—	$p_T^{\rm cut}$
γ_{ss}	$\Gamma(C_1+C_2)$	$\frac{2\alpha_s}{\pi} \Delta\gamma_{ss}(m_i)$	1	$p_T^{\rm cut}$	$p_T^{\rm cut}$
γ_{sc}^i	$-\Gamma C_i$	0	1	$p_T^{\rm cut} \mathcal{R}$	$p_T^{\rm cut} \mathcal{R}$

Profile Functions (1/2)

$$\mu_S^i(\tau_a^i) = (1 + e_S g(\tau))\mu(\tau_a^i) \qquad \mu_J^i(\tau_a^i) = (1 + e_J g(\tau))\big(p_T \mathcal{R}\big)^{\frac{1-a}{2-a}}\big(\mu(\tau_a^i)\big)^{\frac{1}{2-a}}$$

$$\mu(\tau) = \begin{cases} \mu_0 + \alpha \tau^\beta \sqrt{-t}, & \tau < \tau^{\min} \\ \dfrac{p_T\,\tau}{\mathcal{R}^{1-a}}, & \tau > \tau^{\min}, \end{cases}$$

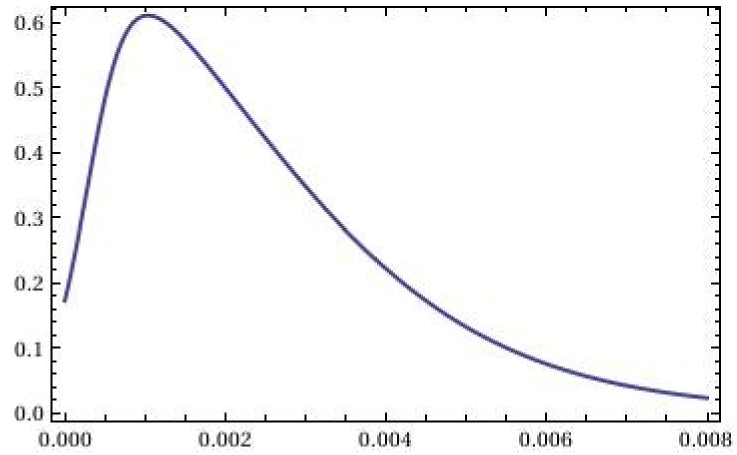
$$\alpha = \frac{p_T}{\beta (\tau^{\min})^{\beta-1} \mathcal{R}^{1-a} \sqrt{-t}}$$

$$\beta = \left(1 - \frac{\mu_0 R^{1-a}}{p_T \tau^{\min}}\right)^{-1},$$

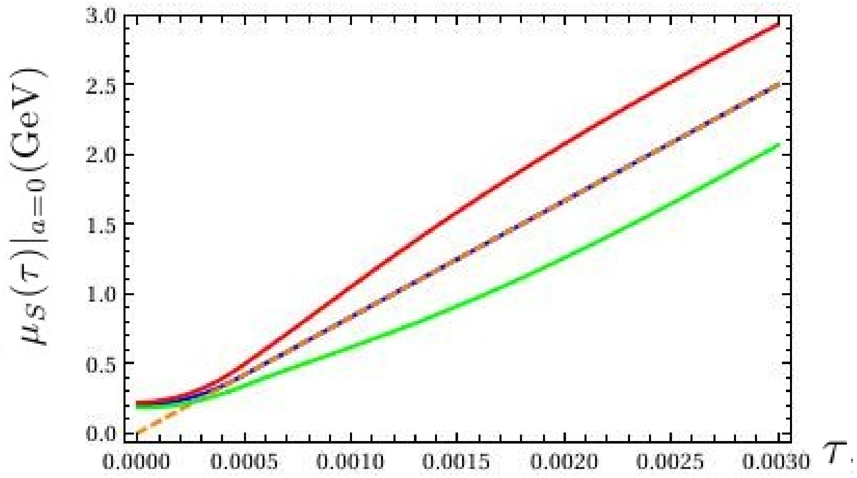
$$g(\tau) = \theta_{\epsilon_1}(\tau - \tau^{\min})\,\theta_{\epsilon_2}(\tau^{\max} - \tau)$$

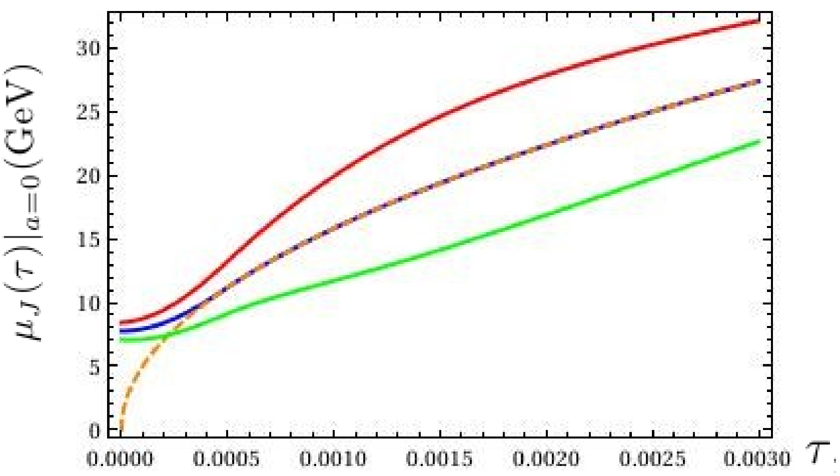
$$\theta_\epsilon(x) \equiv \frac{1}{1 + \exp\left(-x/\epsilon\right)}$$

$$g(\tau) = \theta_{\epsilon_1}(\tau - \tau^{\min})\,\theta_{\epsilon_2}(\tau^{\max} - \tau)$$



Profile Functions (2/2)





$$\tau^{\min} = 2(1-a)\mu_0\mathcal{R}^{1-a}/p_T = .00032(1-a)$$
$$\tau^{\max} = .002$$

and

$$\frac{\epsilon_1}{\tau^{\min}} = \frac{\epsilon_2}{\tau^{\max}} = 10^{-0.1}$$
$$\mu_0 = 200\,\mathrm{MeV}$$

Soft Function

Without Refactorization

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \left[\mathbf{S}_0 \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left(S_{ij}^{\text{incl}} + \sum_{k=1}^{N} S_{ij}^{k}\right) + \text{h.c.}\right]$$

With Refactorization

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \frac{\alpha_s}{4\pi}\frac{1}{2}\left[\mathbf{S}_0\left(\mathbf{S}_s^{(1)}(p_T^{\text{cut}}) + \sum_{k=1,2} S_{sc}^{k(1)}(p_T^{\text{cut}}\mathcal{R})\right) + \text{h.c.}\right] + \mathcal{O}(\alpha_s^2)$$

$$\mathbf{S}_s^{(1)}(p_T^{\text{cut}}) = \frac{4}{\epsilon}\left(\frac{\mu}{p_T^{\text{cut}}}\right)^{2\epsilon} \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left[\mathcal{I}_{ij}^{\text{incl}} + (\delta_{iB} + \delta_{i\bar{B}})(\delta_{jJ_1} + \delta_{jJ_2})\mathcal{I}_{ij}^{i} + \delta_{iB}\delta_{i\bar{B}}(\mathcal{I}_{ij}^{i} + \mathcal{I}_{ij}^{j})\right]$$

$$S_{sc}^{k(1)}(p_T^{\text{cut}}\mathcal{R}) = \frac{4}{\epsilon}\left(\frac{\mu}{p_T^{\text{cut}}}\right)^{2\epsilon} \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left[\delta_{ik}\mathcal{I}_{ij}^{i}\right]$$

Results (1/2)

Soft function after RG Evolution

$$\mathbf{S}(\tau_a^1, \tau_a^2, \mu, \mu_S^1, \mu_S^2, \bar{\mu}_S) = U_S^1(\tau_a^1, \mu, \mu_S^1) U_S^1(\tau_a^2, \mu, \mu_S^2) \left[1 + \left(f_S^1(\tau_a^1; \omega_S^1, \mu_S^1) + f_S^2(\tau_a^2; \omega_S^2, \mu_S^2)\right)\right]$$

$$\times\, \Pi_S^{\mathrm{unmeas}}(\mu, \bar{\mu}_S) \left[\mathbf{\Pi}_S^\dagger(\mu, \bar{\mu}_S) \mathbf{S}^{\mathrm{unmeas}}(\bar{\mu}_S) \mathbf{\Pi}_S(\mu, \bar{\mu}_S)\right]$$

$$f_S^i(\tau; \Omega, \mu) = \frac{\alpha_s C_i}{\pi(1-a)} \left[\psi^{(1)}(-\Omega) - \left(H(-1-\Omega) + \ln \frac{\mu \mathcal{R}^{1-a}}{p_T\, \tau}\right)^2 - \frac{\pi^2}{8}\right]$$

Without s-c Refactorization

$$\mathbf{S}^{\mathrm{unmeas}} = \mathbf{S}_0 + \frac{\alpha_s}{\pi} \left\{ \mathbf{S}_0 \left[\left(\frac{1}{2\epsilon} + \ln \frac{\mu}{p_T^{\mathrm{cut}}} \right) \left(\mathbf{S}^{\mathrm{div}} + \sum_{i=1,2} C_i \ln \mathcal{R} \right) - \frac{1}{2} \sum_{i=1,2} C_i \ln^2 \mathcal{R} \right. \right.$$

$$\left. \left. - \mathbf{T}_1 \cdot \mathbf{T}_2 \ln\left(1 + e^{\Delta y}\right) \ln\left(1 + e^{-\Delta y}\right) \right] + \mathrm{h.c.} \right\} + \mathcal{O}(\alpha_s^2)$$

$$\mathbf{S}^{\mathrm{div}} = \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{n_i \cdot n_j}{2} - y_{\mathrm{cut}} \left(C_B + C_{\bar{B}}\right) - \sum_{i=1,2} C_i \ln(2 \cosh y_i)$$

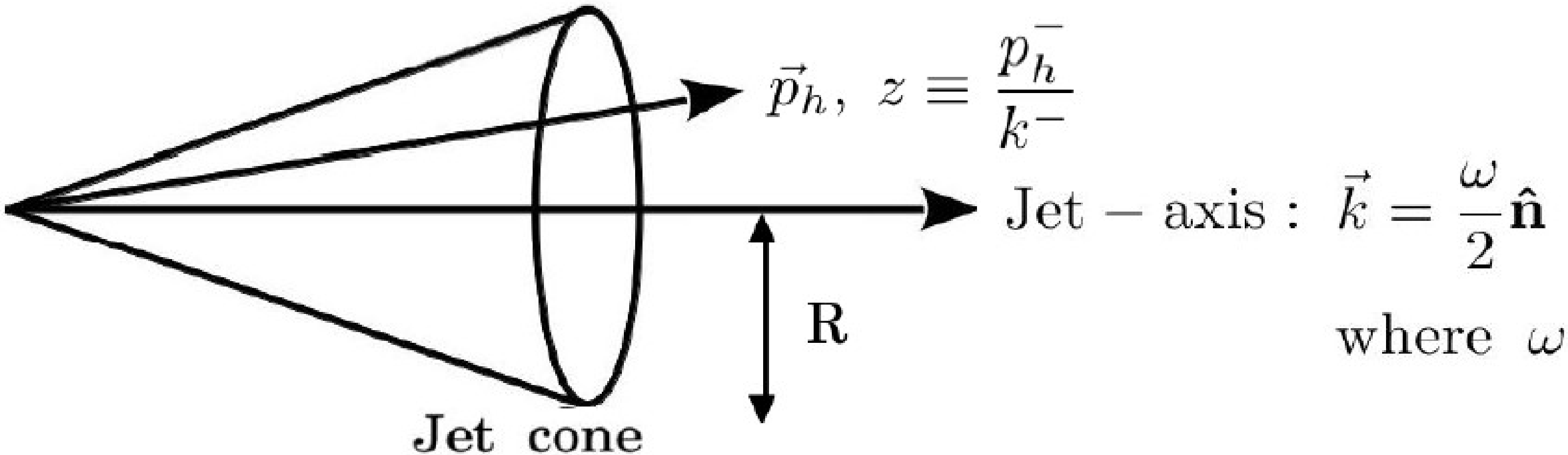
Results (2/2)

With s-c Refactorization

$$
\begin{aligned}
\mathbf{S}^{\mathrm{unmeas}}(\Omega, \mu_{sc}, \mu_{ss}) \equiv \mathbf{S}_0 + \Bigg\{ \mathbf{S}_0 \Bigg[& \frac{\alpha_s(\mu_{ss})}{4\pi} \Bigg(\frac{1}{2}\mathbf{f}_s^2 + \mathbf{f}_s^1 \Big(\ln\frac{\mu_{ss}}{p_T^{\mathrm{cut}}} + H(-\Omega) \Big) \\
& + \mathbf{f}_s^0 \Big(\frac{\pi^2}{6} - \psi^{(1)}(1-\Omega) + \Big(\ln\frac{\mu_{ss}}{p_T^{\mathrm{cut}}} + H(-\Omega) \Big)^2 \Big) \Bigg) \\
+ & \frac{\alpha_s(\mu_{sc})}{4\pi} \Bigg(\frac{1}{2} f_c^2 + f_c^1 \Big(\ln\frac{\mu_{sc}}{p_T^{\mathrm{cut}}\mathcal{R}} + H(-\Omega) \Big) \\
& + f_c^0 \Big(\frac{\pi^2}{6} - \psi^{(1)}(1-\Omega) + \Big(\ln\frac{\mu_{sc}}{p_T^{\mathrm{cut}}\mathcal{R}} + H(-\Omega) \Big)^2 \Big) \Bigg) \Bigg] + \mathrm{h.c.} \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
f_c^0 &= -2(C_1 + C_2) & \mathbf{f}_s^0 &= -f_c^0 \\
f_c^1 &= 0 & \mathbf{f}_s^1 &= 4\mathbf{S}^{\mathrm{div}} \\
f_c^2 &= \frac{\pi^2}{6}(C_1 + C_2) & \mathbf{f}_s^2 &= -8\mathbf{T}_1 \cdot \mathbf{T}_2 \ln(1 + e^{\Delta y}) \ln(1 + e^{-\Delta y}) - f_c^2
\end{aligned}
$$

Applications in heavy meson and quarkonium production



where $\omega = k^- = s/k^+$

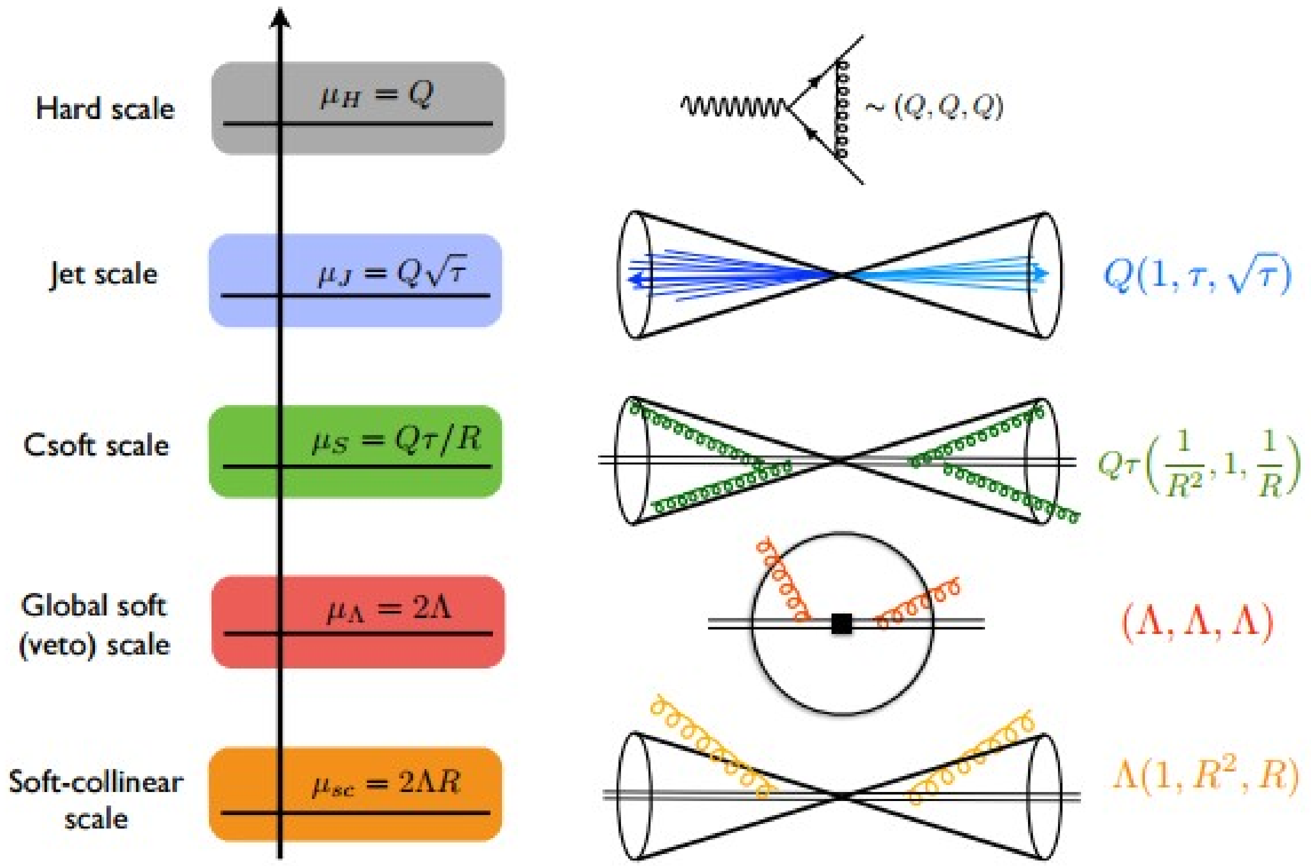
$$\text{Identified Jets:} \quad J_i(p^2,\tau,\mu) \longrightarrow \mathcal{G}_i^h(z,\tau,\mu)$$

$$\text{OPE}: \quad \mathcal{G}(z,\tau,\mu) = \sum_j \left[\mathcal{J}_i^j(\tau,\mu) \bullet D_{j\to h}(\mu)\right](z)$$

$$[g \bullet f](z) = [f \bullet g](z) \equiv \int_z^1 \frac{dx}{x} g\left(\frac{z}{x}\right) f(x)$$

[arXiv:0911.4980]
M. Procura and I. W. Stewart
[arXiv:1111.6605]
M. Procura and W. J. Waalewijn
[arXiv:1101.4953]
A. Jain, M. Procura, and W. J. Waalewijn

Non-Global Logs and S-C Refactorization



Chien, Hornig, and Lee, [arXiv:1509.04287]