Analytical and Monte Carlo Studies of Jets with Heavy Mesons and Quarkonia

Reggie Bain
Duke University

In collaboration with L. Dai, A. Hornig, A.Leibovich, Y. Makris, T. Mehen

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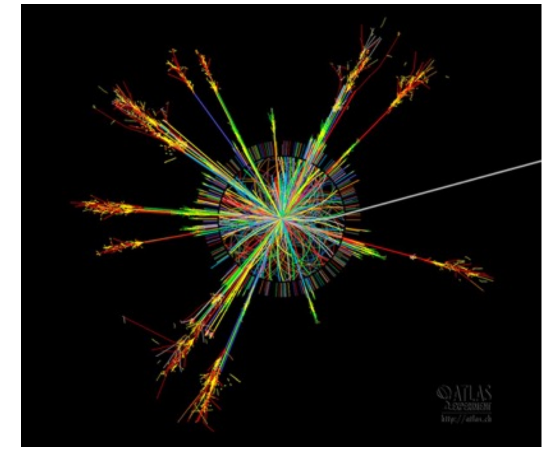
Motivations

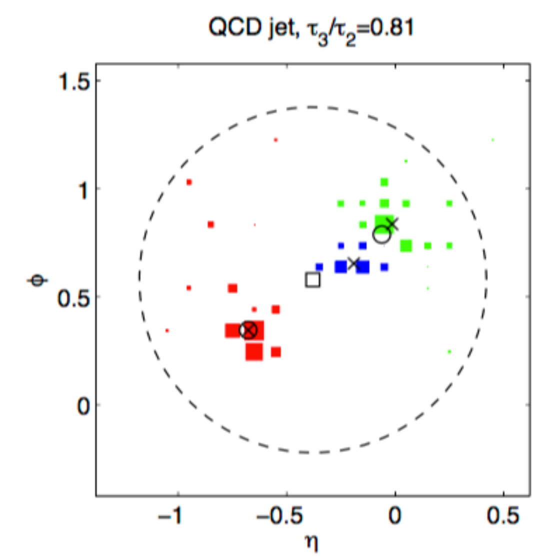
Understand high energy jets at LHC

- ***Testing QCD***
- ***Calculating backgrounds for new physics***



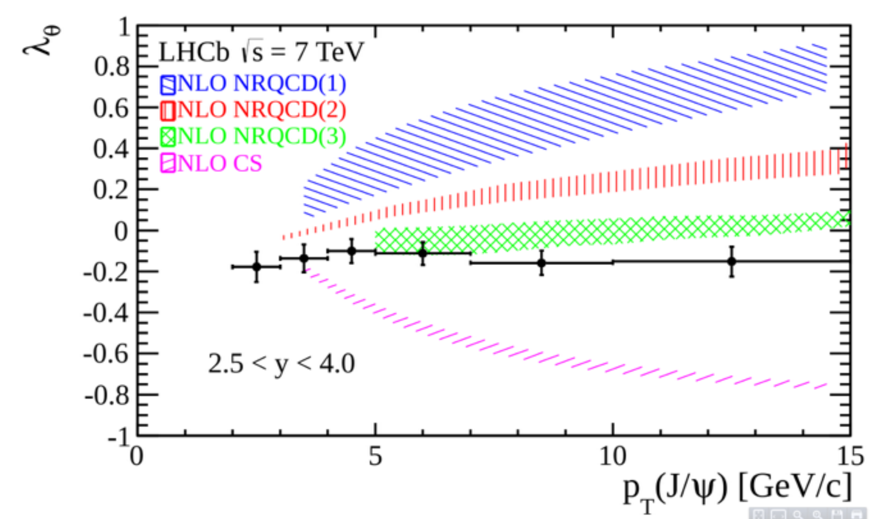
ATLAS Collaboration

Study wealth of jet substructure observables



Thaler, v. Tilberg, arXiv:1011.2268

Elucidate outstanding puzzles in quarkonia production



LHCb Collaboration

Outline

- Fragmenting jet functions w/ angularities
- NLL' cross section calculations
  - $e^+e^- \longrightarrow$ *2 jets with B meson*
  - $e^+e^- \longrightarrow$ *3 jets with J/ψ from gluon*
- Comparisons of NLL' vs. Monte Carlo

Jet Cross-Sections in SCET

Factorization ⟷ Short Distance x Long Distance

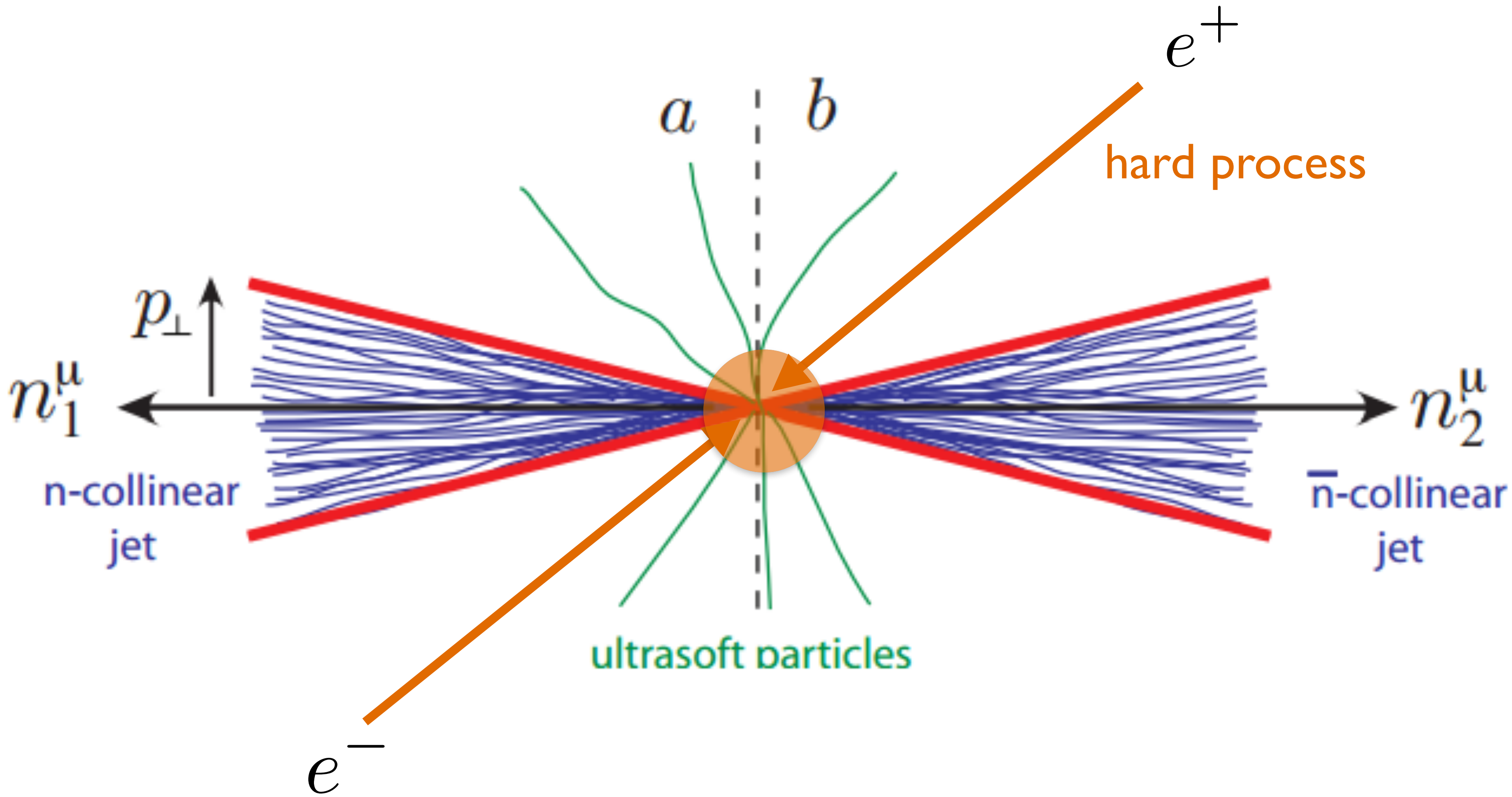


Figure from "Lectures on the Soft Collinear Effective Theory" by Iain W. Stewart, 2013

Factorization Theorem

$$d\sigma = H \times J^1 \otimes J^2 \otimes S$$

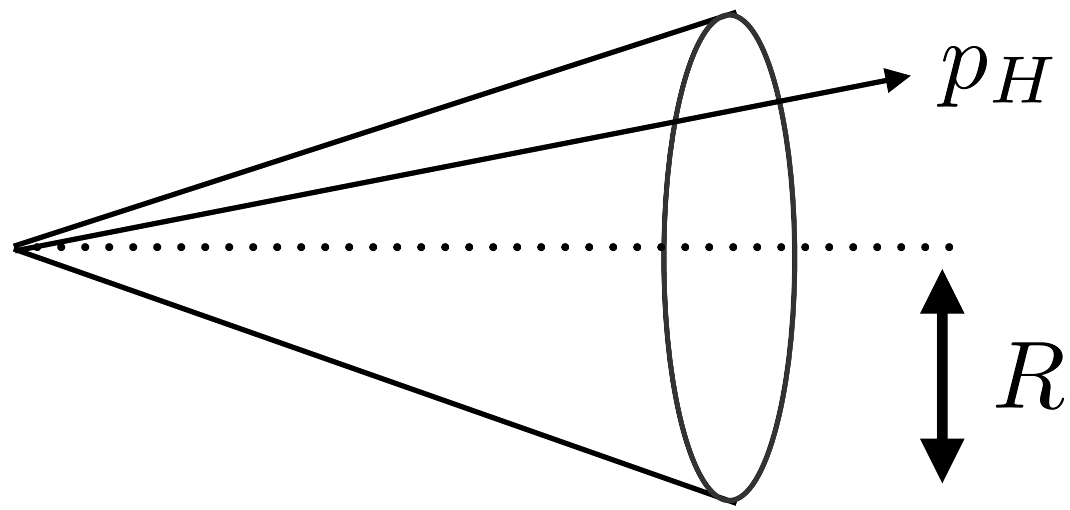
$\left\{ \begin{array}{ll} \text{Hard function} & H(\mu) \\ \text{Jet Functions} & J^{(1)}(\mu) \\ \text{Soft Function} & S(\mu) \end{array} \right.$

Measured

Unmeasured

Fragmenting Jet Functions (FJF's)

Study jets with identified hadron



Jet Inv. Mass s

Plus Momentum $p_H^+ = z p_J^+$

Cone Size R

Study different measured jet observables/hadrons

Measured invariant mass $\mathcal{G}_i^H(s, z, \mu)$

Measured angularity $\mathcal{G}_i^H(\tau_a, z, \mu)$

Procura, Stewart, arXiv:0911.4980

Our observable: Angularities τ_a

Generalization of jet invariant mass

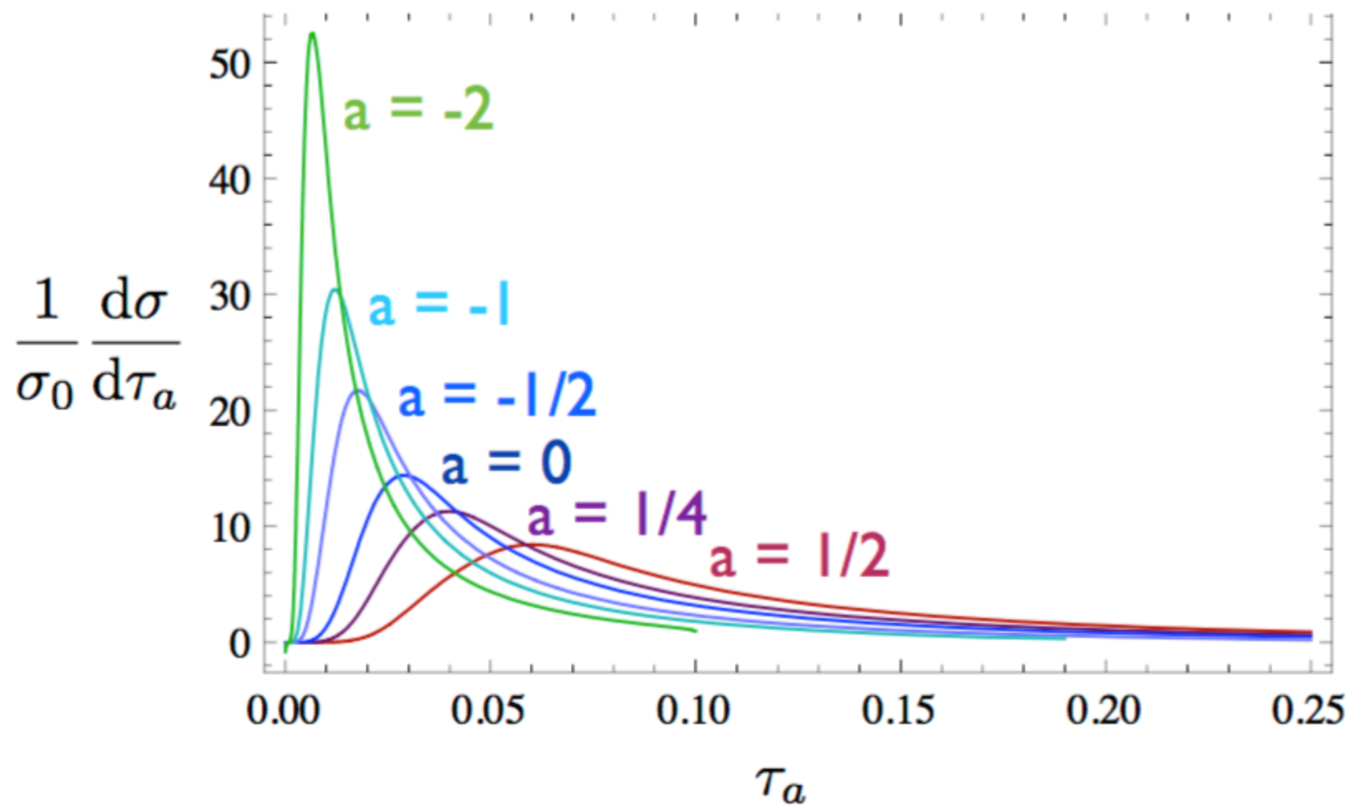
$$\tau_a = \frac{1}{\omega}\sum_i (p_i^+)^{1-a/2}(p_i^-)^{a/2}$$

Sum over jet particles i

$$\omega = \sum_i p_i^- \approx 2E_{jet}$$

a=0 jet invariant mass $s = \omega^2 \tau_0$

a=1 jet broadening



We have good analytic handle on τ_a

Hornig, Lee, Ovanesyan, arXiv:0905.0168
S.D.Ellis, et. al (2010), arXiv:1001.0014

Calculate Cross-Section with FJFs

Jet cross-section $\longrightarrow$ Jet w/ Identified Hadron cross-section

$$J_i(\tau_a, \mu) \to \frac{1}{2(2\pi)^3} \mathcal{G}_i^H(\tau_a, z, \mu) dz$$

Convolution of Matching Coefficients & Fragmentation Functions (FF's)

$$\mathcal{G}_i^H(\tau_a, z, \mu) = \sum_j \left[\mathcal{J}_{ij}(\tau_a, \mu) \bullet D_j^H(\mu) \right](z)$$

where $\quad [f \bullet g](z) \equiv \int_z^1 \frac{dx}{x} f(x) g(z/x)$

Calculate $\mathcal{J}_{ij}(\tau_a, z, \mu)$ perturbatively for different observables

Procura, Stewart, arXiv:0911.4980

Matching Coefficients at NLO

We calculated all 4 NLO (1-loop) $\mathcal{J}_{ij}$ for measured angularities

$$
\begin{aligned}
\frac{\mathcal{J}_{qq}(\omega,z,\tau_a,\mu)}{2(2\pi)^3} = & \frac{C_F\alpha_s}{2\pi}\frac{1}{\omega^2}\Bigg\{\delta(\tau_a)\delta(1-z)\frac{2-a}{1-a}\left(-\frac{\pi^2}{12}+\frac{1}{2}\ln^2\left(\frac{\mu^2}{\omega^2}\right)\right)\\
& +\delta(\tau_a)\left(1-z-\left[\ln\left(\frac{\mu^2}{\omega^2}\right)+\frac{1}{1-a/2}\ln\left(1+\frac{(1-z)^{1-a}}{z^{1-a}}\right)\right]\frac{1+z^2}{(1-z)_+}\right.\\
& \left.+\frac{1-a}{1-a/2}(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+\right)\\
& +\left[\frac{1}{\tau_a}\right]_+\left(\frac{1}{1-a/2}\frac{1+z^2}{(1-z)_+}-\delta(1-z)\frac{2}{1-a}\ln\left(\frac{\mu^2}{\omega^2}\right)\right)\\
& +\frac{2\delta(1-z)}{(1-a)(1-a/2)}\left[\frac{\ln\tau_a}{\tau_a}\right]_+\Bigg\}
\end{aligned}
$$

also… $\mathcal{J}_{qg}$, $\mathcal{J}_{gq}$, $\mathcal{J}_{gg}$

Consistency checks

1. $\lim_{a\to 0}\mathcal{J}_{ij}(\tau_a,z,\mu)=\omega^2\mathcal{J}_{ij}(s,z,\mu)\longrightarrow$ Jain, et. al, arXiv:1101.4953

2. $J_i(s,\mu)=\frac{1}{2(2\pi)^3}\sum_j\int_0^1 dz\,z\,\mathcal{J}_{ij}(s,z,\mu)\longrightarrow$ S.D.Ellis, et. al, arXiv:1001.0014

First steps: e^+e^- collisions

R. Bain, L. Dai, A. Hornig, A.Leibovich, Y. Makris, T. Mehen

$e^+e^- \to b\bar{b}$ vs. Monte Carlo

$\hookrightarrow$ B jet

$e^+e^- \to b\,\bar{b}\,g$ vs. Monte Carlo

$\hookrightarrow$ J/ψ jet

Goals

1. Study z, τ_0 distributions
2. pp $\longrightarrow$ B, J/ψ

Cross Section for 2 jets & B^+/B^0

Re-summed to NLL' using renormalization group (RG)

FF for B's at jet scale (DGLAP)

$$d\sigma(\tau_a, z) \equiv \frac{1}{\sigma_0}\frac{d\sigma^{(b)}}{d\tau_a dz} = H_2(\mu_H) \times S^{\text{unmeas}}(\mu_\Lambda) \times J_{\bar{n}}^{(\bar{b})}(\mu_{J_{\bar{n}}}) \times$$
$$\times \sum_j \left\{ \left(\frac{\Theta(\tau_a)}{\tau_a^{1+\Omega}}\right) \left[\delta_{bj}\delta(1-z)\left(1 + f_S(\tau_a, \mu_{S^{\text{meas}}})\right) + f_{\mathcal{J}}^{bj}(\tau_a, z, \mu_{J_n})\right] \bullet \frac{D_{j\to B}(z, \mu_{J_n})}{2(2\pi)^3} \right\}_+$$
$$\times\, \Pi(\mu, \mu_H, \mu_\Lambda, \mu_{J_{\bar{n}}}, \mu_{J_n}, \mu_{S^{\text{meas}}})$$

coupled z & τ_a

RG evolution factor

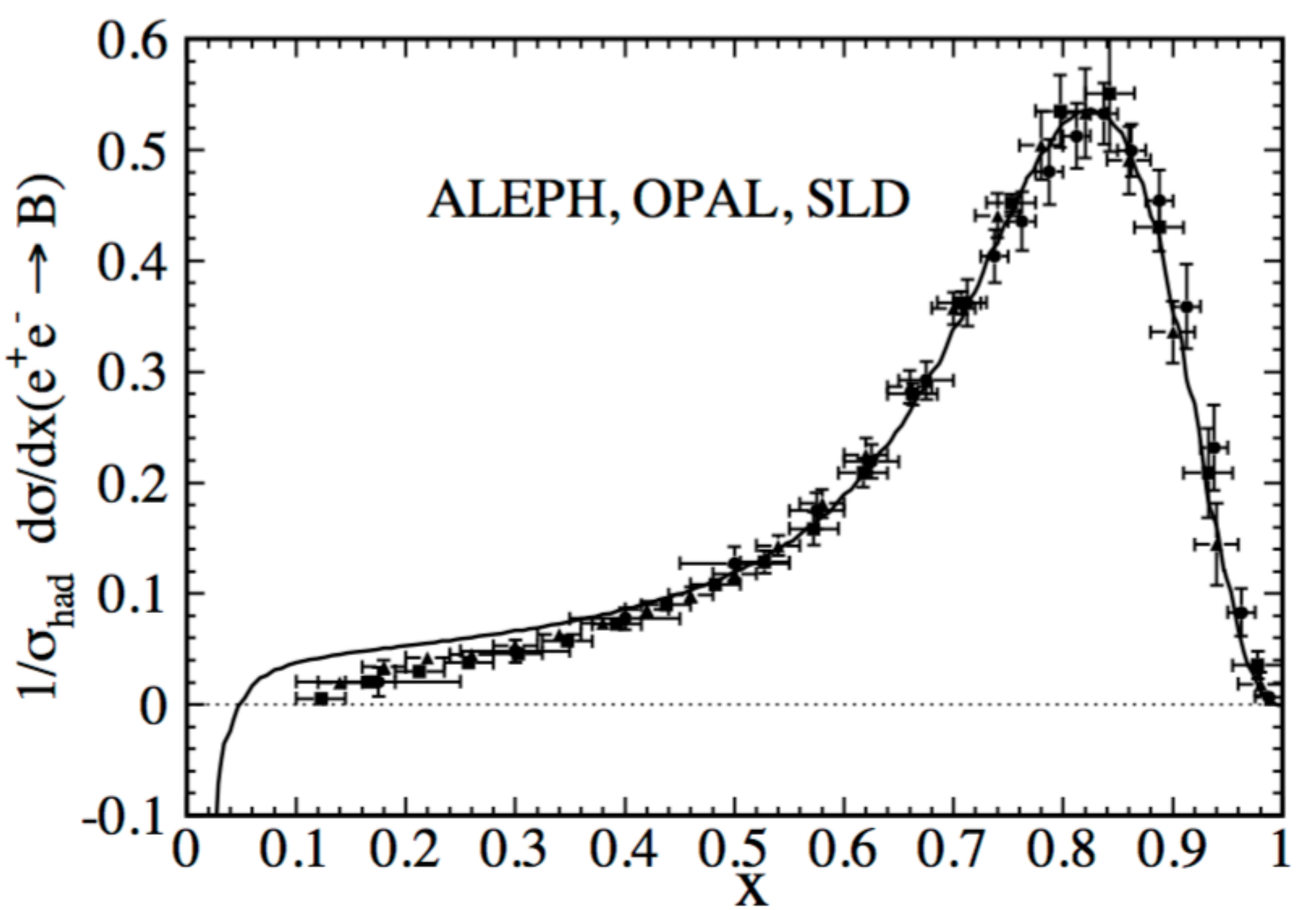
Coupling of z and τ_a dependence appears first at NLO

Evaluate each piece at characteristic scale, evolve up to hard scale

b quark Fragmentation Function

Fit power model to LEP data

Inclusive Cross-Section vs. z



$$D(x, \mu_0) = N x^\alpha (1-x)^\beta$$

$$N = 4684.1$$

$$\alpha = 16.87$$

$$\beta = 2.028$$

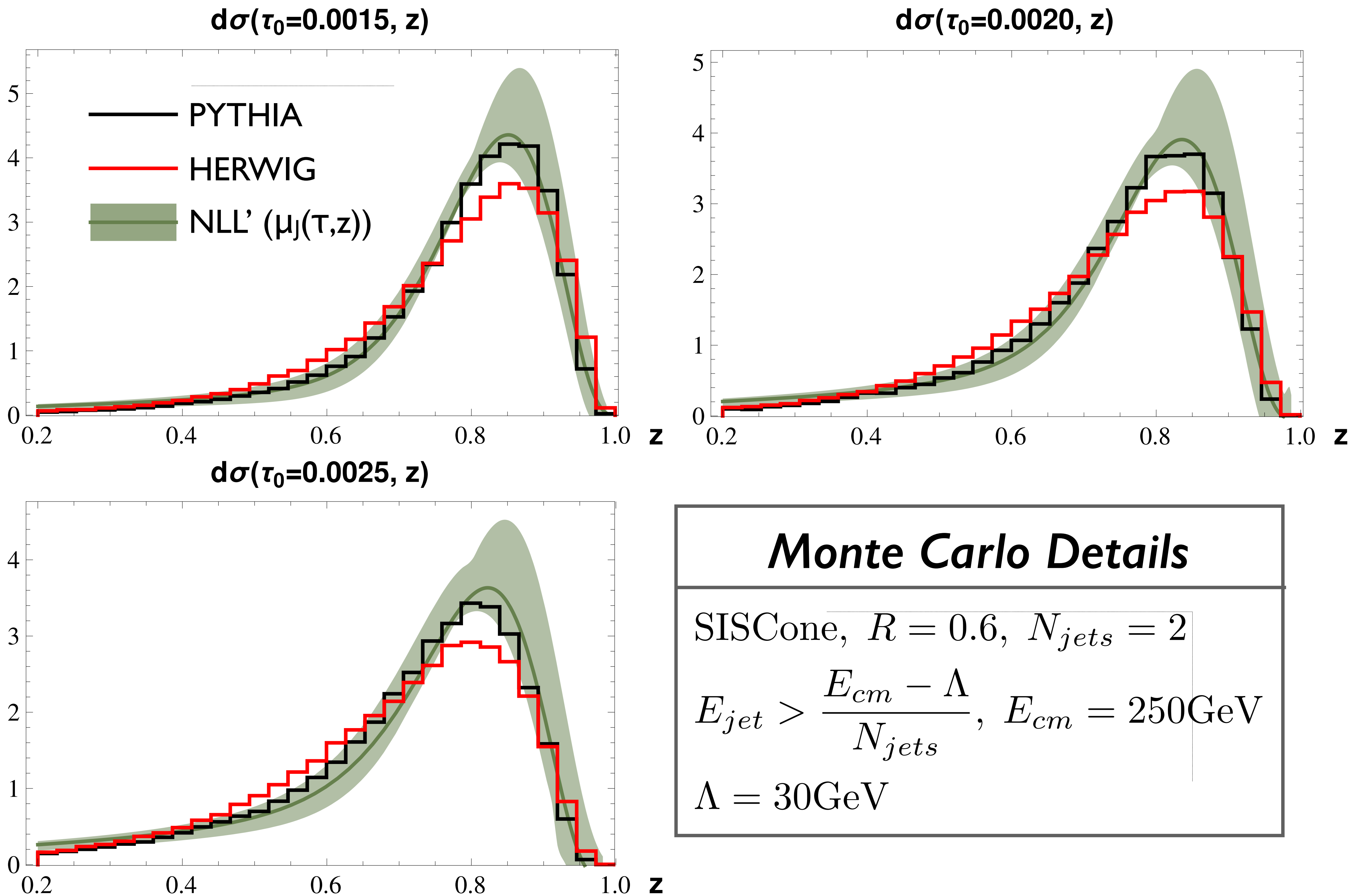
$$\mu_0 = m_b = 4.5 GeV$$

$$\chi^2_{dof} = 1.495$$

Kniehl, et al. arXiv:0705.4392

NLL' vs. Monte Carlo (B^+/B^0)

z distributions for fixed τ_a match well



Monte Carlo Details

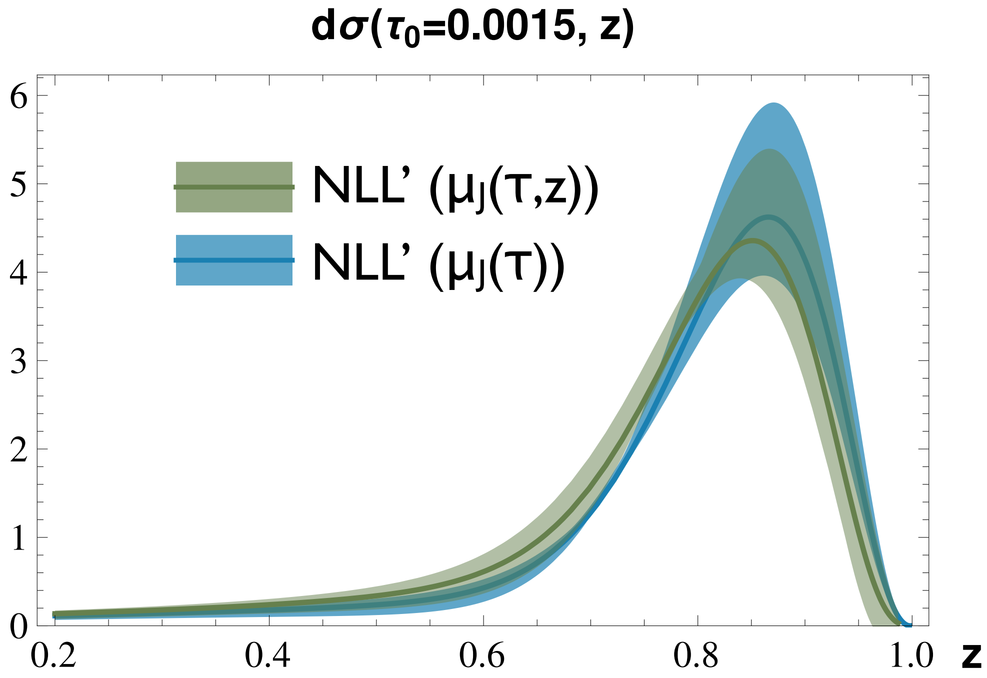
$\text{SISCone}, \ R = 0.6, \ N_{jets} = 2$

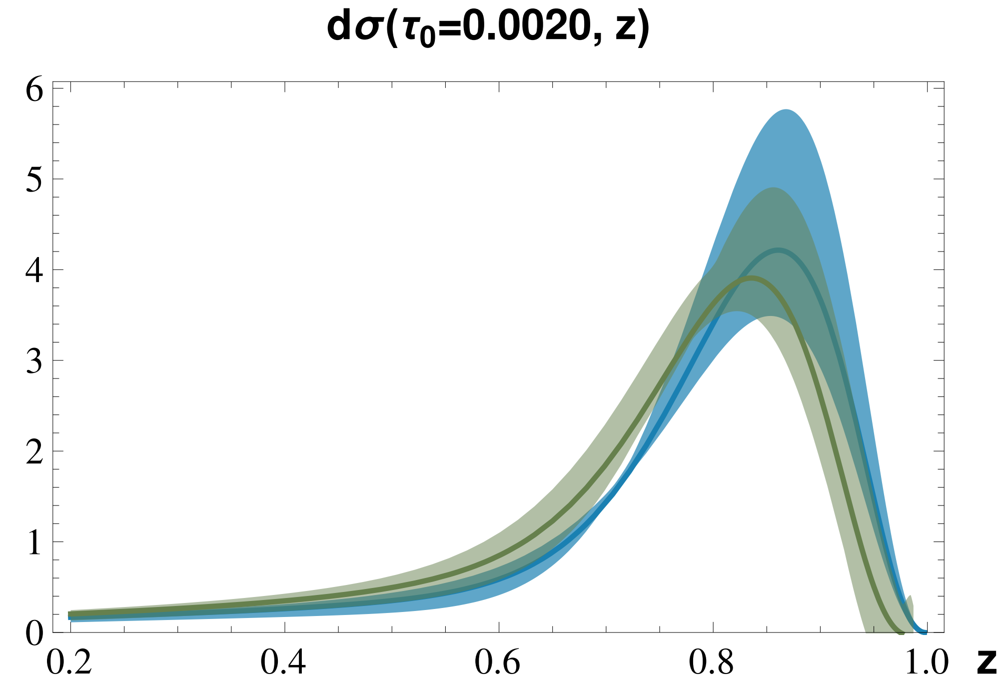
$E_{jet} > \dfrac{E_{cm} - \Lambda}{N_{jets}}, \ E_{cm} = 250\text{GeV}$

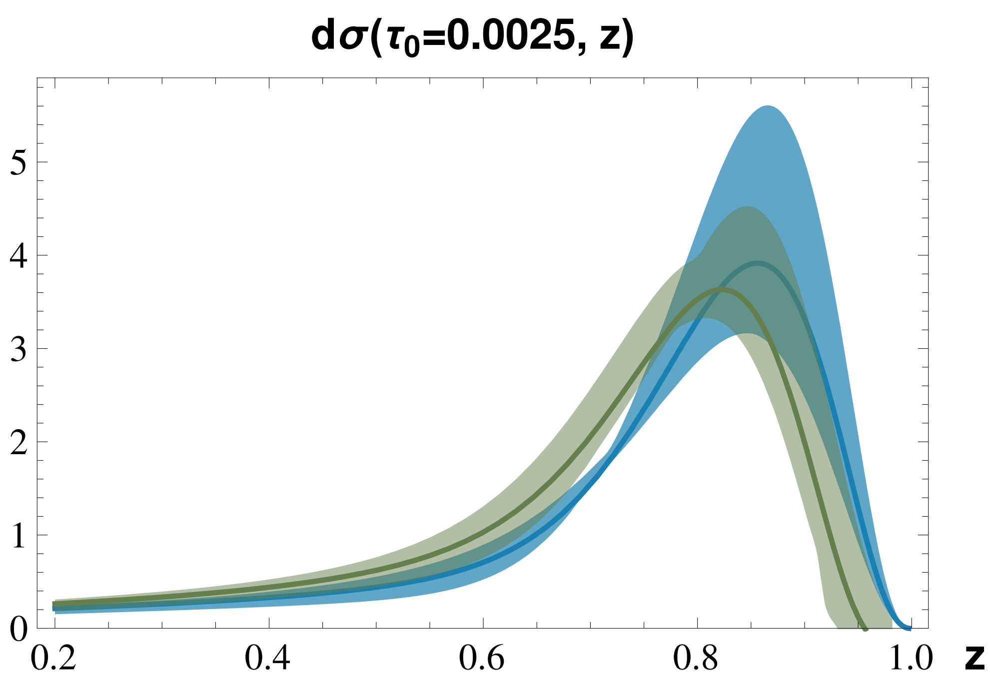
$\Lambda = 30\text{GeV}$

Minimizing Large Logs of 1-z

Use z-dependent measured jet scale in FJF ⟶ decreases uncertainty







Reorganize Logs of z

$\left[f_{\mathcal{J}}^{ij} \bullet D\right](z)$ contains $\log\left(\frac{\mu}{\mu_J(\tau, z)}\right)$

where

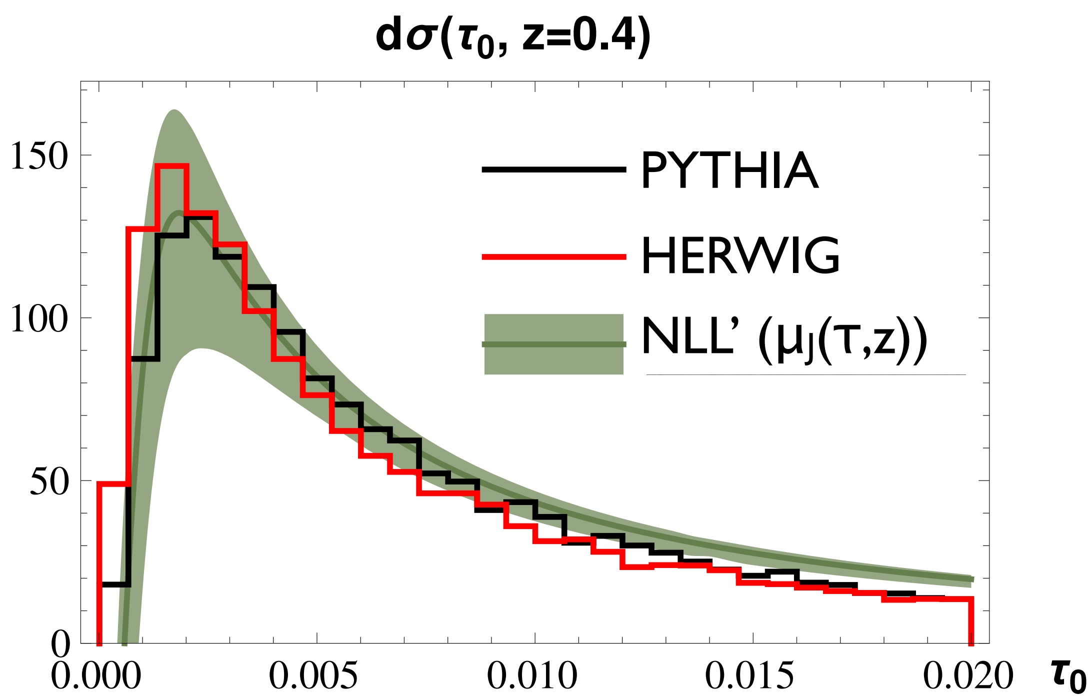
$$\mu_J(z) = \omega\tau^{1/(2-a)}(1-z)^{(1-a)/(2-a)}$$

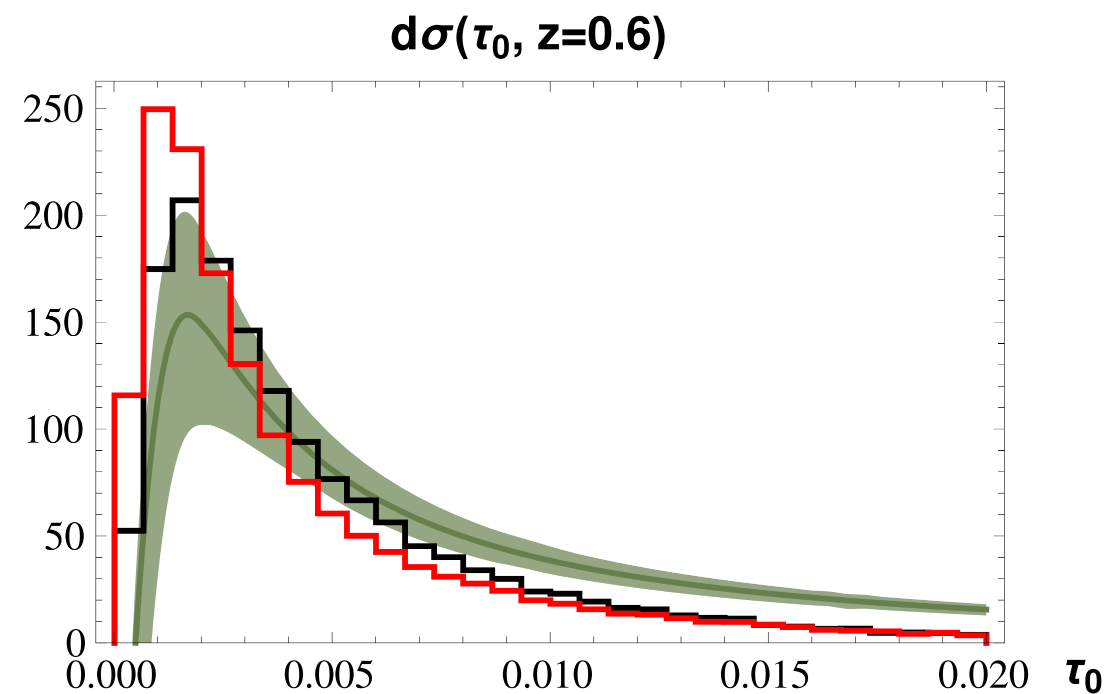
Abbate, Fickinger, Hoang, Mateu, Stewart, arXiv:1006.3080
Hornig, Makris, Mehen, arXiv:1601.01319
Ligeti, Stewart, Tackmann, arXiv:0807.1926

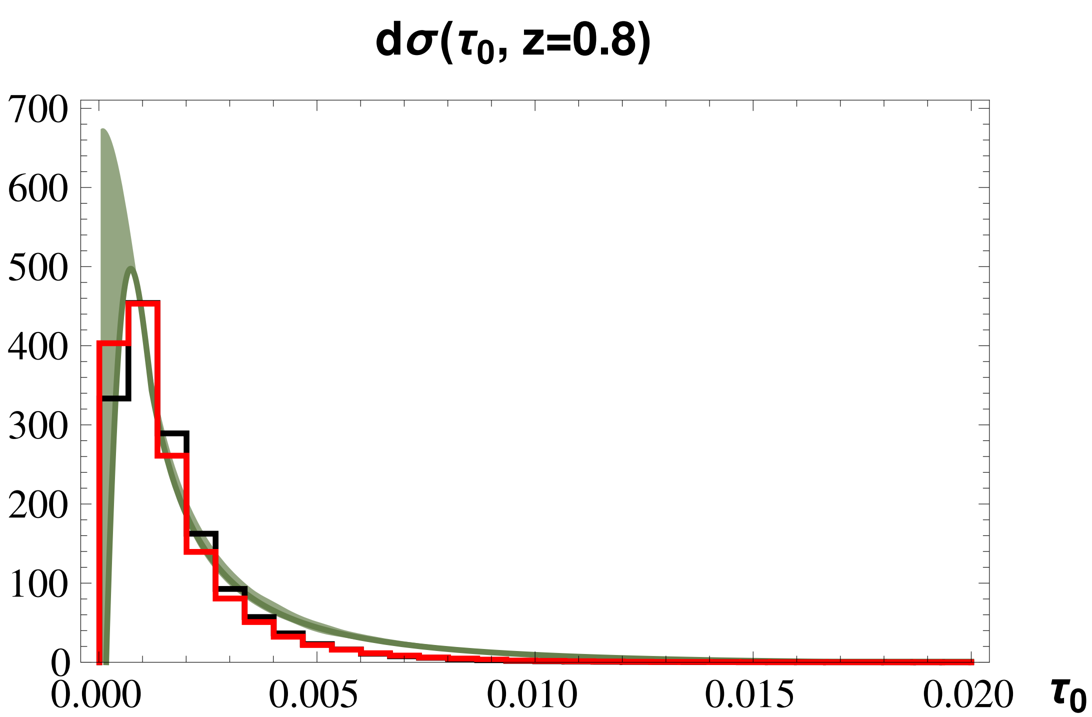
Procura, Waalewijn, arXiv:1111.6605

NLL' vs. Monte Carlo (B^+/B^0)

τ_0 distributions for fixed z also show good agreement







Monte Carlo Details

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$\Lambda = 30\text{GeV}$

Next steps

- NLL' and Monte Carlo match well for B mesons in jets

- Use FJF's to probe quarkonium production?

- Baumgart, et al. $\longrightarrow$ discriminate between J/ψ production mechanisms with FJFs

- Calculate 3 jet cross-section with J/ψ, compare with MC…

Cross Section for 3 jets & J/ψ

Re-summed to NLL' using renormalization group (RG)

NRQCD FF for J/ψ's

$$\frac{1}{\sigma_0}\frac{d\sigma^{(g)}}{d\tau_a dz} = H_3(\mu_H) \times S^{\text{unmeas}}(\mu_\Lambda) \times J_{n_1}^{(\bar{b})}(\mu_{J_{n_1}}) \times J_{n_2}^{(b)}(\mu_{J_{n_2}})$$
$$\times \sum_i \left\{ \left(\frac{\Theta(\tau_a)}{\tau_a^{1+\Omega}}\right) \left[\delta_{gi}\delta(1-z)(1+f_S(\tau_a, z, \mu_{S^{\text{meas}}})) + f_{\mathcal{J}}^{gi}(\tau_a, z, \mu_{J_{n_3}})\right] \bullet \frac{D_{i\to J/\psi}(z, \mu_{J_{n_3}})}{2(2\pi)^3} \right\}_+$$
$$\times \Pi(\mu, \mu_H, \mu_\Lambda, \mu_{J_{n_1}}, \mu_{J_{n_2}}, \mu_{J_{n_3}}, \mu_{S^{\text{meas}}}),$$

coupled z & τ_a

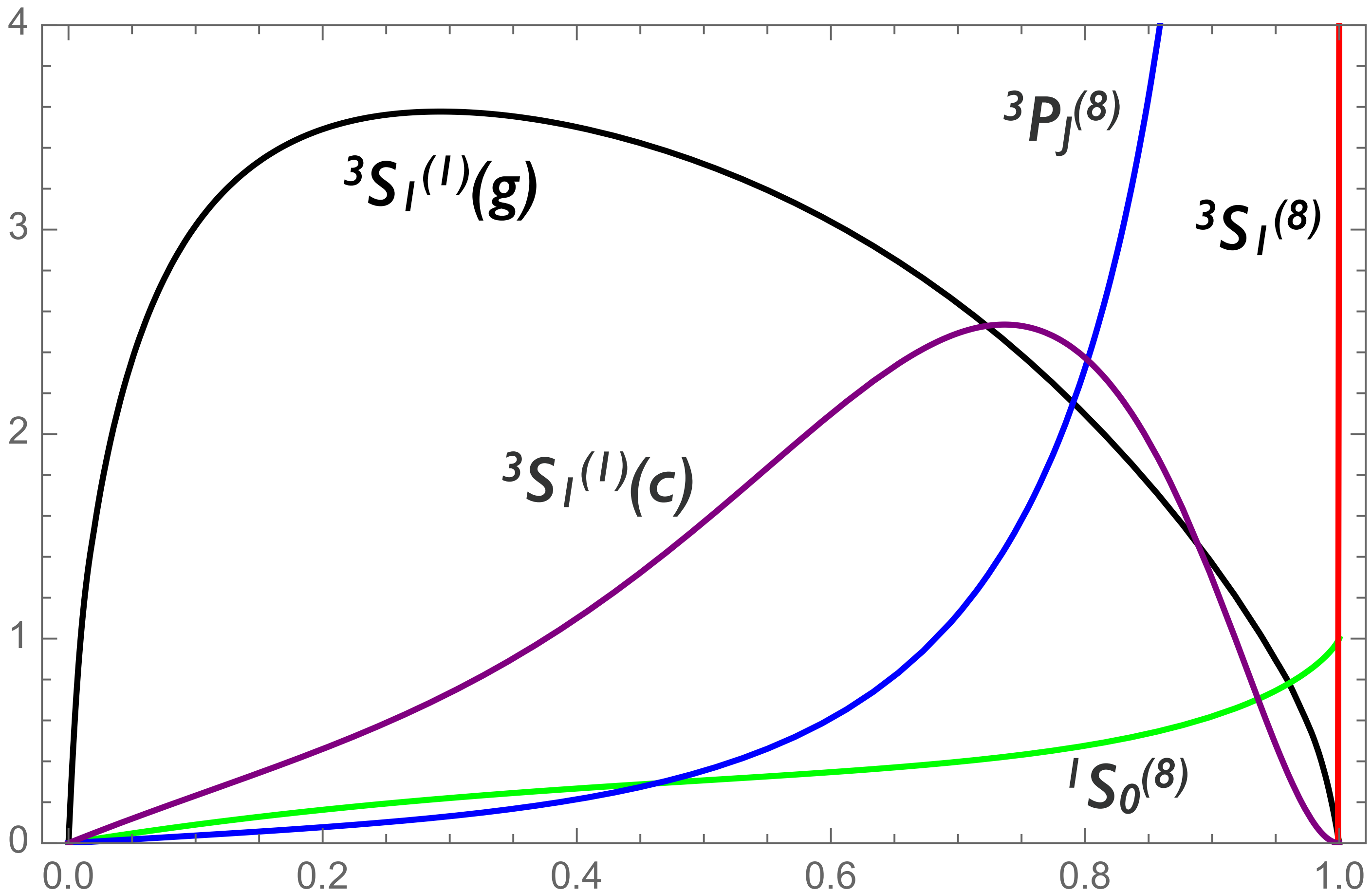
RG evolution factor

Similar to cross-section for B's with new FF and 3 jets

Want to study pp $\longrightarrow$ focus on g fragmentation

NRQCD Fragmentation Functions

Fragmentation Function vs. z of J/ψ



NRQCD Factorization

$$D_{g\to J/\psi} = \sum_n D^{(n)}_{g\to J/\psi}\langle \mathcal{O}^{J/\psi}(n)\rangle$$

with $n = {}^{2S+1}L_J^{(1,8)}$

Examples of $\alpha_s(2m_c)$ & z dependence

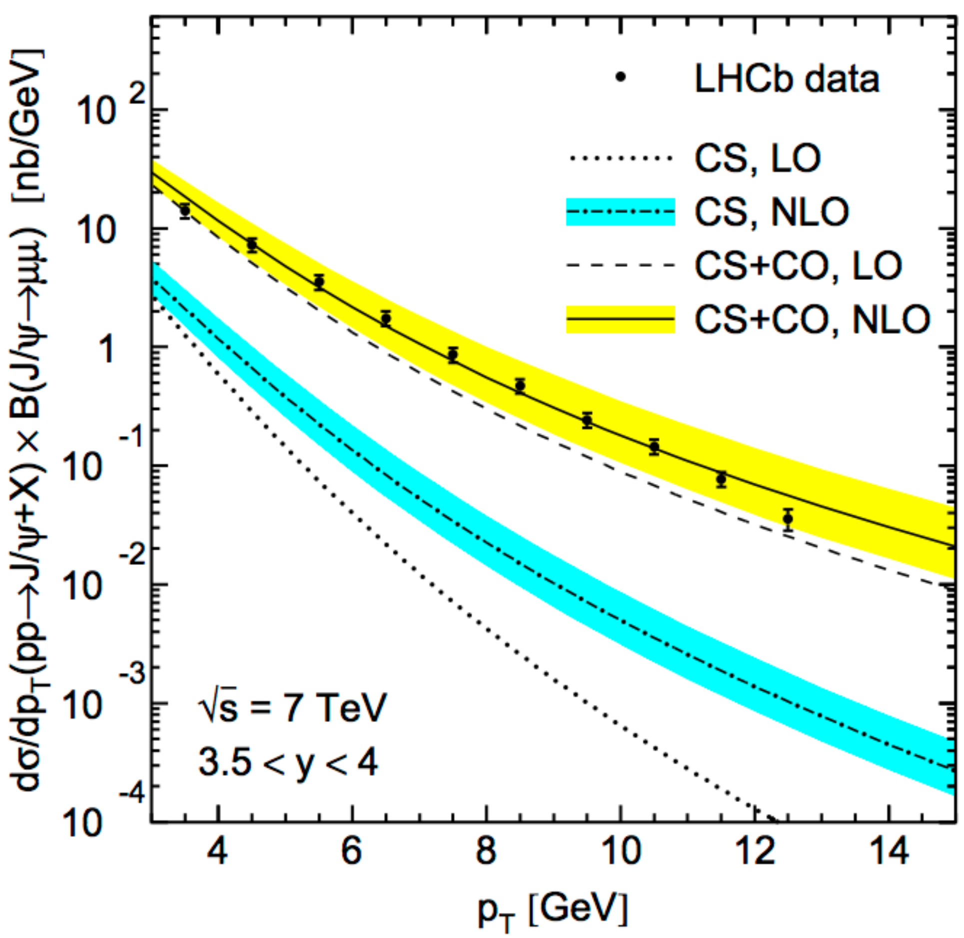
$$D^{^3S_1^{(8)}}_{g\to J/\psi}(z,2m_c) = \frac{\pi\alpha_s(2m_c)}{24m_c^3}\delta(1-z)$$

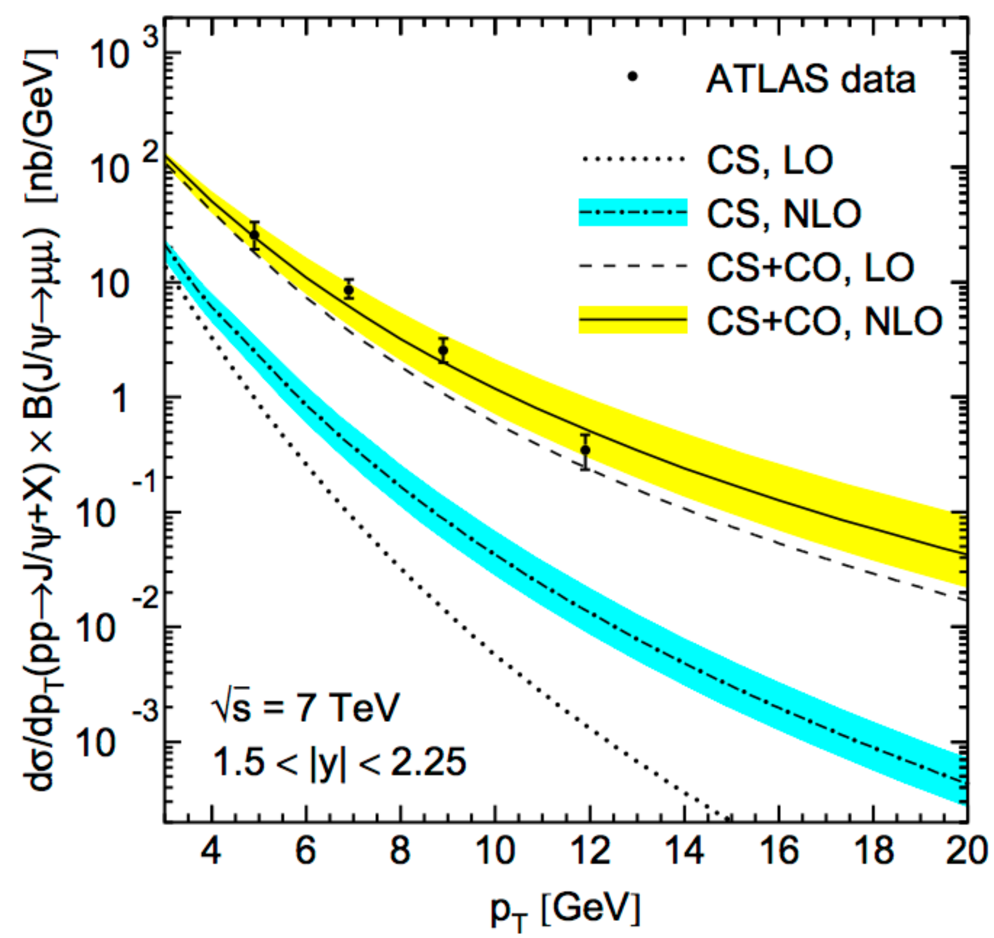
$$D^{^1S_0^{(8)}}_{g\to J/\psi}(z,2m_c) = \frac{5\alpha_s(2m_c)}{96m_c^3}\left(3z-2z^2+2(1-z)\log(1-z)\right)$$

Braaten, Chen, hep-ph/9610401
Braaten, Chen, hep-ph/9604237
Braaten, Yuan, hep-ph/9302307

Extract LDME's from World's Data

Need CS+CO at NLO to fit data from various experiments





Fit to world data (2/26 plots shown) to $e^+e^-,\ \gamma\gamma,\ \gamma p,\ p\bar{p}, pp \to J/\psi + X$

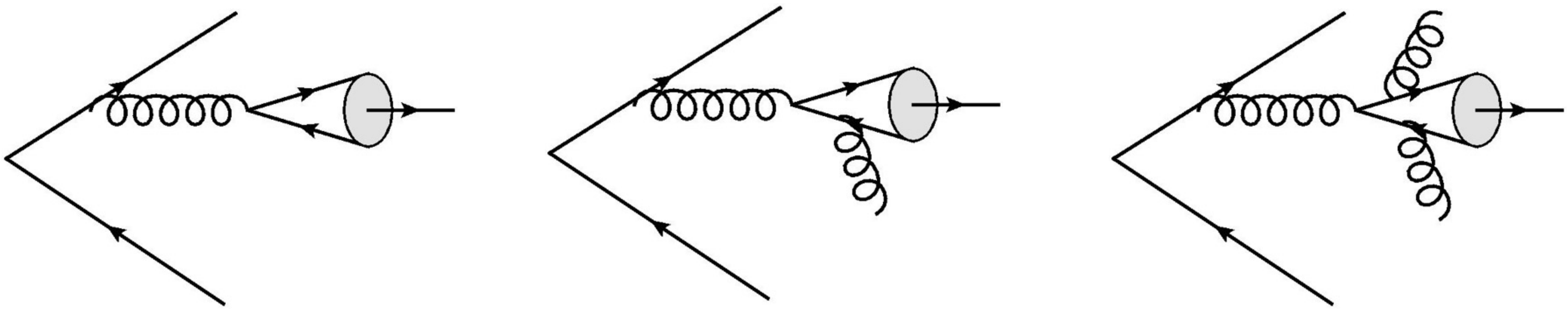
$\langle \mathcal{O}^{J/\psi}(^3S_1^{(1)}) \rangle$	$\langle \mathcal{O}^{J/\psi}(^3S_1^{(8)}) \rangle$	$\langle \mathcal{O}^{J/\psi}(^1S_0^{(8)}) \rangle$	$\langle \mathcal{O}^{J/\psi}(^3P_J^{(8)}) \rangle / m_c^2$
1.32 GeV^3	2.24 $\times 10^{-3}$ GeV^3	4.97 $\times$ 10^{-2} GeV^3	-7.16 $\times 10^{-3}$ GeV^3

Buttenschon, Kniehl (2011), arXiv:1105.0820

Default MadGraph + PYTHIA

1. MadOnia: Create J/psi in hard process

$e^+e^- \to b\,\bar{b}\,c\,\bar{c}\left[{}^3S_1^{(8)}\right]$ $\quad e^+e^- \to b\,\bar{b}\,g\,c\,\bar{c}\left[{}^1S_0^{(8)}\right]$ $\quad e^+e^- \to b\,\bar{b}\,g\,g\,c\,\bar{c}\left[{}^3S_1^{(1)}\right]$

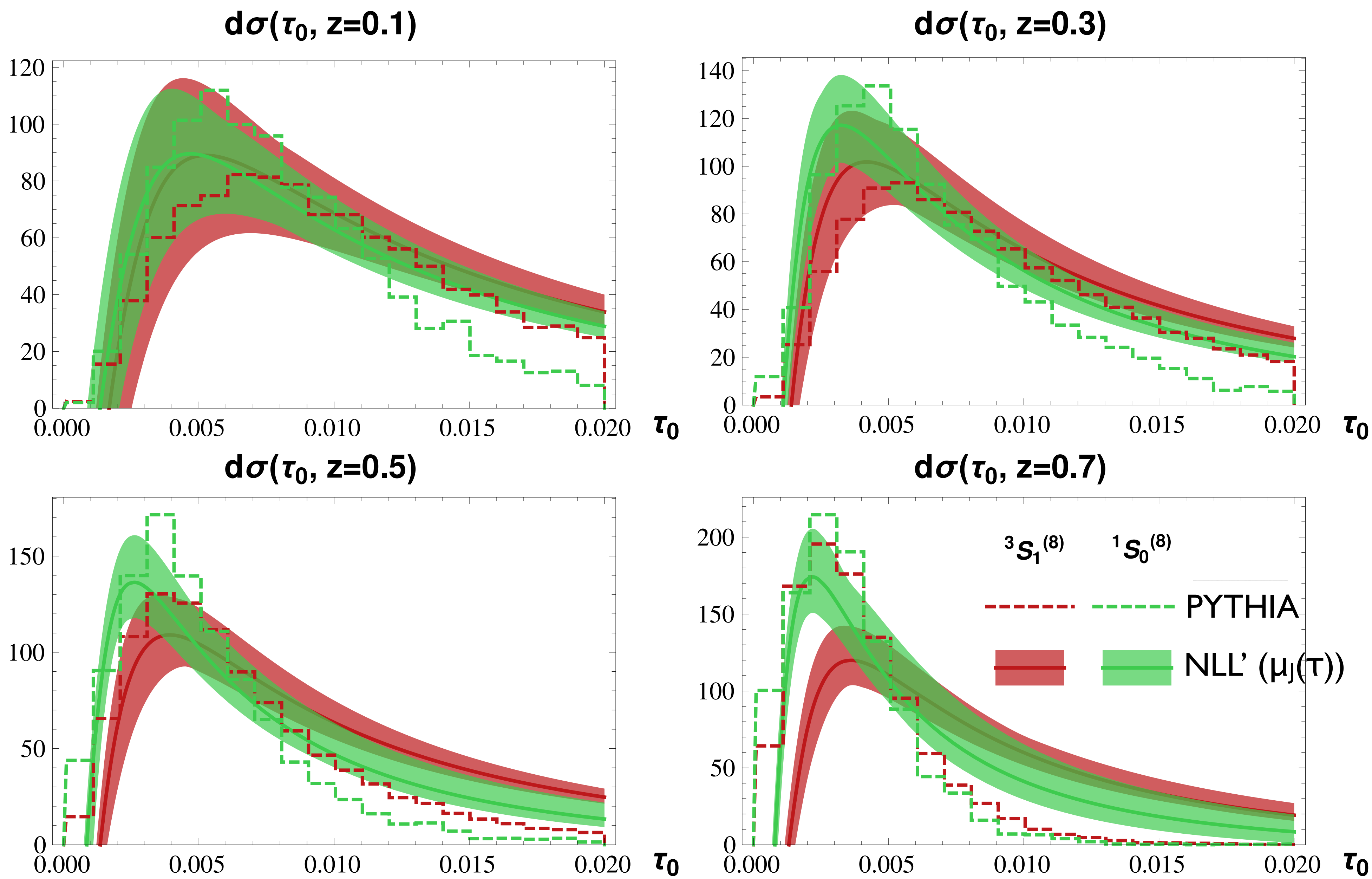


2. PYTHIA $\longrightarrow$ Parton shower + hadronization

3. RIVET $\longrightarrow$ Reconstruct jets + implement cuts

NLL' vs. PYTHIA (J/ψ)

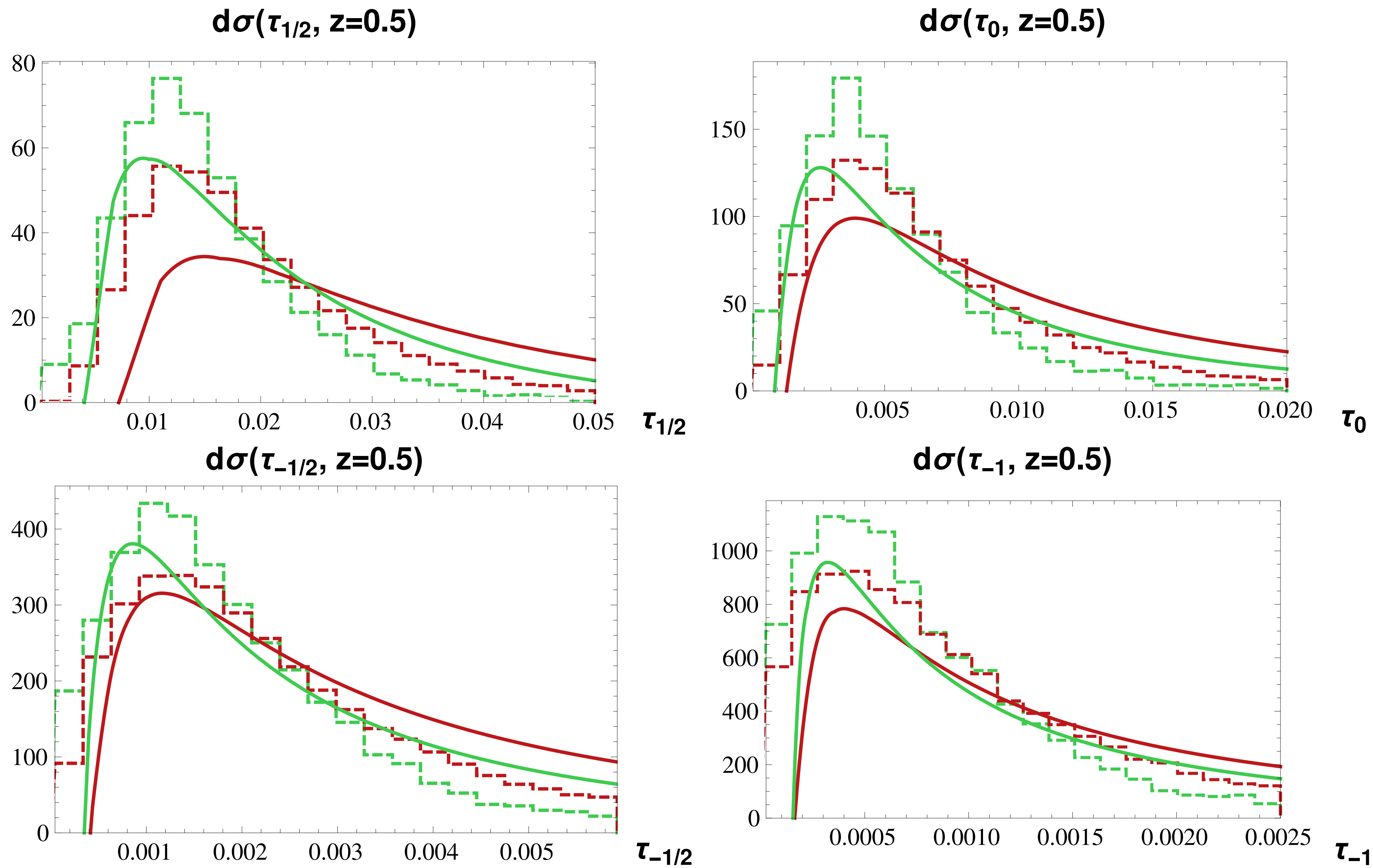
Monte Carlo/NLL' τ_a distributions for fixed z's show similarities



As z ⟶ 0 we see less dependence on production mechanism

NLL' vs. PYTHIA (J/ψ)

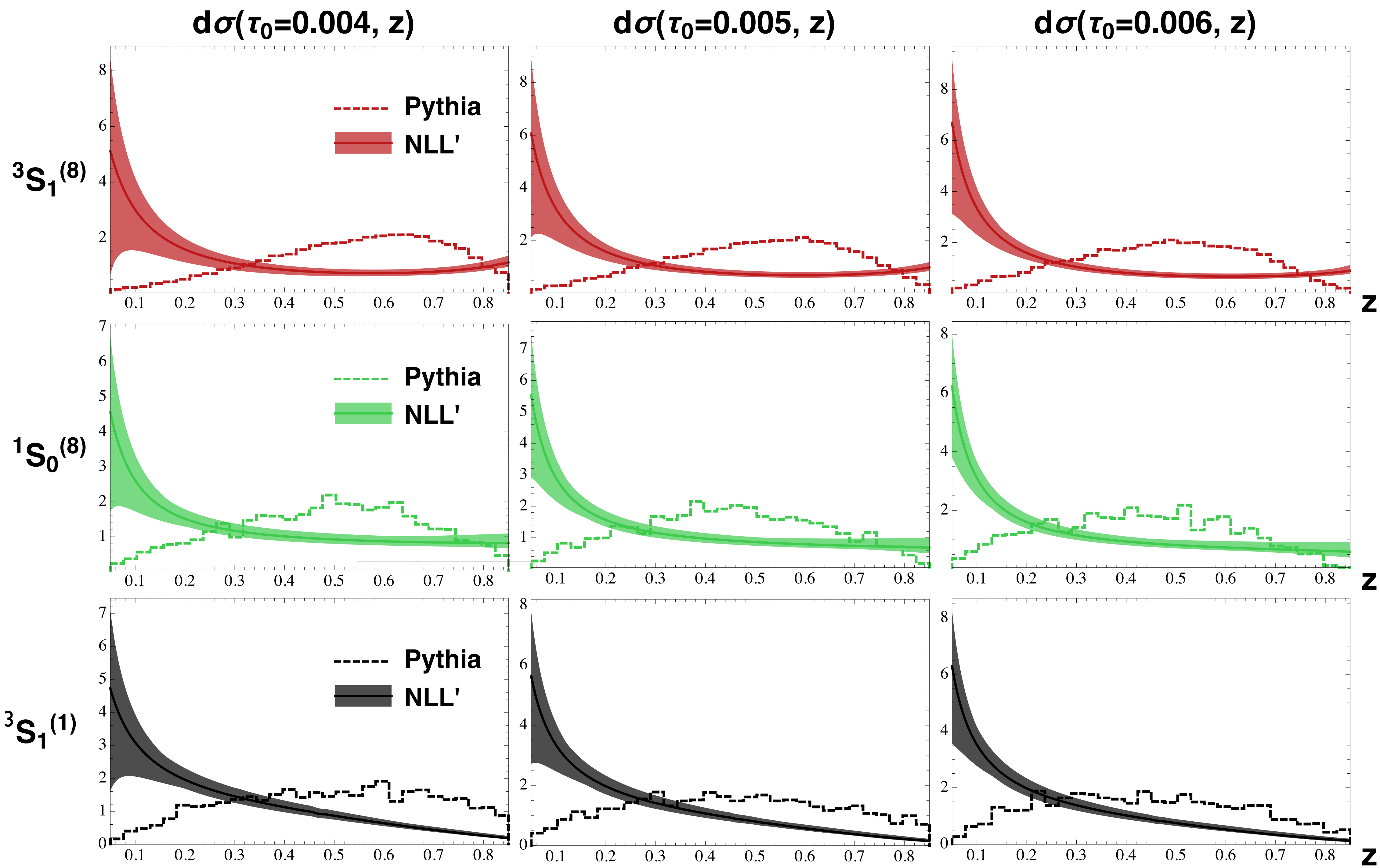
Monte Carlo/NLL' τ_a distributions for different a's also show similarities



More discriminating power for larger a (a < 1 in $\mathrm{SCET_I}$)

Comparing NLL' & PYTHIA

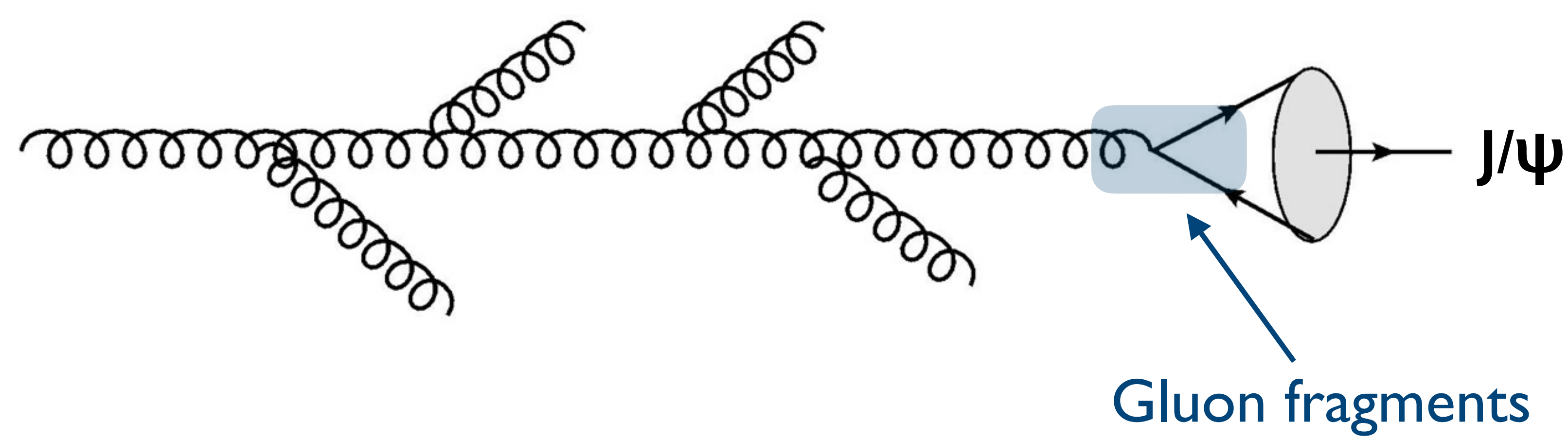
PYTHIA yields much harder z distributions



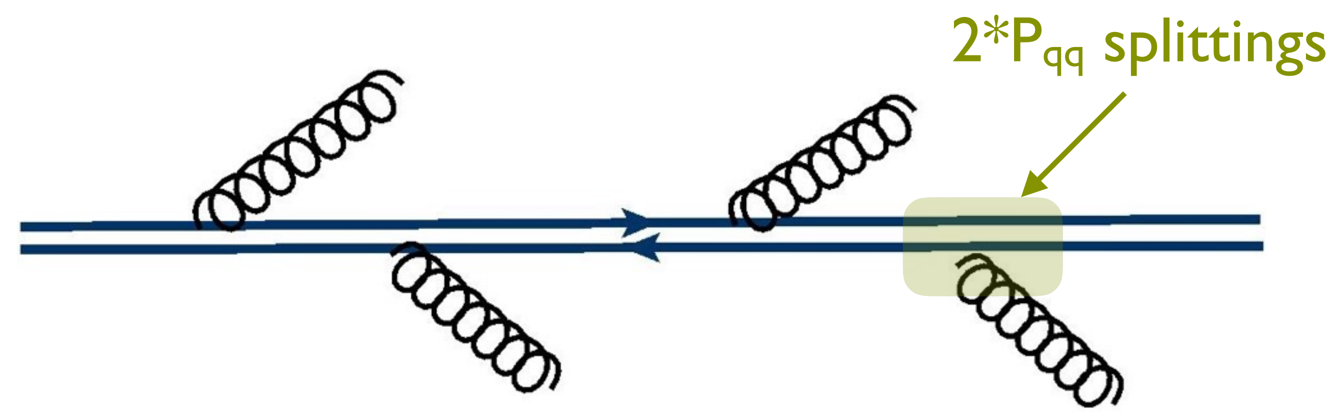
Gluon Fragmentation and PYTHIA

PYTHIA's picture of showering off onia different from theory

Analytic



PYTHIA

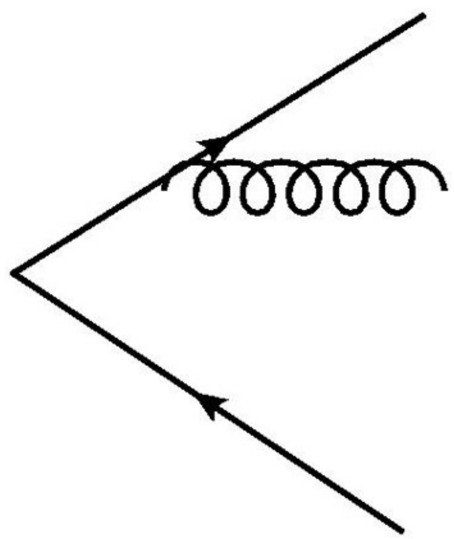


Monte carlo z distributions much harder than analytic

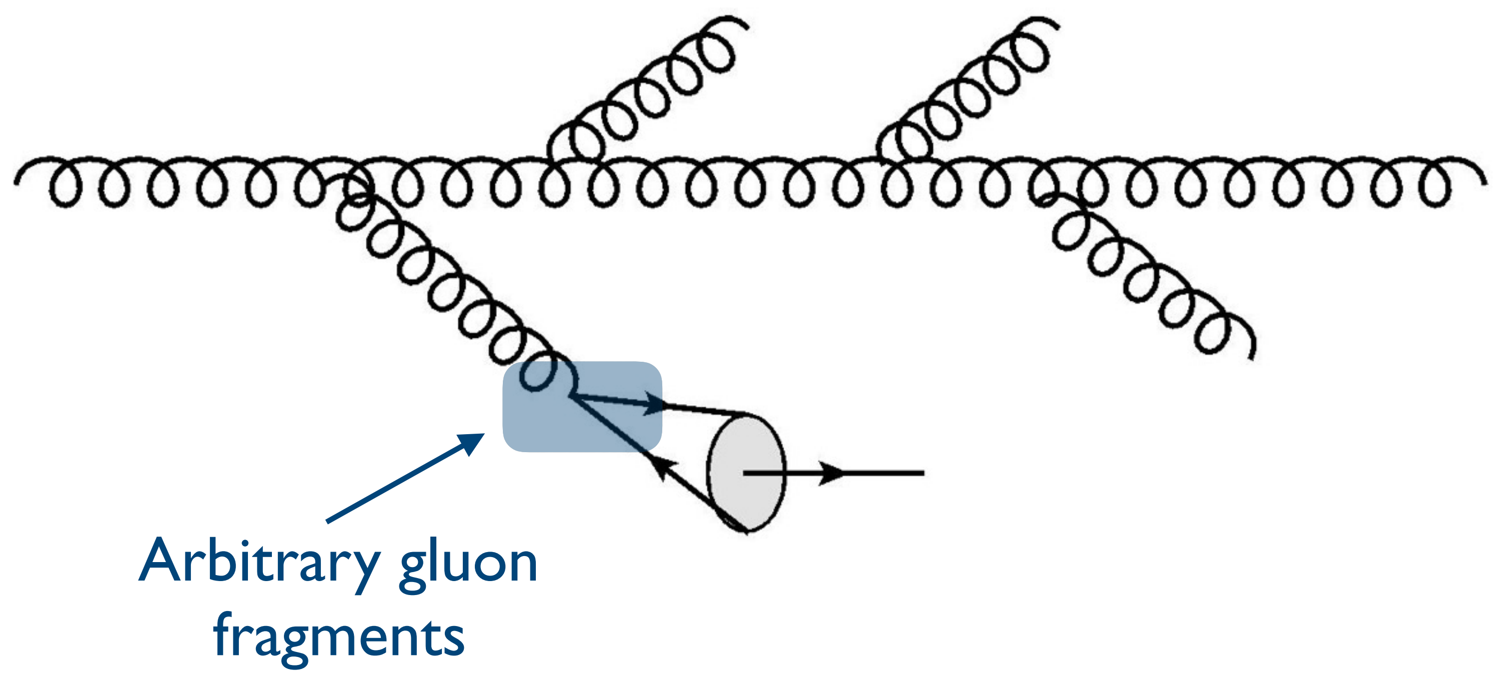
Gluon Fragmentation Improved PYTHIA (GFIP)

Madgraph 5

$$e^+e^- \to b\,\bar{b}\,g$$



PYTHIA + Convolution

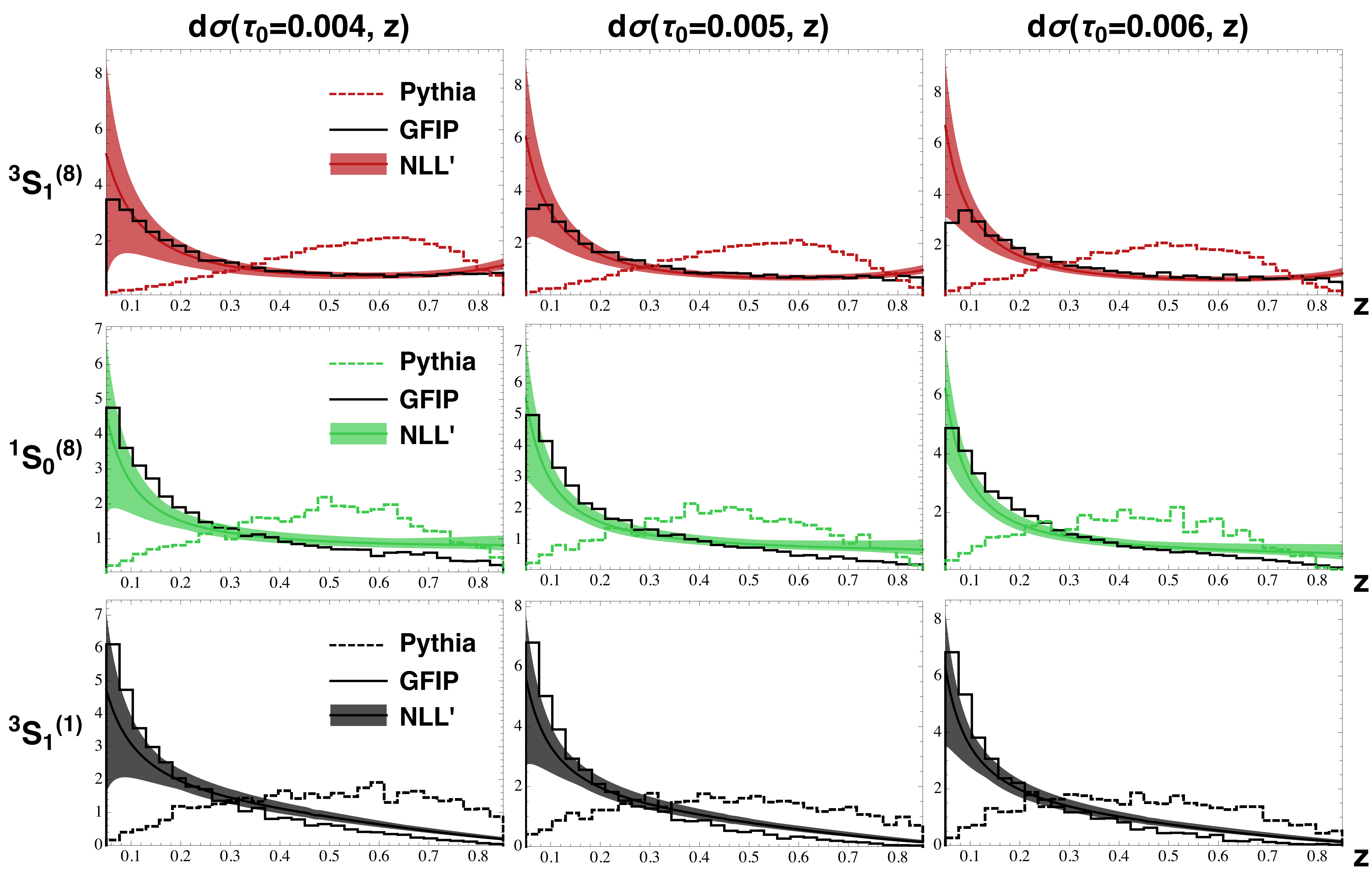


2. PYTHIA ⟶ No hadronization, adjust shower pT cutoff

3. Convolve NRQCD FFs w/ random final state gluon

Comparing NLL', PYTHIA, and GFIP

GFIP shows far better agreement w/ NLL'



Conclusions

- New calculation: FJF for measured angularities
- Our calculation fits B production in Monte Carlo ($d\sigma/d\tau dz$)
- Default Monte Carlo J/ψ seems to lack proper onia showering
- GFIP shows improvements in z-distributions

Future Work

- Proper modification of Pythia to fix showering of quarkonia

- Calculate cross-section for pp w/ measured angularity

- Extend to other jet observables

Thank you!

Backup Slides

Extra Details on Scales/FJF's

Characteristic Scales in B meson case

Function (F)	H_2	$J_{\bar{n}}^{\bar{b}}$	$S^{\rm unmeas}$	$\mathcal{J}(\tau, z)$	$S^{\rm meas}(\tau)$
Scale (μ_F)	$E_{\rm cm}$	$\omega_{\bar{n}} r$	$2\Lambda r^{1/2}$	$\omega_n \tau^{1/(2-a)}(1-z)^{(1-a)/(2-a)}$	$\omega_n \tau / r^{1-a}$

Previous Studies on FJF's

Different identified hadrons/measured observables

X. Liu, arXiv:1011.3872
Jain, Procura, Waalewijn, arXiv: 1101.4953

Jain, Procura, Waalewijn, arXiv:1110.0839

Procura, Waalewijn, arXiv:1111.6605

Jain, Procura, Waalewijn, B. Shotwell, arXiv:1207.4788

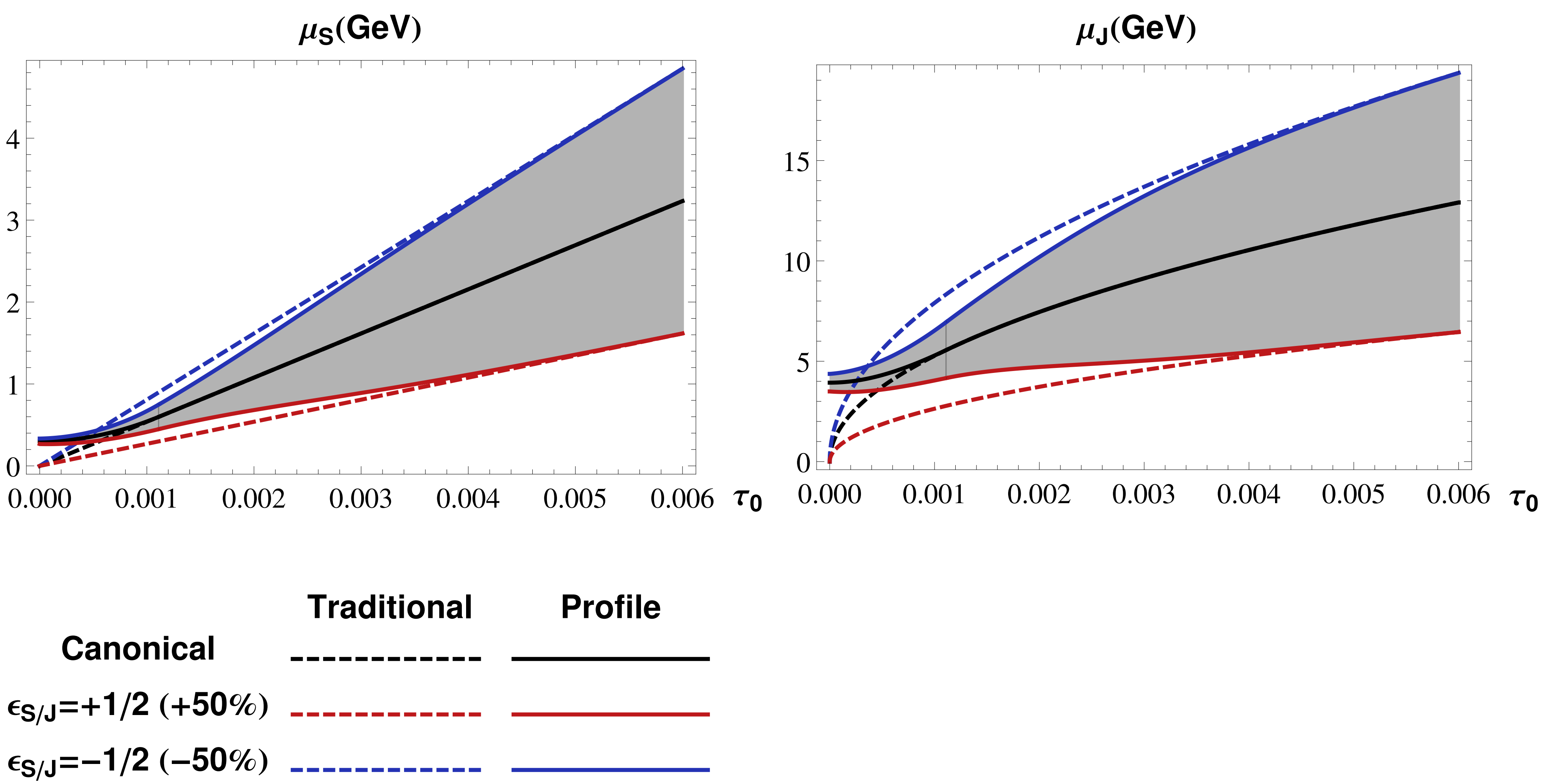
Bauer, Mereghetti, arXiv:1312.5605

Baumgart, Mehen, Leibovich, Rothstein, arXiv:1406.2295

Chien, Z.-B. Kang, F. Ringer, I. Vitev and H. Xing, arXiv:1512.06851

Profile Functions

Abbate, Fickinger, Hoang, Mateu, Stewart, arXiv:1006.3080
Ligeti, Stewart, Tackmann, arXiv:0807.1926
Hornig, Makris, Mehen, arXiv:1601.01319



$$\mu_S^{PF}(\tau) = \left[1 + \epsilon_S \frac{g(\tau)}{g(1)}\right] \times \begin{cases} \mu_0 + \alpha\tau^\beta; & 0 < \tau < \tau_{min} \\ \omega\tau/r^{(1-a)}; & \tau_{min} \leq \tau \end{cases}$$

$$\mu_J^{PF}(\tau) = \left[1 + \epsilon_J \frac{g(\tau)}{g(1)}\right] \times \begin{cases} (\omega r)^{(1-a)/(2-a)}(\mu_0 + \alpha\tau^\beta)^{1/(2-a)}; & 0 < \tau < \tau_{min} \\ \omega\tau^{1/(2-a)}; & \tau_{min} \leq \tau \end{cases},$$

Reorganizing Log(1-z)

Convolution in z

$$\frac{1}{T_{ij}}\frac{2\pi}{\alpha_s(\mu)}f_{\mathcal{J}}^{ij}(\tau,z,\mu)\bullet D(z) = \delta_{ij}\ f_1(\tau,z,\mu)\ D(z) - \int_z^1 dx\ \ f_2(\tau,x,\mu)\Big(\frac{\bar{P}_{ji}(x)}{x}\circ D\left(\frac{z}{x}\right)\Big)$$
$$+\int_z^1 dx\ \ \left[c_{ij}(x) - \frac{1}{1-a/2}\ln\left(1+\left(\frac{1-x}{x}\right)^{1-a}\right)\frac{\bar{P}_{ji}(x)}{x}\right]\circ D\left(\frac{z}{x}\right),$$

Definitions of functions

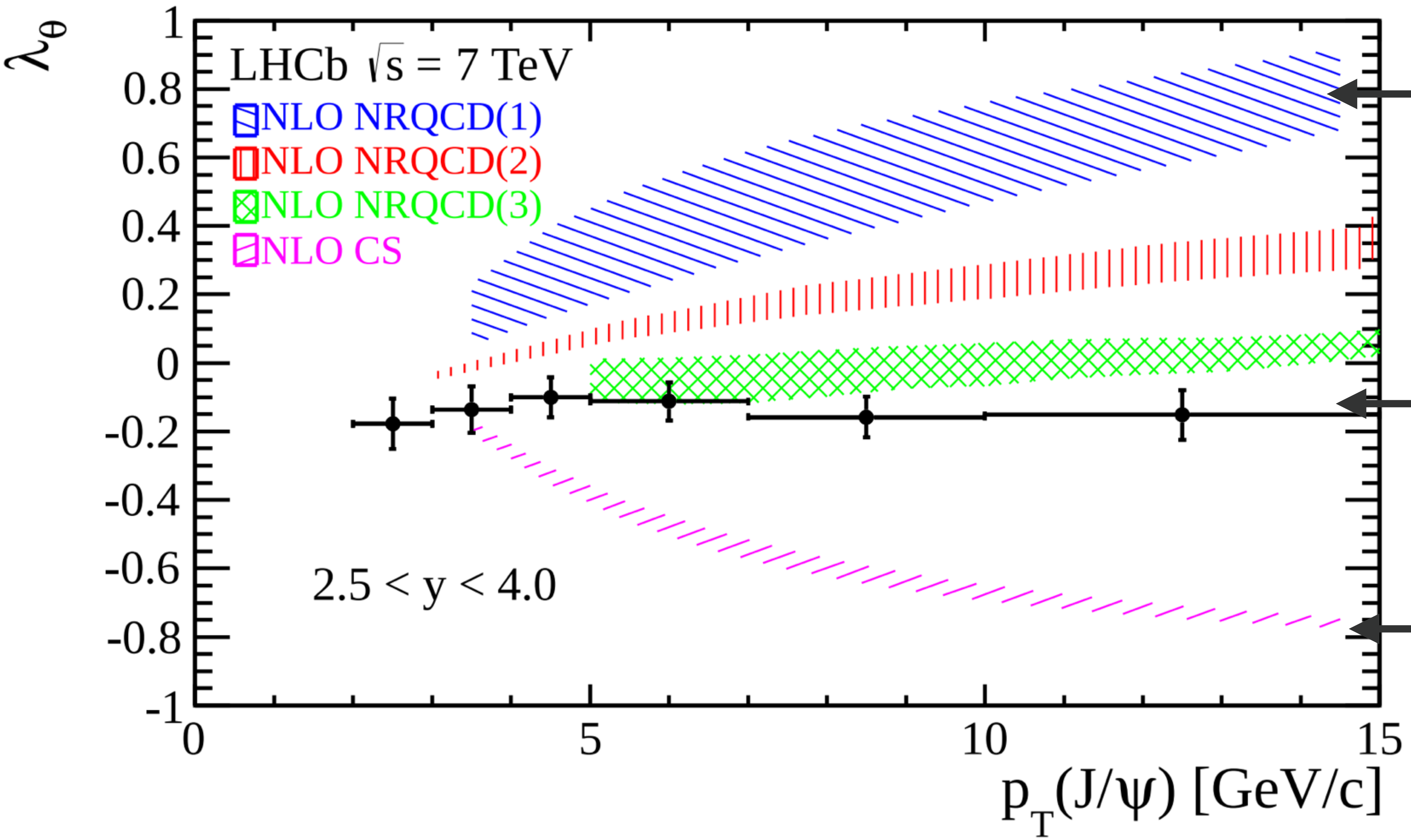
$$f_1(\tau,z,\mu) = \frac{1-a/2}{1-a}\Big(f_2(\tau,z,\mu)\Big)^2 + \frac{a(1-a/4)}{(1-a)(1-a/2)}\frac{\pi^2}{6} - \frac{1}{(1-a)(1-a/2)}\psi^{(1)}(-\Omega)$$

$$f_2(\tau,z,\mu) = 2\ln\left(\frac{\mu}{\mu_J(\tau,z)}\right) + \frac{1}{1-a/2}H(-1-\Omega),$$

z dependent scale

$$\mu_J(\tau,z) = \omega\tau^{1/(2-a)}(1-z)^{(1-a)/(2-a)}$$

Polarization Problem



LDME from global fits

LHCb data

CSM

λ_θ = +1 (trans.), 0 (unpol.), -1 (long.)

θ = J/ψ and μ+ momentum polar angle

Blue = No feed down, pT > 3 GeV; Buttenschon et. al (2012)

Red = Chi_cJ and Psi(2S) feed down, pT > 7 GeV; Gong et al. (2013)

Green = No feed down, pT > 7 GeV; Chao et. al (2012)

Magenta = Color singlet at NLO; Buttenschon et al (2012)

Terms that Arise at NLL'

Measured jet function contribution (NLO/NLL')

$$f_{\mathcal{J}}^{ij}(\tau, z, \mu) = T_{ij}\,\frac{\alpha_s(\mu)}{2\pi}\Bigg(c_0^{ij}(z,\mu) + c_1^{ij}(z,\mu)\Big(\ln\tau - H(-1-\Omega)\Big) + c_2\delta_{ij}\delta(1-z)\Big(\frac{(\ln\tau - H(-1-\Omega))^2 + \pi^2/6 - \psi^{(1)}(-\Omega)}{2}\Big)\Bigg).$$

Measured soft function contribution (NLO/NLL')

$$f_S(\tau,\mu) = -\frac{\alpha_s(\mu)C_F}{\pi}\,\frac{1}{1-a}\left\{\left[\ln\frac{\mu\tan^{1-a}\frac{R}{2}}{\omega\tau} + H(-1-\Omega)\right]^2 + \frac{\pi^2}{6} - \psi^{(1)}(-\Omega)\right\},$$

Apply to Heavy Quarkonium?

Non-relativistic QCD Factorization Formalism

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X) \langle \mathcal{O}^{J/\psi}(n) \rangle$$

Expand in α_s

Scaling in v

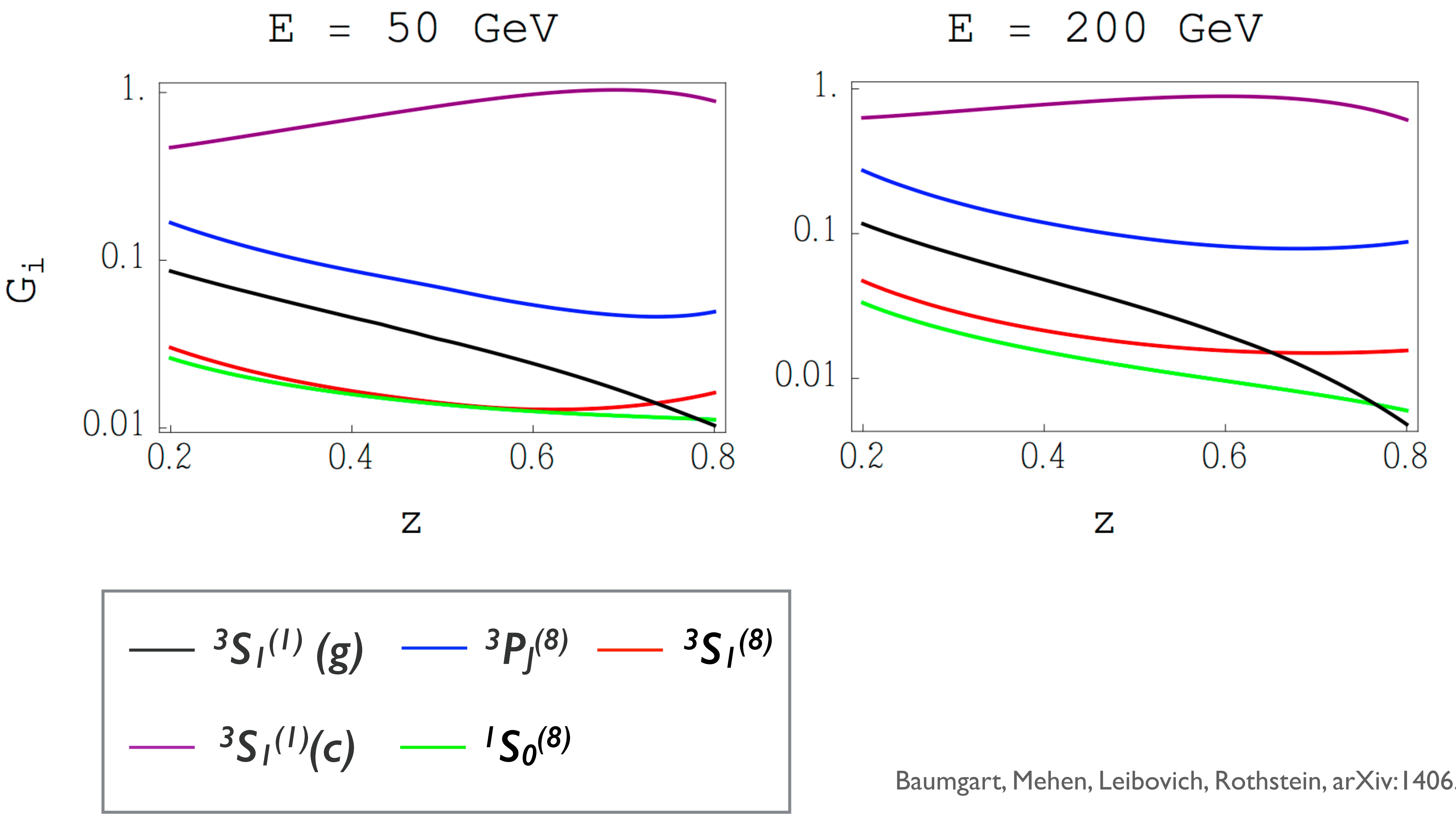
NRQCD Power Counting in α_s, v		
Mechanism	$d_n(z)$	$\langle \mathcal{O}_n^H \rangle$
${}^3S_1^{(1)}$	α_s^3	v^3
${}^3S_1^{(8)}$	α_s	v^7
${}^1S_0^{(8)}$	α_s^2	v^7
${}^3P_J^{(8)}$	α_s^2	v^7

with $n = {}^{2S+1}L_J^{(1,8)}$

Bodwin, Braaten, Lepage

FJF's and Quarkonia Production

Discriminating power between NRQCD production mechanisms



Baumgart, Mehen, Leibovich, Rothstein, arXiv:1406.2295

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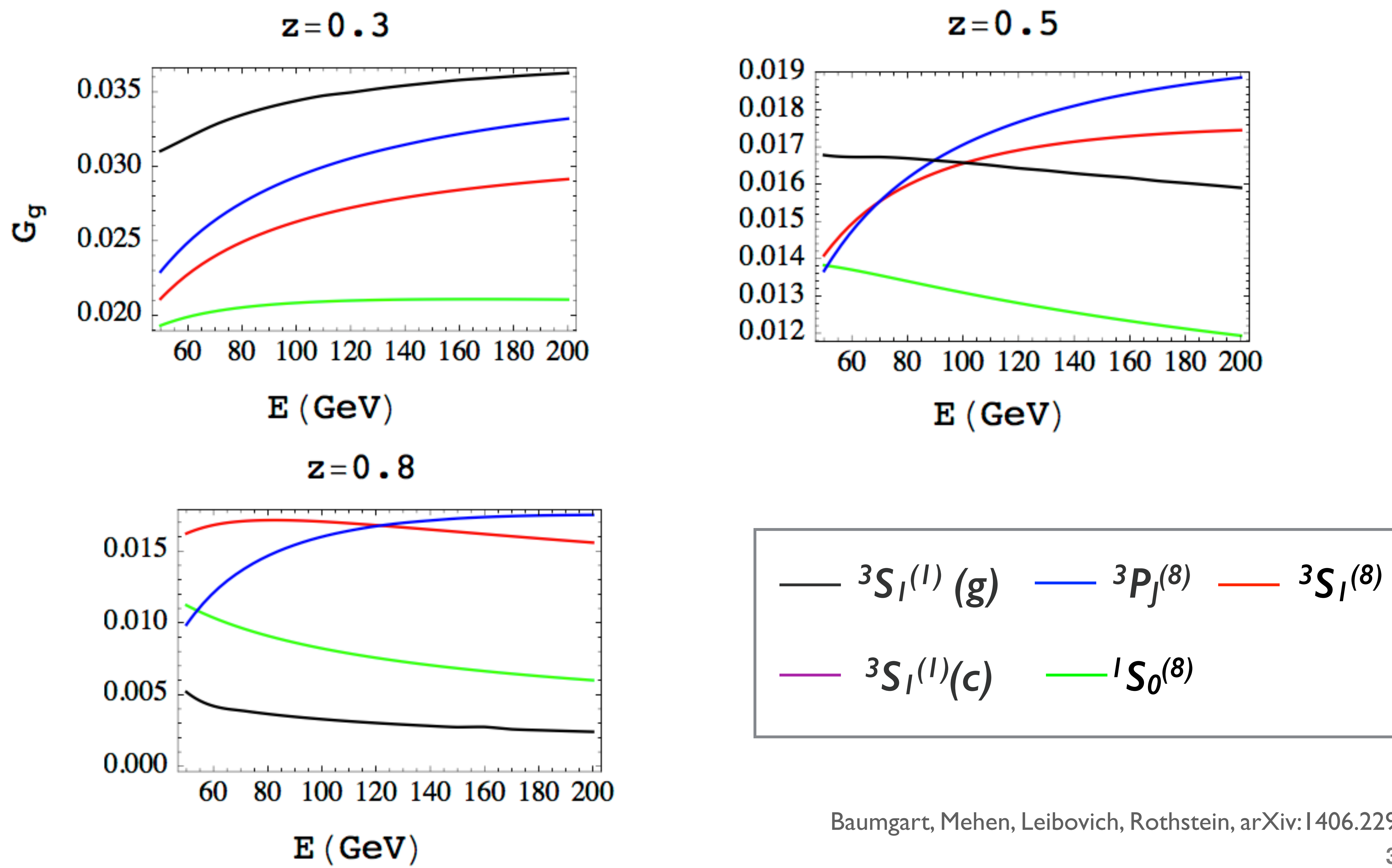
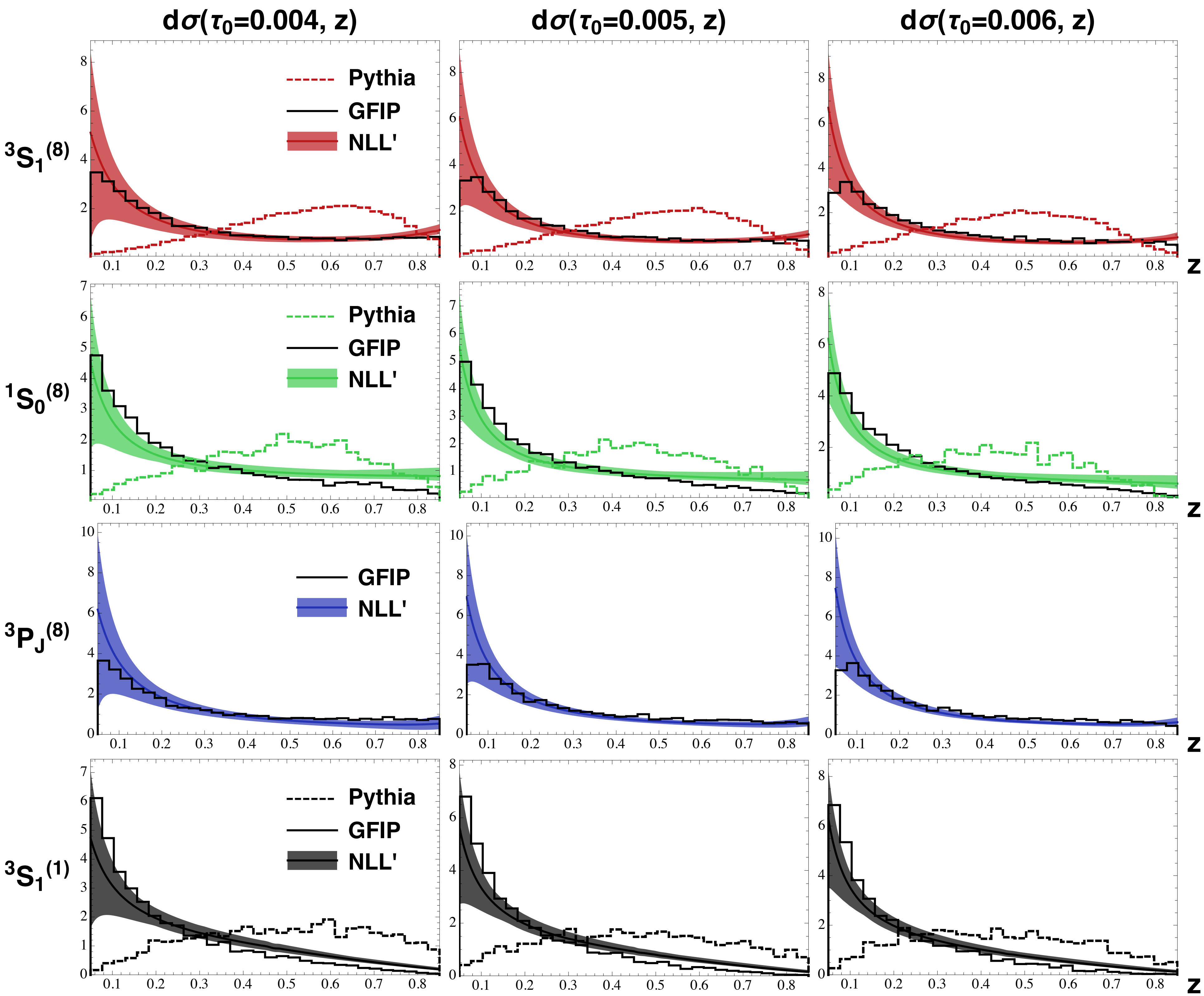


Baumgart, Mehen, Leibovich, Rothstein, arXiv:1406.2295

Comparing NLL', PYTHIA, and GFIP

GFIP shows far better agreement w/ NLL'



Details of Calculation

Hard, Soft Unmeasured, and Unmeasured Jet Functions

$$H_2(\mu) = 1 - \frac{\alpha_s(\mu)C_F}{2\pi}\left[8 - \frac{7\pi^2}{6} + \ln^2\frac{\mu^2}{\omega^2} + 3\ln\frac{\mu^2}{\omega^2}\right]$$

$$S^{\mathrm{unmeas}}(\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left[\ln^2\frac{\mu^2}{4\Lambda^2} - \ln^2\frac{\mu^2}{4\Lambda^2 r^2} - \frac{\pi^2}{3}\right]$$

$$J_{\bar{n}}^{(\bar{b})}(\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}J^q_{\mathrm{alg}}(\mu).$$

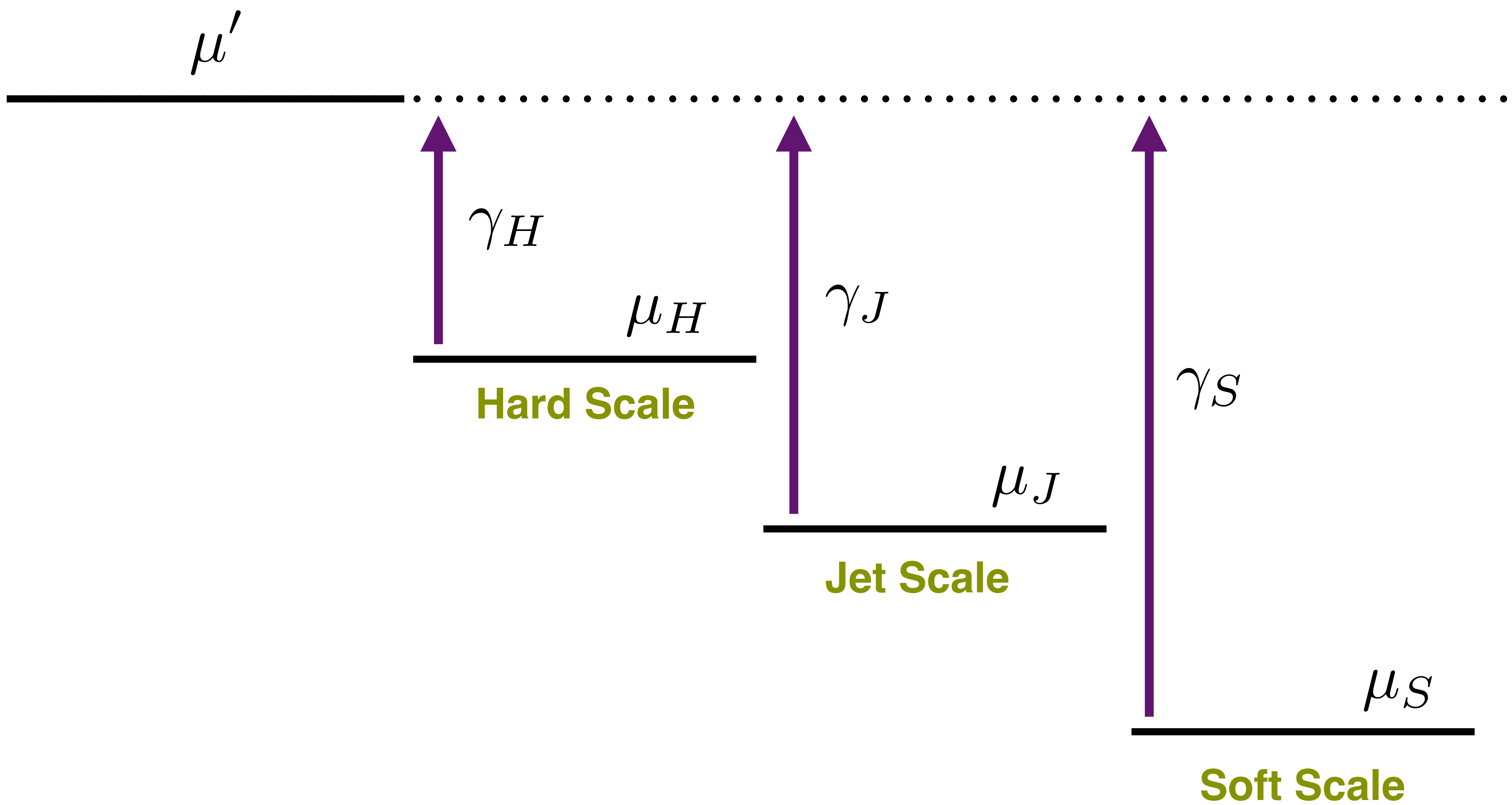
RG Evolution Factor

$$\Pi(\mu, \mu_H, \mu_\Lambda, \mu_{J_{\bar{n}}}, \mu_{J_n}, \mu_{S^{\mathrm{meas}}}) = \prod_{i=H, J_{\bar{n}}, S^{\mathrm{unmeas}}} \exp(K_i(\mu,\mu_i))\left(\frac{\mu_i}{m_i}\right)^{\omega_i(\mu,\mu_i)}$$

$$\times\frac{1}{\Gamma(-\Omega(\mu_{J_n}, \mu_{S^{\mathrm{meas}}}))} \times \prod_{i=J_n, S^{\mathrm{meas}}} \exp(K_i(\mu,\mu_i) + \gamma_E\omega_i(\mu,\mu_i))\left(\frac{\mu_i}{m_i}\right)^{j_i\omega_i(\mu,\mu_i)}$$

Resummation of Logarithms

Evolve each function to common scale using RG



Resumming Logarithms

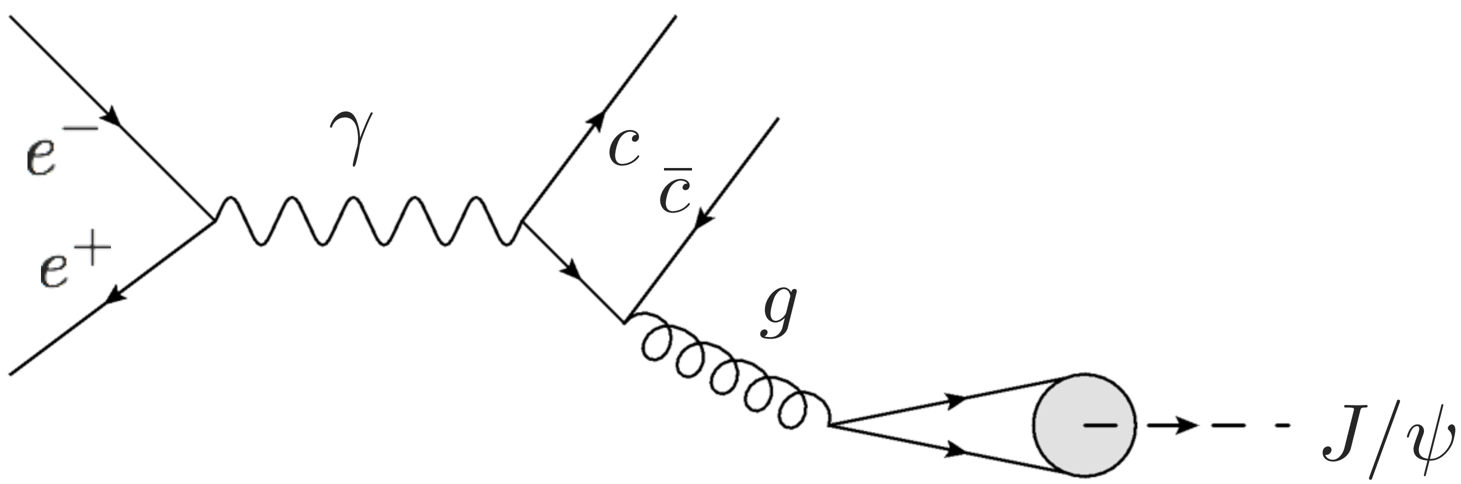
$$
\begin{aligned}
A = a &+ \alpha\left(b_1 + b_2 \log\left(\frac{\mu}{\mu_0}\right)\right) \\
&+ \alpha^2\left(c_1 + c_2 \log\left(\frac{\mu}{\mu_0}\right) + c_3 \log^2\left(\frac{\mu}{\mu_0}\right)\right) \\
&+ \alpha^3\left(d_1 + d_2 \log\left(\frac{\mu}{\mu_0}\right) + d_3 \log^2\left(\frac{\mu}{\mu_0}\right) + d_4 \log^3\left(\frac{\mu}{\mu_0}\right)\right) \\
&+ \alpha^4\left(e_1 + e_2 \log\left(\frac{\mu}{\mu_0}\right) + e_3 \log^2\left(\frac{\mu}{\mu_0}\right) + e_4 \log^3\left(\frac{\mu}{\mu_0}\right) + e_5 \log^4\left(\frac{\mu}{\mu_0}\right)\right) \\
&+ \ldots
\end{aligned}
$$

N^{3}LL **NNLL** **NLL** **LL**

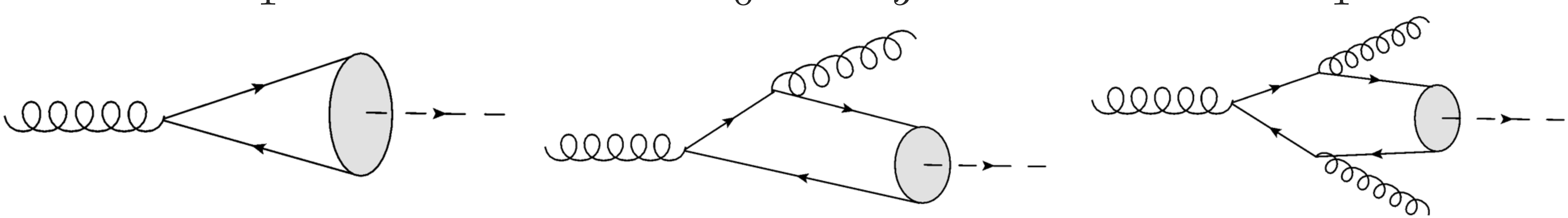
$$
N^{n-m}LL \sim \sum \alpha_s^n \log^m\left(\frac{\mu}{\mu_0}\right)
$$

J/ψ Production Mechanisms

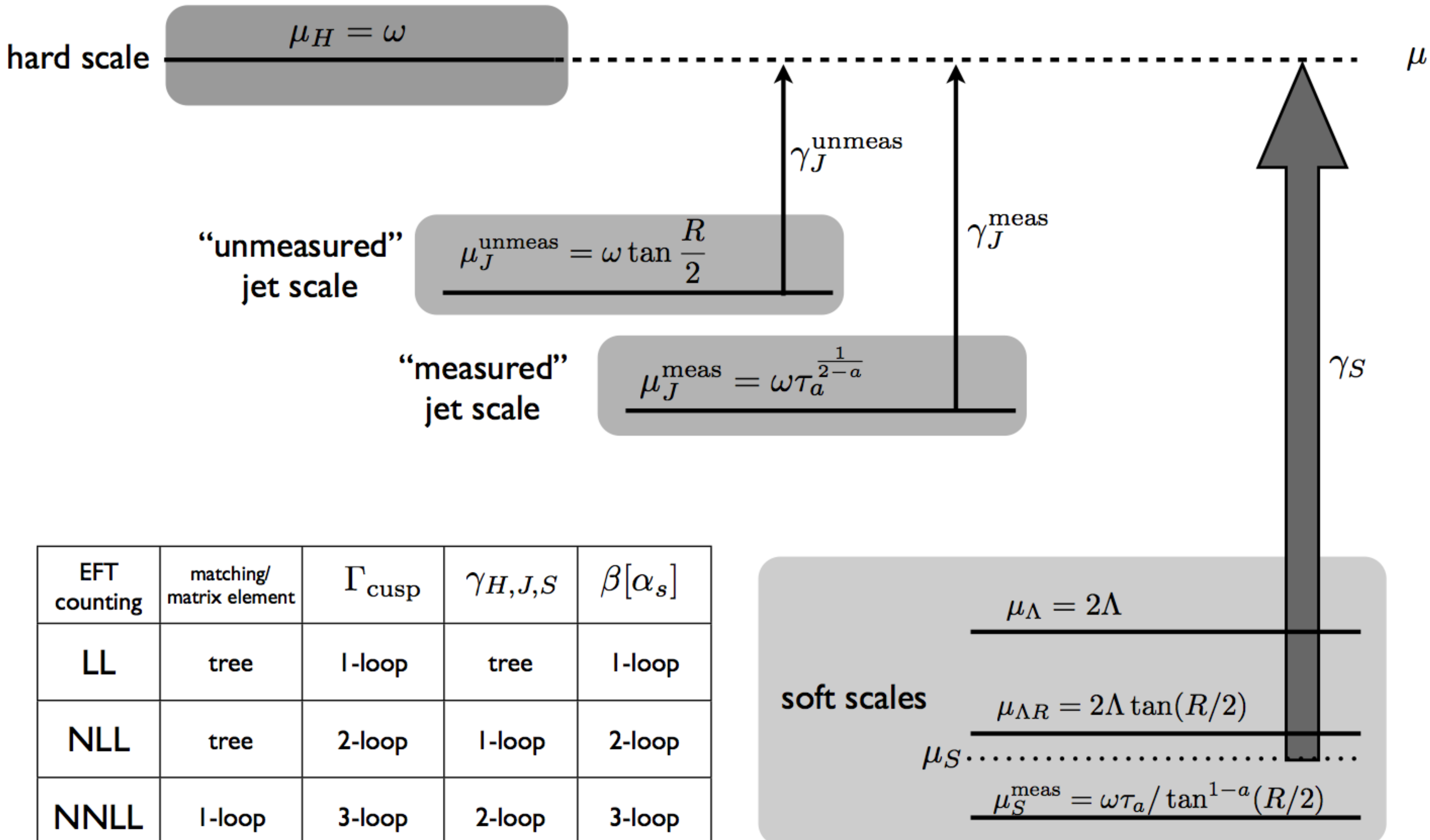
Diagrams for each singlet/octet channels



$^3S_1^{(8)}$ $\quad$ $^1S_0^{(8)}$ $\;$ $^3P_J^{(8)}$ $\quad$ $^3S_1^{(1)}$



Characteristic Scales in Factorization Theorem

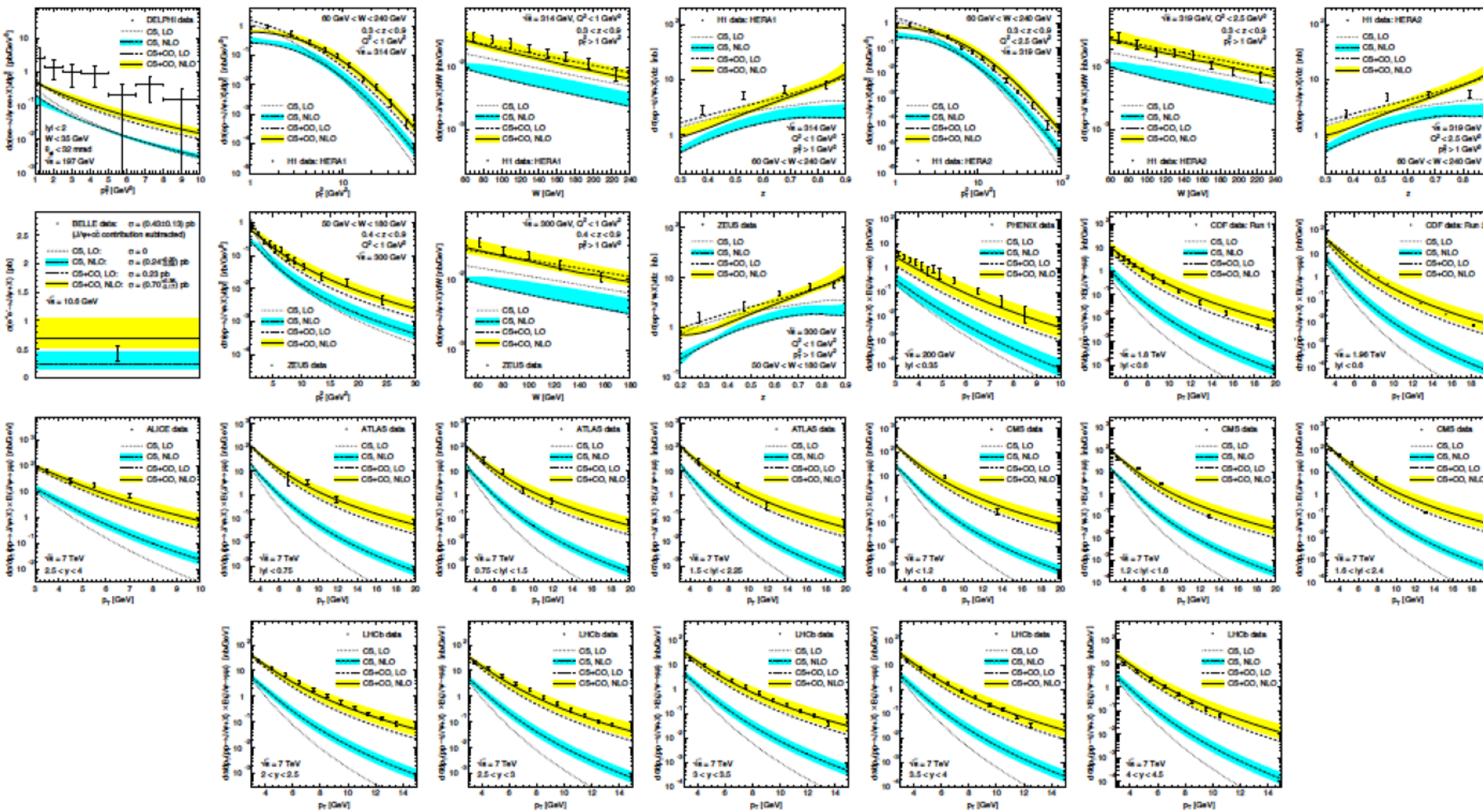


EFT counting	matching/ matrix element	Γ_{cusp}	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop

S.D.Ellis, et. al, arXiv:1001.0014

Global Fits to World's Data

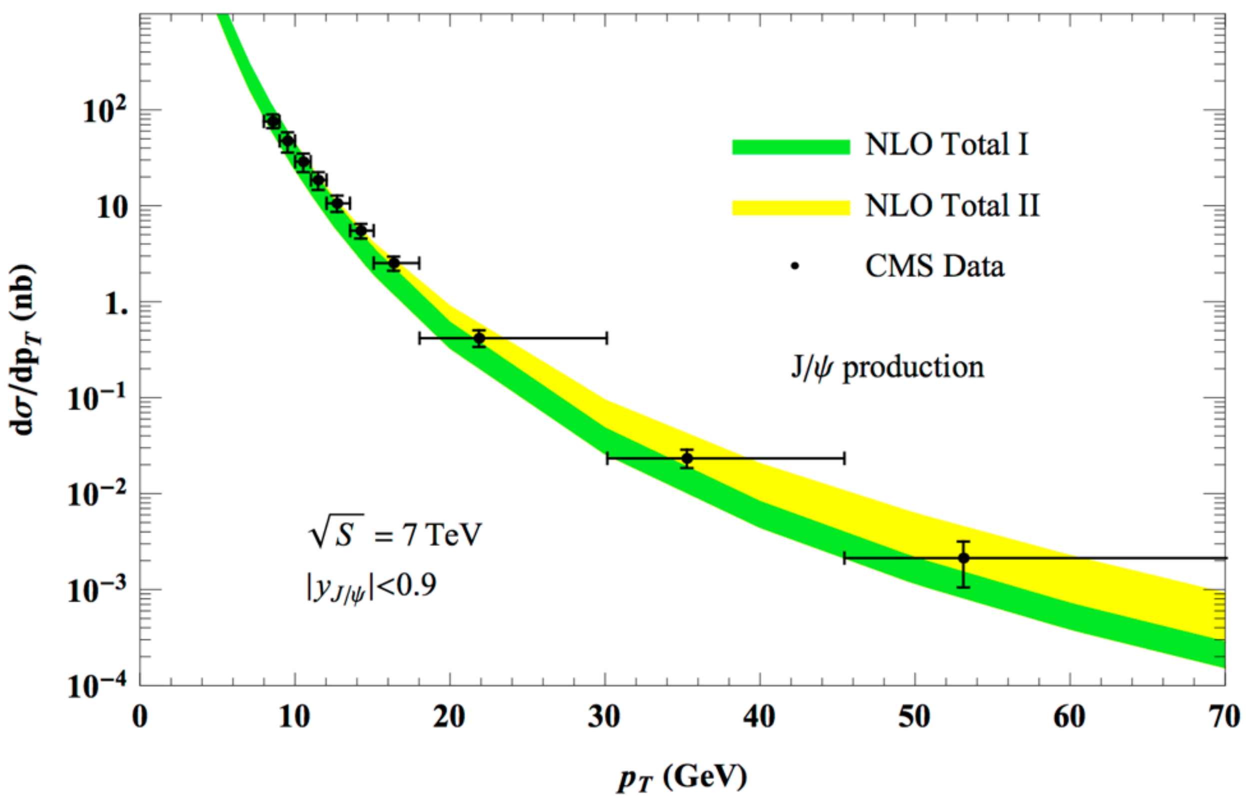
Fit done on 194 data points, 26 data sets

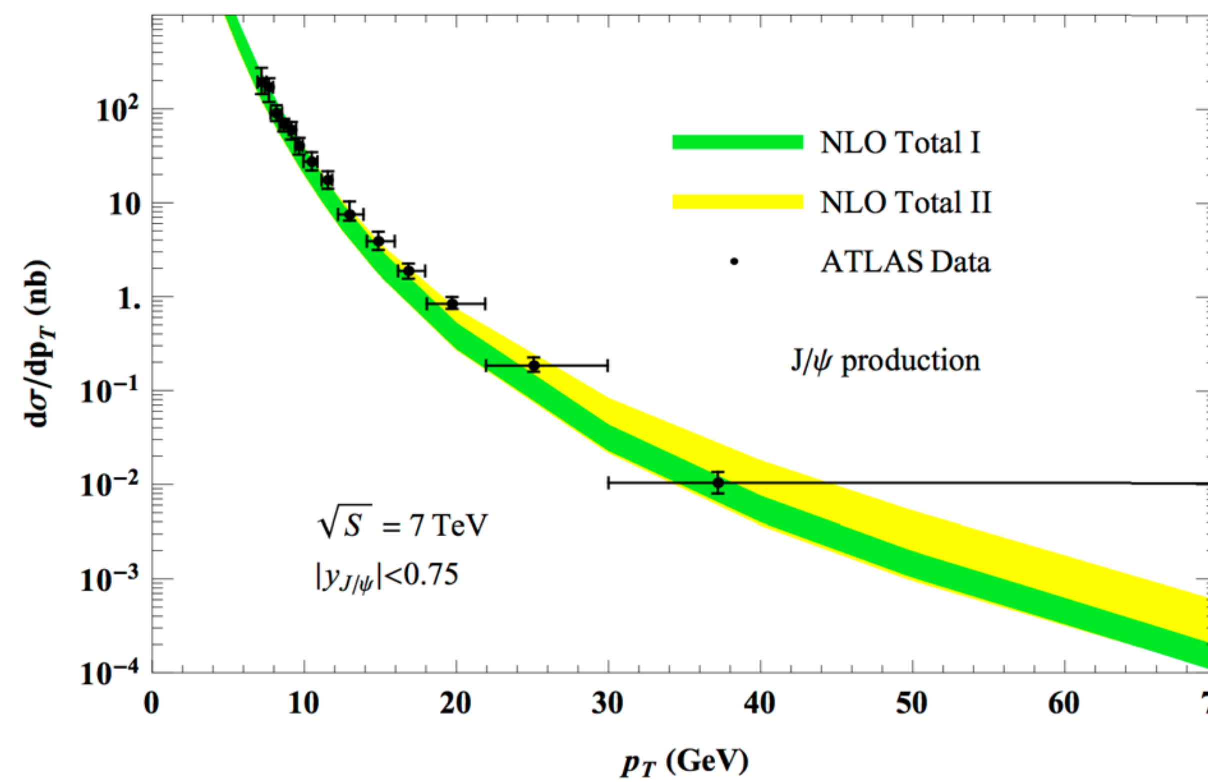


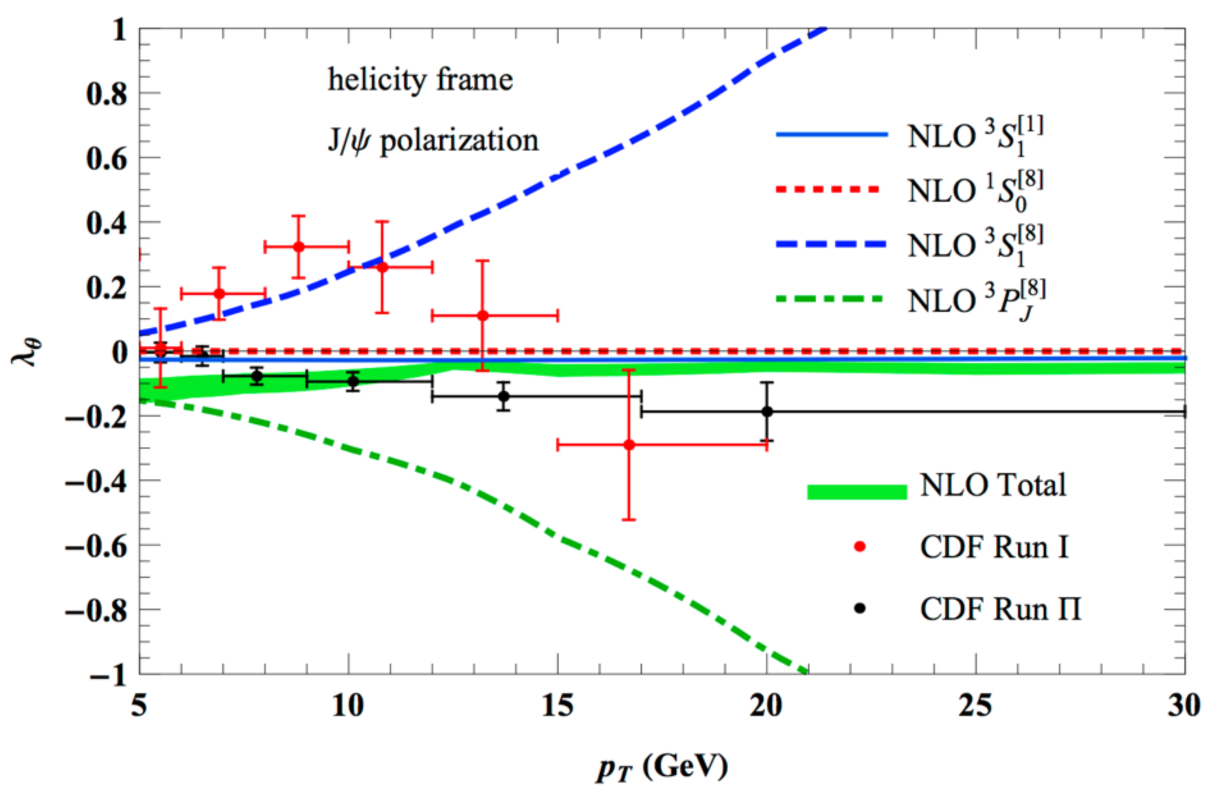
Buttenschon, Kniehl (2011), arXiv:1105.0820

Attempts to Fix Polarization Problem

Simultaneous NLO fit to CMS, ATLAS high p_t production, polarization







Chao, et al. (2012),arXiv:1201.2675

$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2}GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2}GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle/m_c^2$ 10^{-2}GeV^3
1.16	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
1.16	0	1.4	2.4
1.16	11	0	0

Inconsistent with global fits!

Deriving the Cross Section

Measure Hadron z and Jet τ

$$\frac{1}{\sigma_0}\frac{d\sigma^{(i)}}{d\tau_a dz} = H(\mu)S^{unmeas}(\mu)J^{(1)}_{\omega_1}(\mu)\sum_j \left[\left(S^{meas}(\mu)\otimes\frac{\mathcal{J}_{ij}(\mu)}{2(2\pi)^3}\right)(\tau_a)\bullet D_j^H(\mu)\right](z)$$

Convolutions of the form

$$[f\otimes g](\tau_a) \equiv \int d\tau' f(\tau-\tau')g(\tau')$$

$$[f\bullet g](z) \equiv \int_z^1 \frac{dx}{x} f(x)g(z/x)$$

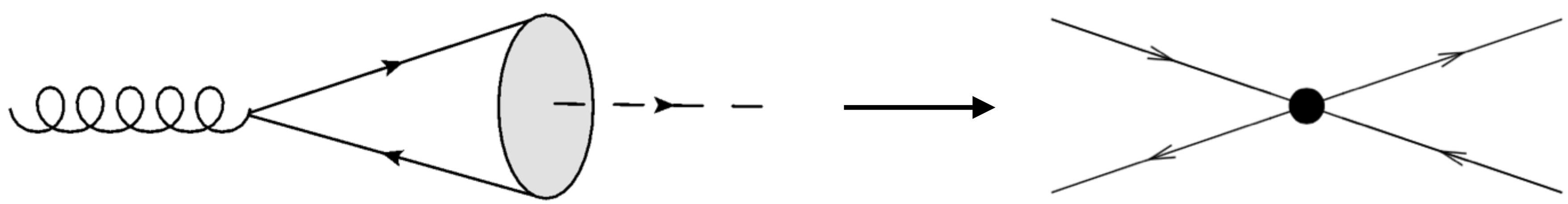
Definitions

$$\bar{\omega}_H = \prod_{i=1}^N \omega_i^{\mathbf{T}_i^2/\mathbf{T}^2}$$

$$\mathbf{T}^2 = \sum_{i=1}^N \mathbf{T}_i^2$$

NRQCD Fragmentation Functions

Matching QCD and NRQCD



Perturbatively Calculable Frag. Functions

$$D_{g\to J/\psi}^{^3S_1^{(8)}}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{24m_c^3}\langle\mathcal{O}^{J/\psi}(^3S_1^{(8)})\rangle\delta(1-z)$$

Braaten, Chen, hep-ph/9610401

Braaten, Chen, hep-ph/9604237

Braaten, Yuan, hep-ph/9302307

Definitions of Operators

QCD Fragmentation Function

$$D_q^h(z) = z \int \frac{dx^+}{4\pi} e^{ik^- x^+/2} \frac{1}{4N_c} \mathrm{Tr} \sum_X \langle 0| \not{\bar{n}} \psi(x^+, 0, 0_\perp) |Xh\rangle \langle Xh| \bar{\psi}(0) |0\rangle \Big|_{p_h^\perp = 0}$$

SCET Fragmentation Function

$$D_q^h(\frac{p_h^-}{\omega}, \mu) = \pi\omega \int dp_h^+ \frac{1}{4N_c} \mathrm{Tr} \sum_X \not{\bar{n}} \langle 0| \delta_{\omega,\bar{\mathcal{P}}} \delta_{0,\mathcal{P}_\perp} \chi_n(0) |Xh\rangle \langle Xh| \bar{\chi}_n(0) |0\rangle$$

SCET Jet Function

$$J(p^\mu) = \frac{1}{8\pi N(\bar{n}\cdot p)} \sum_X \int d^4x e^{ipx} \mathrm{Tr} \left[\langle \Omega| \bar{\chi}_n(x) |X_n\rangle \langle X_n| \not{\bar{n}} \chi_n(0) |\Omega\rangle\right]$$

SCET Fragmenting Jet Function

$$\mathcal{G}_{q,\mathrm{bare}}^h(s, z) = \int \mathrm{d}^4y\, e^{\mathrm{i}k^+y^-/2} \int \mathrm{d}p_h^+ \sum_X \frac{1}{4N_c} \mathrm{tr}\left[\frac{\not{\bar{n}}}{2} \langle 0|[\delta_{\omega,\overline{\mathcal{P}}} \delta_{0,\mathcal{P}_\perp} \chi_n(y)]|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]$$