

# NNLL resummation for transverse thrust

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in collaboration with

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# Event Shapes

- measure geometrical properties of energy flow
- mostly used in lepton collisions
- precise extraction of  $\alpha_s$

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- mostly used in lepton collisions
- precise extraction of  $\alpha_s$
- definitions for hadron collision exists as well
- use only momenta transverse to beam
- useful to study jet substructure and underlying event
- NLL resummation is available in automated framework CAESAR

[Banfi, Salam, Zanderighi]

# Transverse Thrust

defined in analogy to thrust:

$$T_{\perp} = \max_{\vec{n}_{\perp}} \frac{\sum |\vec{p}_{m\perp} \cdot \vec{n}_{\perp}|}{\sum |\vec{p}_{m\perp}|} \quad \tau_{\perp} = 1 - T_{\perp}$$

Goal: resum singular terms in dijet limit  $\tau_{\perp} \rightarrow 0$  at N<sup>2</sup>LL using SCET

# Transverse Thrust

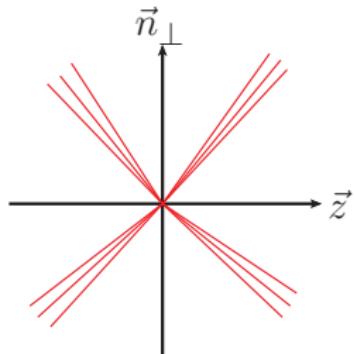
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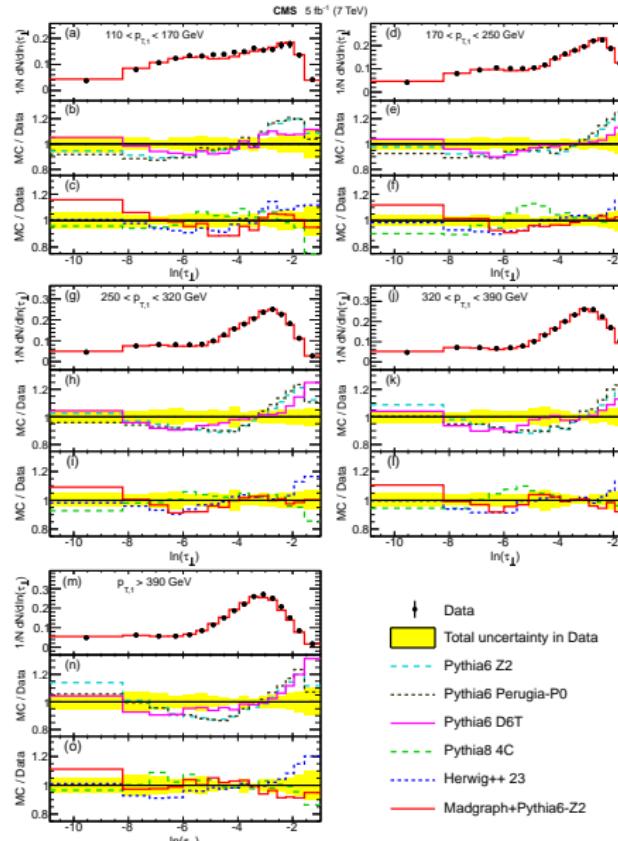
Goal: resum singular terms in dijet limit  $\tau_{\perp} \rightarrow 0$  at N<sup>2</sup>LL using SCET

$\tau_{\perp} \rightarrow 0$  limit also contains non-singular configurations with all particles in the same plane

$\rightsquigarrow$  larger corrections when matching to fixed order



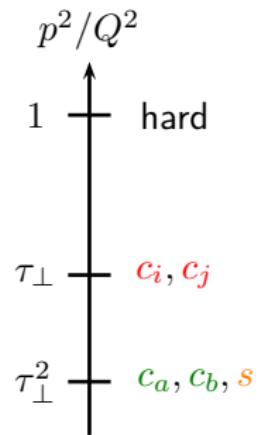
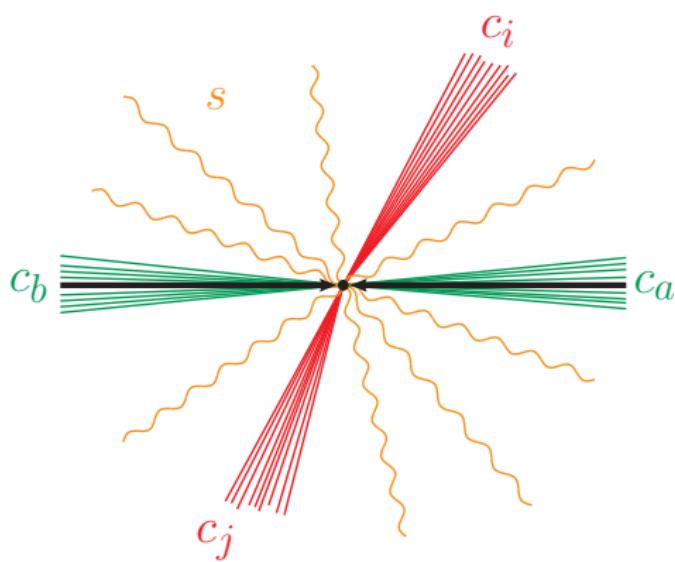
# Transverse Thrust at LHC



[CMS]

# Momentum Modes

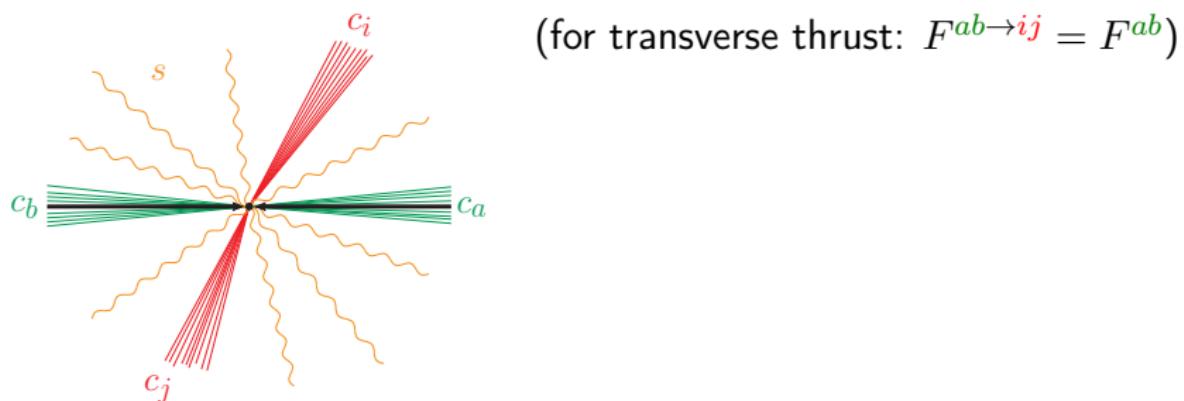
consider transverse thrust in  $pp \rightarrow 2\text{jets}$ :



# Factorisation Formula

partonic cross section  $ab \rightarrow ij$  for recoil-free transverse event shape:

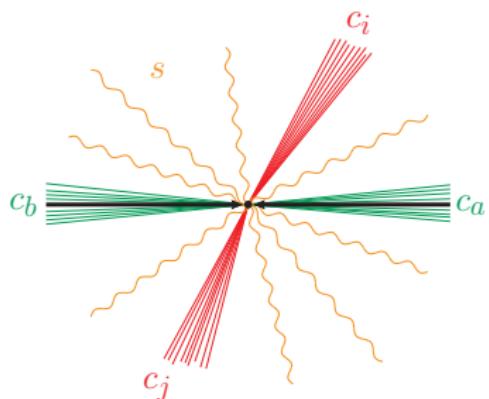
$$\tilde{t}(\kappa) \sim H_{IJ}^{ab \rightarrow ij} \left( \frac{Q^2}{\kappa^2} \right)^{-F^{ab \rightarrow ij}(\kappa)} \tilde{S}_{JI}^{ab \rightarrow ij}(\kappa) \tilde{B}_a(\kappa) \tilde{B}_b(\kappa) \tilde{J}_i(\kappa) \tilde{J}_j(\kappa)$$



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(for transverse thrust:  $F^{ab \rightarrow ij} = F^{ab}$ )

needed for N<sup>2</sup>LL:

- 1-loop hard, soft, jet, and beam function
- 3-loop cusp anomalous dimension
- 2-loop anomalous dimensions
- 2-loop anomaly exponent

# RG and Factorisation Constraints

$$\tilde{t}(\kappa) \sim H_{IJ}^{ab \rightarrow ij} \left( \frac{Q^2}{\kappa^2} \right)^{-F^{ab \rightarrow ij}(\kappa)} \tilde{S}_{JI}^{ab \rightarrow ij}(\kappa) \tilde{B}_a(\kappa) \tilde{B}_b(\kappa) \tilde{J}_i(\kappa) \tilde{J}_j(\kappa)$$

Factorisation requires:

$$F^{ab \rightarrow ij} = F^{ab} + F^{ij} = \frac{C_a + C_b}{2} F_\perp + \frac{C_i + C_j}{2} F'_\perp$$

RG invariance requires:

$$\gamma_{H^{ab \rightarrow ij}} + \gamma_{S^{ab \rightarrow ij}} + \gamma_{B_a} + \gamma_{B_b} + \gamma_{J_i} + \gamma_{J_j} = 0$$

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$$e^+ e^- \rightarrow ij : \quad \gamma_{H^{ij}} + \gamma_{S^{ij}} + \gamma_{J_i} + \gamma_{J_j} = 0$$

$$ab \rightarrow e^+ e^- : \quad \gamma_{H^{ab}} + \gamma_{S^{ab}} + \gamma_{B_a} + \gamma_{B_b} = 0$$

# Anomalous Dimension of Beam Function

consider  $q\bar{q} \rightarrow e^+e^-$ :

$$\gamma_{H^{q\bar{q}}} = -\gamma_{S^{q\bar{q}}} - \gamma_{B_q} - \gamma_{B_{\bar{q}}}$$

- hard anomalous dimension is known:  $\gamma_{H^{q\bar{q}}} = 2\gamma^q$
- soft function is scaleless:  $\gamma_{S^{q\bar{q}}} = 0$

$$\rightsquigarrow \gamma_{B_q} = \gamma^q$$

consider  $gg \rightarrow H \rightarrow \gamma\gamma$  to obtain  $\gamma_{B_g} = \gamma^g$

# Anomalous Dimensions of Soft and Jet Function

consider  $e^+e^- \rightarrow q\bar{q}$  and  $e^+e^- \rightarrow gg$ :

$$\gamma_{H^{ij}} = -\gamma_{S^{ij}} - \gamma_{J_i} - \gamma_{J_j}$$

for  $F^{ij} = 0$ :

- $\gamma_{H^{q\bar{q}}} = 2\gamma^q$ ,  $\gamma_{H^{gg}} = 2\gamma^g$
- $\rightsquigarrow \gamma_{J_q} = -\gamma^q - \frac{1}{2}\gamma_{S^{q\bar{q}}}$ ,  $\gamma_{J_g} = -\gamma^g - \frac{1}{2}\gamma_{S^{gg}}$
- extract  $\gamma_{S^{q\bar{q}}}$  from fixed order result, e.g. EVENT2 [Catani, Seymour]
- $\gamma_{S^{gg}}$  is related to  $\gamma_{S^{q\bar{q}}}$  by Casimir scaling (at two loops)
- $\gamma_{S^{ab \rightarrow ij}}$  for other channels is determined by RG invariance

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- compute  $\gamma_{S^{q\bar{q}}}$ , see yesterday's talk by Rudi Rahn
- $\gamma_{S^{gg}}$  is related to  $\gamma_{S^{q\bar{q}}}$  by Casimir scaling (at two loops)
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- define observable with known anomaly that
  - ▶ agrees with  $\tau_{\perp}$  for one emission
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$\tau_{\perp}$  in  $q\bar{q} \rightarrow e^+e^-$ :

$$\begin{aligned}\mathcal{T}_{\perp} &= \left| \vec{p}_{m\perp} \right| - \left| \vec{p}_{m\perp} \cdot \vec{n}_{\perp} \right| \\ \mathcal{S}_{\perp} &= \left| \sum \vec{p}_{m\perp} \right| - \left| \sum \vec{p}_{m\perp} \cdot \vec{n}_{\perp} \right|\end{aligned}$$

- compute  $\mathcal{S}_{\perp}$  from known Drell-Yan results [Becher, Neubert]
- $d_2^{\text{DY}} - d_2^{\perp}$  is determined by rapidity divergences of soft function

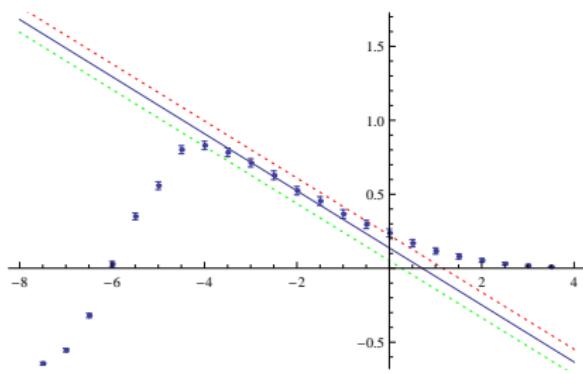
# Anomaly Coefficient

$$F_{\perp}^{q\bar{q}}(L_{\perp}) = \frac{\alpha_s}{4\pi} C_F \Gamma_0 L_{\perp} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_F \left( \Gamma_0 \beta_0 \frac{L_{\perp}^2}{2} + \Gamma_1 L_{\perp} + d_2^{\perp} \right)$$

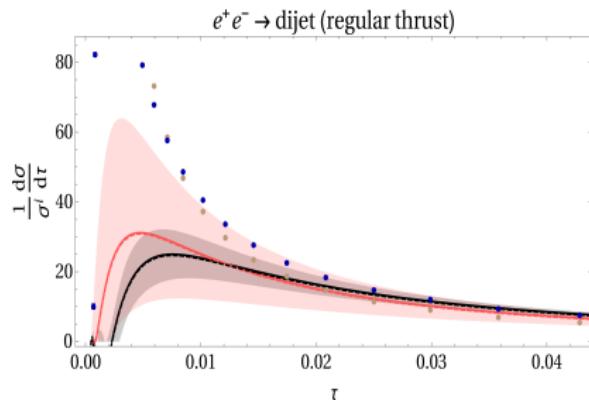
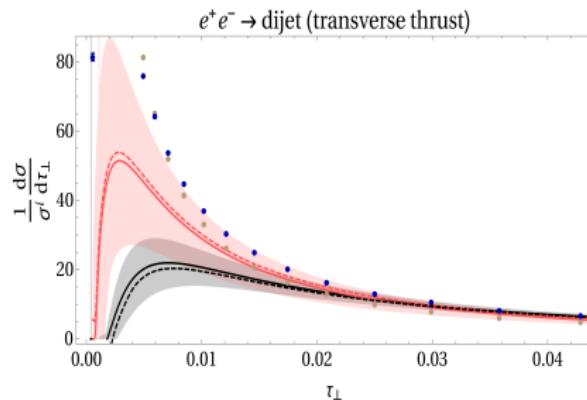
from numerical evaluation of tree-level two-emission soft amplitude:

$$d_2^{\perp} = (182.3 \pm 0.1) C_A + (-51.881 \pm 0.006) T_F n_f$$

compare with DYNNLO [Grazzini] as cross-check:



# Results: Transverse vs. Regular Thrust



NLL

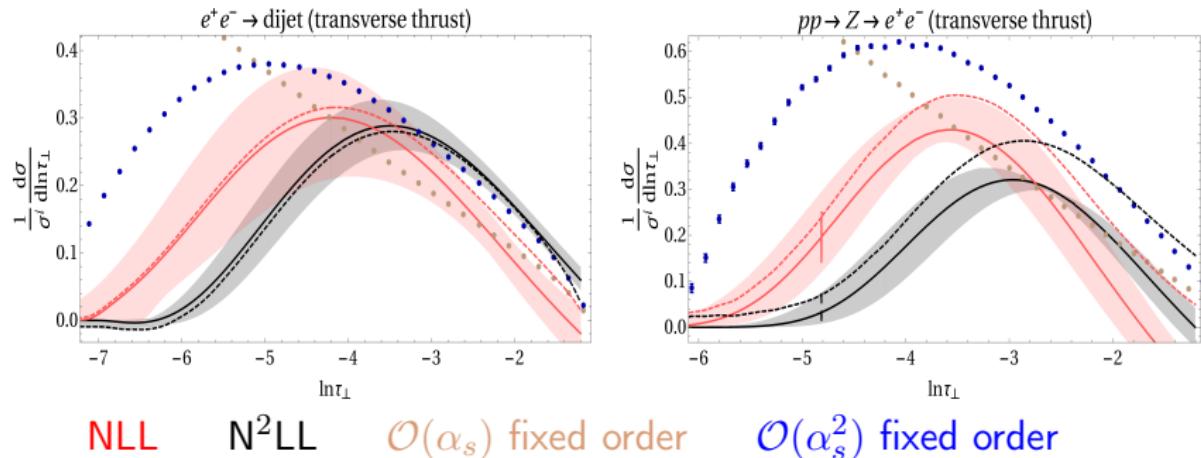
$N^2LL$

$\mathcal{O}(\alpha_s)$  fixed order

$\mathcal{O}(\alpha_s^2)$  fixed order

$$\mu_{\text{soft}} = 4M_Z\tau_{\perp}, \mu_{\text{jet}} = 2M_Z\sqrt{\tau_{\perp}}$$

# Results: Transverse Thrust



$$\mu_{\text{soft}} = 4M_Z\tau_\perp, \mu_{\text{jet}} = 2M_Z\sqrt{\tau_\perp}, \mu_{\text{beam}} = 2e^{4G/\pi}M_Z\tau_\perp$$

MSTW 2008 NNLO PDFs,  $\alpha_S(M_Z) = 0.11707$

- we determined all ingredients for NNLL resummation of transverse thrust by exploiting universality properties of factorised cross section
- we find large corrections at NNLL
- transverse thrust in  $pp \rightarrow e^+e^-$  could be used as a probe of underlying event
- the method can be applied to other observables
- combination with automated computation of soft, jet, and beam functions would allow for an automated resummation framework