

# Uncertainties in the atmospheric lepton fluxes

**Felipe C. Penha**

Hans Dembinski, Thomas K. Gaisser

The Bartol Research Institute

Newark - DE

July 14, 2016

## **Support**

CAPES (Processo BEX 5348/14-5)

CNPq (Processo 142180/2012-2)

FAEPEX (Solicitacao 2490/16)

Bartol Research Institute.

# Outline

- Introduction
- Part I - Atmospheric fluxes
  - Primary cosmic ray spectrum
  - Cascade equations
  - Generalization ( $\mu$  and  $\nu_\mu$ )
- Part II - Uncertainties
  - Hadronic
  - Hadronic & Primary

Overlaps w/ **Prof. Thomas Lohse** lecture on Astroparticle Physics.

# Introduction

- Our focus: semi-analytical calculations for conventional muonic leptons:  $\mu$  and  $\nu_\mu$

## Conventional component

- Main contribution, up to 1 PeV:

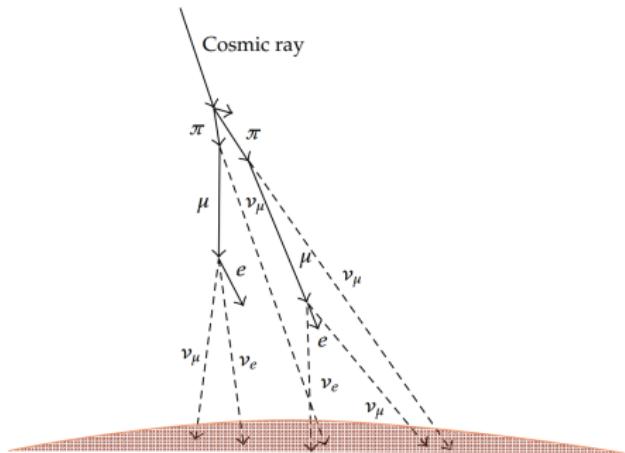
$$\begin{aligned}\pi^+ &\rightarrow \mu^+ + \nu_\mu & 100\% \\ K^+ &\rightarrow \mu^+ + \nu_\mu & 63.5\%\end{aligned}$$

## Prompt component

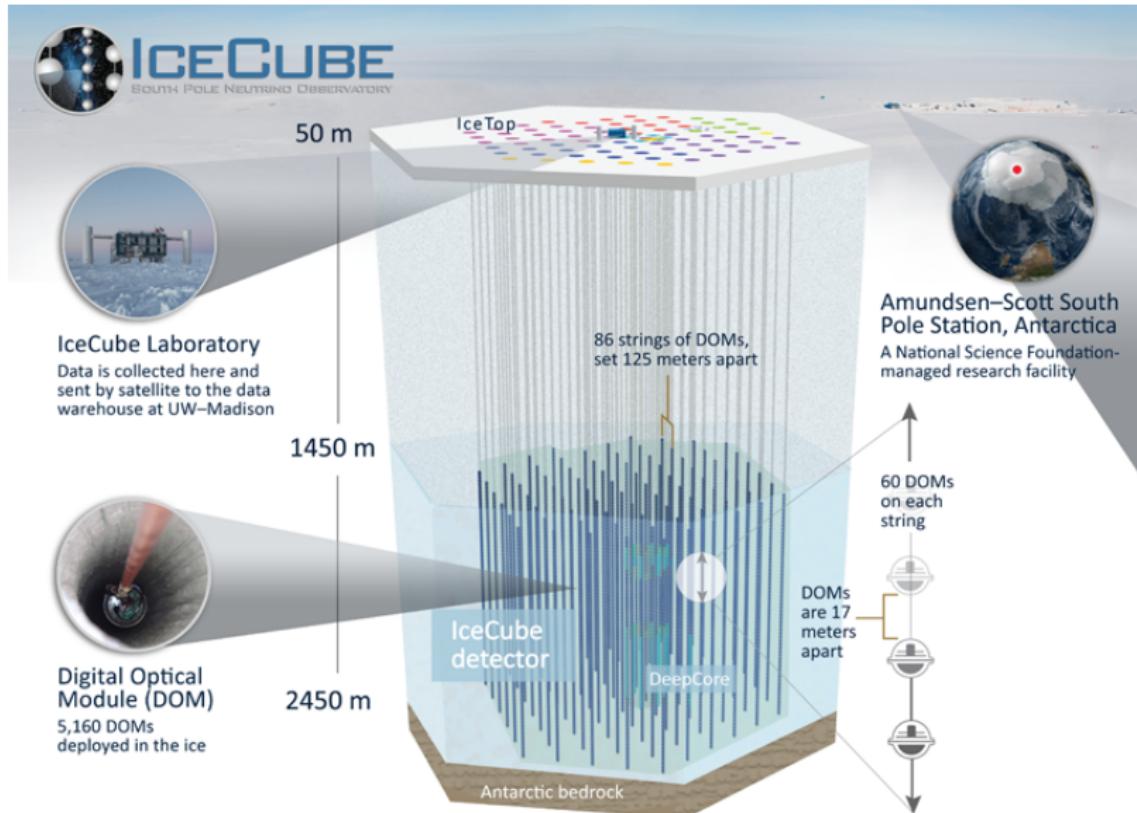
- Contribution from hadrons with  $c\tau^0 < 1 \text{ cm}$

$$\begin{aligned}D^+ &\rightarrow \bar{K}^0 + \mu^+ + \nu_\mu & 9.2\% \\ D^0 &\rightarrow \bar{K}^- + \mu^+ + \nu_\mu & 3.3\%\end{aligned}$$

⋮



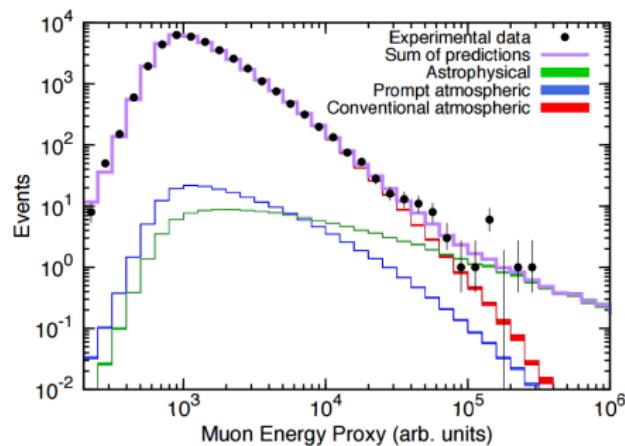
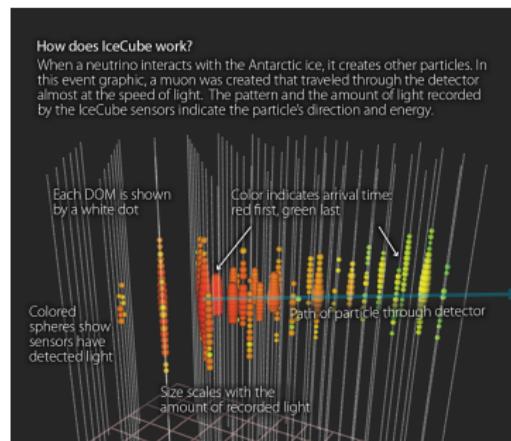
# Introduction



# Introduction

Atm.  $\nu_\mu$  fluxes above  $\sim 100$  TeV are

- background to the astrophysical signal at IceCube
- sensitive to CR primary spectrum features



[IceCube collab. Phys.Rev.Lett. 115 \(2015\) no.8, 081102](#)

# Introduction

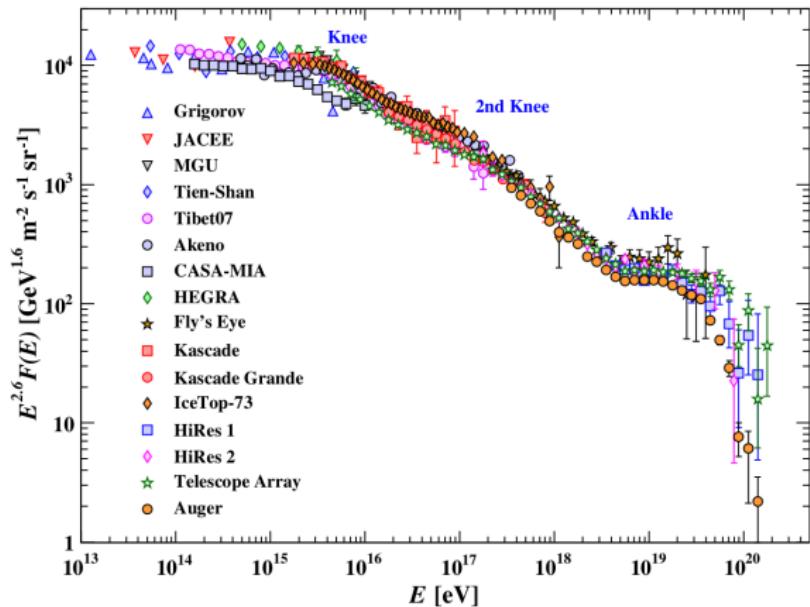
## Objectives:

- Understand well the fluxes in terms of the primary cosmic rays
- Understand the following sources of uncertainties:
  - Hadronic data/models
  - Primary cosmic rays data/models

# Part I

## Atmospheric fluxes

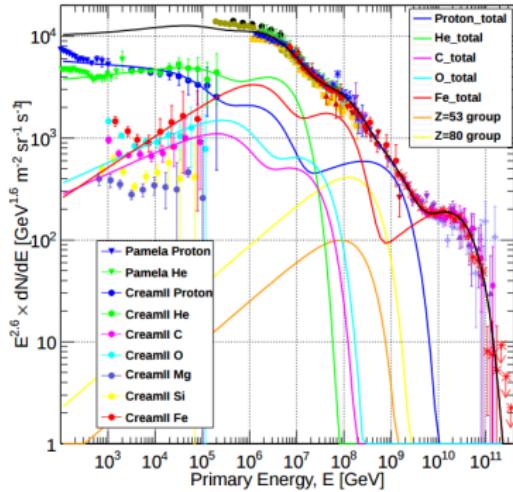
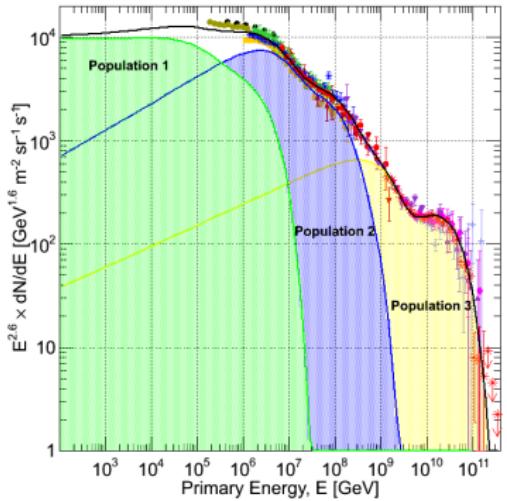
# Primary cosmic ray spectrum



# Primary cosmic ray spectrum

$$\varphi^{(ij)}(E) = a_{ij} E^{-\gamma_{ij}-1} \exp\left(-\frac{E}{Z_i e R_j}\right) \quad i \rightarrow \text{nuclei}, \quad j \rightarrow \text{pop.}$$

Power law Expon.cutoff



## Primary cosmic ray spectrum

- nuclei  $\rightarrow$  nucleons ( $p, n$ )
- Assume independ. nucleons  
in a classical picture

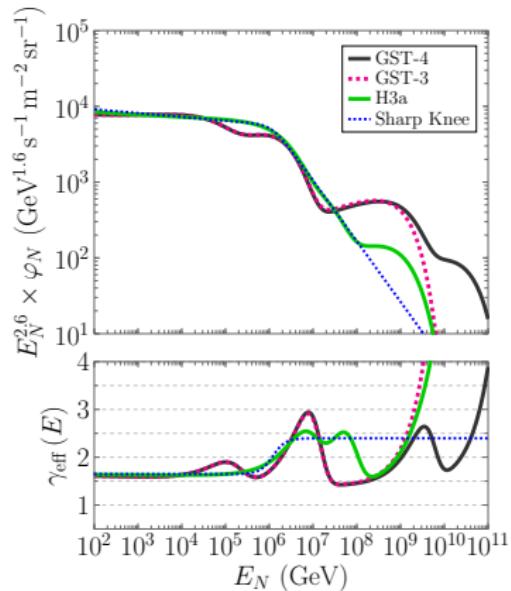
$$\varphi_N^{(ij)} = A_i^2 \varphi^{(ij)}(A_i E)$$

# Primary cosmic ray spectrum

- nuclei → nucleons ( $p, n$ )
- Assume independ. nucleons in a classical picture

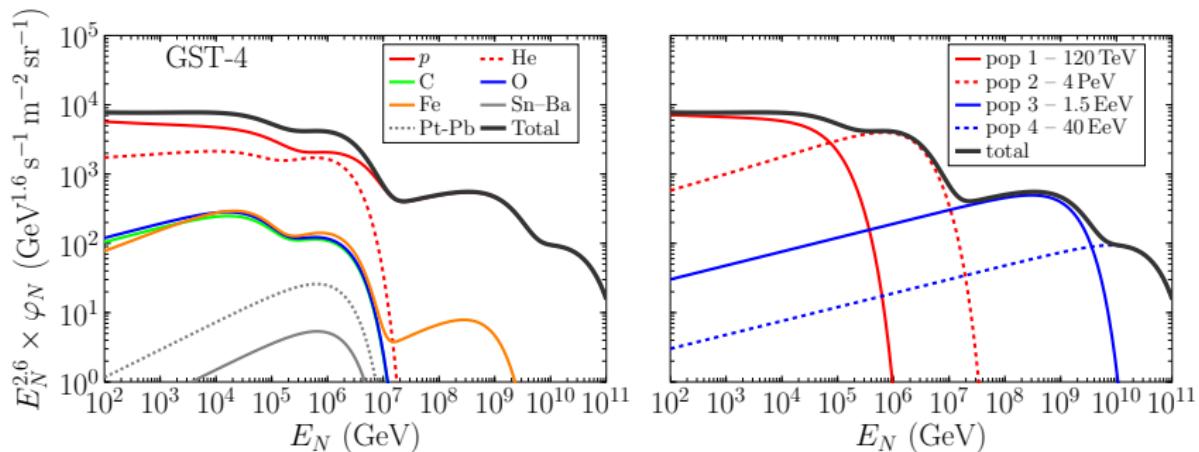
$$\varphi_N^{(ij)} = A_i^2 \varphi^{(ij)}(A_i E)$$

- 3 main models:
  - **H3a:** 3 pop, 5 nuclei
  - **GST-3:** 3 pop, 7 nuclei
  - **GST-4:** 4 pop, 7 nuclei



# Primary cosmic ray spectrum

■ **GST-4** nucleon spectrum, fully decomposed:



# Cascade equations

- General cascade equation:

$$\begin{aligned}\frac{d\Phi_a}{dX}(E, X) &= - \left[ \frac{1}{\lambda_a(E)} + \frac{1}{d_a(E)} \right] \Phi_a(E, X) \\ &\quad + \sum_b \int_E^\infty dE_b \frac{F_{ba}(E, E_b)}{E} \frac{\Phi_b(E_b, X)}{\lambda_b(E_b)} \\ &\quad + \sum_k \Gamma_{ka} \int_{E_{\min}}^{E_{\max}} dE_k \frac{dn_{ka}(E, E_k)}{dE} \frac{\Phi_k(E_k, X)}{d_k(E_k)}\end{aligned}$$

- Slant depth  $[\text{g cm}^{-2}]$ :

$$X(\vec{r}_0, \vec{r}) \equiv \int_r^{r_0} dr' \rho(h')$$

# Cascade equations

- Decay length, assuming isothermal atmosphere:

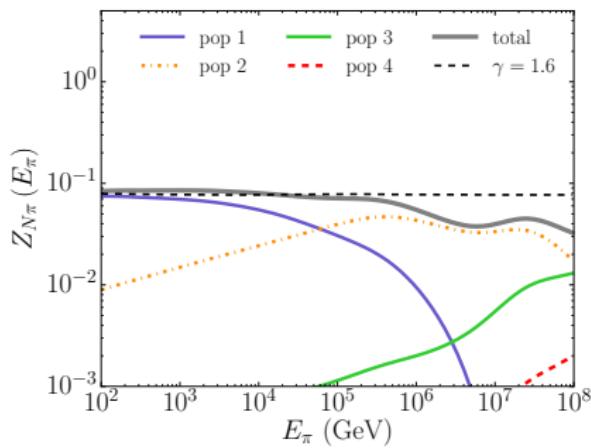
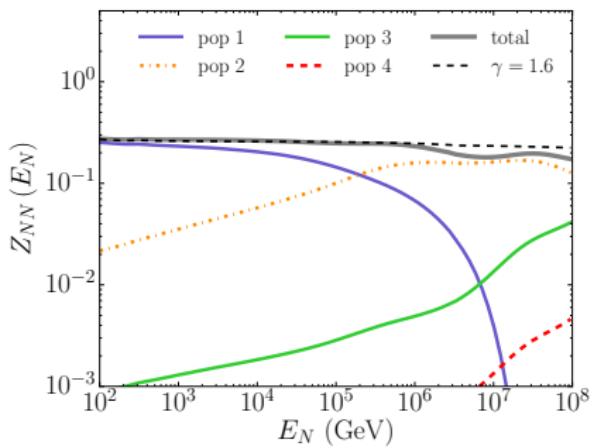
$$\frac{1}{d_a(E)} \approx \frac{\epsilon_a}{EX \cos \theta^*}, \quad \text{with } \epsilon_a \equiv \frac{m_a c h_0}{\tau_a^{(0)}}$$

Particle	Quark content	$mc^2$ (MeV)	$c\tau^{(0)}$ (cm)	$\epsilon$ (GeV)
$\pi$	$u\bar{d}$	139.6	780.5	115
$K$	$u\bar{s}$	493.7	371.2	851
$D$	$c\bar{d}$	$1.870 \times 10^3$	$3.118 \times 10^{-2}$	$3.84 \times 10^7$

# Generalization

- Sibyll2.3 & CRMC for computing the production distr.'s

$$Z_{Ns}^{(ij)}(E) \approx \int_0^1 \frac{dx}{x} \frac{\lambda_N(E)}{\lambda_N\left(\frac{E}{x}\right)} \left[ \frac{\varphi_N^{(ij)}\left(\frac{E}{x}\right)}{\varphi_N^{(ij)}(E)} \right] \frac{dn_{Ns}}{dx}\left(\frac{E}{x}, x\right) \quad Z_{Ns}(E) = \frac{\sum_{ij} \varphi_N^{(ij)}(E) Z_{Ns}^{(ij)}(E)}{\varphi_N(E)}$$



# Conventional muonic leptons

- From 2-body decay kinematics ( $\pi$  and  $K$  decays):

$$\frac{dn_{sl}}{dE_s} \approx \frac{1}{1 - r_s} \frac{1}{E_s}, \quad r_s \equiv \frac{m_l^2}{m_s^2}$$

Thus,

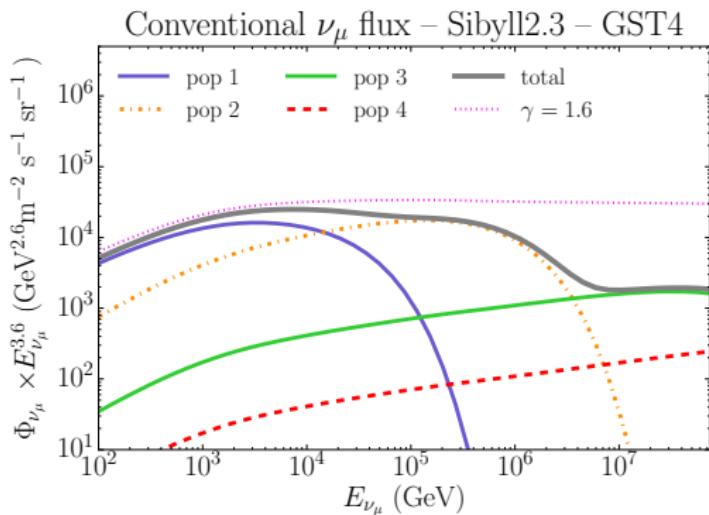
$$\Phi_{sl}^{(ij)}(E, \cos \theta^*) = \int_{+\infty}^0 dX \int_{E_{\min}}^{E_{\max}} dE_s \frac{1}{1 - r_s} \frac{1}{E_s} \frac{\Gamma_{sl} \epsilon_s}{E_s X \cos \theta^*} \Phi_s^{(ij)}(E_s, X)$$

$$\Phi_{sl}^{\text{HE}(ij)}(E, \cos \theta^*) \approx \left[ \frac{\Lambda_s}{\Lambda_s - \Lambda_N} \right] \log \left( \frac{\Lambda_s}{\Lambda_N} \right) \frac{\Gamma_s \epsilon_s}{\cos \theta^*} \varphi_N^{(ij)}(E) \left[ \frac{Z_{Ns}^{(ij)}(E)}{1 - Z_{NN}^{(ij)}(E)} \right] \frac{Z_{sl}^{\text{HE}(ij)}(E)}{E}$$

"primary"    scattering    decay

# Conventional muonic leptons

- Representations of solutions in terms of  
**upper incomplete  $\Gamma$ 's: series & continued fraction**



# Part II

## Uncertainties

## Hadronic uncertainties

- The **conventional muonic lepton fluxes** are functions of

$$Z = \left( Z_{NN}^{(11)}, Z_{N\pi}^{(11)}, Z_{NK}^{(11)}, Z_{NN}^{(12)}, Z_{N\pi}^{(12)}, Z_{NK}^{(12)}, \dots \right).$$

- We assume constant  $Z_{\pi\pi} = 0.271$  and  $Z_{KK} = 0.223$ .

## Hadronic uncertainties

- The conventional muonic lepton fluxes are functions of

$$Z = \left( Z_{NN}^{(11)}, Z_{N\pi}^{(11)}, Z_{NK}^{(11)}, Z_{NN}^{(12)}, Z_{N\pi}^{(12)}, Z_{NK}^{(12)}, \dots \right).$$

- We assume constant  $Z_{\pi\pi} = 0.271$  and  $Z_{KK} = 0.223$ .
- Assume that each hadronic model is a **statistical sample**

Sample Mean:  $\bar{Z}_{Nk}^{(ij)} = \frac{1}{M} \sum_{\alpha=1}^M Z_{Nk}^{(ij\alpha)}, \quad \bar{Z} = \frac{1}{M} \sum_{\alpha=1}^M Z^{(\alpha)}$

# Hadronic uncertainties

- The conventional muonic lepton fluxes are functions of

$$Z = \left( Z_{NN}^{(11)}, Z_{N\pi}^{(11)}, Z_{NK}^{(11)}, Z_{NN}^{(12)}, Z_{N\pi}^{(12)}, Z_{NK}^{(12)}, \dots \right).$$

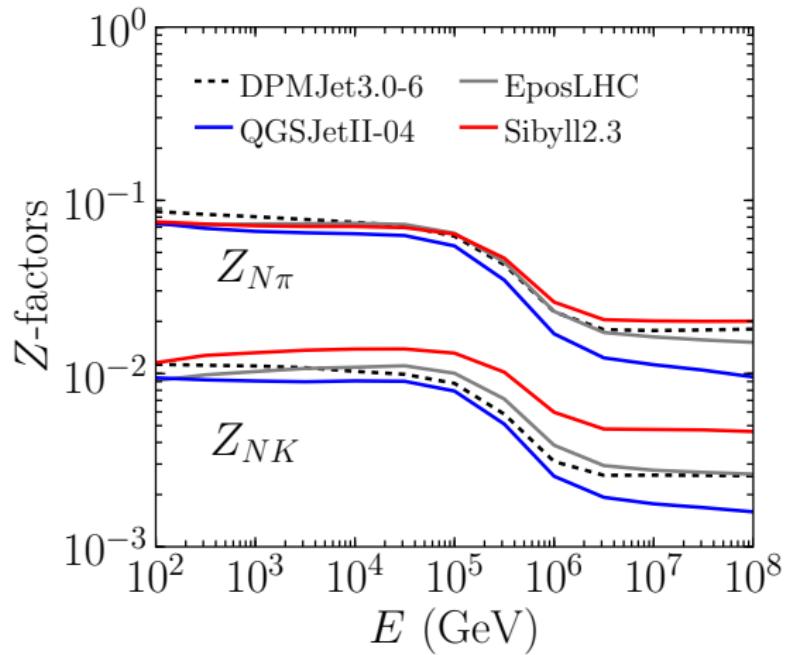
- We assume constant  $Z_{\pi\pi} = 0.271$  and  $Z_{KK} = 0.223$ .
- Assume that each hadronic model is a **statistical sample**

**Sample Mean:**  $\overline{Z}_{Nk}^{(ij)} = \frac{1}{M} \sum_{\alpha=1}^M Z_{Nk}^{(ij\alpha)}, \quad \overline{Z} = \frac{1}{M} \sum_{\alpha=1}^M Z^{(\alpha)}$

**Sample covariance elements:**

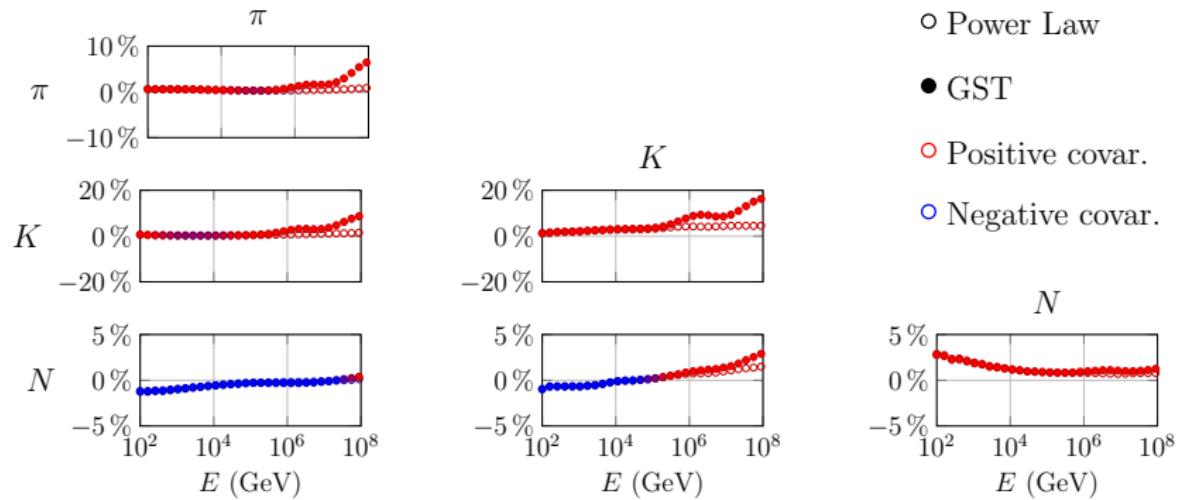
$$\left[ s_{pk}^{(i_1 i_2 j_1 j_2)} \right]^2 \equiv \frac{1}{M-1} \sum_{\alpha=1}^M \left[ Z_{Np}^{(i_1 j_1 \alpha)} - \overline{Z}_{Np}^{(i_1 j_1)} \right] \left[ Z_{Nk}^{(i_2 j_2 \alpha)} - \overline{Z}_{Nk}^{(i_2 j_2)} \right]$$

# Hadronic uncertainties



# Hadronic uncertainties

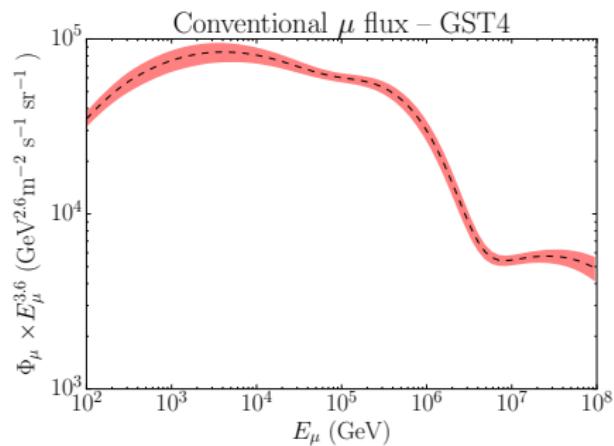
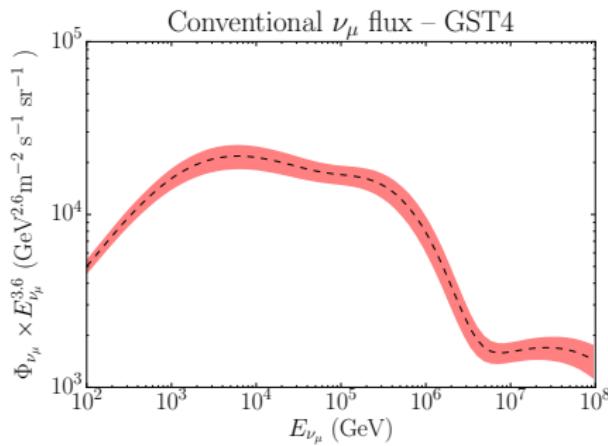
$$s_{pk}^2 \equiv \frac{1}{M-1} \sum_{\alpha=1}^M [Z_{Np} - \bar{Z}_{Np}] [Z_{Nk} - \bar{Z}_{Nk}]$$



# Hadronic uncertainties

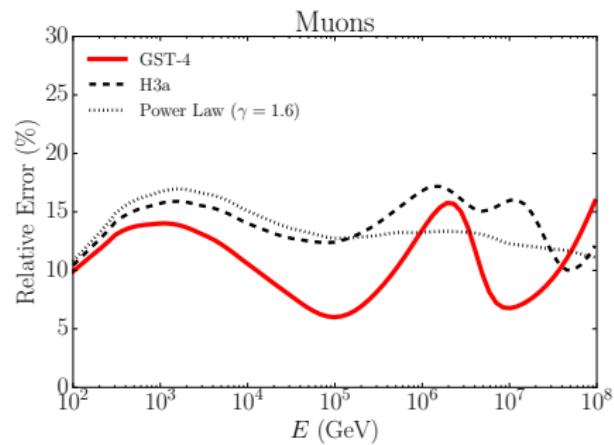
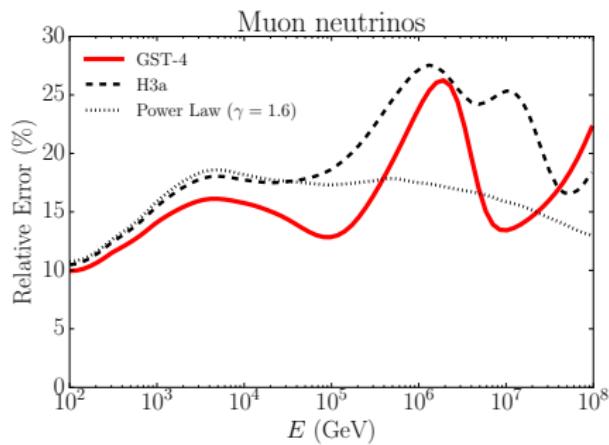
- Error propagation fully decomposed:

$$\sigma^2 = \sum_{i_1, i_2} \sum_{j_1, j_2} \sum_{p, k} \left[ s_{pk}^{(i_1 i_2 j_1 j_2)} \right]^2 q_p^{(i_1 j_1)} q_k^{(i_2 j_2)}, \quad q_k^{(ij)} \equiv \frac{\partial \Phi_I^{(ij)}}{\partial Z_k^{(ij)}} (\bar{Z})$$



# Hadronic uncertainties

## ■ Relative errors



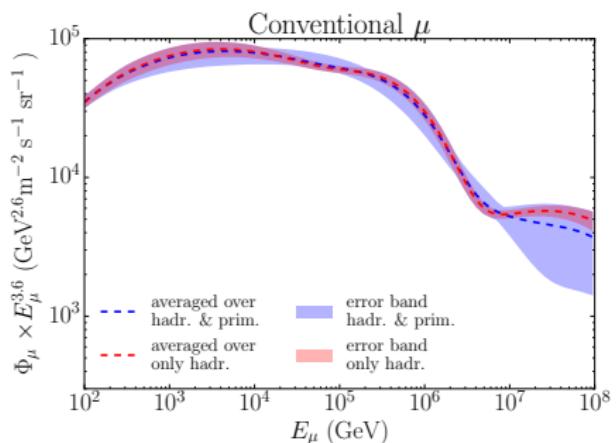
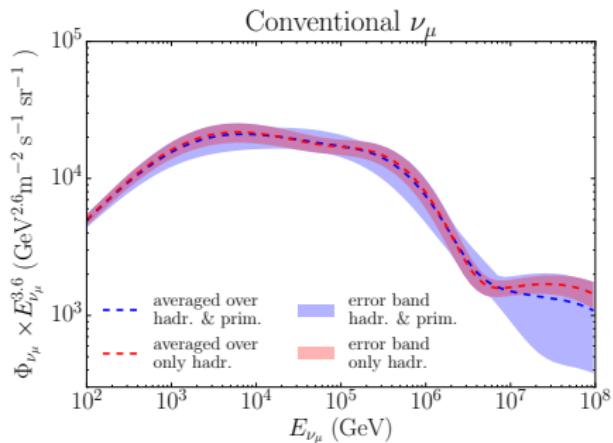
## Hadronic & Primary uncertainties

- We do not have a clear way to vary the parameters (amplitudes, spectral indices, composition, rigidity cutoffs )
- We choose to use H3a, GST-3, and GST-4 to set limits:

$$\Phi^{1\sigma\downarrow} = \min \left[ \Phi_{\text{H3a}}^{1\sigma\downarrow}, \Phi_{\text{GST3}}^{1\sigma\downarrow}, \Phi_{\text{GST4}}^{1\sigma\downarrow} \right], \quad \text{max for } 1\sigma \uparrow$$

$$\langle \Phi \rangle = \frac{1}{3} [\Phi_{\text{H3a}}(\bar{Z}) + \Phi_{\text{GST3}}(\bar{Z}) + \Phi_{\text{GST4}}(\bar{Z})]$$

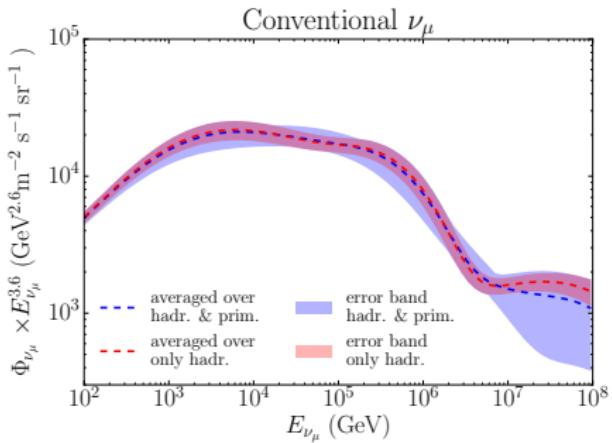
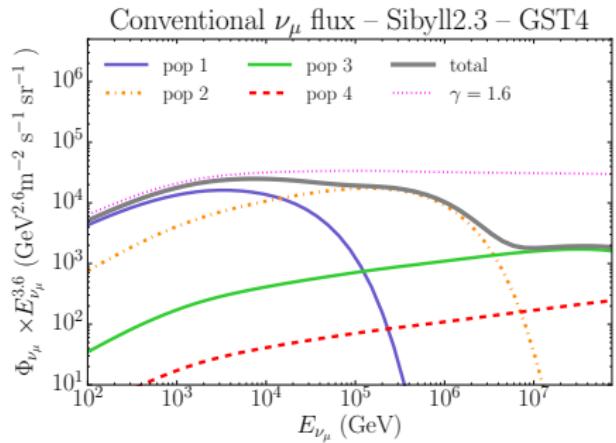
# Hadronic & Primary uncertainties



\*Red curves use GST-4

# Conclusion

- Generalized conventional muonic lepton fluxes
- Use generalized conv fluxes to derive error propagation
- Hadronic & Primary uncertainties



# Acknowledgements

## **Support:**

- CAPES (Processo BEX 5348/14-5)
- CNPq (Processo 142180/2012-2)
- FAEPEX (Solicitacao 2490/16)
- Bartol Research Institute at the University of Delaware

## **I thank for useful discussions:**

- Orlando Luis Goulart Peres
- Todor Stanev, Thomas Gaisser, Serap Tilav, David Seckel
- Mary Hall Reno
- Ralph Engel, Felix Rienh – provided Sibyll2.3 files





