HIGGS COUPLINGS : TWO – LOOP CORRECTIONS AND DECOUPLING PROPERTIES

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I Introduction

II $\phi^0 b\bar{b}$ Couplings

III $\phi^0 gg$ Couplings

IV Summary

in collaboration with D. Noth, M. Mühlleitner, H. Rzehak

I INTRODUCTION

MSSM

- 2 Higgs doubletts $\xrightarrow{\text{ESB}}$ 5 Higgs bosons: $h, H, A, H^\pm$

- LO: 2 input parameters: $M_A, \text{tg}\beta = \frac{v_2}{v_1}$

- radiative corrections $\propto m_t^4 \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \quad \to \boxed{M_h \lesssim 135 \text{ GeV}}$

Haber
Carena,. . .
Heinemeyer,. . .
Zhang
Slavich,. . .
Harlander,. . .
. . .

- Yukawa couplings: $\text{tg}\beta\uparrow \quad \Rightarrow \quad g_u^\phi\downarrow \quad g_d^\phi\uparrow \quad g_V^\phi\downarrow$

- LHC: $gg \to \phi$ dominant for $\text{tg}\beta \lesssim 10$
  $gg \to \phi b\bar{b}$ dominant for $\text{tg}\beta \gtrsim 10$

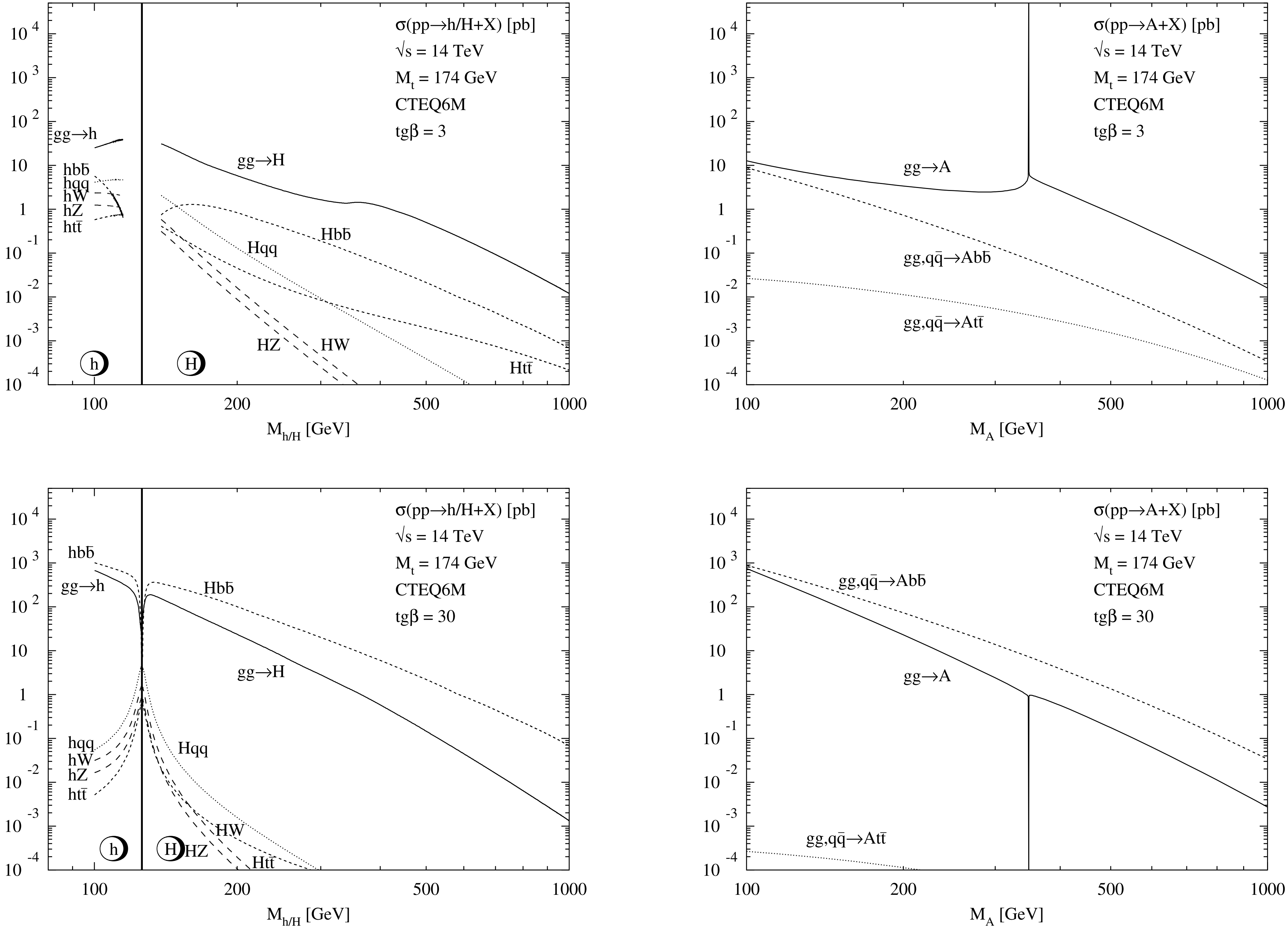
F

$h \to \gamma\gamma, b\bar{b}$
$H, A \to \tau^+\tau^-, \mu^+\mu^-$
$H^\pm \to \tau\nu_\tau$
and $VV \to h, H \to \tau^+\tau^-$

Kauer, . . .

- $h \to b\bar{b}$ in SUSY production

Paige, . . .


σ(pp→h/H+X) [pb]
√s = 14 TeV
M_t = 174 GeV
CTEQ6M
tgβ = 3
gg→h
hbb̄
hqq
hW
hZ
htt̄
gg→H
Hbb̄
Hqq
HZ
HW
Htt̄
h
H
M_h/H [GeV]
σ(pp→A+X) [pb]
√s = 14 TeV
M_t = 174 GeV
CTEQ6M
tgβ = 3
gg→A
gg,qq̄→Abb̄
gg,qq̄→Att̄
M_A [GeV]
σ(pp→h/H+X) [pb]
√s = 14 TeV
M_t = 174 GeV
CTEQ6M
tgβ = 30
hbb̄
gg→h
Hbb̄
gg→H
hqq
hW
hZ
htt̄
Hqq
HW
HZ
Htt̄
h
H
M_h/H [GeV]
σ(pp→A+X) [pb]
√s = 14 TeV
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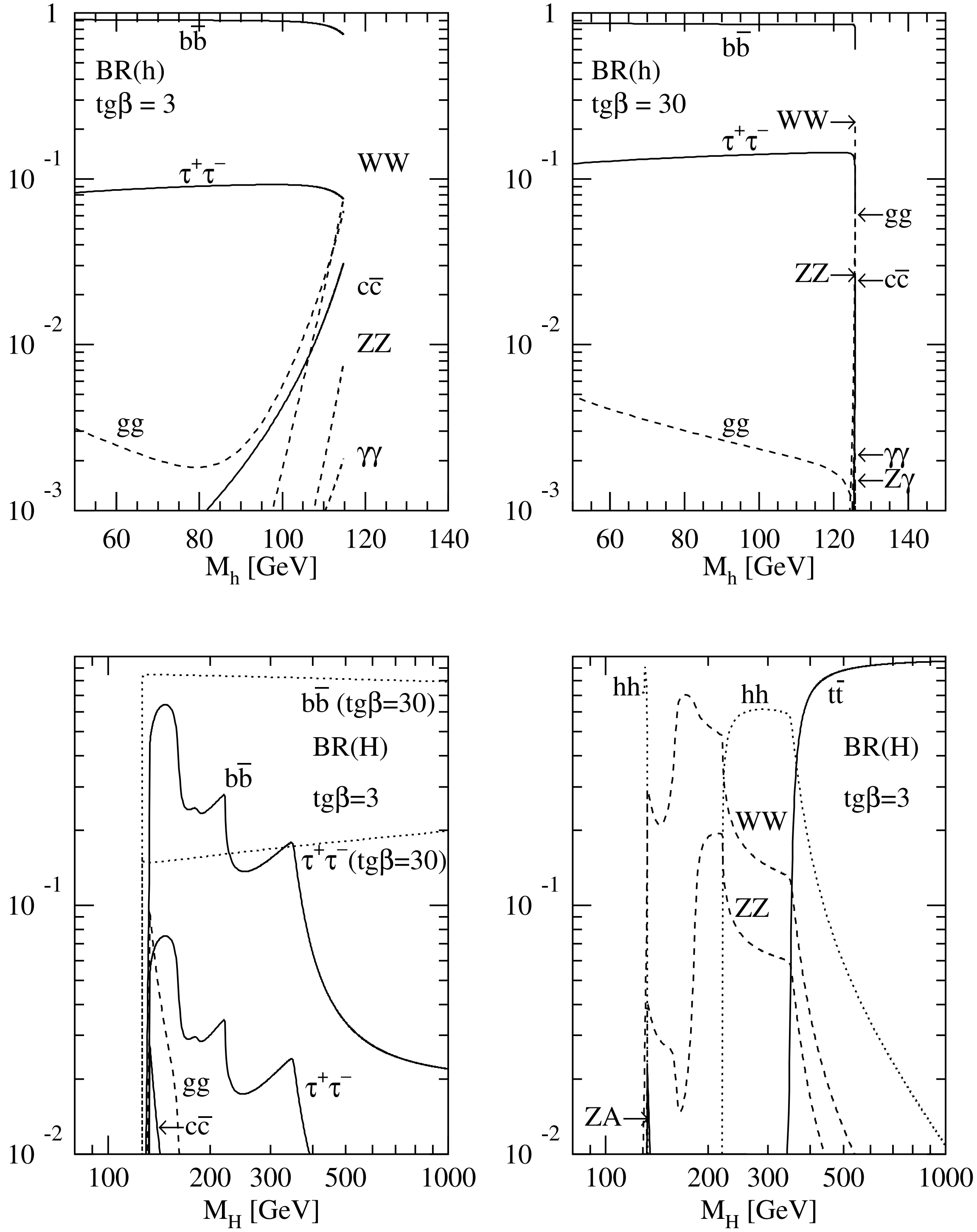
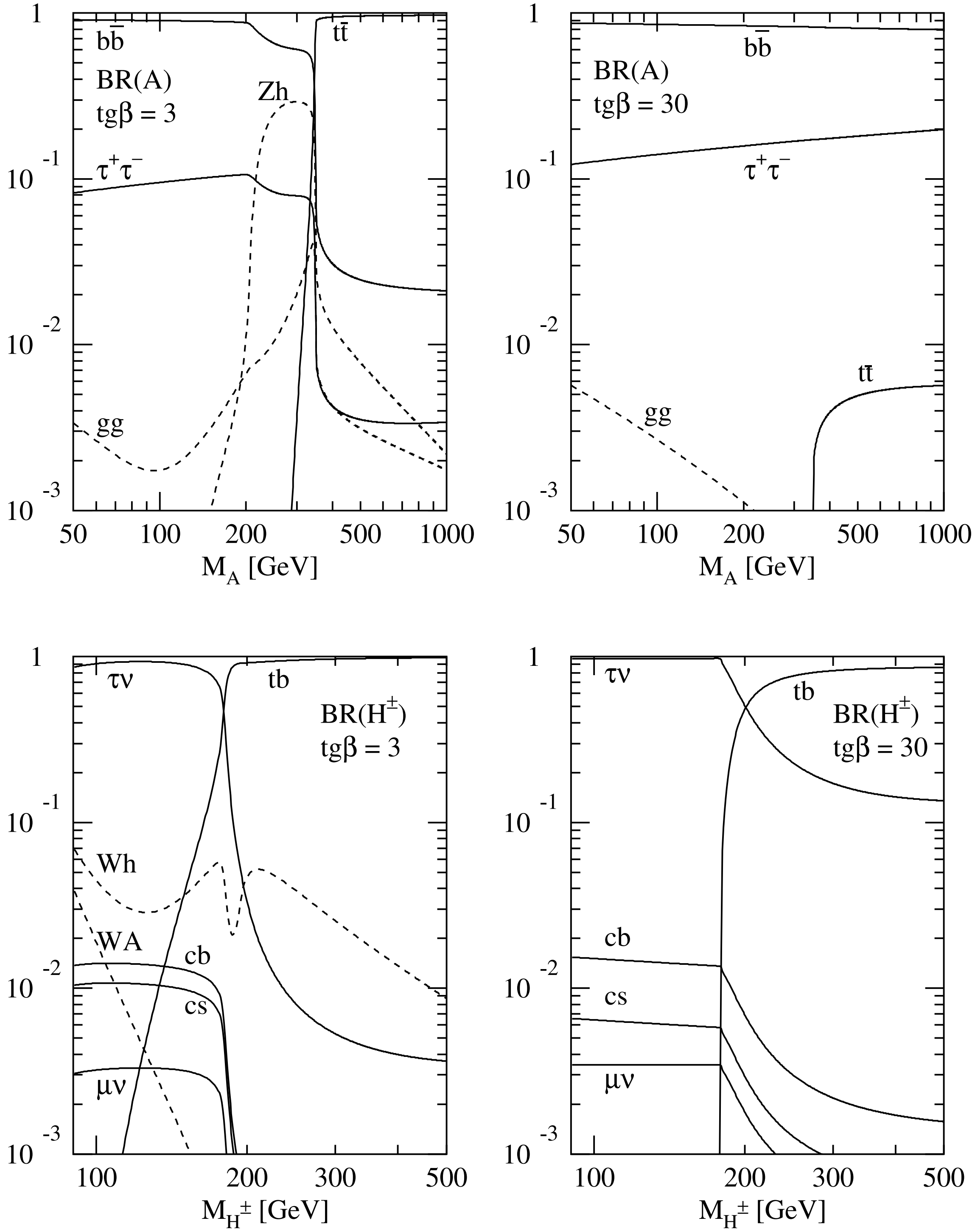
CTEQ6M
tgβ = 30
gg,qq̄→Abb̄
gg→A
gg,qq̄→Att̄
M_A [GeV]


BR(h)
tgβ = 3
bb̄
τ+τ−
WW
cc̄
ZZ
gg
γγ
BR(h)
tgβ = 30
WW→
←gg
ZZ→
←cc̄
←γγ
←Zγ
M_h [GeV]
bb̄ (tgβ=30)
BR(H)
tgβ=3
τ+τ− (tgβ=30)
gg
←cc̄
hh
tt̄
ZZ
ZA→
M_H [GeV]

HDECAY



BR(A)
tgβ = 3
bb̄
Zh
tt̄
τ+τ−
gg
BR(A)
tgβ = 30
MA [GeV]
BR(H±)
tgβ = 3
τν
tb
Wh
WA
cb
cs
μν
BR(H±)
tgβ = 30
MH± [GeV]

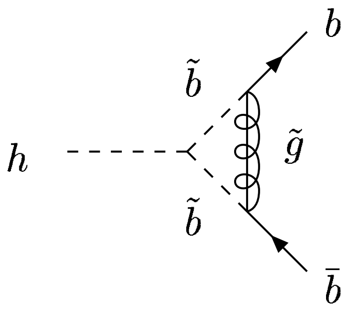
HDECAY

II $\phi^0 b\bar{b}$ COUPLINGS

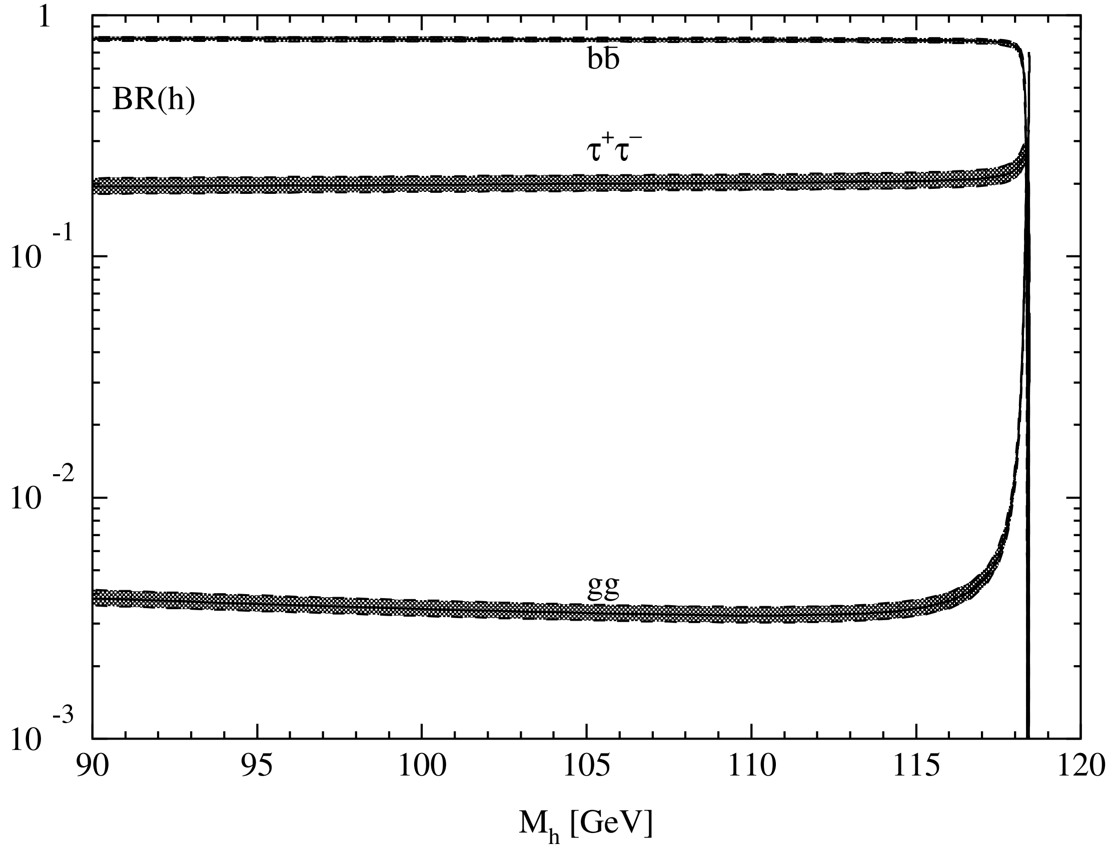
Braaten, Leveille
Drees, Hikasa
Kataev, . . .
Chetyrkin, . . .
etc.

- QCD corrections to $\phi^0 \to b\bar{b}$ known to NNLO
- large SUSY–QCD corrections to $\phi^0 \to b\bar{b}$ $(\Delta\Gamma/\Gamma \sim 10\%)$

Hall, . . .
Carena, . . .
Nierste, . . .
Guasch, . . .
etc.



$$+ \cdots \qquad \propto \frac{\alpha_s}{\pi} \frac{M_{\tilde{g}} \mu \, \mathrm{tg}\beta}{m_{\tilde{b}}^2}$$



Guasch, Häfliger, S.

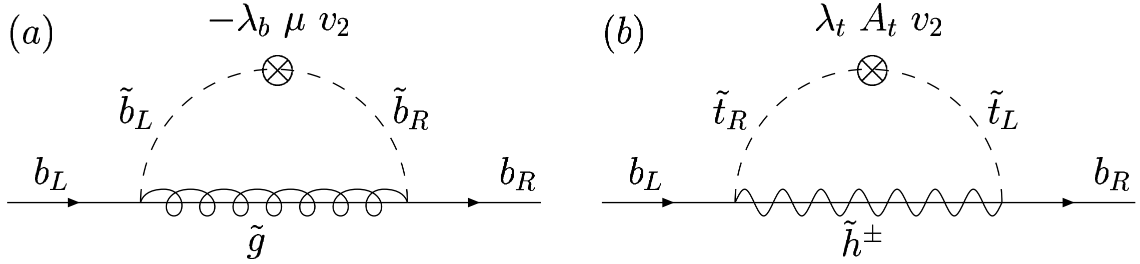
SUSY-QCD Corrections to $b\bar{b}\phi^0$ $[\Delta \lesssim 1\%]$

$$
\begin{aligned}
\mathcal{L}_{eff} &= -\lambda_b \overline{b_R} \left[\phi_1^0 + \frac{\Delta m_b}{\mathrm{tg}\beta} \phi_2^{0*} \right] b_L + h.c. \quad \text{valid to all orders in } \Delta m_b \\
&= -m_b \bar{b} \left[1 + i\gamma_5 \frac{G^0}{v} \right] b - \frac{m_b/v}{1+\Delta m_b} \bar{b} \left[g_b^h \left(1 - \frac{\Delta m_b}{\mathrm{tg}\alpha\ \mathrm{tg}\beta} \right) h \right. \\
&\qquad \left. + g_b^H \left(1 + \Delta m_b \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} \right) H - g_b^A \left(1 - \frac{\Delta m_b}{\mathrm{tg}^2\beta} \right) i\gamma_5 A \right] b
\end{aligned}
$$

$$
\begin{aligned}
\Delta m_b &= \Delta m_b^{QCD(1)} + \Delta m_b^{elw(1)} \\
\Delta m_b^{QCD(1)} &= \frac{2}{3} \frac{\alpha_s(\mu_R)}{\pi} M_{\tilde{g}}\ \mu\ \mathrm{tg}\beta\ I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, M^2_{\tilde{g}}) \\
\Delta m_b^{elw(1)} &= \frac{\lambda_t^2(\mu_R)}{(4\pi)^2}\ \mu\ A_t\ \mathrm{tg}\beta\ I(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}, \mu^2) \\
I(a,b,c) &= -\frac{ab \log\frac{a}{b} + bc \log\frac{b}{c} + ca \log\frac{c}{a}}{(a-b)(b-c)(c-a)}
\end{aligned}
$$

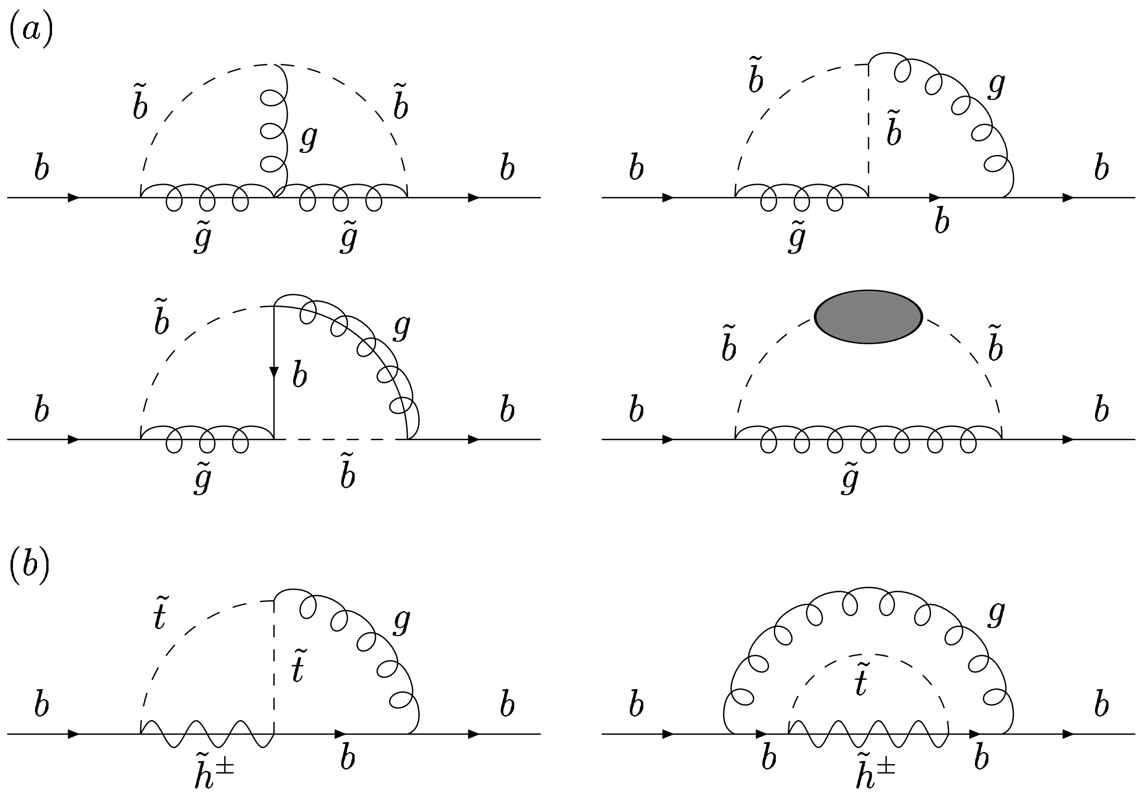
$\Rightarrow$ resummed Yukawa couplings

Carena, Garcia, Nierste, Wagner
Guasch, Häfliger, S.



- LET: $v_2 \to \sqrt{2}\phi_2^{0*}$

Ellis,. . .
Shifman,. . .



- 2-loop self-energies @ vanishing momentum

- dimensional regularization in $n = 4 - 2\epsilon$ dimensions

- integration by parts: reduction to 1-point functions

$$A_0(m) = \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m^2}$$

and one 2-loop master integral

$$T_{134}(m_1, m_3, m_4) = \int \frac{d^n k}{(2\pi)^n} \frac{d^n q}{(2\pi)^n} \frac{1}{(k^2 - m_1^2)[(k-q)^2 - m_3^2](q^2 - m4^2)}$$

- α_s, λ_t: $\overline{\text{MS}}$ scheme [5 flavours]
  masses, A_t: on-shell

- dim. reg. violates SUSY: anomalous counter terms

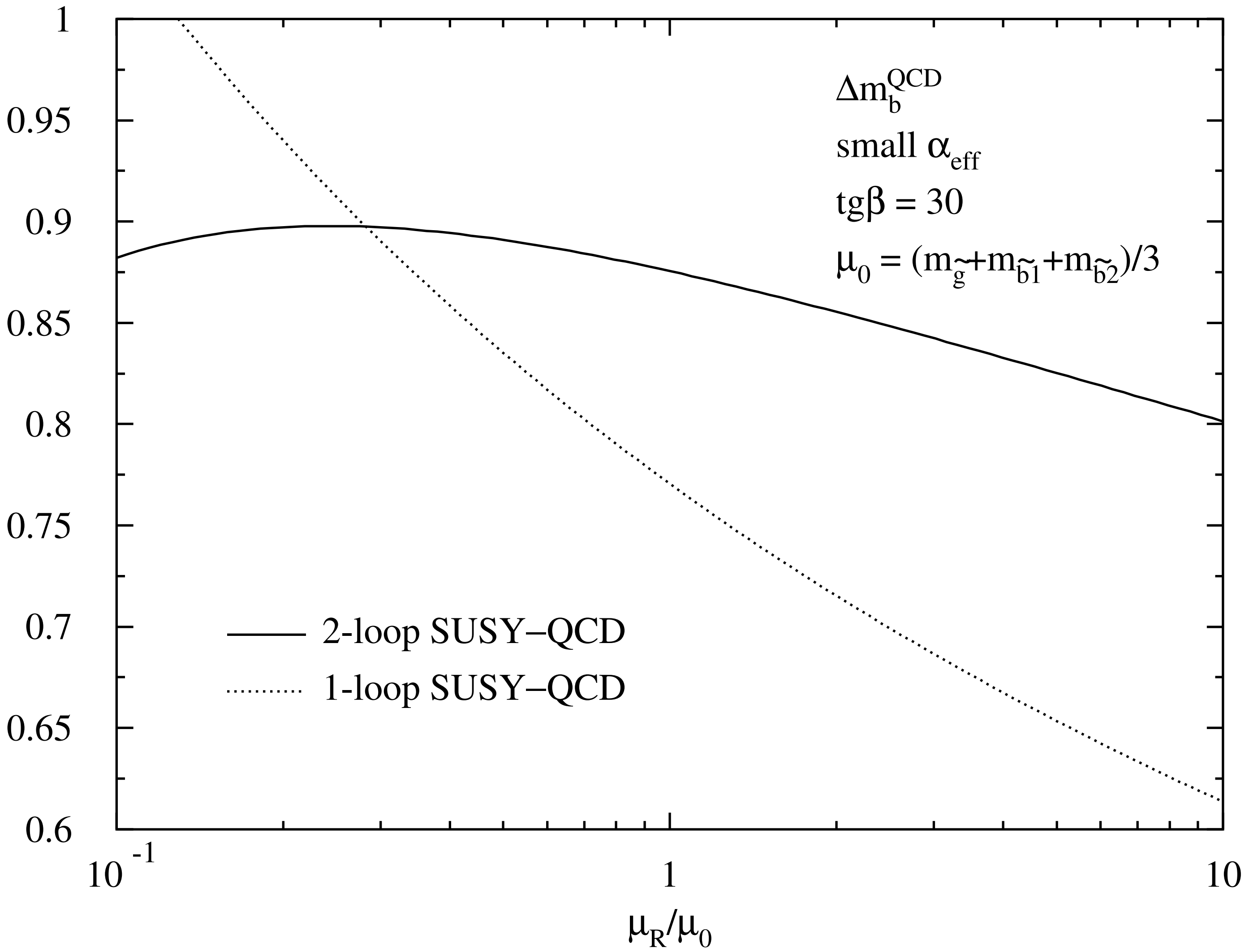
$$\begin{aligned}
\widehat{g}_s &= g_s \left[1 + \left(\frac{C_A}{6} - \frac{C_F}{8}\right) \frac{\alpha_s}{\pi}\right] \\
\lambda_{Hbb} &= \lambda_{H\tilde{b}\tilde{b}} \left[1 + \frac{C_F}{4} \frac{\alpha_s}{\pi}\right] = \lambda_{\tilde{H}\tilde{b}b} \left[1 + \frac{3}{8} C_F \frac{\alpha_s}{\pi}\right]
\end{aligned}$$

Martin, Vaughn

small α_{eff} scenario

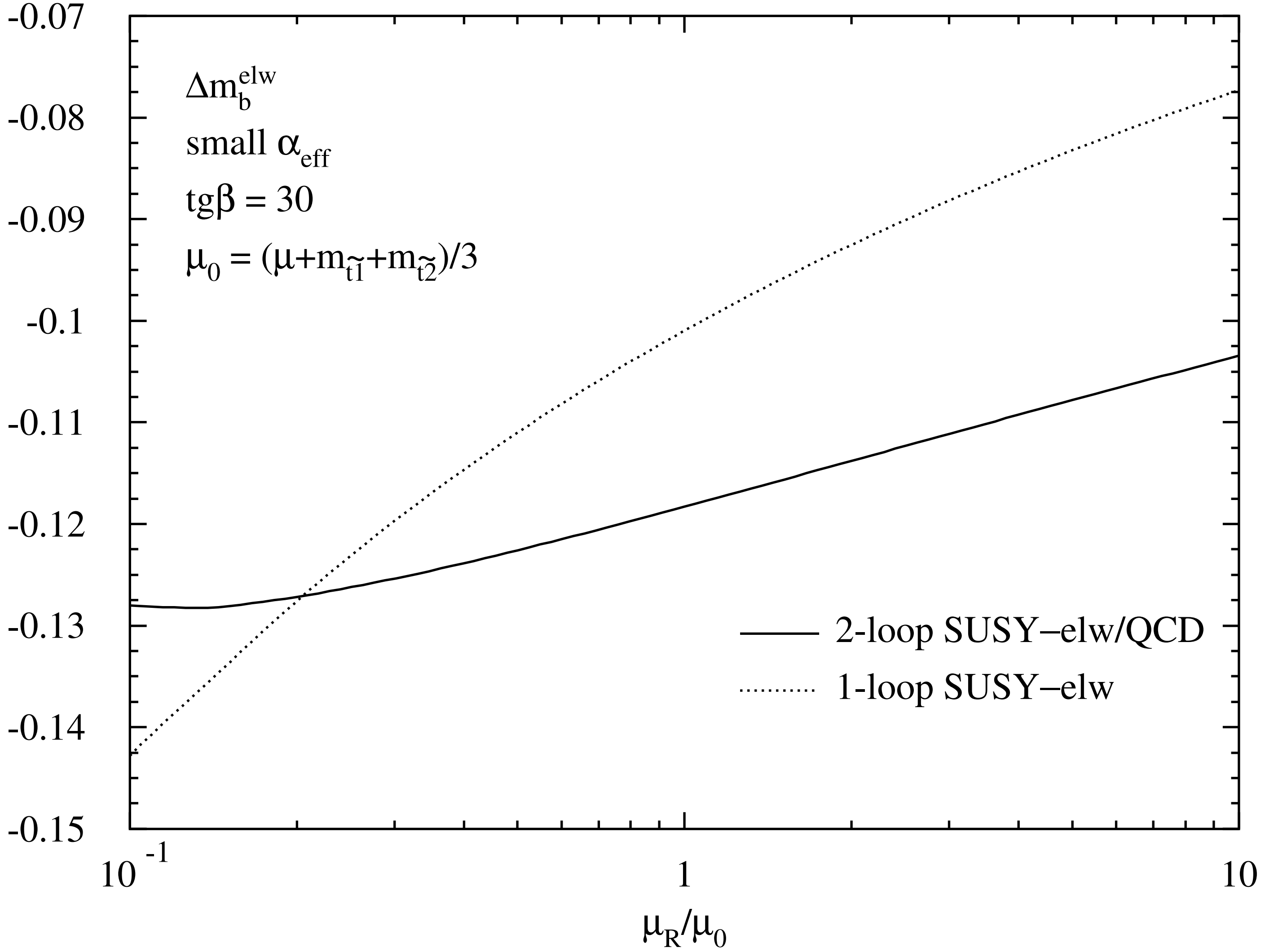
$$
\begin{aligned}
\mathrm{tg}\beta &= 30 \\
M_{\tilde{Q}} &= 800 \text{ GeV} \\
M_{\tilde{g}} &= 500 \text{ GeV} \\
M_2 &= 500 \text{ GeV} \\
A_b = A_t &= -1.133 \text{ TeV} \\
\mu &= 2 \text{ TeV}
\end{aligned}
$$

$$
\begin{aligned}
m_{\tilde{t}_1} &= 679 \text{ GeV} \qquad & m_{\tilde{t}_2} &= 935 \text{ GeV} \\
m_{\tilde{b}_1} &= 601 \text{ GeV} \qquad & m_{\tilde{b}_2} &= 961 \text{ GeV}
\end{aligned}
$$



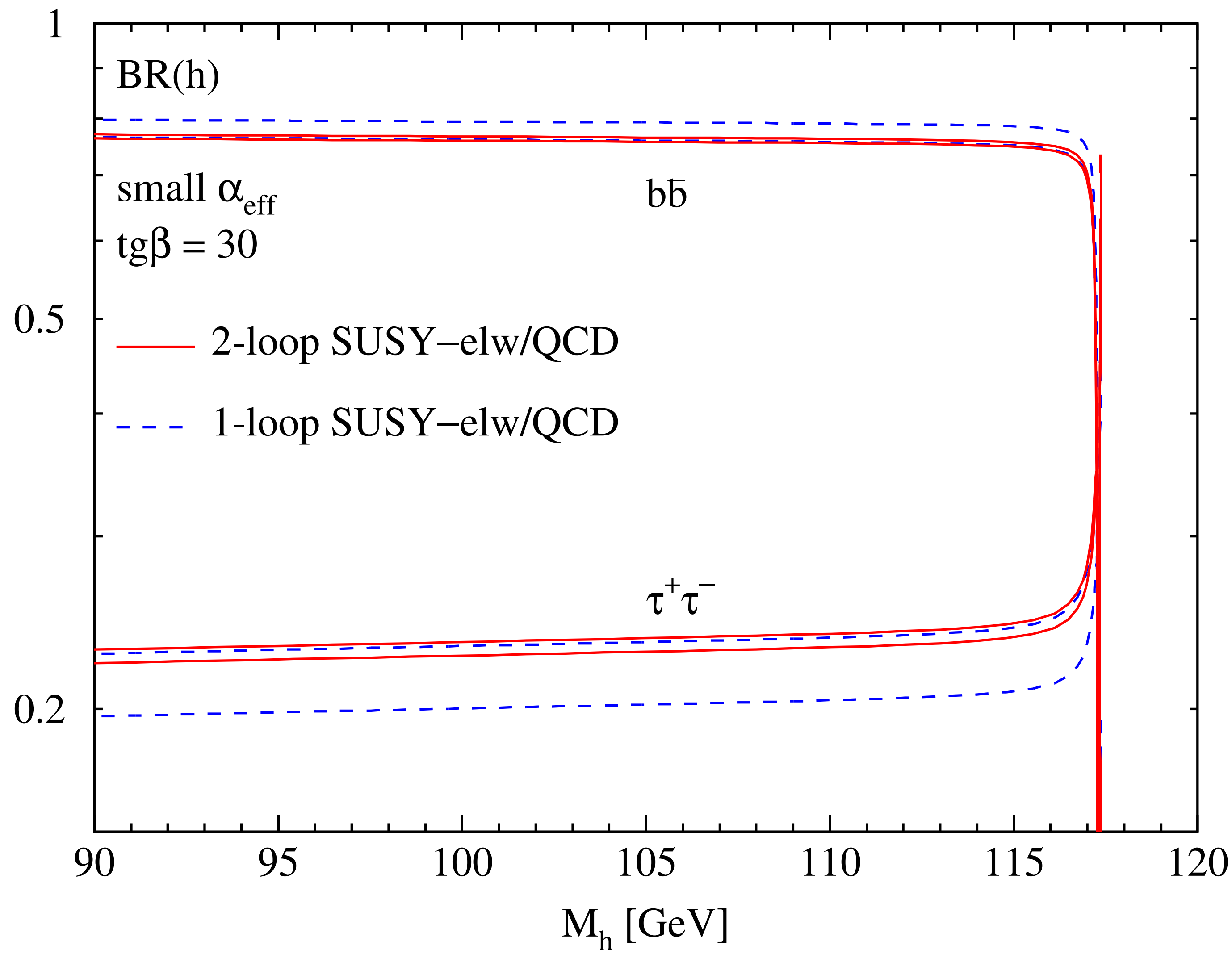
Δm_b^{QCD}
small α_{eff}
$tg\beta = 30$
$\mu_0 = (m_{\tilde{g}}+m_{\tilde{b}1}+m_{\tilde{b}2})/3$
2-loop SUSY−QCD
1-loop SUSY−QCD
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
10^{-1}
1
10
μ_R/μ_0

Noth, S.



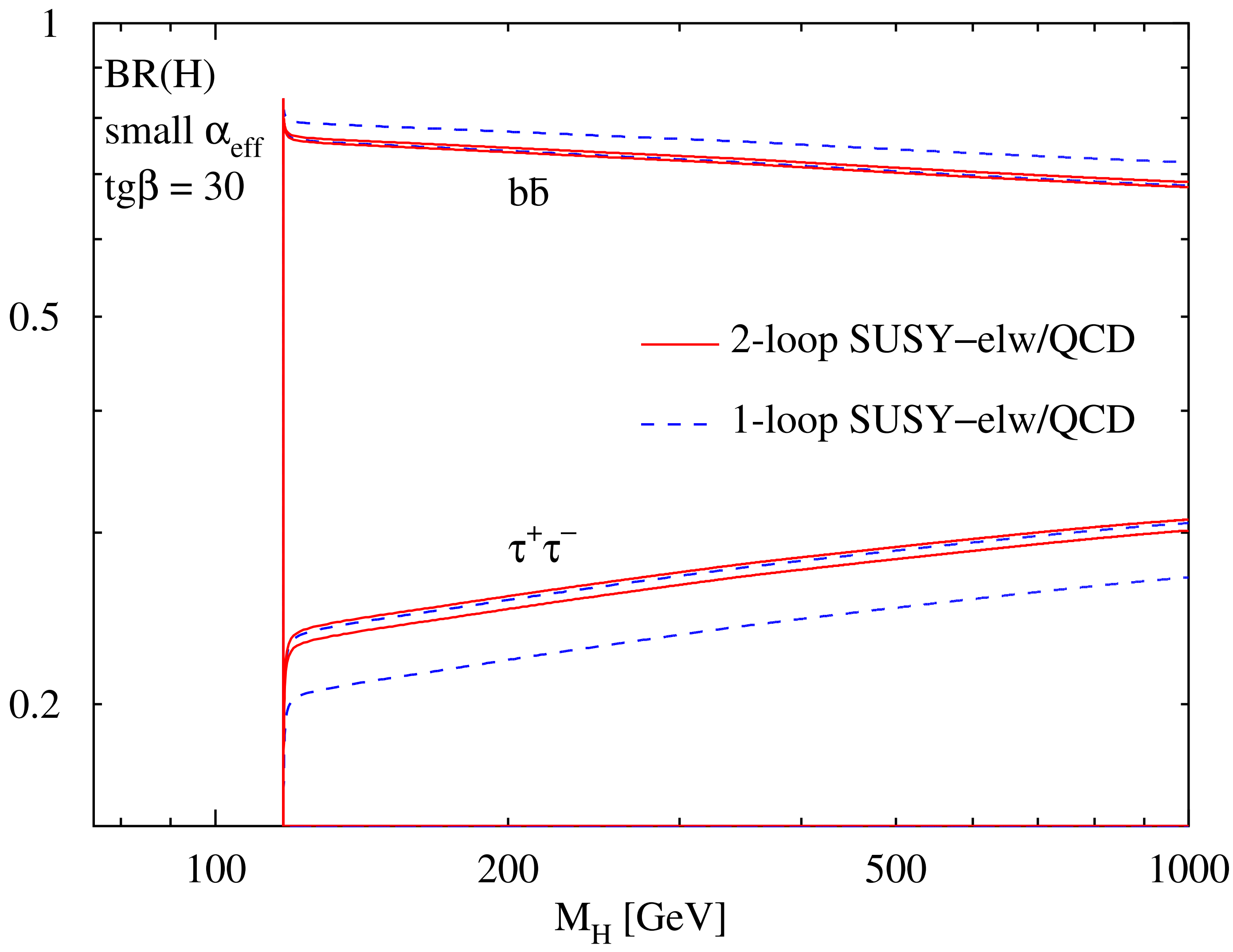
Δm_b^{elw}
small α_{eff}
tgβ = 30
$\mu_0 = (\mu + m_{\tilde{t}1} + m_{\tilde{t}2})/3$
2-loop SUSY−elw/QCD
1-loop SUSY−elw
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
10^{-1}
1
10
μ_R/μ_0

Noth, S.



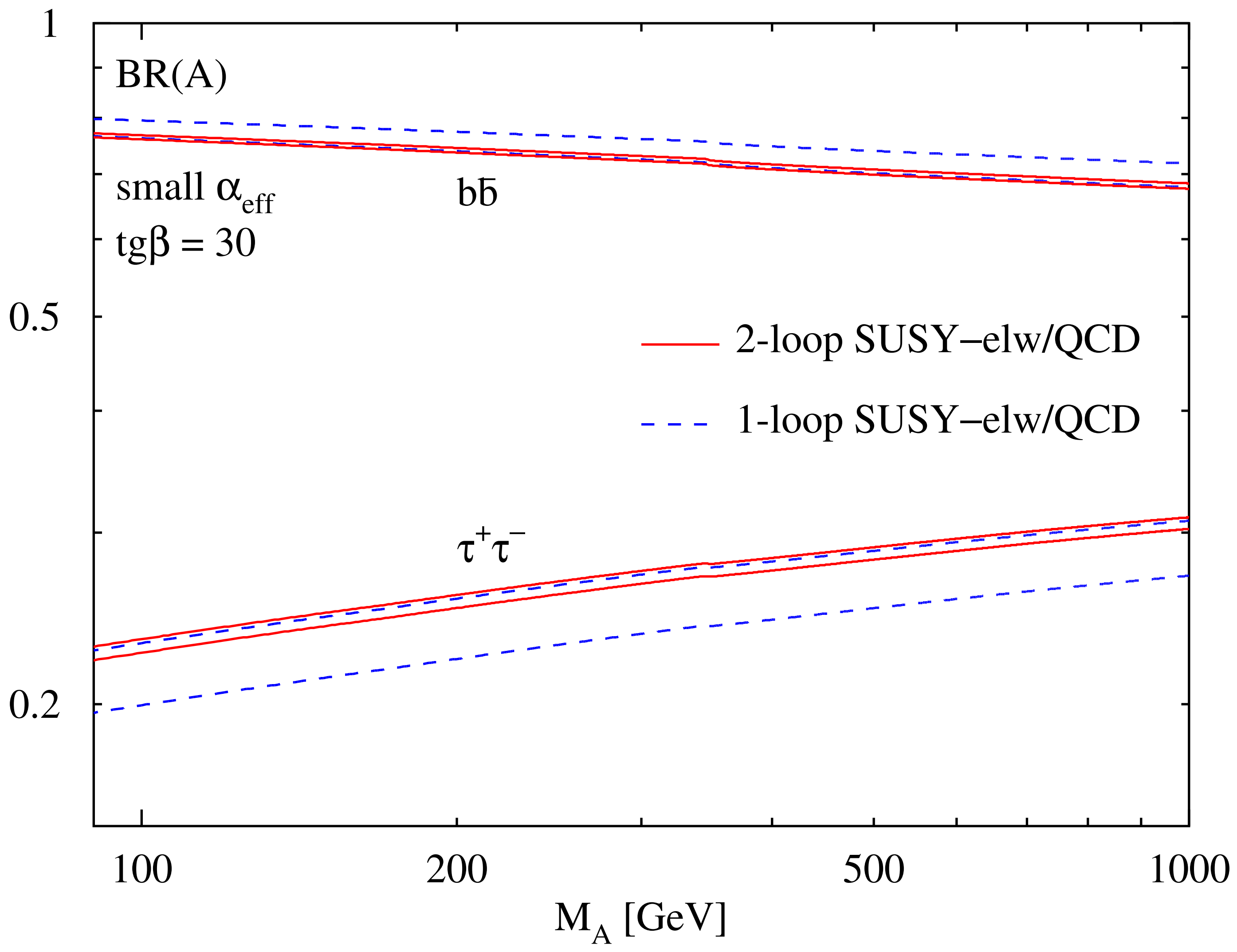
BR(h)
small α_{eff}
tgβ = 30
$b\bar{b}$
$\tau^+\tau^-$
2-loop SUSY−elw/QCD
1-loop SUSY−elw/QCD
1
0.5
0.2
90
95
100
105
110
115
120
M_h [GeV]

Noth, S.



BR(H)
small α_{eff}
tgβ = 30
$b\bar{b}$
$\tau^+\tau^-$
2-loop SUSY−elw/QCD
1-loop SUSY−elw/QCD
1
0.5
0.2
100
200
500
1000
M_H [GeV]

Noth, S.

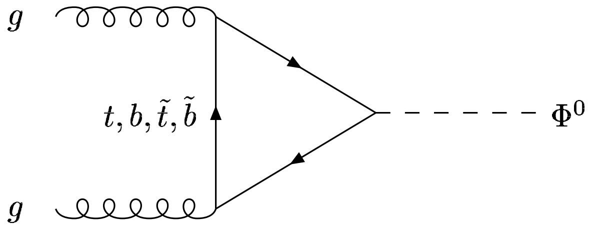


1
BR(A)
small α_{eff}
$tg\beta = 30$
$b\bar{b}$
0.5
2-loop SUSY−elw/QCD
1-loop SUSY−elw/QCD
$\tau^+\tau^-$
0.2
100
200
500
1000
M_A [GeV]

Noth, S.

III $\phi^0 gg$ COUPLINGS

Gluon fusion: $pp \to gg \to h/H/A$



Georgi,. . .

Gamberini,. . .

- third generation dominant [$\tilde{t}, \tilde{b}$: $m_{\tilde{q}} \lesssim$ 400 GeV]

- two-loop QCD corrections: $\sim 10 \ldots 100\%$
  [moderate for large tg$\beta \leftarrow b$-loop]

SDGZ
Dawson, Kauffman
Harlander, Kant
Aglietti, Boncani, Degrassi, Vicini
Anastasiou, Beerli, Bucherer, Daleo, Kunszt

- tg$\beta \lesssim$ 5: limit $m_t \gg M_\phi$ approximation for K-factor [$\Delta \lesssim$ 25%]

- NNLO calculated for $m_t \gg M_\phi \Rightarrow +$ 20–30%

Harlander, Kilgore
Anastasiou, Melnikov
Ravindran,. . .

- NNNLO estimated $m_t \gg M_\phi$ $\Rightarrow$ scale stabilization

scale dependence: $\Delta \lesssim 10 - 15\%$

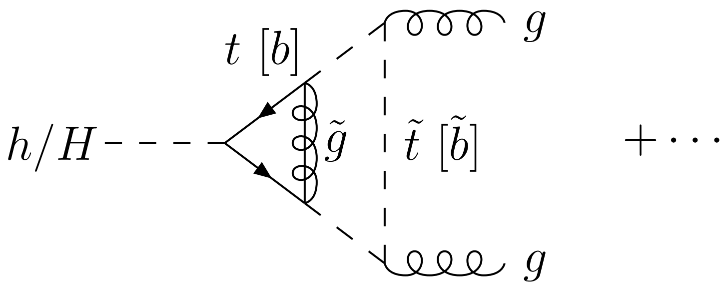
Catani, de Florian, Grazzini, Nason
Moch, Vogt
Ravindran

• two-loop QCD corrections to squark loops: $\sim 10 \ldots 100\%$

Dawson, Djouadi, S.
Mühlleitner, S.
Aglietti, Boncani, Degrassi, Vicini
Anastasiou, Beerli, Bucherer, Daleo, Kunszt

• genuine SUSY–QCD corrections: limit heavy SUSY masses $\to \mathcal{O}(10\%)$

Harlander, Steinhauser
Hofmann



after renormalization for $M_{\tilde{g}} \gg m_i$: log $M_{\tilde{g}}$
$\leftarrow$ Appelquist–Carazzone ??

full result recently

Anastasiou, Beerli, Daleo

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \frac{\mathcal{H}}{v} \left\{ \sum_Q g_Q^{\mathcal{H}} \left[1 + \frac{11}{4} \frac{\alpha_s}{\pi} \right] + \sum_{\tilde{Q}} \frac{g_{\tilde{Q}}^{\mathcal{H}}}{4} \left[1 + C_{SQCD} \frac{\alpha_s}{\pi} \right] \right\}$$

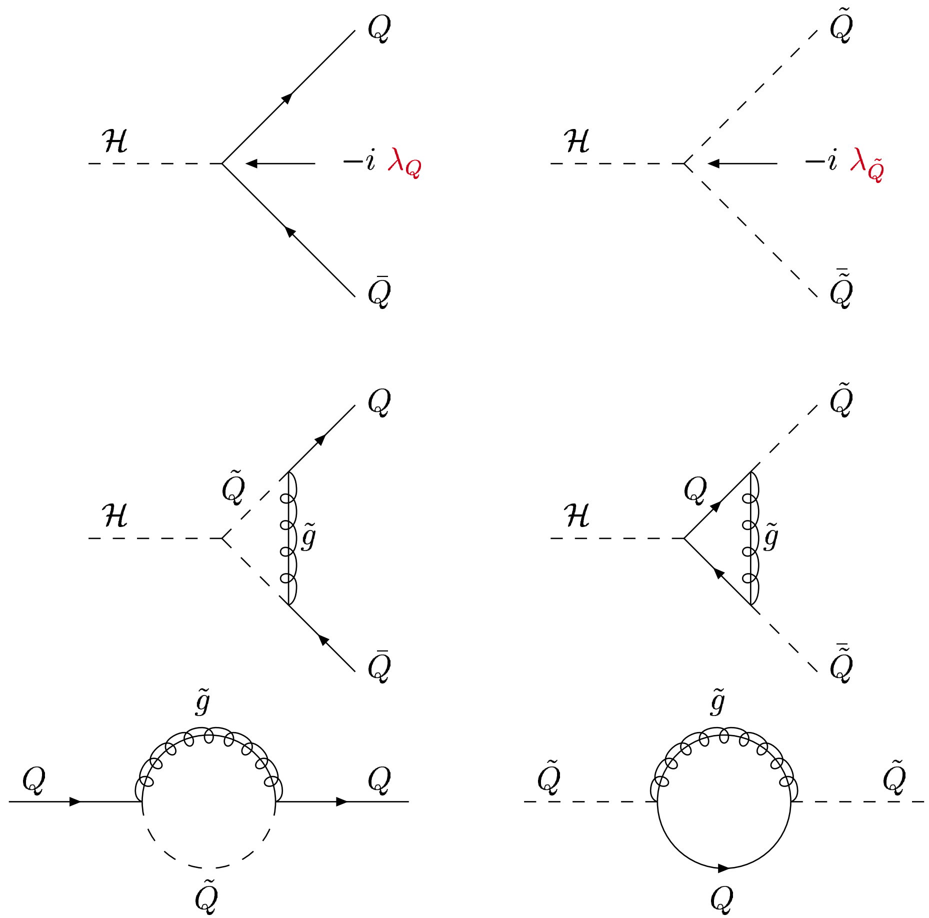
• Harlander,Steinhauser: mass degenerate squarks, no mixing, supersymmetric renormalization

$M_{\tilde{g}} \gg m_{\tilde{Q}}, m_Q$:

$$C_{SQCD}^{HS} = \frac{11}{2} - \frac{4}{3} \log \frac{M_{\tilde{g}}^2}{m_{\tilde{Q}}^2} - 2 \log \frac{m_{\tilde{Q}}^2}{m_Q^2}$$

[SUSY: $g_{\tilde{Q}}^{\mathcal{H}} = 2 g_Q^{\mathcal{H}} \frac{m_Q^2}{m_{\tilde{Q}}^2}$]

• $M_{\tilde{g}} \gg m_{\tilde{Q}}, m_Q$: supersymmetry lost due to decoupled gluino $\longrightarrow$ integrate gluinos out



- no mixing @ LO:

$$\lambda_Q = g_Q^{\mathcal{H}} \frac{m_Q}{v}$$

$$\lambda_{\tilde{Q}} = 2 g_Q^{\mathcal{H}} \frac{m_Q^2}{v} = \kappa \lambda_Q^2$$

$$\kappa = 2 \frac{v}{g_Q^{\mathcal{H}}}$$

• SUSY beyond LO: $\overline{MS}$ couplings [$\mu_R > M_{\tilde{g}}$]

$$\bar{\lambda}_{\tilde{Q}}(\mu_R) = \kappa \bar{\lambda}^2_Q(\mu_R)$$

• $\mu_R < M_{\tilde{g}}$: (i) threshold corrections
(ii) different RGEs [decoupled $\tilde{g}$]

• $\mu_R < M_{\tilde{g}}$: momentum subtracted coupling → threshold correction:

$$\bar{\lambda}_{Q,MO}(M_{\tilde{g}}) = \bar{\lambda}_Q(M_{\tilde{g}}) \left\{ 1 - \frac{3}{8} C_F \frac{\alpha_s(M_{\tilde{g}})}{\pi} \right\}$$

→ different RGEs:

$$\begin{aligned} \mu_R^2 \frac{\partial \bar{\lambda}_Q(\mu_R)}{\partial \mu_R^2} &= -\frac{C_F}{2} \frac{\alpha_s(\mu_R)}{\pi} \bar{\lambda}_Q(\mu_R) && [\mu_R > M_{\tilde{g}}] \\ \mu_R^2 \frac{\partial \bar{\lambda}_{Q,MO}(\mu_R)}{\partial \mu_R^2} &= -\frac{3}{4} C_F \frac{\alpha_s(\mu_R)}{\pi} \bar{\lambda}_{Q,MO}(\mu_R) && [\mu_R < M_{\tilde{g}}] \end{aligned}$$

• analogously for $\lambda_{\tilde{Q}}$:

(i) threshold correction:

$$\bar{\lambda}_{\tilde{Q},MO}(M_{\tilde{g}}) = \bar{\lambda}_{\tilde{Q}}(M_{\tilde{g}}) \left\{ 1 + \frac{3}{4} C_F \frac{\alpha_s(M_{\tilde{g}})}{\pi} \right\}$$

(ii) different RGEs:

$$\begin{aligned} \mu_R^2 \frac{\partial \bar{\lambda}_{\tilde{Q}}(\mu_R)}{\partial \mu_R^2} &= -C_F \frac{\alpha_s(\mu_R)}{\pi} \bar{\lambda}_{\tilde{Q}}(\mu_R) && [\mu_R > M_{\tilde{g}}] \\ \mu_R^2 \frac{\partial \bar{\lambda}_{\tilde{Q},MO}(\mu_R)}{\partial \mu_R^2} &= -\frac{C_F}{2} \frac{\alpha_s(\mu_R)}{\pi} \bar{\lambda}_{\tilde{Q},MO}(\mu_R) && [\mu_R < M_{\tilde{g}}] \end{aligned}$$

• relation to quark pole mass: Gray, Broadhurst, Grafe, Schilcher

$$g_Q^{\phi} \frac{m_Q}{v} = \bar{\lambda}_{Q,MO}(m_Q) \left\{ 1 + C_F \frac{\alpha_s(m_Q)}{\pi} \right\}$$

$$2 g_Q^{\mathcal{H}} \frac{m_Q^2}{v} = \bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}}) \left\{ 1 + C_F \frac{\alpha_s}{\pi} \left(\log \frac{M_{\tilde{g}}^2}{m_{\tilde{Q}}^2} + \frac{3}{2} \log \frac{m_{\tilde{Q}}^2}{m_Q^2} + \frac{1}{2} \right) \right\}$$

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \frac{\mathcal{H}}{v} \left\{ \sum_Q g_Q^{\mathcal{H}} \left[1 + \frac{11}{4} \frac{\alpha_s}{\pi} \right] + \sum_{\tilde{Q}} \frac{g_{\tilde{Q}}^{\mathcal{H}}}{4} \left[1 + C_{SQCD} \frac{\alpha_s}{\pi} \right] \right\}$$

$$g_{\tilde{Q}}^{\mathcal{H}} = v \frac{\bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}})}{m_{\tilde{Q}}^2}$$

$$\Delta C_{SQCD} = \frac{4}{3} \log \frac{M_{\tilde{g}}^2}{m_Q^2} + 2 \log \frac{m_{\tilde{Q}}^2}{m_Q^2} + \frac{2}{3} \qquad \Rightarrow \qquad \boxed{C_{SQCD} = \frac{37}{6}}$$

- solution to RGEs $[\beta_0 = (33 - 2N_F - N_{\tilde{F}})/12]$

$$\boxed{\bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}}) = 2 g_Q^{\mathcal{H}} \frac{m_Q^2}{v} \frac{1 + \frac{3}{2} C_F \frac{\alpha_s(M_{\tilde{g}})}{\pi}}{1 + 2 C_F \frac{\alpha_s(m_Q)}{\pi}} \left(\frac{\alpha_s(M_{\tilde{g}})}{\alpha_s(m_{\tilde{Q}})} \right)^{\frac{C_F}{\beta_0}} \left(\frac{\alpha_s(m_{\tilde{Q}})}{\alpha_s(m_Q)} \right)^{\frac{3C_F}{2\beta_0}}}$$

- no $\tilde{q}$ loops to $gg \to A$ at LO $\Rightarrow$ no log $M_{\tilde{g}}$

Harlander, Hofmann

V *SUMMARY*

- Higgs boson searches @ Tevatron, LHC, ILC major endeavours

- MSSM: large NLO SUSY corrections to bottom Yukawa couplings [$\leftarrow \Delta m_b$]

- NNLO corrections to Δm_b: $\mathcal{O}(10\%)$, $\Delta \sim \mathcal{O}(10\%) \to \mathcal{O}(1\%)$

- $\mathcal{H}gg$ coupling: decoupling of gluinos for large $M_{\tilde{g}}$: consistent with Appelquist–Carazzone theorem if properly renormalized $\to$ effective Lagrangian