

All-order corrections in production of (gluon fusion) Higgs boson plus jets

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Zürich
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The impact of higher order corrections...

- Significant perturbative corrections

... also from **hard** emissions

S. Höcker: Incl. H @NNLO: Significant contribution from multi jet states (unless **vetoed**)

L. Reina: Ass. $W/Z + b\bar{b}$: Estimate of ren. scale uncertainty larger at NLO than at LO, except if extra jets are **vetoed**

⋮

- Propose to **calculate to all orders** the source of these large corrections, in order to **stabilise the perturbative calculation** rather than removing the contributions by vetoes (sensitive to HO corrections).
- **Flexible** implementation necessary for realistic analyses

What, Why, How?

What?

Develop a framework for reliably calculating many-parton rates inclusively (both real and virtual corrections to create ensemble of 2, 3, 4, ... parton rates) in a flexible way (jets, W+jets, Higgs+jets, ...) **without relying on soft/collinear approximation of parton shower**

Why?

$(n+1)$ -jet rate not necessarily small compared to n -jet rate
Need inclusive perturbative corrections for realistic jet studies

How?

Factorisation of QCD Amplitudes in the High Energy Limit.
New Technique. Validation. Use H +dijet as example.

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Previous studies of Higgs Boson plus jets

Necessary to understand multi-emission topologies in order to cleanly extract WBF signal (c. jet veto, angular dist. of jets, . . .)

- hjj @full NLO: Increase in cross section over LO estimate of factors 1.2-1.3 or 1.7-1.8 depending on cuts (note: discussion of K -factors not really useful for a multi-scale problem).

J. Campbell, K. Ellis, G. Zanderighi

- hjj @LO+parton showers: Focus on effects of soft and collinear radiation to all orders. Find significant effects beyond NLO.

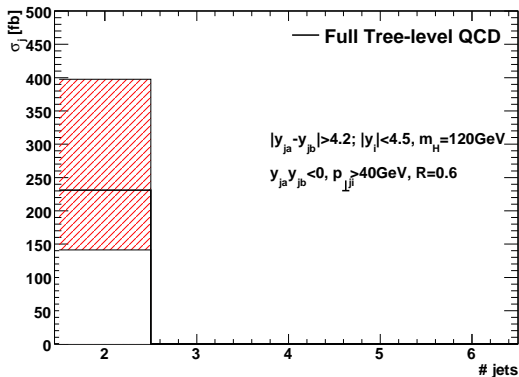
V. Del Duca, G. Klämke, D. Zeppenfeld, M.L. Mangano, M. Moretti, F. Piccinini, R. Pittau, A.D. Polosa

Will focus on **developing a framework** which captures a part of the **perturbative series to all orders (not relying** on soft and collinear factorisation) - and **compare it order by order** to the full result where known.

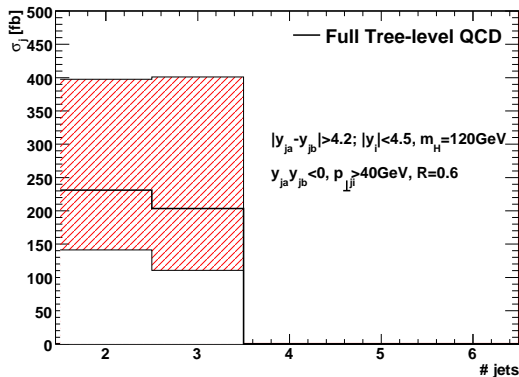
All Order Resummation Necessary?

Are tree-level (or generally fixed order) calculation always sufficient?

Higgs Boson plus n jets at the LHC at leading order

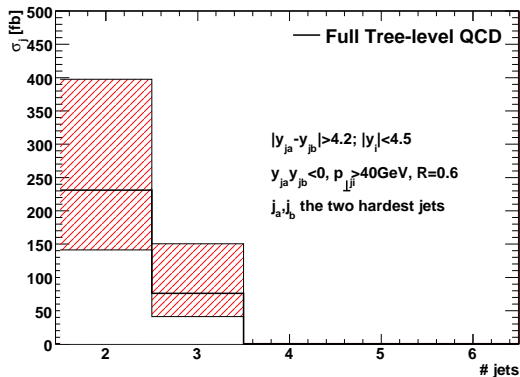


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Indication that we need to go further! However, fixed order tools **exhausted** (full $2 \rightarrow 3$ with a massive leg at two loops **untenable!**).

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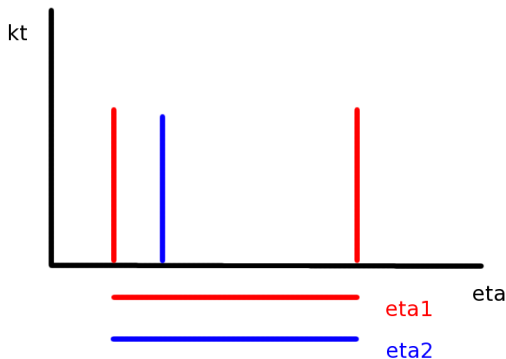
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Could require that the two jets passing the cuts are also the two hardest jets. This reduces the three-jet phase space and the higher order corrections. Sensitivity to pert. corrections?

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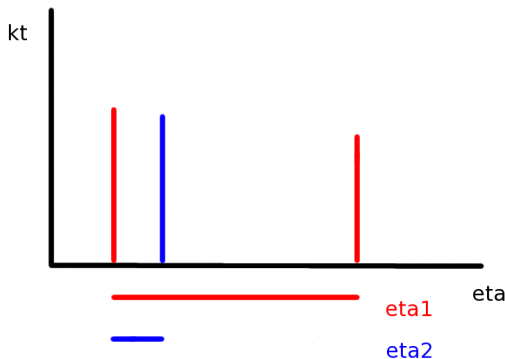
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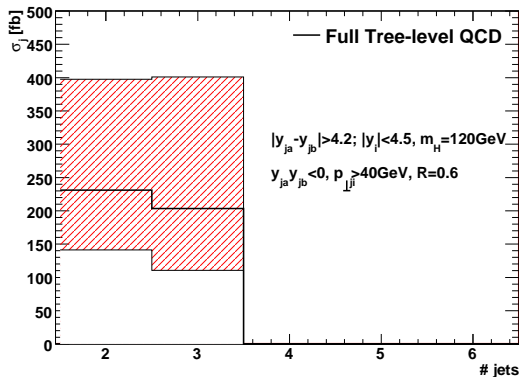


Rapidity span between hardest jets very sensitive to small perturbative corrections

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Higgs Boson plus n jets at the LHC at leading order



The method we develop will be applicable to both set of cuts, but crucially will allow a **stabilisation** of the perturbative series by **resummation**

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:

Soft limit \rightarrow **eikonal approximation** \rightarrow enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

This factorisation only **becomes exact** in a region **outside** the reach of any collider. . . **also** control sub-asympt. terms

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Regge and High Energy Factorisation of Amplitudes

In the High Energy Limit, $2 \rightarrow 2$ **scattering amplitudes** are **dominated** by the **t -channel exchange** of the particle of the **highest spin** allowed by the scattering theory

$$\mathcal{M}^{p_a p_b \rightarrow p_1 p_2} \xrightarrow{\text{Regge limit}} \hat{\mathbf{S}}^{\hat{\alpha}(\hat{t})} \gamma(\hat{t})$$

Regge (1959)

$$\hat{s} = (p_a + p_b)^2, \hat{t} = (p_a - p_1)^2, \text{Regge limit: } \hat{s} \rightarrow \infty, \hat{t} \text{ fixed.}$$

Multi-particle generalisation?

$$\mathcal{M}^{p_a p_c \rightarrow p_{a'} p_b p_{c'}} \xrightarrow{\text{Multi Regge limit}} \hat{\mathbf{s}}_1^{\hat{\alpha}(\hat{t}_1)} \hat{\mathbf{s}}_2^{\hat{\alpha}(\hat{t}_2)} \gamma(\hat{t}_1, \hat{t}_2, \frac{\mathbf{s}_{12}}{\mathbf{s}_1 \mathbf{s}_2})$$

MRK: $\hat{s}_{12}, \hat{s}_1, \hat{s}_2 \rightarrow \infty, t_1, t_2$ fixed

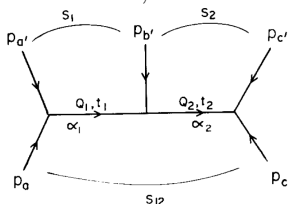
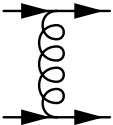
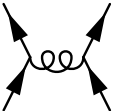
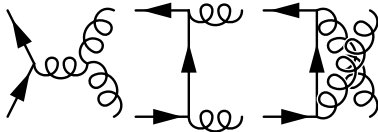


Fig. 2.1. Five-particle diagram showing notation.

Brower, DeTAR, Weis (1974)

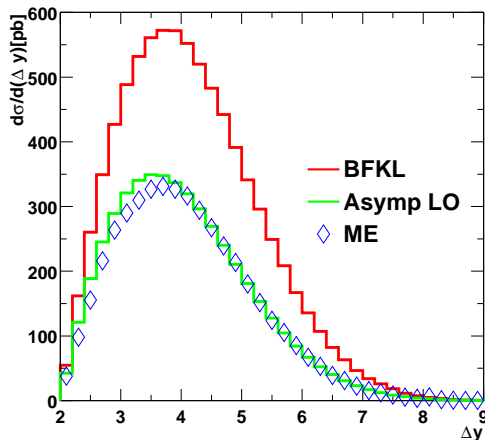
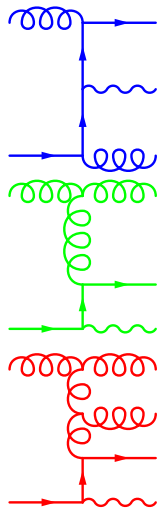
High Energy Factorisation - t -channel dominance

Process	Diagrams	$\overline{\sum} \mathcal{M} ^2 / g^4$
$qq' \rightarrow qq'$		$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q\bar{q} \rightarrow q'\bar{q}'$		$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$q\bar{q} \rightarrow gg$		$\frac{32}{27} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$

High Energy Limit: $|\hat{t}|$ fixed, $\hat{s} \rightarrow \infty$

t -channel dominance

Example: W + n -jet production at the LHC



$$\Delta y = y_{j_2} - y_{j_1}, y_W, y_{j_2} \geq 1, y_{j_1} \leq -1$$

Fadin, Kuraev, Lipatov

etc.

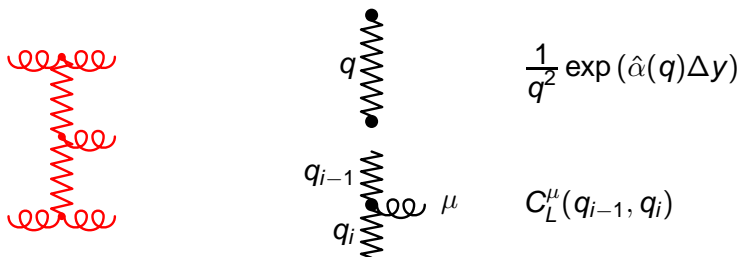
etc.

All these contributions can be calculated using **effective vertices** and propagators for the **reggeized gluon**.

General form proved using s-channel unitarity and a set of bootstrap relations NLL: Fadin, Fiore, Kozlov, Reznichenko

FKL formalism (Fadin, Kuraev, Lipatov)

FKL: Identification of the **dominant contributions** to the **perturbative series** for processes with two large (perturbative) and disparate energy scales $\hat{s} \gg |\hat{t}|$ (\hat{s} : E_{cm}^2 , \hat{t} : p_{\perp}^2)



$$C_L^\mu(q_{i-1}, q_i) = \left[-(q_i + q_{i+1})^{\mu_i} + p_a^\mu \left(\frac{q_i^2}{k_i \cdot p_1} + 2 \frac{k_i \cdot p_b}{p_a \cdot p_b} \right) - p_b \left(\frac{q_{i+1}^2}{k_i \cdot p_b} + 2 \frac{k_i \cdot p_a}{p_a \cdot p_b} \right) \right]$$

(note $C^\mu k_\mu = 0$. Subasymptotic controlled by requirement of gauge invariance). Framework exact in the limit of Multi Regge Kinematic (MRK)

$$y_0 \gg y_1 \gg \dots \gg y_n, \quad |k_{i\perp}| \approx |k_{j\perp}|, \quad q_i^2 \approx q_j^2$$

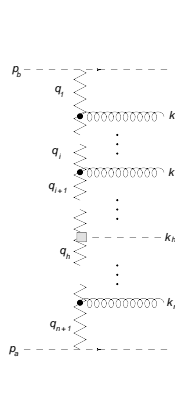
Higgs Boson plus $n \geq 2$ jets in the HE limit



Extract the effective GG-Higgs Boson vertex using Hgggg-amplitudes (also checks factorisation)

Only four diagrams contribute to the process Higgs Boson plus 3 jets in the High Energy Limit!

The Scattering Amplitude



$$i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} = 2i\hat{s} \cdot \left(ig_s f^{ad_0 c_1} g_{\mu_a \mu_0} \right) \cdot \prod_{i=1}^j \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left(ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \cdot \left(\frac{1}{q_h^2} \exp[\hat{\alpha}(q_h^2)(y_j - y_h)] C_H(q_{j+1}, q_h) \right) \cdot \prod_{i=j+1}^n \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y'_{i-1} - y'_i)] \left(ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \cdot \frac{1}{q_{n+1}^2} \exp[\hat{\alpha}(q_{n+1}^2)(y'_n - y_b)] \left(ig_s f^{bd_{n+1} c_{n+1}} g_{\mu_b \mu_{n+1}} \right)$$

Have: **exact** result in the **very exclusive limit** of **infinite separation** between **all particles**

Want: **inclusive** cross sections...

Improving the Framework

Start again from the FKL amplitudes:

$$i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} = 2i\hat{s} \dots \prod_{i=1}^j \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left(ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \dots$$

Result exact in the MRK limit. Control the sub-asymptotic behaviour by imposing

- ① Position of Divergences
- ② Gauge invariance (also in sub-MRK region)

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- 2 Gauge invariance (also in sub-MRK region)

The full scattering amplitude is divergent for several momentum configurations, for which the use of MRK approximations of invariants would render the amplitude finite. However, we choose to re-instate several of these divergences by using **the full momentum dependence of all invariants**.

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Using full expression for propagators **automatically takes into account the dominant source of NLL corrections** to *any* logarithmic accuracy. NLL corrections to Lipatov Vertex C^μ starts to address the dependence on longitudinal momenta between two neighbouring partons. We can restore the **full** propagator between all gluons.

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Choose form of Lipatov vertex satisfying $C^\mu k_\mu = 0$. The gauge dependent terms are suppressed in the HE limit (and thus not controlled by such considerations), but we are seeking a form which works **everywhere**. Requirement of Gauge invariance severely constrains the sub-asymptotic terms.

Improving the Framework

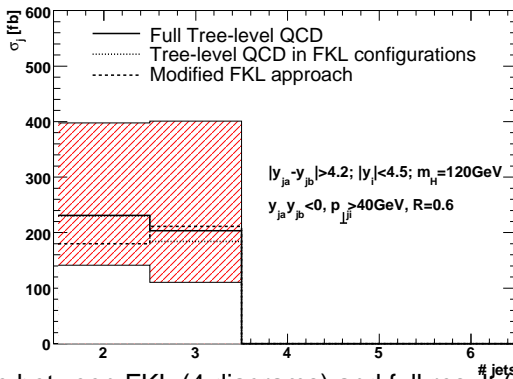
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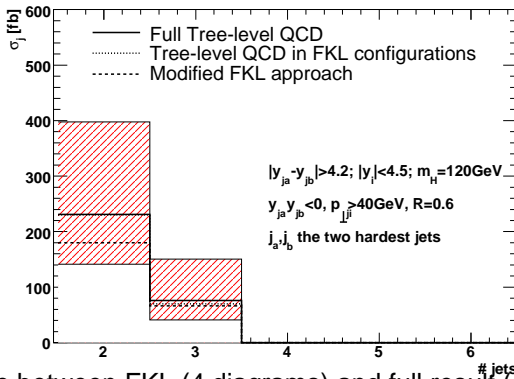
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- 2 Gauge invariance (also in sub-MRK region)

These two **constrains the subasymptotic** form of the amplitude (and obviously does not alter the asymptotic form). Approximates the full results well where known. **Sufficiently simple** to allow an **all-order resummation**.



Difference between FKL (4 diagrams) and full result (10^3 diagrams) is much less than the renormalisation and factorisation scale uncertainty. Repair with matching corrections.

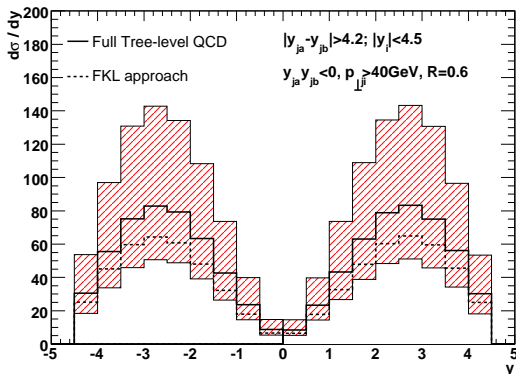
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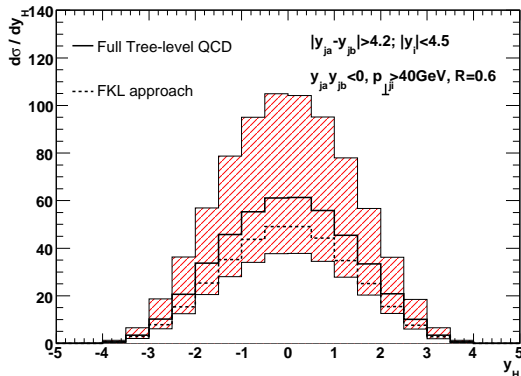
Fixed Order Compⁿ: FKL vs Full Matrix Element

Rapidities of forward/backward jet in hjj



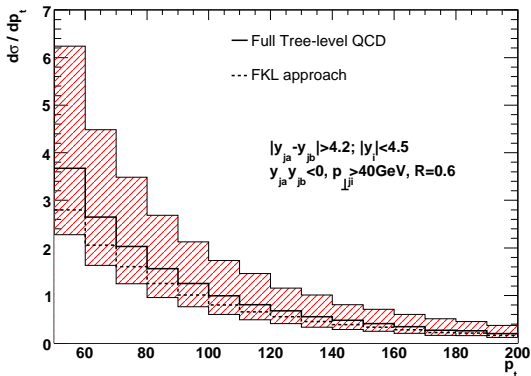
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Rapidity of Higgs Boson in hjj



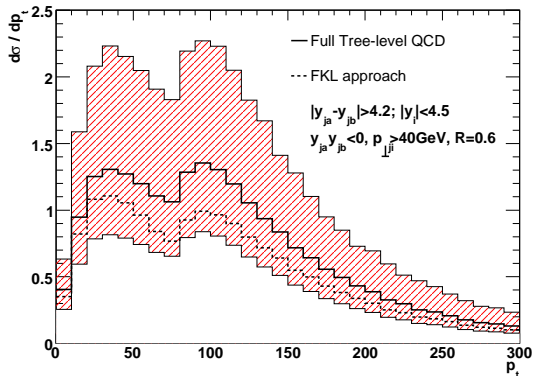
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Transverse momentum of jets in hjj



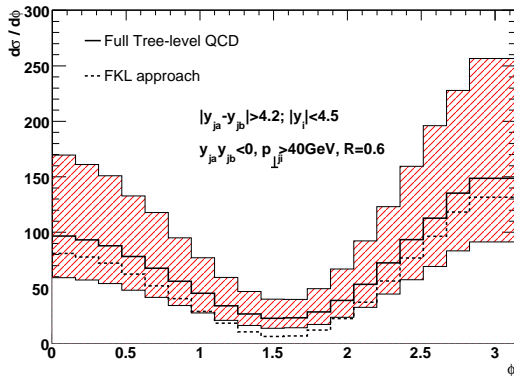
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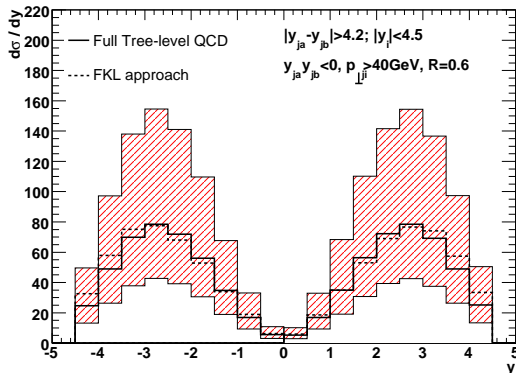
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Azimuthal angle between jets in hjj



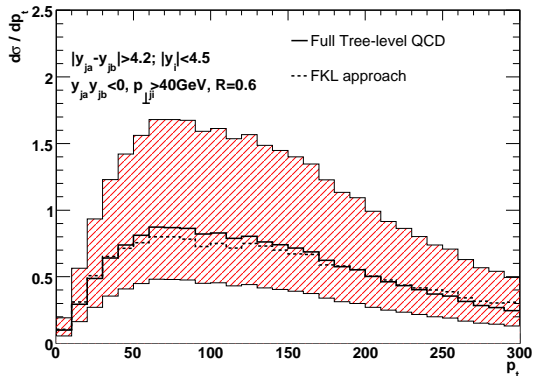
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Rapidities of forward/backward jet in $hjjj$



Fixed Order Compⁿ: FKL vs Full Matrix Element

Transverse momentum of Higgs Boson in hjjj



Beyond validation. . .

Have so far demonstrated that the terms we can take into account reproduce the full tree level results to within 10 – 25% where ever these are known - and reproduce distributions.

Can calculate this approximation for the tree-level Higgs Boson plus n -parton amplitude, and include also the corresponding virtual corrections. Can thereby form the inclusive *any*-parton sample (i.e. LO: only H+2 partons, NLO: H+2 and H+3 partons, ...)

Fully exclusive in all particles - Can perform any analysis using your favourite jet algorithm

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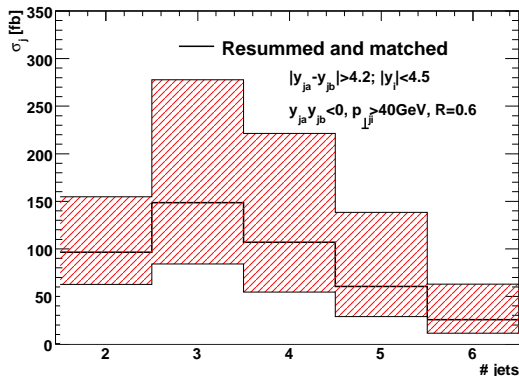
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FKL All Order Resummation Incl. Matching

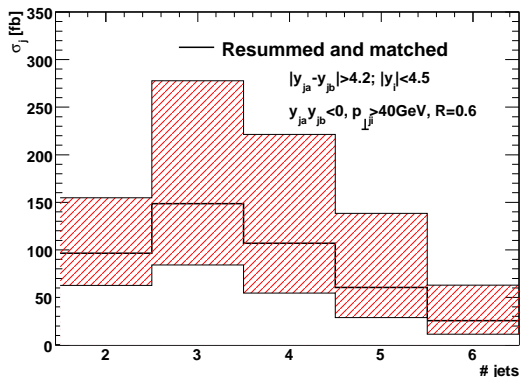
$$\sigma_{hjj}^{LO} : 230\text{fb}; \sigma_{hjj}^{\text{resummed+matched}} : 435\text{fb} (K = 1.9)$$



Can sum over n -parton inclusive samples (both real and virtual contributions included). Matching to the tree level n -parton matrix elements.

FKL All Order Resummation Incl. Matching

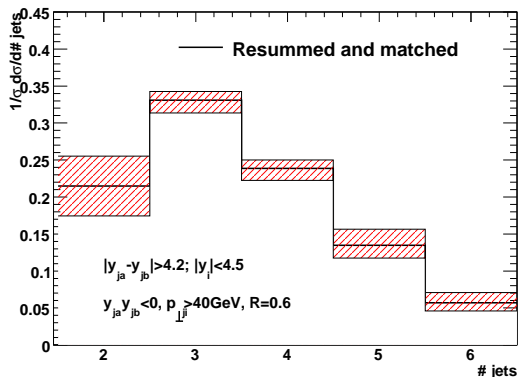
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Significant jet activity - 10% – 20% increase in inclusive cross section compared to NLO

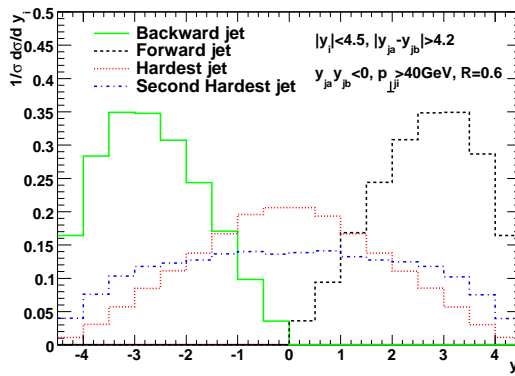
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Significant jet activity - relative jet counts stable against scale variation

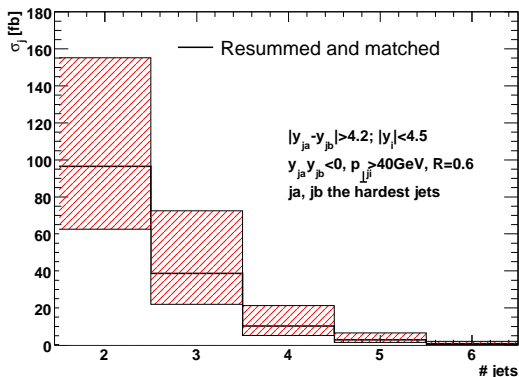
Rapidities of jets



Rapidities of forward/backward jet similar to LO distributions; hardest and next-to-hardest jets much more central (as expected)

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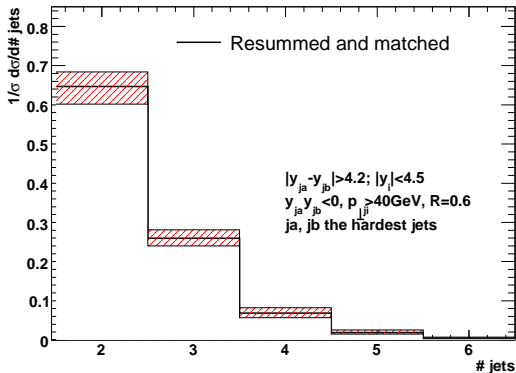
$$\sigma_{hjj}^{LO} : 230\text{fb}; \sigma_{hjj}^{\text{resummed+matched}} : 149\text{fb} (K = 0.65)$$



Most events are rejected because of the central jet activity - cross section reduced compared to NLO value

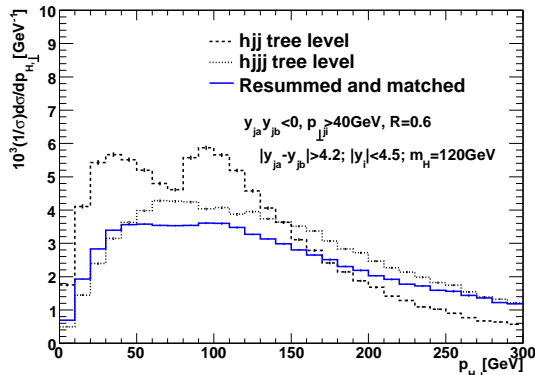
FKL All Order Resummation Incl. Matching

$$\sigma_{hjj}^{LO} : 230\text{fb}; \sigma_{hjj}^{\text{resummed+matched}} : 149\text{fb} (K = 0.65)$$



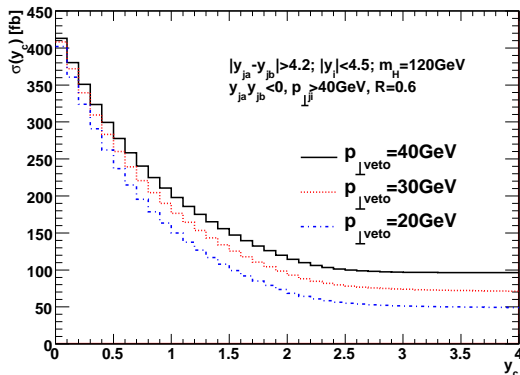
Most events are rejected because of the central jet activity - cross section reduced compared to NLO value

Transverse Momentum Spectrum of the Higgs Boson



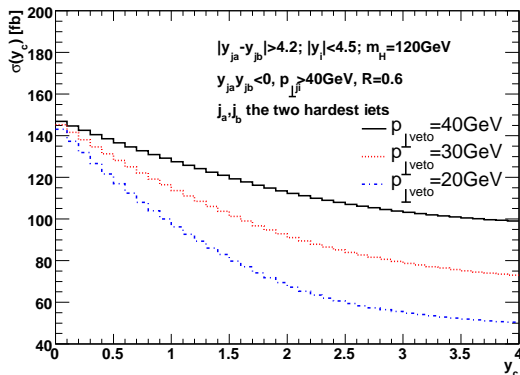
Strong features of Higgs boson transverse momentum spectrum (caused by strong azimuthal correlation coupled with cuts on jets) disappears at higher orders.

Central rapidity jet veto



$$\forall j \in \{\text{jets with } p_{j\perp} > p_{\perp,\text{veto}}\} \setminus \{a, b\} : \left| y_j - \frac{y_a + y_b}{2} \right| > y_c$$

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$$\forall j \in \{\text{jets with } p_{j\perp} > p_{\perp,\text{veto}}\} \setminus \{a, b\} : \left| y_j - \frac{y_a + y_b}{2} \right| > y_c$$

Azimuthal decorrelation

$$A_\phi = \frac{\sigma(\phi_{j_a j_b} < \pi/4) - \sigma(\pi/4 < \phi_{j_a j_b} < 3\pi/4) + \sigma(\phi_{j_a j_b} > 3\pi/4)}{\sigma(\phi_{j_a j_b} < \pi/4) + \sigma(\pi/4 < \phi_{j_a j_b} < 3\pi/4) + \sigma(\phi_{j_a j_b} > 3\pi/4)}$$

Results from lowest order:

$A_\phi > 0$ (CP-even), $A_\phi \approx 0$ (CP-blind), $A_\phi < 0$ (CP-odd)

Inclusive cuts	A_ϕ	Hardest cuts	A_ϕ
LO 2-jet	0.456	LO 2-jet	0.456
Resummed, = 2-jet	0.437	Resummed, = 2-jet	0.436
LO 3-jet	0.203	LO 3-jet	0.374
Resummed	0.133	Resummed	0.372

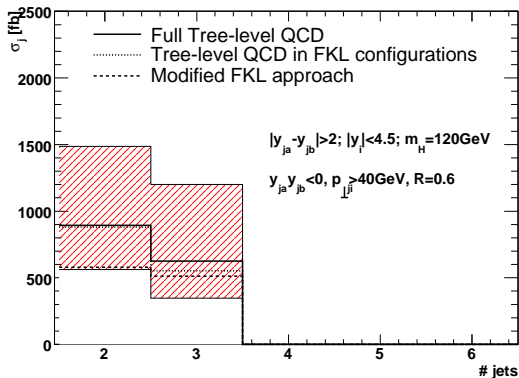
Significant azimuthal decorrelation from higher orders real radiation - most stable when hardest jets used for tagging (since multi-jet events in this case are practically vetoed)

Summary

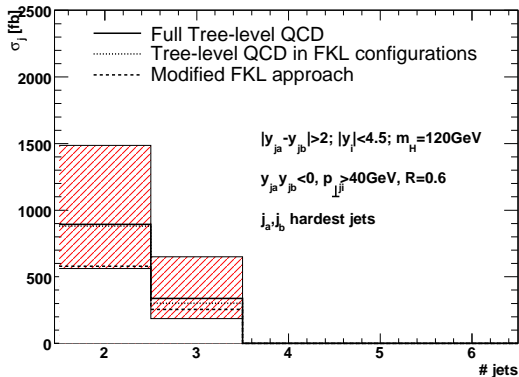
Summary

- Emerging framework for the study of processes with multiple hard jets
- Working implementation, including matching to the known fixed order results; public code available:
<http://andersen.web.cern.ch/andersen/MJEV>
- Impact many studies: jet correlations,...
- Les Houches Interface to study effects of showering

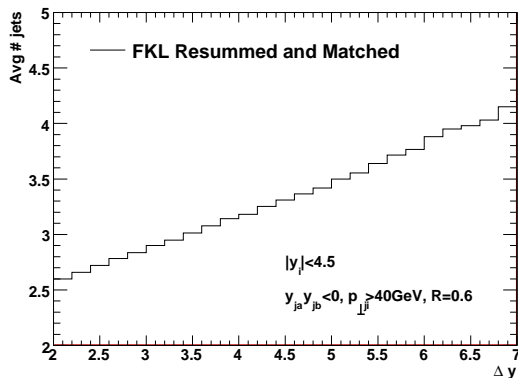
Relaxed Rapidity Cuts



Relaxed Rapidity Cuts



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Relaxed Rapidity Cuts

