Effective Field-Theory Methods for Collider Physics: Higgs Production and More

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T. Becher, MN: arXiv:0901.0722 (today)
V. Ahrens, T. Becher, MN and L. Yang: arXiv:0808.3008 and 0809.4283

... and More

T. Becher, MN: arXiv:0901.0722

IR singularities of QCD amplitudes

- On-shell parton scattering amplitudes in gauge theories contain IR divergences from soft and collinear loop momenta
- Cancel between virtual and real corrections
- Nevertheless interesting:
    - resummation of large Sudakov logarithms remaining after cancellation
    - check on multi-loop calculations
    - better handle on real-emission graphs

Catani's fomula (1998)

- Specifies structure of IR singularities for an n-parton amplitude at 2-loop order: Catani 1998

$$\left[1 - \frac{\alpha_s}{2\pi}\,\boldsymbol{I}^{(1)}(\epsilon) - \left(\frac{\alpha_s}{2\pi}\right)^2 \boldsymbol{I}^{(2)}(\epsilon) + \ldots\right] |\mathcal{M}_n(\epsilon, \{p\})\rangle = \text{finite}$$

with

$$\boldsymbol{I}^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_i \left(\frac{1}{\epsilon^2} + \frac{g_i}{\boldsymbol{T}_i^2}\,\frac{1}{\epsilon}\right) \sum_{j\neq i} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2} \left(\frac{\mu^2}{-s_{ij}}\right)^{\epsilon}$$

$$\boldsymbol{I}^{(2)}(\epsilon) = \frac{e^{-\epsilon\gamma_E}\,\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(K + \frac{\beta_0}{2\epsilon}\right) \boldsymbol{I}^{(1)}(2\epsilon) - \frac{1}{2}\,\boldsymbol{I}^{(1)}(\epsilon)\left(\boldsymbol{I}^{(1)}(\epsilon) + \frac{\beta_0}{\epsilon}\right) + \boldsymbol{H}^{(2)}_{\text{R.S.}}(\epsilon)$$

unspecified

- Derivation using factorization properties and IR evolution equation for form factor

Sterman, Tejeda-Yeomans 2003

SCET approach

Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke et al. 2002

- Effective theory for n-jet processes contains n different types of collinear fields, interacting only via soft fields

- Hard modes (Q ~ √s) are integrated out and absorbed into Wilson coefficients: Bauer, Schwartz 2006

$$\mathcal{H}_n = \sum_i \mathcal{C}_{n,i}(\mu)\, O_{n,i}^{\rm ren}(\mu)$$

- Scale dependence controlled by RGE:

$$\frac{d}{d\ln\mu}\,|\mathcal{C}_n(\{p\},\mu)\rangle = \mathbf{\Gamma}(\mu,\{p\})\,|\mathcal{C}_n(\{p\},\mu)\rangle$$

anomalous dimension matrix

On-shell parton scattering amplitudes

- On-shell parton scattering amplitudes have no IR scales, and so loop matrix elements of bare SCET operators vanish
- One obtains:

renormalization factor
(minimal subtraction of IR poles)

$$|\mathcal{C}_n(\{p\},\mu)\rangle = \lim_{\epsilon\to 0} \boldsymbol{Z}^{-1}(\epsilon,\{p\},\mu)\,|\mathcal{M}_n(\epsilon,\{p\})\rangle$$

Becher, MN 2009

where

$$\boldsymbol{\Gamma} = -\frac{d\ln \boldsymbol{Z}}{d\ln\mu}$$

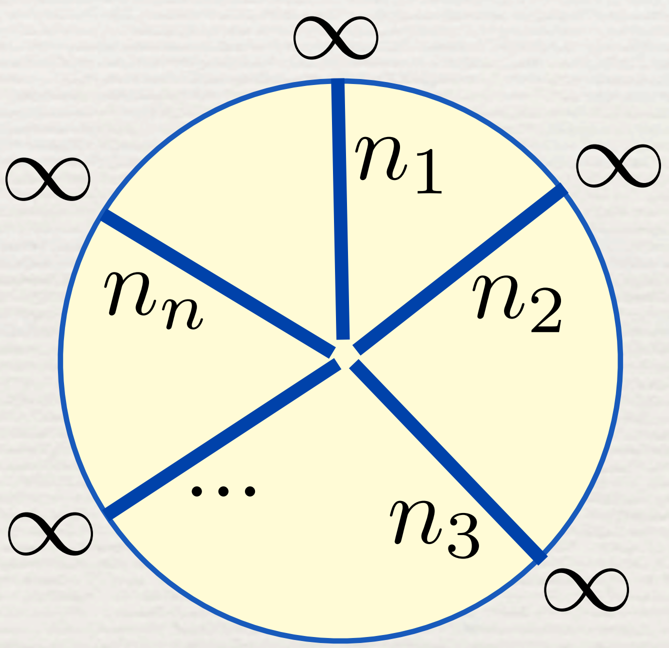
- IR poles of scattering amplitudes mapped onto UV poles of n-jet SCET operators
- Multiplicative subtraction, controlled by RG!

Conjecture for anomalous dimension

Conjecture for anomalous dimension

- SCET decoupling transformation removes soft interactions among collinear fields and absorbs them into soft Wilson lines

$$\boldsymbol{S}_i = \mathbf{P} \exp \left[ig \int_{-\infty}^{0} dt \, n_i \cdot A_a(tn_i) \, T_i^a \right]$$



- For n-jet operator one gets:

$$\langle 0 | \boldsymbol{S}_1 \dots \boldsymbol{S}_n | 0 \rangle$$

- Use powerful theorems on renormalization of Wilson loops and non-abelian exponentation

Brandt et al. 1981, 1982; Frenkel, Taylor 1984; Korchemsky, Radyushkin 1986, 1987

Conjecture for anomalous dimension

- Based on these results, we propose the exact form: Becher, MN 2009

cusp anomalous dimension

$$\boldsymbol{\Gamma} = \sum_{(i,j)} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \Gamma_{\text{cusp}}(\alpha_s) \, \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

$(p_i + p_j)^2$ quark/gluon anomalous dimensions

  - simplest, most beautiful form possible (only two-parton correlations)
  - consistent with two-loop soft anom. dim.
    Mert Aybat, Dixon, Sterman 2006
  - predicts relation between cusp anomalous dimensions of quarks and gluons, which has been tested to three-loop order
    Moch, Vermaseren, Vogt 2004

Obtain Z factor by integration

- Result:

d-dimensional β-function

$$\ln \boldsymbol{Z} = \int\limits_0^{\alpha_s} \frac{d\alpha}{\alpha} \frac{1}{2\epsilon - \beta(\alpha)/\alpha} \left[\boldsymbol{\Gamma}(\alpha) - \Gamma'(\alpha) \int\limits_{\alpha_s}^{\alpha} \frac{d\alpha'}{\alpha'} \frac{1}{2\epsilon - \beta(\alpha')/\alpha'} \right]$$

where

$$\Gamma' = \frac{\partial}{\partial \ln \mu} \boldsymbol{\Gamma} = -\Gamma_{\text{cusp}}(\alpha_s) \sum_i C_i$$

- Perturbative expansion:

all coefficients known!

$$\begin{aligned} \ln \boldsymbol{Z} &= \frac{\alpha_s}{4\pi} \left(\frac{\Gamma'_0}{4\epsilon^2} + \frac{\boldsymbol{\Gamma}_0}{2\epsilon} \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[-\frac{3\beta_0 \Gamma'_0}{16\epsilon^3} + \frac{\Gamma'_1 - 4\beta_0 \boldsymbol{\Gamma}_0}{16\epsilon^2} + \frac{\boldsymbol{\Gamma}_1}{4\epsilon} \right] \\ &+ \left(\frac{\alpha_s}{4\pi} \right)^3 \left[\frac{11\beta_0^2 \Gamma'_0}{72\epsilon^4} - \frac{5\beta_0 \Gamma'_1 + 8\beta_1 \Gamma'_0 - 12\beta_0^2 \boldsymbol{\Gamma}_0}{72\epsilon^3} + \frac{\Gamma'_2 - 6\beta_0 \boldsymbol{\Gamma}_1 - 6\beta_1 \boldsymbol{\Gamma}_0}{36\epsilon^2} + \frac{\boldsymbol{\Gamma}_2}{6\epsilon} \right] + \dots \end{aligned}$$

exponentiation yields **Z** factor at 3 loops!

Checks

- Comparison with Catani's formula at two loops yields explicit expression for 1/ε pole term:

$$\boldsymbol{H}^{(2)}_{\text{R.S.}} = \frac{1}{16\epsilon} \sum_i \left(\gamma_1^i - \frac{\Gamma_1^{\text{cusp}}}{\Gamma_0^{\text{cusp}}}\, \gamma_0^i + \frac{\pi^2}{16}\, \beta_0\, C_i \right)$$

$$+ \frac{i f_{abc}}{4\epsilon} \sum_{(i,j,k)} T_i^a\, T_j^b\, T_k^c \ln\frac{-s_{ij}}{-s_{jk}} \ln\frac{-s_{jk}}{-s_{ki}} \ln\frac{-s_{ki}}{-s_{ij}}$$

- Non-trivial color structure only arises since his operators are not defined in a minimal scheme
- Confirms conjecture for this term Bern, Dixon, Kosower 2004

Checks

- Expression for IR pole terms agrees with all known results:
  - 3-loop quark and gluon form factors, which determine the functions $\gamma^{q,g}(\alpha_s)$
    Moch, Vermaseren, Vogt 2005
  - 2-loop 3-jet qqg amplitude Garland, Gehrmann et al. 2002
  - 2-loop 4-jet amplitudes Anastasiou, Glover et al. 2001 Bern, De Freitas, Dixon 2002, 2003
  - 4-loop 4-jet amplitudes in N=4 super Yang-Mills theory in planar limit Anastasiou et al. 2003 Bern et al. 2005, 2007

Potential applications

- Resummation of Sudakov logarithms for hard scattering functions at N^3LL in closed form
- Generalization to include massive partons
- Improved understanding and treatment of real-emission graphs
- Great simplicity of our result hints at universal origin of IR singularities, disconnected from genuine dynamics of scattering amplitudes

Potential applications

- Evolution of hard-scattering coefficients is first step in complete analysis of resummation for hadron collider processes near partonic thresholds
- Will now consider Higgs production as the simplest case of such a complete analysis

$$gg \to H + X_{\mathrm{soft}}$$

EFT-based resummation for Higgs production

V, Ahrens, T. Becher, MN, L. Yang:

arXiv:0808.3008 and 0809.4283

Fixed-order cross section

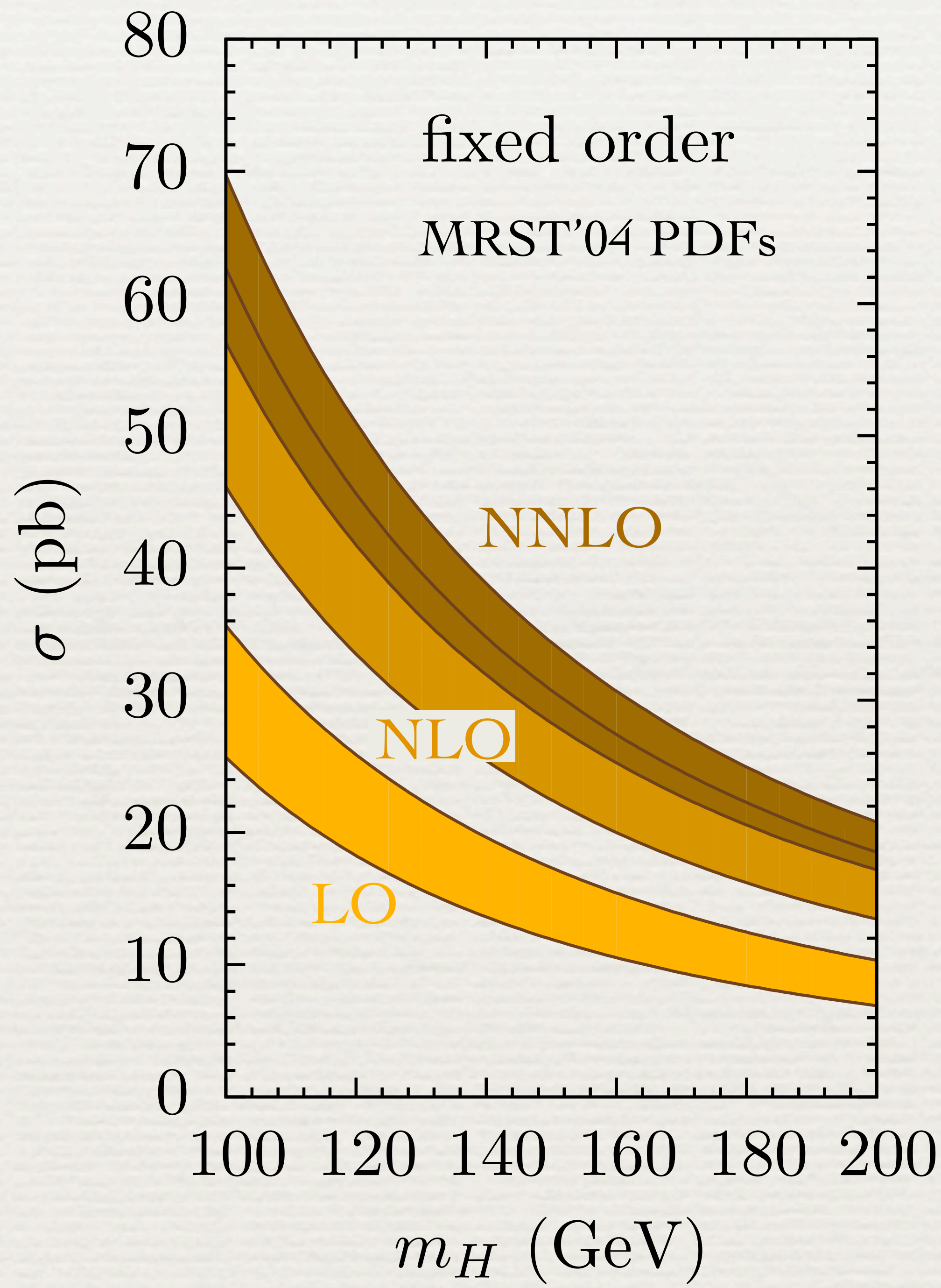
$$\sigma = \sigma_0 \sum_{i,j} \int_\tau^1 \frac{dz}{z}\, C_{ij}(z, m_t, m_H, \mu_f)\, ff_{ij}(\tau/z, \mu_f)$$

- Here $\tau = M_H^2/s$ and $z = M_H^2/\hat{s}$
- Hard scattering kernels are convoluted with parton luminosities

$$ff_{ij}(y, \mu) = \int_y^1 \frac{dx}{x}\, f_{i/N_1}(x, \mu)\, f_{j/N_2}(y/x, \mu)$$

- Cross section is dominated by leading terms near partonic threshold $z \to 1$ (empirical obs.)
- Perform soft-gluon resummation at N^3LL order plus matching to fixed-order result at NNLO (state of the art)

Large higher-order corrections



- Corrections are large: 70% at NLO + 30% at NNLO [130% and 80% if PDFs and α_s are held fixed]
- Only C_{gg} contains leading singular terms, which give 90% of NLO and 94% of NNLO correction
- Contributions of C_{qg} and C_{qq} are small: -1% and -8% of the NLO correction

Harlander, Kilgore 2002; Anastasiou, Melnikov 2002
Ravindran, Smith, van Neerven 2003

Effective theory analysis

- Separate contributions associated with different scales, turning a multi-scale problems into a series of single scale problems
- Evaluate each contribution at its natural scale, leading to improved perturbative behavior
- Use renormalization group to evolve contributions to an arbitrary factorization scale, thereby exponentiating (resumming) large corrections

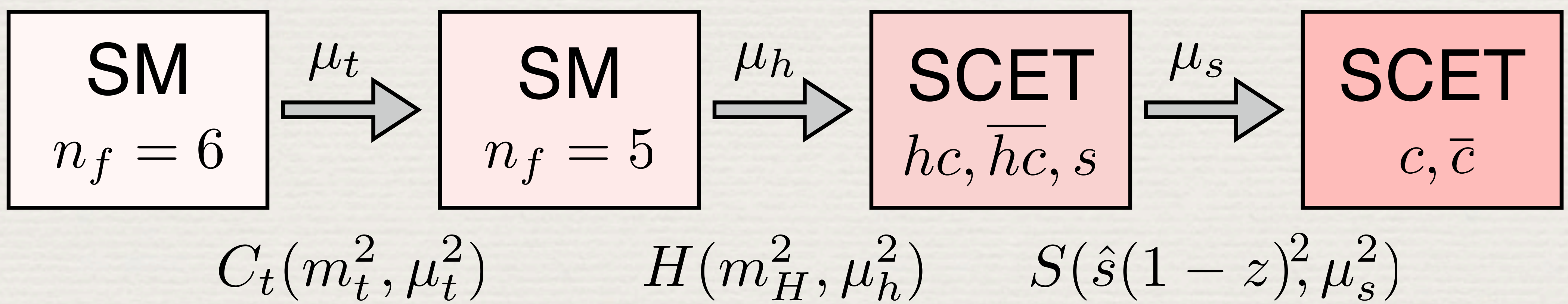
When this is done consistently, large K-factors should never arise, since no large perturbative corrections should be left unexponentiated!

Scale hierarchy

- We will analyze the Higgs cross section assuming the scale hierarchy ($z = M_H^2/\hat{s}$)

$$2m_t \gg m_H \sim \sqrt{\hat{s}} \gg \sqrt{\hat{s}}(1-z) \gg \Lambda_{\rm QCD}$$

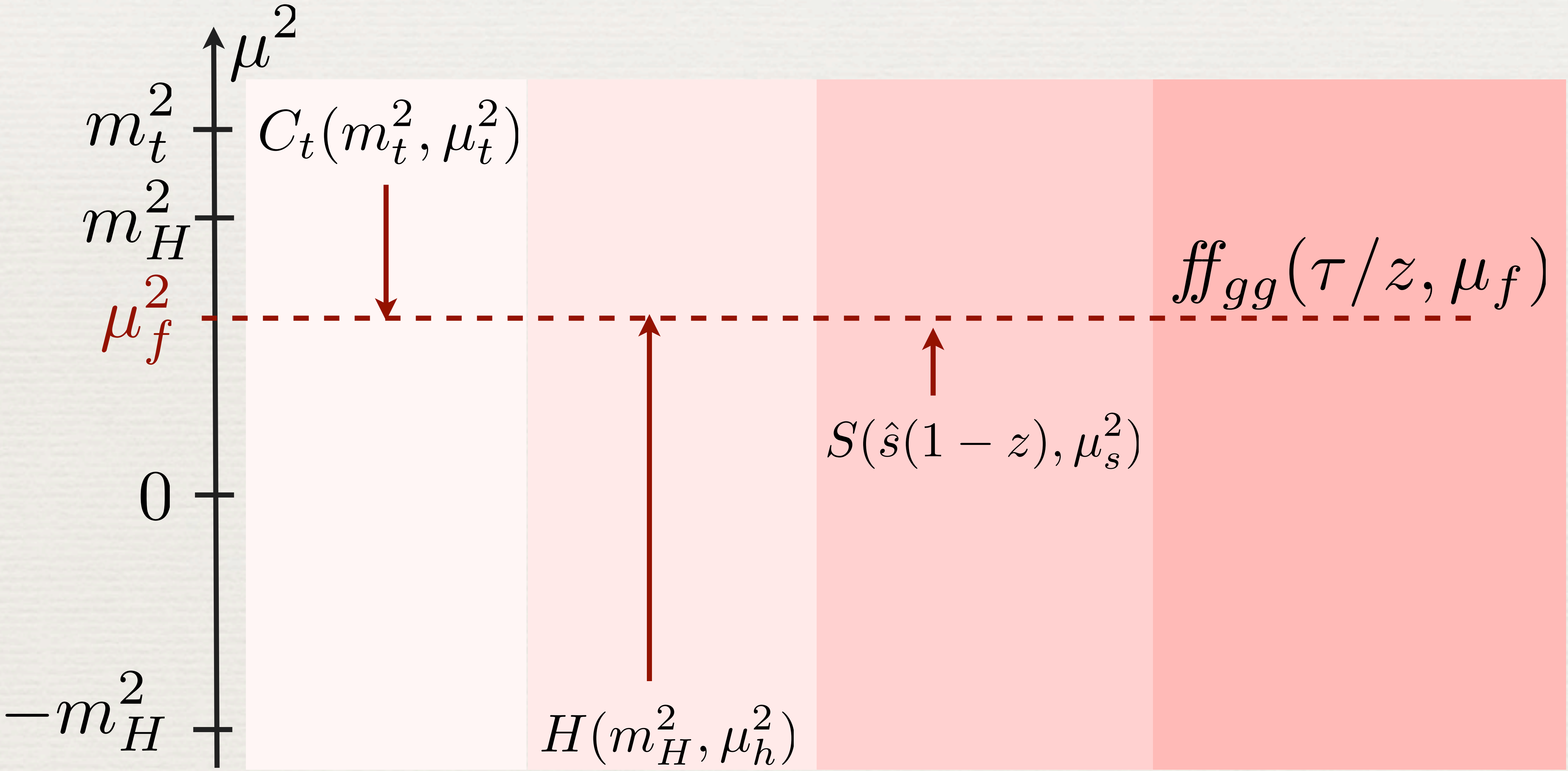
- Treating one scale at a time leads to a sequence of effective theories:



- Effects associated with each scale absorbed into matching coefficients

Scale hierarchy

- Evaluate each part at its characteristic scale and evolve to a common scale using RGEs:



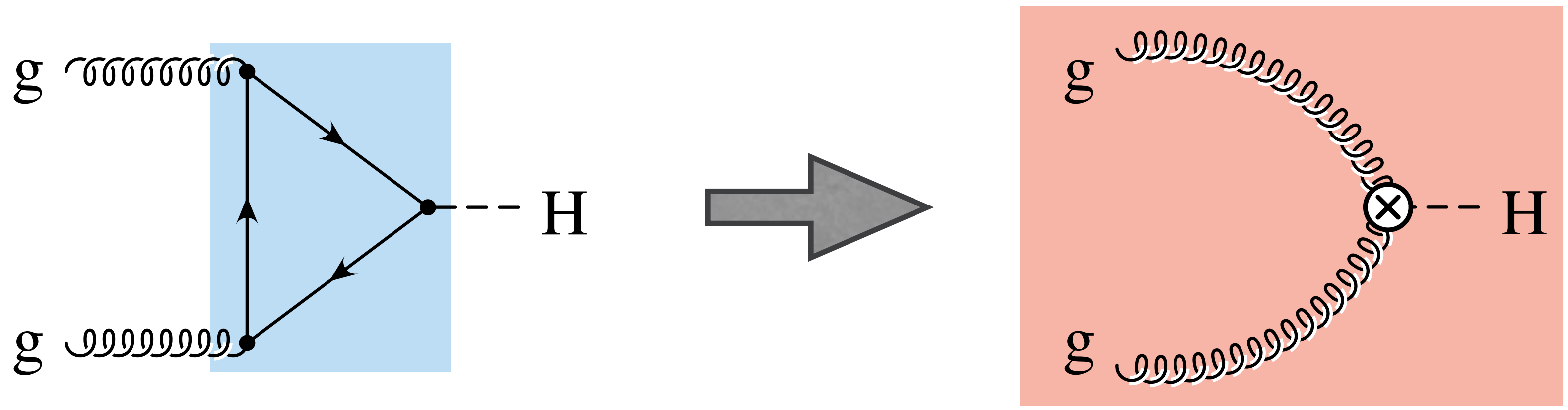
Advantages over standard approach

- Resummation directly in momentum space avoids Landau-pole ambiguity (Mellin inversion)
- Equivalent to Mellin-moment approach up to power corrections

Catani, de Florian, Grazzini, Nason 2003
Moch, Vogt 2005; Laenen, Magnea 2005;
Idilbi et al. 2005, 2006; Ravindran 2006

- Following EFT philosophy literally automatically resums class of large perturbative effects related to time-like kinematics of Higgs production, strongly reducing the K-factor to about 1.3 at

First step: integrate out the top



$$\mathcal{L}_{\rm eff} = C_t(m_t^2,\mu^2)\,\frac{H}{v}\,\frac{\alpha_s(\mu^2)}{12\pi}\,G_{\mu\nu,a}\,G_a^{\mu\nu}$$

- Matching coefficient exhibits good convergence at natural scale choice $\mu \approx m_t$:

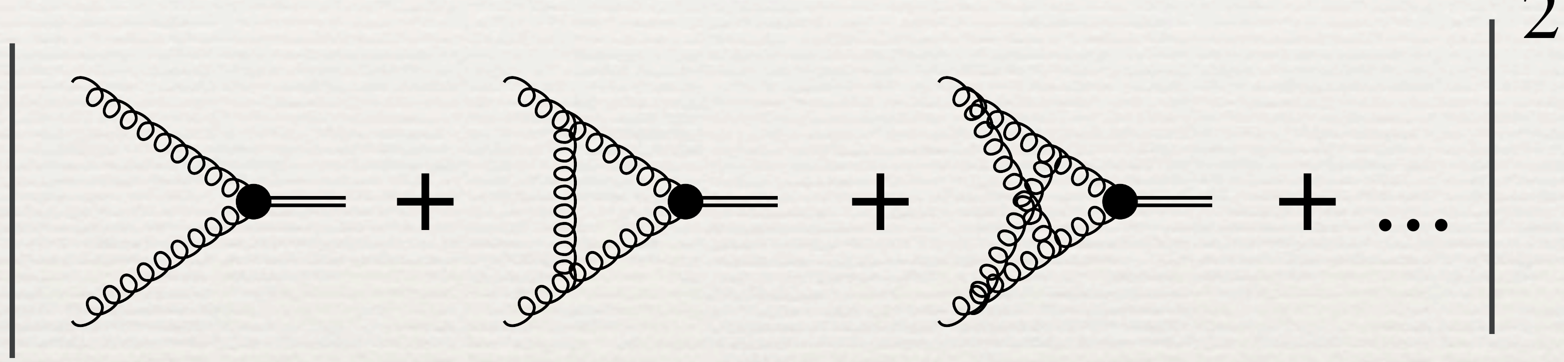
$$C_t(m_t^2,\mu) = 1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\frac{2777}{18} - 19\ln\frac{m_t^2}{\mu^2} + n_f\left(-\frac{67}{6} - \frac{16}{3}\ln\frac{m_t^2}{\mu^2}\right)\right] + \ldots$$

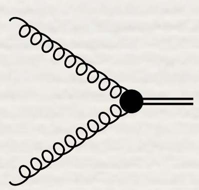
$$\approx 1 + 0.09 + 0.007 + \ldots \quad \text{for } \mu = m_t$$

Kramer, Laenen, Spira 1996; Chetyrkin, Kniehl, Steinhauser 1997

Second step: hard contributions H

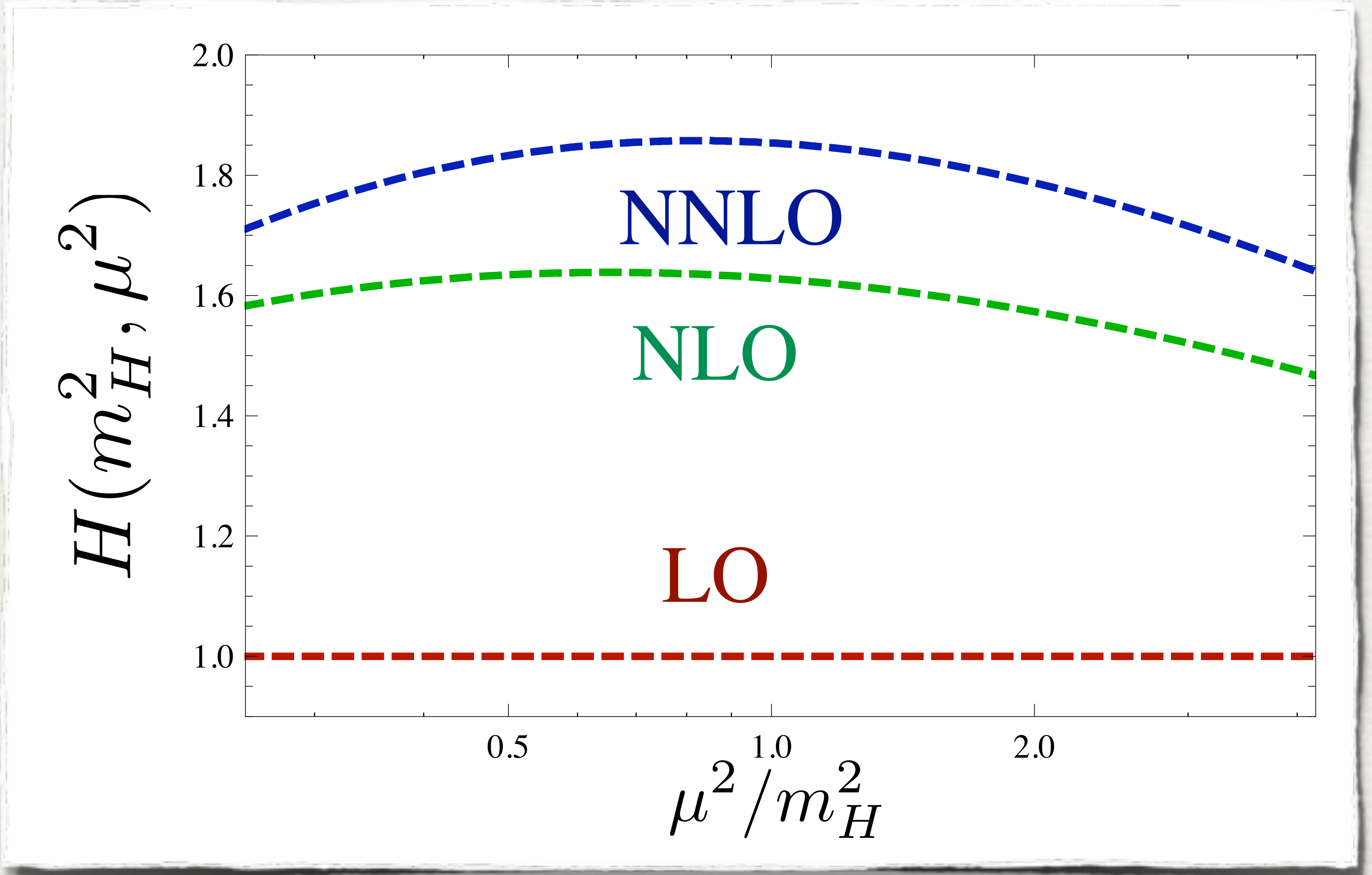
- Separate the contributions of the hard scale $\hat{s}$ from the soft scale $\hat{s}(1-z)^2$:

$H =$ 

where  $= C_t(m_t^2, \mu^2)\,\frac{H}{v}\,\frac{\alpha_s(\mu^2)}{12\pi}\,G_{\mu\nu,a}\,G_a^{\mu\nu}$

- H is the on-shell gluon form factor squared
- Simplest example of an on-shell QCD scattering amplitude!

Choice of the hard scale



- Matching corrections to hard function appear to be huge for any choice of scale !?!
- Break-down of EFT ?

Scalar form factor

- Hard function $H(m_H^2, \mu^2) = \left|C_S(-m_H^2 - i\epsilon, \mu^2)\right|^2$
- Scalar form factor

$$C_S(Q^2, \mu^2) = 1 + \sum_{n=1}^{\infty} c_n(L) \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^n, \quad L = \ln(Q^2/\mu^2)$$

$$c_1(L) = C_A \left(-L^2 + \frac{\pi^2}{6}\right)$$

Sudakov double logarithm

- Perturbative expansions:

space-like: $C_S(Q^2, Q^2) = 1 + 0.393\, \alpha_s(Q^2) - 0.152\, \alpha_s^2(Q^2) + \ldots$

time-like: $C_S(-q^2, q^2) = 1 + 2.75\, \alpha_s(q^2) + (4.84 + 2.07i)\, \alpha_s^2(q^2)$

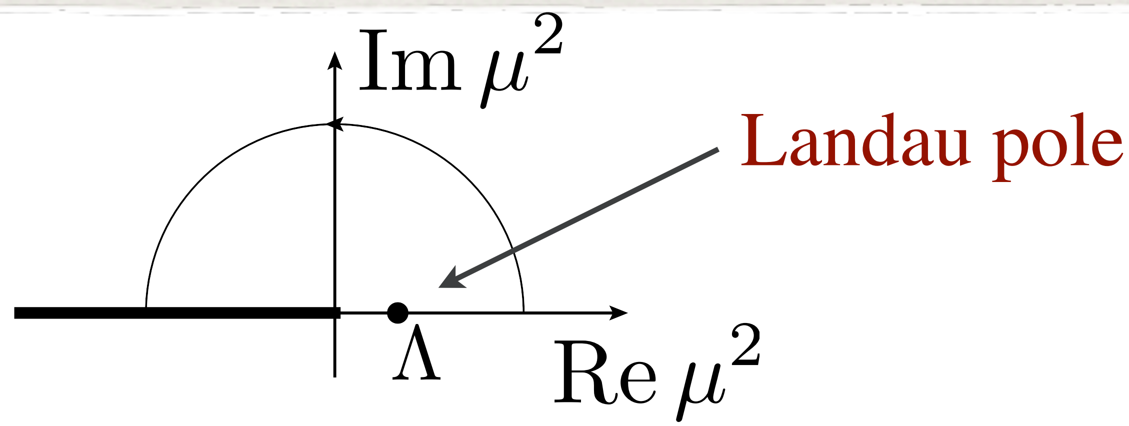
Solution

- Reason: $L \to \ln q^2/\mu^2 - i\pi$ and double logarithms give rise to π^2 terms
- Can avoid the large values of L by choosing a time-like matching scale $\mu^2 = -q^2$:

$$C_S(-q^2, -q^2) = 1 + 0.393\,\alpha_s(-q^2) - 0.152\,\alpha_s^2(-q^2) + \ldots$$

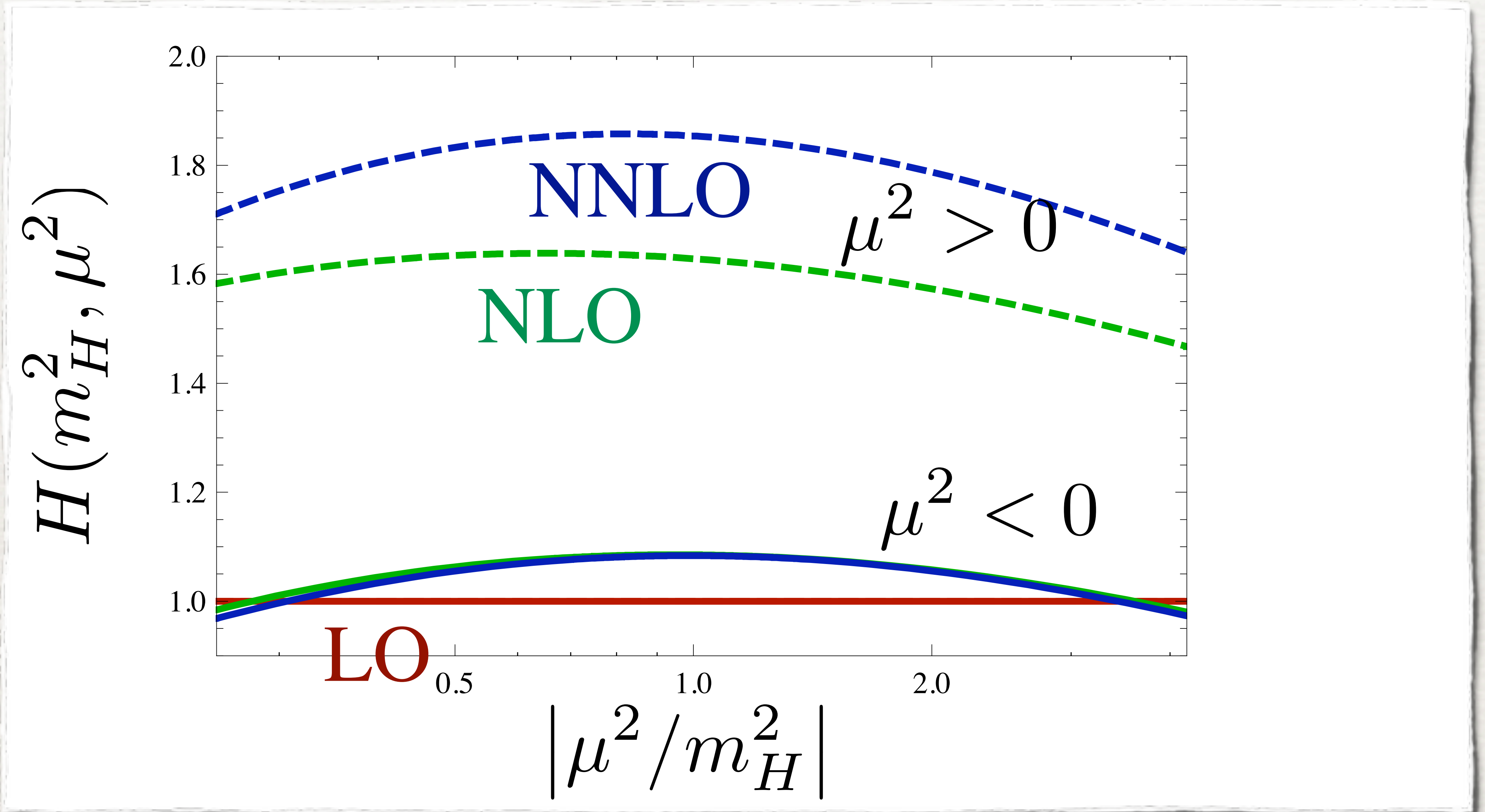
- Note: RG-evolution defines $\alpha_s(\quad^2)$ for *any* μ

$$\alpha_s(\mu^2) \sim \frac{1}{\ln(\mu^2/\Lambda^2)}$$



$$\alpha_s\left[-(120\,\text{GeV})^2 + i\epsilon\right] \approx 0.108 - 0.025i \qquad \alpha_s\left[(120\,\text{GeV})^2\right] \approx 0.114$$

Time-like vs. space-like μ²



- Convergence is very much better for $\mu^2 < 0$
- Evaluate H for $\mu^2 < 0$, where convergence is good, and use RG to evolve to other scales

RG evolution of hard function

- Hard function fulfills RG equation

$$\frac{d}{d\ln\mu} C_S(-m_H^2 - i\epsilon, \mu^2) = \left[\Gamma^A_{\text{cusp}}(\alpha_s)\,\ln\frac{-m_H^2 - i\epsilon}{\mu^2} + \gamma^S(\alpha_s)\right] C_S(-m_H^2 - i\epsilon, \mu^2)$$

produces Sudakov double log's

- Neglecting single logs and running of α_s (approximation for illustration only):

$$C_S(-m_H^2, \mu^2) = \exp\left(C_A \frac{\alpha_s}{4\pi} \ln^2 \frac{-m_H^2}{\mu^2}\right) \times C_S(-m_H^2, -m_H^2)$$

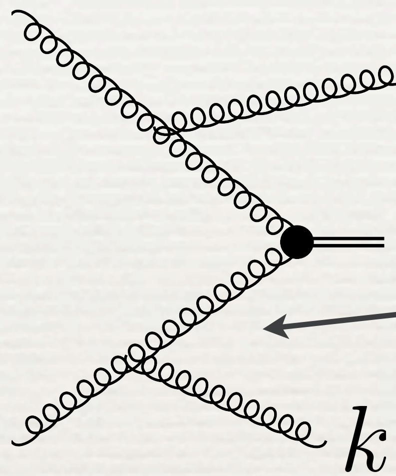
$$H(m_H^2, \mu^2 = +m_H^2) = \exp\left(C_A \frac{\alpha_s}{2\pi}\pi^2\right) \times |C_S(-m_H^2, -m_H^2)|^2$$

$$\approx 1.7$$

explains large K-factor!

Third step: soft contribution S

$p_2 = x_2 E\,\bar{n} = x_2 E\,(1, 0, 0, -1)$



$$\frac{1}{(p_1 - k)^2} = -\boxed{\frac{1}{2p_1 \cdot k}}$$

$p_1 = x_1 E\,n = x_1 E\,(1, 0, 0, 1)$

- Soft radiation involves eikonal propagators and is described by Wilson lines along n and $\bar{n}$

$$\begin{aligned} S_n(x) &= \exp\left\{ ig \int_{-\infty}^{0} ds\, n \cdot A(x + sn) \right\} \\ &= 1 + ig \int \frac{d^d k}{(2\pi)^d} \boxed{\frac{i}{n \cdot k}} n \cdot \tilde{A}(k)^{-ikx} + \dots \end{aligned}$$

Soft function $S(\sqrt{\hat{s}}(1-z),\mu)$

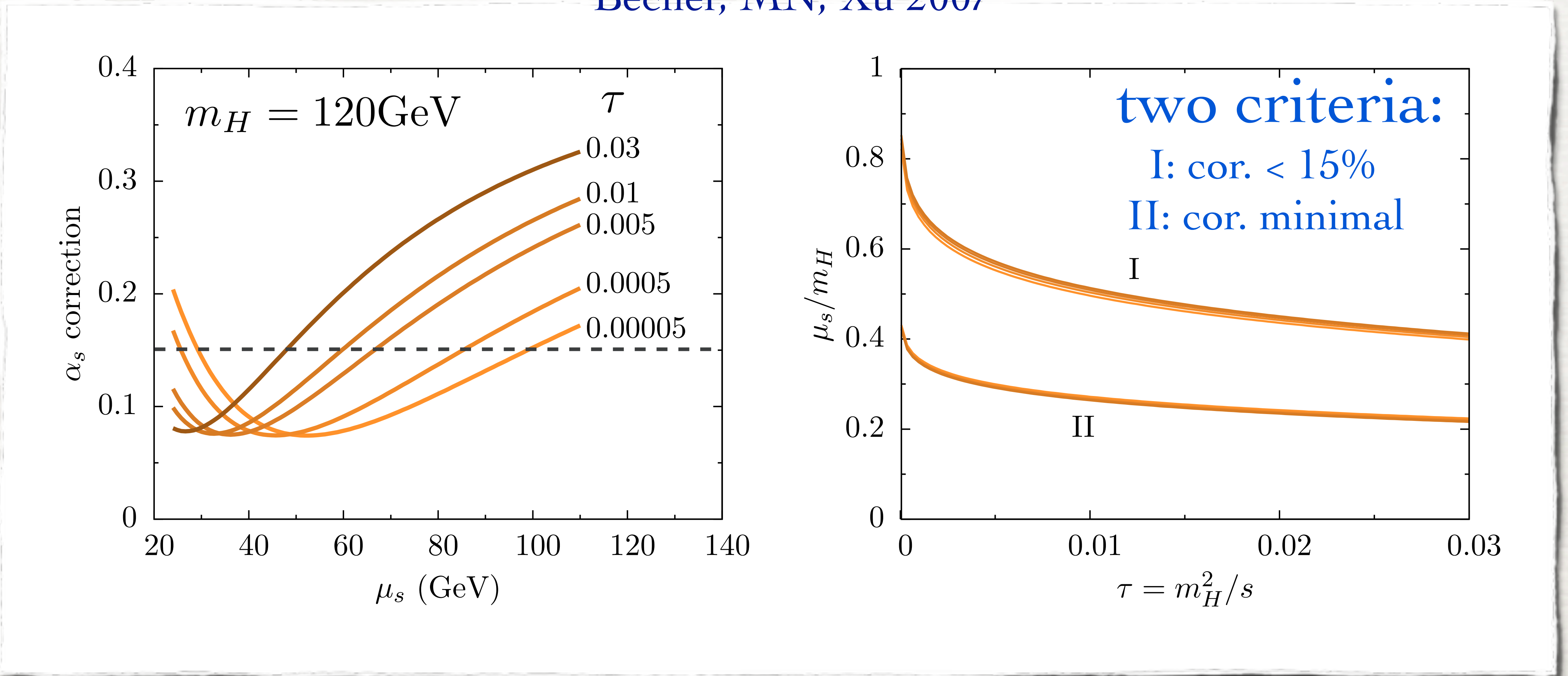
- Could avoid large logarithms by choosing the scale $\mu = \sqrt{\hat{s}}(1-z)$, but z is integrated up to 1
  - ill-defined convolution due to Landau-pole
- Instead choose scale such that the convolution integral

$$\int_\tau^1 \frac{dz}{z} S(\sqrt{\hat{s}}(1-z),\mu)\, ff_{gg}(\tau/z)$$

does not receive large corections

Choice of the soft scale

Becher, MN, Xu 2007



- Good perturbative behavior with $\mu_s \approx m_H/2$
- Indicates that soft-gluon resummation is not a parametrically large effect!

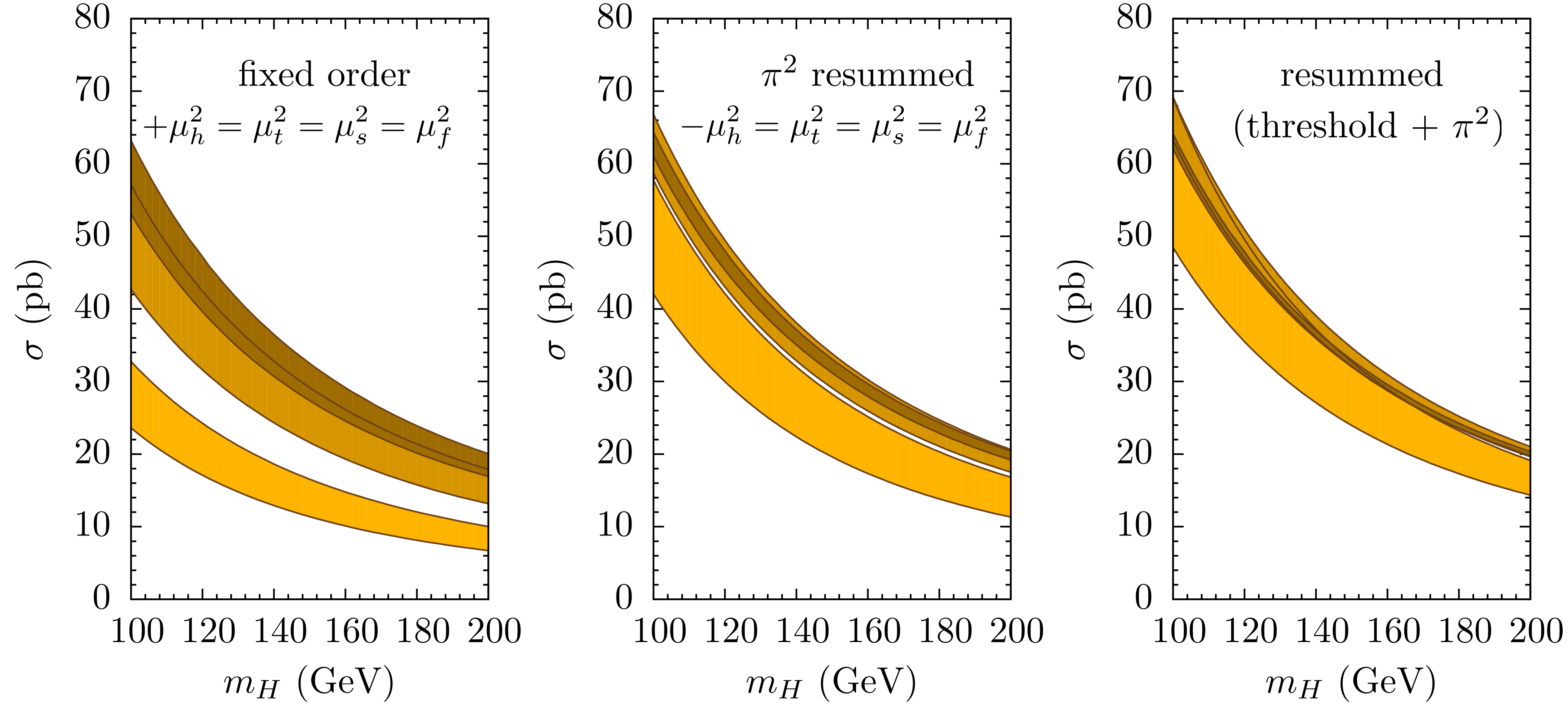
Resummed kernel (in z space)

$$C(z, m_t, m_H, \mu_f) = \left[C_t(m_t^2, \mu_t^2)\right]^2 \left|C_S(-m_H^2 - i\epsilon, \mu_h^2)\right|^2 U(m_H, \mu_t, \mu_h, \mu_s, \mu_f)$$
$$\times \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \widetilde{s}_{\text{Higgs}}\left(\ln \frac{m_H^2(1-z)^2}{\mu_s^2 z} + \partial_\eta, \mu_s^2\right) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

- Contribution of all scales separated, evolution factor U evolves from one scale to another
- Have performed matching to 2-loops, evolution to 3-loop accuracy

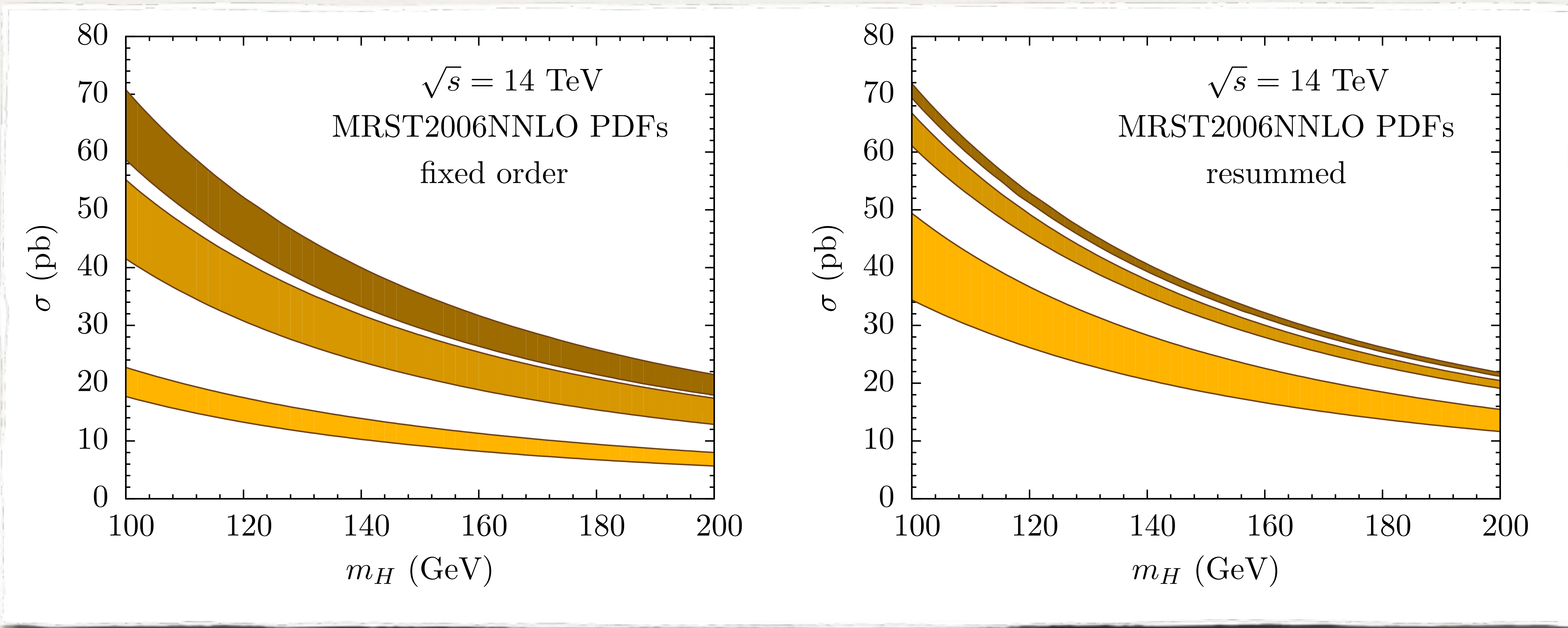
Phenomenological results

Cross sections at the LHC



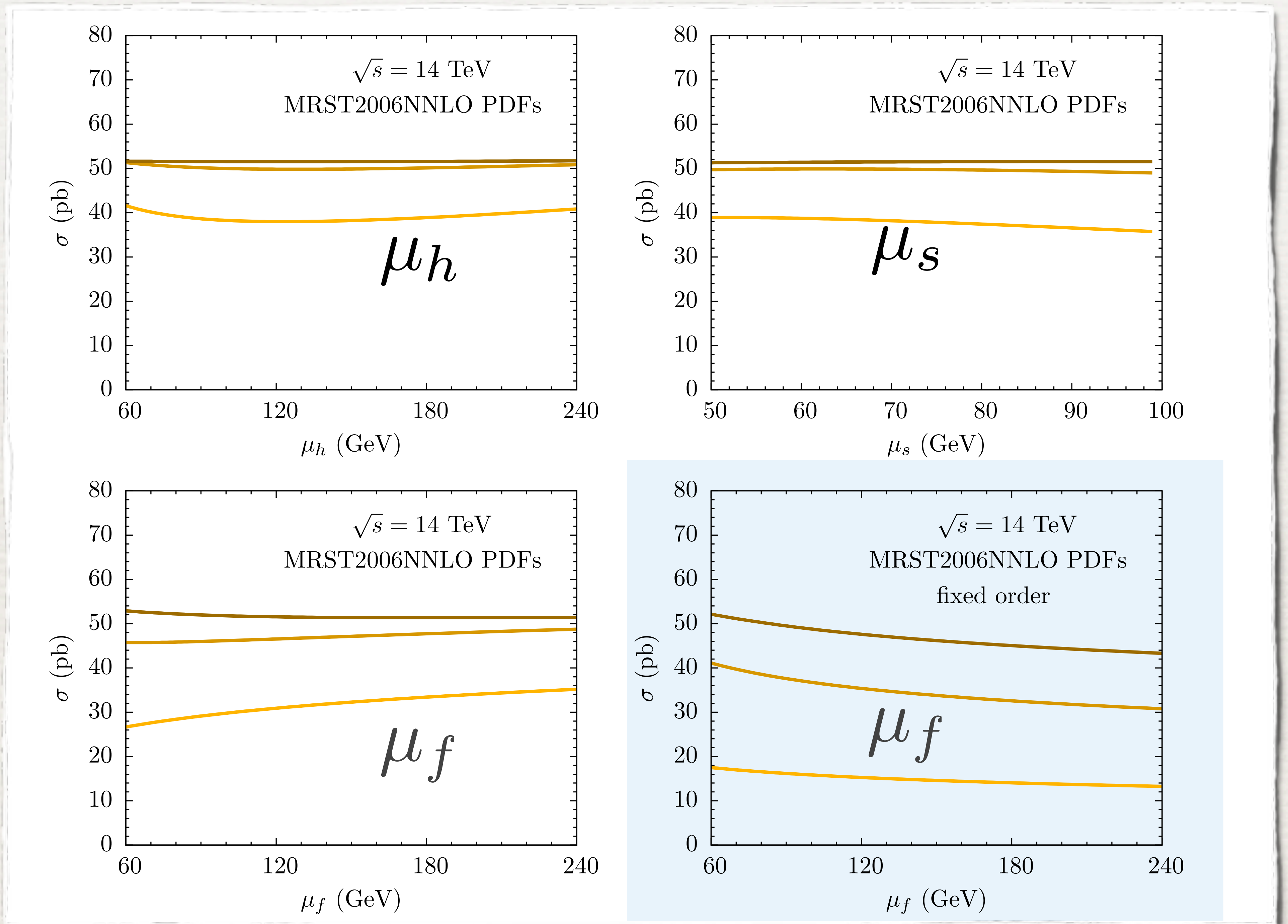
- Here use different MRST PDFs at each order: 2001LO, 2004NLO, 2004NNLO
- Faster convergence, smaller scale dependence after

Cross sections at the LHC



- Here use same PDFs in all orders
- With MRST2006NNLO result is ~10% higher than for MRST2004NNLO (higher value α_s=0.191, more low-x glue)

Scale dependence for m_H = 120 GeV



- Excellent stability at NNLO (negligible dependence on μ_t is not shown)

Comparison of different effects

	$\mu_h^2 > 0$		$\mu_h^2 < 0$	
	fixed order	threshold	π^2-enhanced	threshold + π^2
LO	$15.2^{+2.3+0.3}_{-2.0-0.3}$	$17.5^{+3.3+0.3}_{-2.7-0.3}$	$26.9^{+3.8+0.4}_{-3.7-0.5}$	$31.0^{+5.7+0.5}_{-4.8-0.6}$
NLO	$35.3^{+5.8+0.5}_{-4.5-0.6}$	$37.5^{+3.7+0.6}_{-1.1-0.7}$	$44.9^{+2.9+0.7}_{-3.2-0.8}$	$46.5^{+2.7+0.8}_{-1.2-0.8}$
NNLO	$47.5^{+4.6+0.8}_{-4.2-0.8}$	$48.5^{+2.5+0.8}_{-0.5-0.8}$	$51.5^{+1.7+0.9}_{-1.5-0.9}$	$51.5^{+1.4+0.9}_{-0.3-0.9}$

pdf uncertainty

scale uncertainty

cross section at LHC in pb for $m_H = 120$ GeV

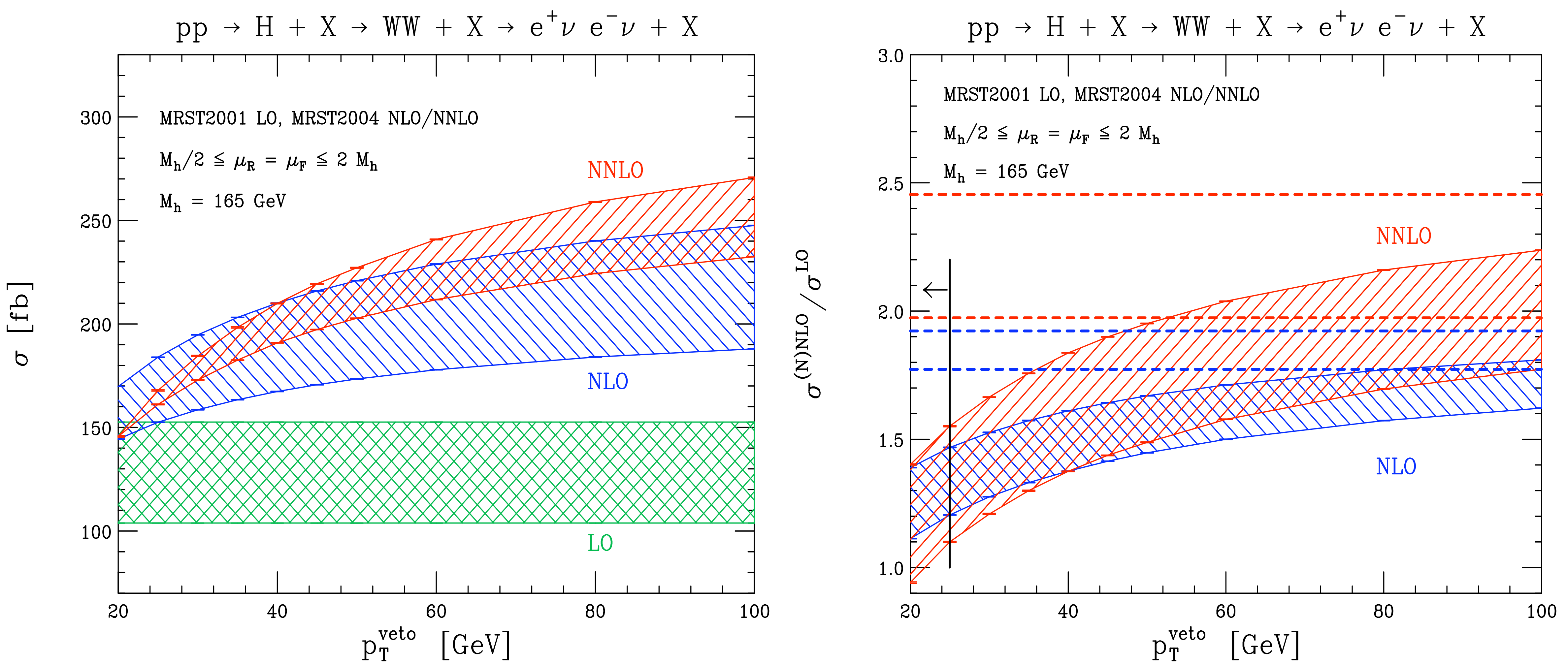
- additional uncertainty from α_s (approx. 6%)
- threshold resummation only has a small effect
- both resummations increase the cross section
- 8.4% increase over fixed-order NNLO result!
  (13% for Tevatron)

Summary

- Effective field theory (SCET) methods offer interesting new perspective on collider physics
- All-order understanding of IR singularities of on-shell n-parton scattering amplitudes!
- Intuitive understanding of factorization and resummation in momentum space
- Well-behaved perturbative results for important processes (Higgs production, Drell-Yan process, W and Z production, ...)

Backup Slide

Jet veto



$$|\eta| < 2.5 \text{ and } p_T^{\text{jet}} > p_T^{\text{veto}}$$

Anastasiou, Dissertori, Stöckli 2007