QCD resummation for Higgs production

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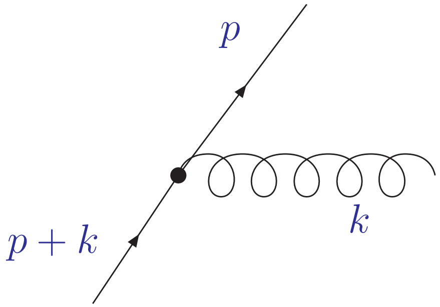
– *Workshop on Higgs Boson Phenomenology*, ETH and University of Zurich, Jan 7 - 9, 2009, Zurich –

Setting the stage

- Large logarithmic corrections soft/collinear regions of phase space



$$\frac{1}{(p+k)^2} \quad = \quad \frac{1}{2p\cdot k} \quad = \quad \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \quad \longrightarrow \quad \alpha_s \int dE_g \, d\theta_{qg} \, \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\longrightarrow \quad \alpha_s \, \ln^2(\ldots)$$

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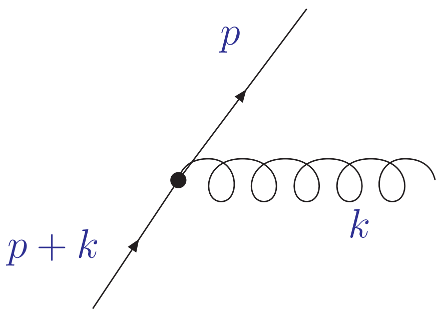

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$$\longrightarrow \alpha_s \ln^2(\dots)$$

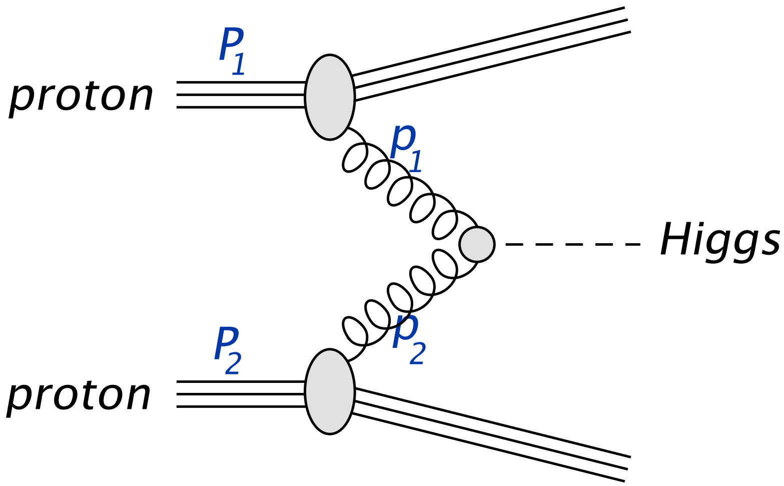
- Resummation
  - reorganize perturbative expansion $\longrightarrow$ stability
  - generating functional for higher orders of perturbation theory

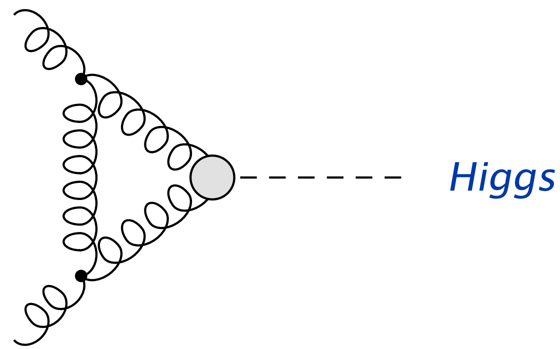
$$\begin{aligned}\mathcal{O} &= 1+\alpha\left(\ln^2+\ln+1\right)+\alpha^2\left(\ln^4+\ln^3+\ln^2+\ln+1\right)+\dots \\ &= \left(1+\alpha\, 1+\alpha^2\, 1+\dots\right)\exp\left(\alpha\, \ln^2+\alpha\, \ln+\alpha^2\, \ln+\dots\right)\end{aligned}$$

- Classical examples
  - QED form factor $x \to Q^2/m_e^2$ Sudakov '54
  - Deep-inelastic scattering $x \to 1$ Sterman '87; Catani, Trentadue '89

QCD factorization

- LHC: most prominent “signal”





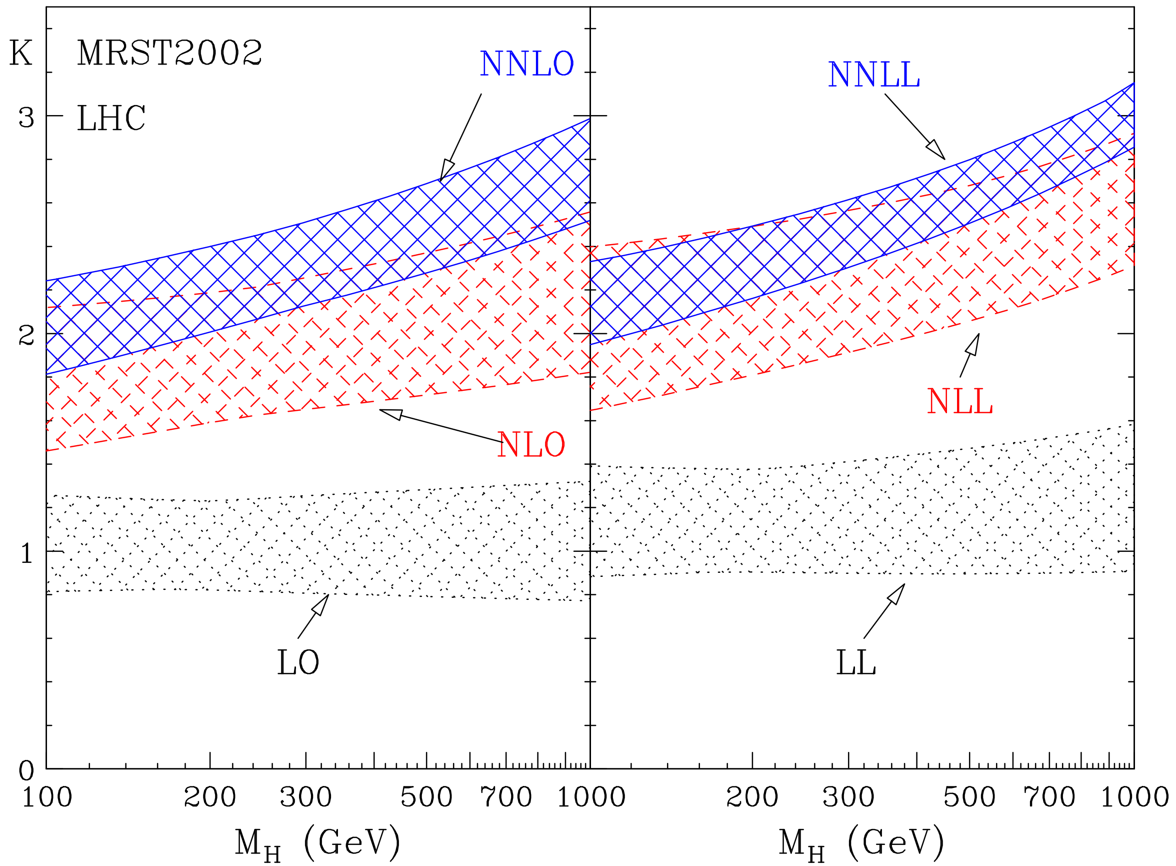
- Higgs production dominant in gg-fusion

- QCD factorization
  - hard parton cross section $\hat{\sigma}_{ij \to H}$
  - parton luminosity $f_i \otimes f_j$

$$\sigma_{pp \to H} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to H} \left(\alpha_s(\mu^2), Q^2, \mu^2 \right)$$

  - hard scale Q, factorization scale μ

Total Higgs cross section and resummation



- Cross section at LHC with scale variation:
  fixed order predictions (left) and resummed perturbation series (right)
  - NNLO corrections

    Harlander, Kilgore ‘02; Anastasiou, Melnikov ‘02; Ravindran, Smith, van Neerven ‘03
  - NNLL resummation

    Catani, Grazzini, de Florian, Nason ‘03

Parton cross section at large N/ large x

- Cross section $\hat{\sigma}_{ij\to H}$ (gg $\to$ Higgs at large m_t) depends on $x = \dfrac{M_H^2}{s}$
  - $x \simeq 1$ close to threshold
  - in large x-limit have large logarithms at n$^{\rm th}$-order

$$\alpha_s^n \left(\frac{\ln^{2n-1}(1-x)}{1-x} \right)_+ \longleftrightarrow \alpha_s^n \ln^{2n}(N)$$

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- Threshold resummation in Mellin space

$$\hat{\sigma}^{\,N} \; = \; (1 + \alpha_s \, g_{01} + \alpha_s^2 \, g_{02} + \ldots) \cdot \exp\left(G^N\right) \; + \; \mathcal{O}(N^{-1} \ln^n N)$$

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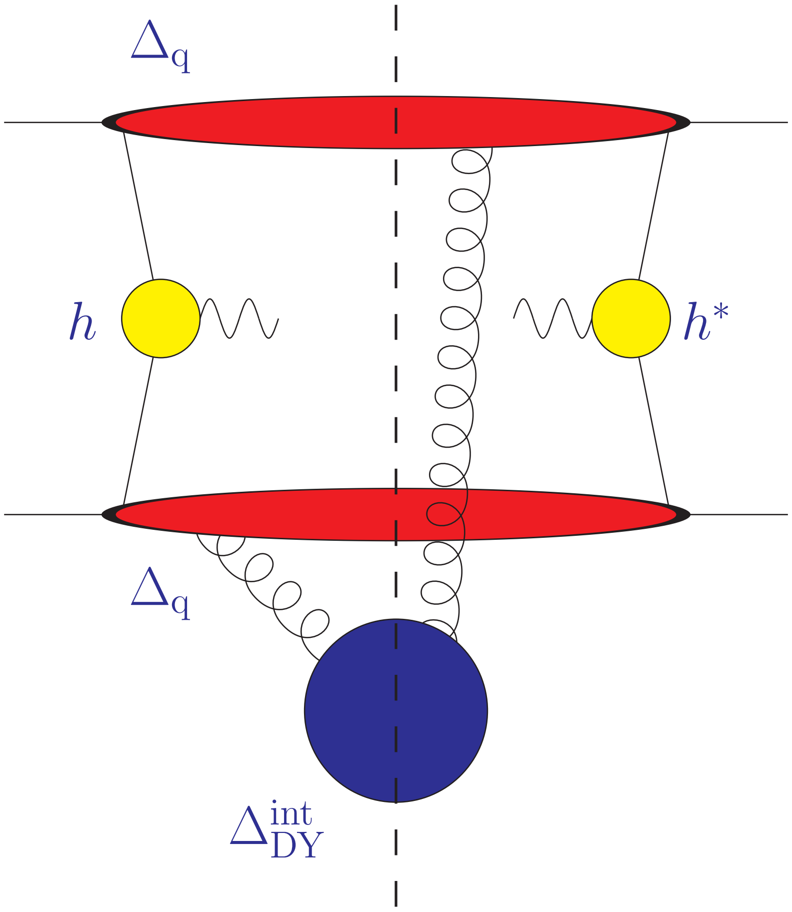
- Exponent for Higgs, Drell-Yan and DIS

$$\begin{aligned} G^N_{\rm H} &= 2 \ln \Delta_{\rm g} + \ln \Delta^{\rm int}_{\rm H} \\ G^N_{\rm DY} &= 2 \ln \Delta_{\rm q} + \ln \Delta^{\rm int}_{\rm DY} \\ G^N_{\rm DIS} &= \ln \Delta_{\rm q} + \ln J_{\rm q} + \ln \Delta^{\rm int}_{\rm DIS} \end{aligned}$$

Soft and collinear factorization

- Separation of parton dynamics in different regions
  Sterman ‘87; Catani, Trentadue ‘89; Contopanagos, Laenen, Sterman ‘97
  - operator definitions for different functions
  - renormalization group equations from factorization



The radiative factors

- Renormalization group equations for functions $\Delta_{\rm p}$, $\Delta_{\rm H}^{\rm int}$, $\Delta_{\rm DY}^{\rm int}$ and $J_{\rm p}$
  - well-known exponentiation from factorization in soft/collinear limit
  - well-known anomalous dimensions

    Vogt '00; Catani, Grazzini, de Florian, Nason '03; S.M., Vermaseren, Vogt '05

- $\Delta_{\rm p}$: soft collinear radiation off initial state parton p
  - cusp anomalous dimension $A_{\rm p}$

$$\ln \Delta_{\rm p} \quad = \quad \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \int_{\mu_f^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_{\rm p} \left(\alpha_s(q^2)\right)$$

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- $\Delta^{\rm int}_{\rm DY/H}$ process dependent gluon emission at large angles
  - $D_{\rm p}$ for $\Delta^{\rm int}_{\rm DY/H}$
- $J_{\rm p}$: collinear emission from “unobserved” final state parton p
  - $B_{\rm p}$ for $J_{\rm p}$

Accuracy under control

- Control over logarithms $\ln(N)$ with $\lambda = \beta_0 \alpha_s \ln(N)$ to N^{k}LL accuracy

$$G^N = \ln(N) g_1(\lambda) + g_2(\lambda) + \alpha_s g_3(\lambda) + \alpha_s^2 g_4(\lambda) + \ldots$$

  - $g_1(\lambda)$: LL Sterman '87; Appell, Mackenzie, Sterman '88
  - $g_2(\lambda)$: NLL Catani Trenatdue '89
  - $g_3(\lambda)$: NNLL or N^{2}LL Vogt '00; Catani, Grazzini, de Florian, Nason '03
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Strategies

- Evaluate resummed cross section (requires matching to fixed order)

$$\hat{\sigma}^{\mathrm{res}} \quad = \quad \int\limits_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \frac{dN}{2\pi \mathrm{i}} \, x^{-N} \left(\hat{\sigma}^N - \hat{\sigma}^N \bigg|_{\mathrm{N^kLO}} \right) + \hat{\sigma}^{\mathrm{N^kLO}}$$

- Use resummed G^N as generating functional of perturbation theory
  - predict towers of large logarithms at fixed orders

Drell-Yan process in soft limit

- N^{3}LO results for Drell-Yan process in $q\bar{q}$-annihilation with $x = M^2/s$
  S.M., Vogt '05
  - check at NNLO Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - cross check on $\frac{1}{(1-x)_+}$-term at N^{3}LO Laenen, Magnea '05

$$
\begin{aligned}
c_3^{\mathrm{DY}} \;=\;& \left(\frac{\ln^5(1-x)}{1-x}\right)_+ \left\{512 C_F^3\right\} + \dots + \\
& \frac{1}{(1-x)_+}\Big\{ C_A^2 C_F \Big[-\frac{594058}{729} + \frac{98224}{81}\zeta_2 + \frac{40144}{27}\zeta_3 - \frac{2992}{15}\zeta_2^2 \\
& -\frac{352}{3}\zeta_2\zeta_3 - 384\,\zeta_5\Big] + C_A C_F^2 \Big[\frac{25856}{27} - \frac{12416}{27}\zeta_2 + \frac{26240}{9}\zeta_3 + \frac{1408}{3}\zeta_2^2 \\
& -1472\,\zeta_2\zeta_3\Big] - C_F^3 \Big[4096\,\zeta_3 + 6144\,\zeta_2\zeta_3 - 12288\,\zeta_5\Big] - C_F n_f^2 \Big[\frac{3712}{729} \\
& -\frac{640}{27}\zeta_2 - \frac{320}{27}\zeta_3\Big] + C_A C_F n_f \Big[\frac{125252}{729} - \frac{29392}{81}\zeta_2 - \frac{2480}{9}\zeta_3 + \frac{736}{15}\zeta_2^2\Big] \\
& -C_F n_f \Big[6 - \frac{1952}{27}\zeta_2 + \frac{5728}{9}\zeta_3 + \frac{1472}{15}\zeta_2^2\Big] \Big\}
\end{aligned}
$$

Higgs production in soft limit

- N^{3}LO results for Higgs production in gluon-gluon fusion with $x = M^2/s$
  S.M., Vogt '05
  - check at NNLO
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03

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& -\frac{23200}{3}\,\zeta_2\zeta_3 + 11904\,\zeta_5 \big] + C_A^2 n_f \big[\frac{125252}{729} - \frac{34768}{81}\,\zeta_2 - \frac{7600}{9}\,\zeta_3 \\
& -\frac{544}{15}\,\zeta_2^2 \big] + C_A C_F n_f \big[\frac{3422}{27} - 32\,\zeta_2 - \frac{608}{9}\,\zeta_3 - \frac{64}{5}\,\zeta_2^2 \big] \\
& -C_A n_f^2 \big[\frac{3712}{729} - \frac{640}{27}\,\zeta_2 - \frac{320}{27}\,\zeta_3 \big] \Big\}
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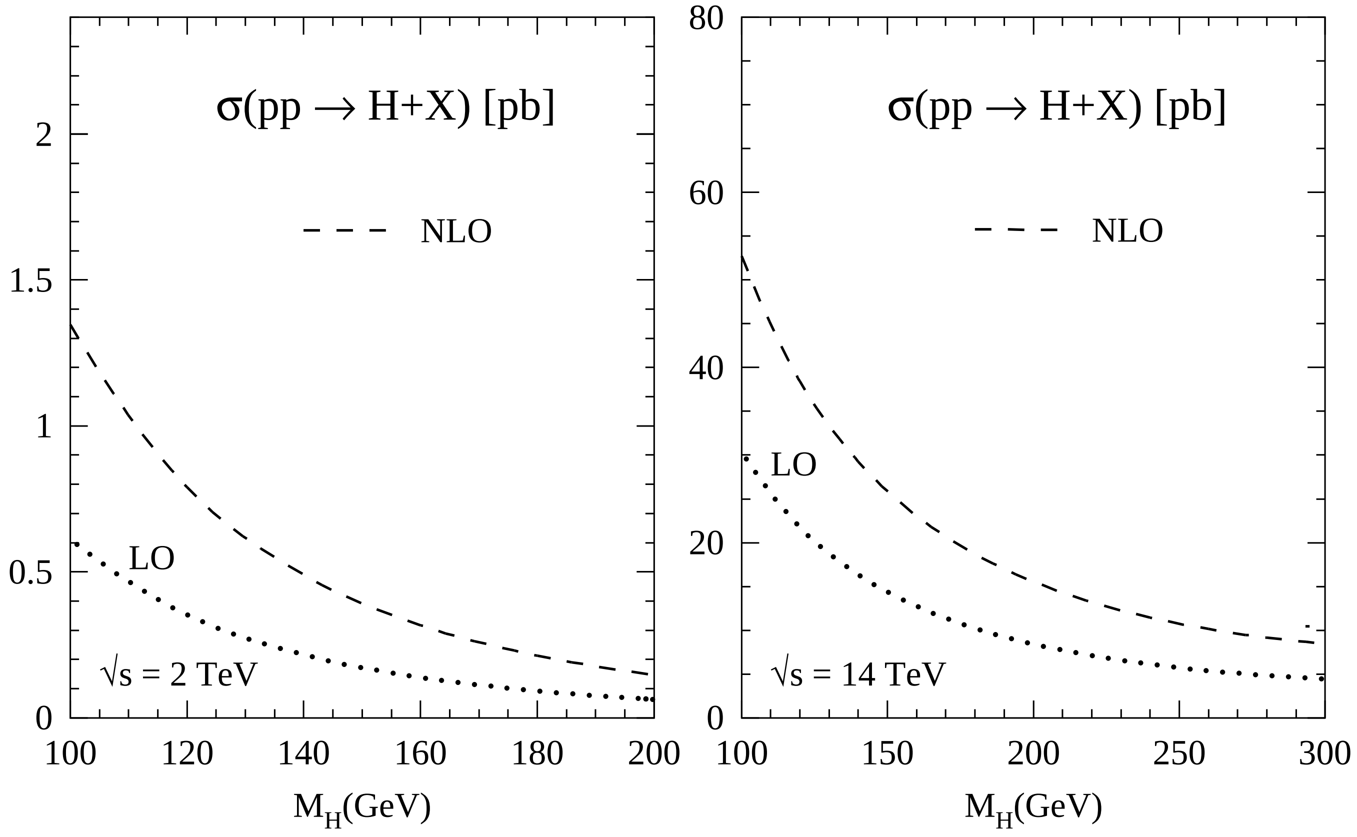
Cross section Higgs production



- QCD corrections to total cross section at Tevatron (left) and LHC (right)

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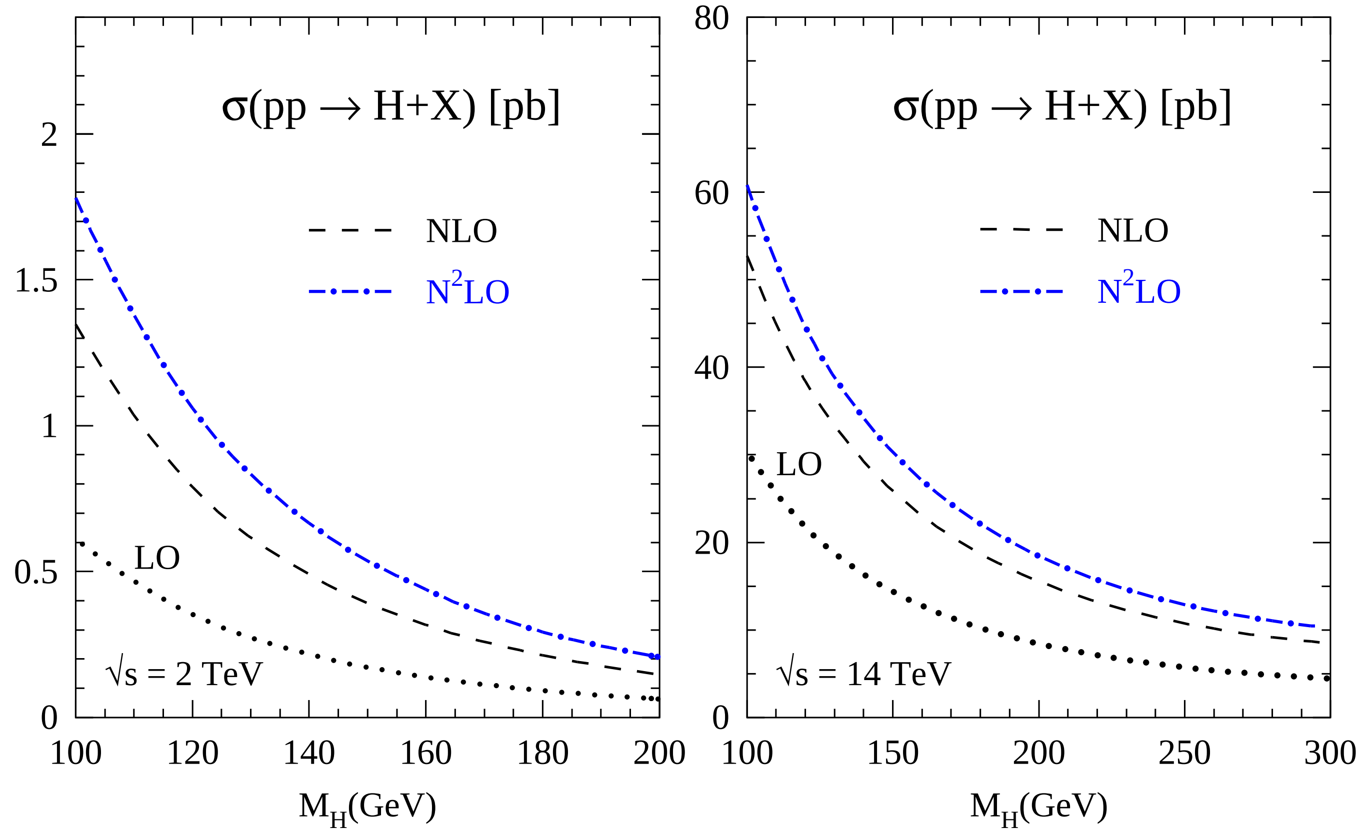

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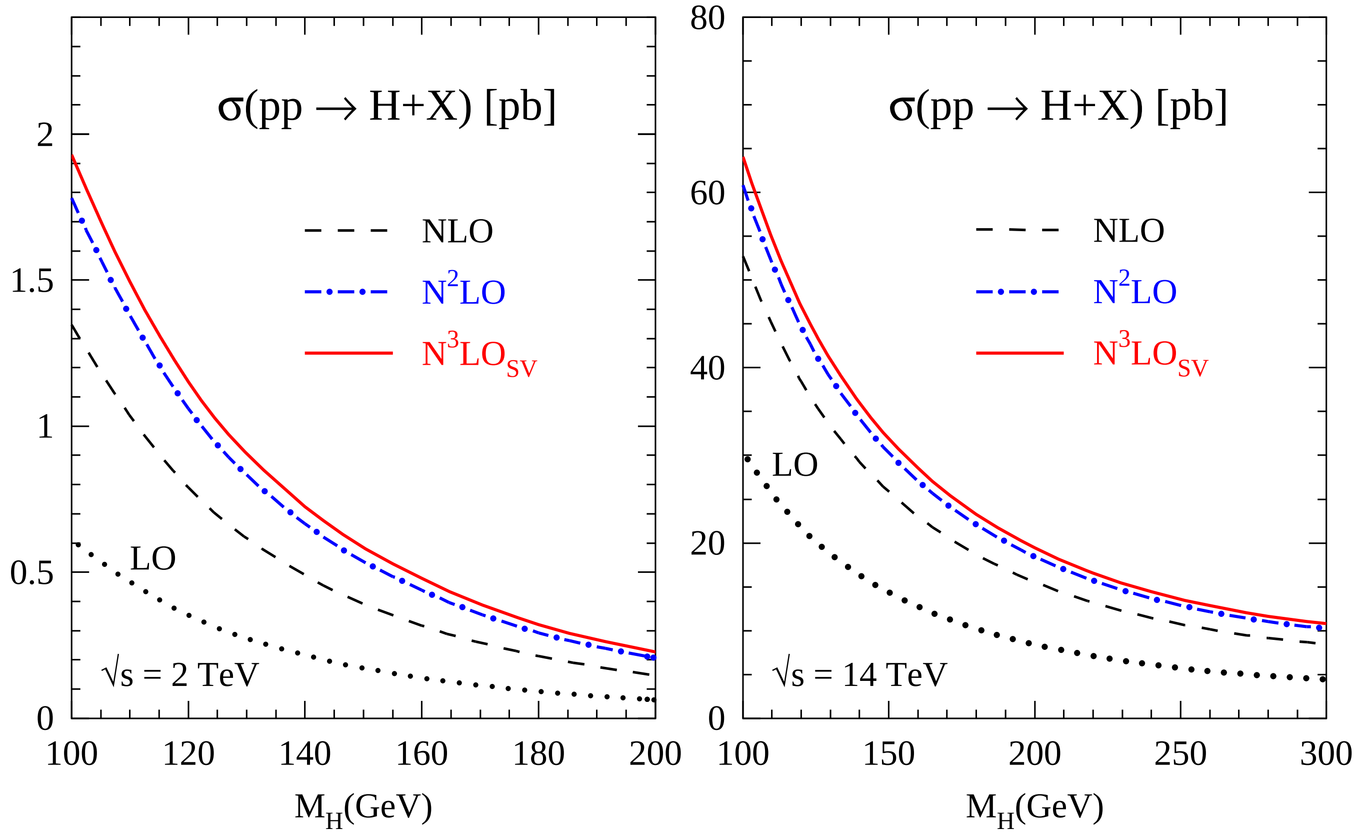

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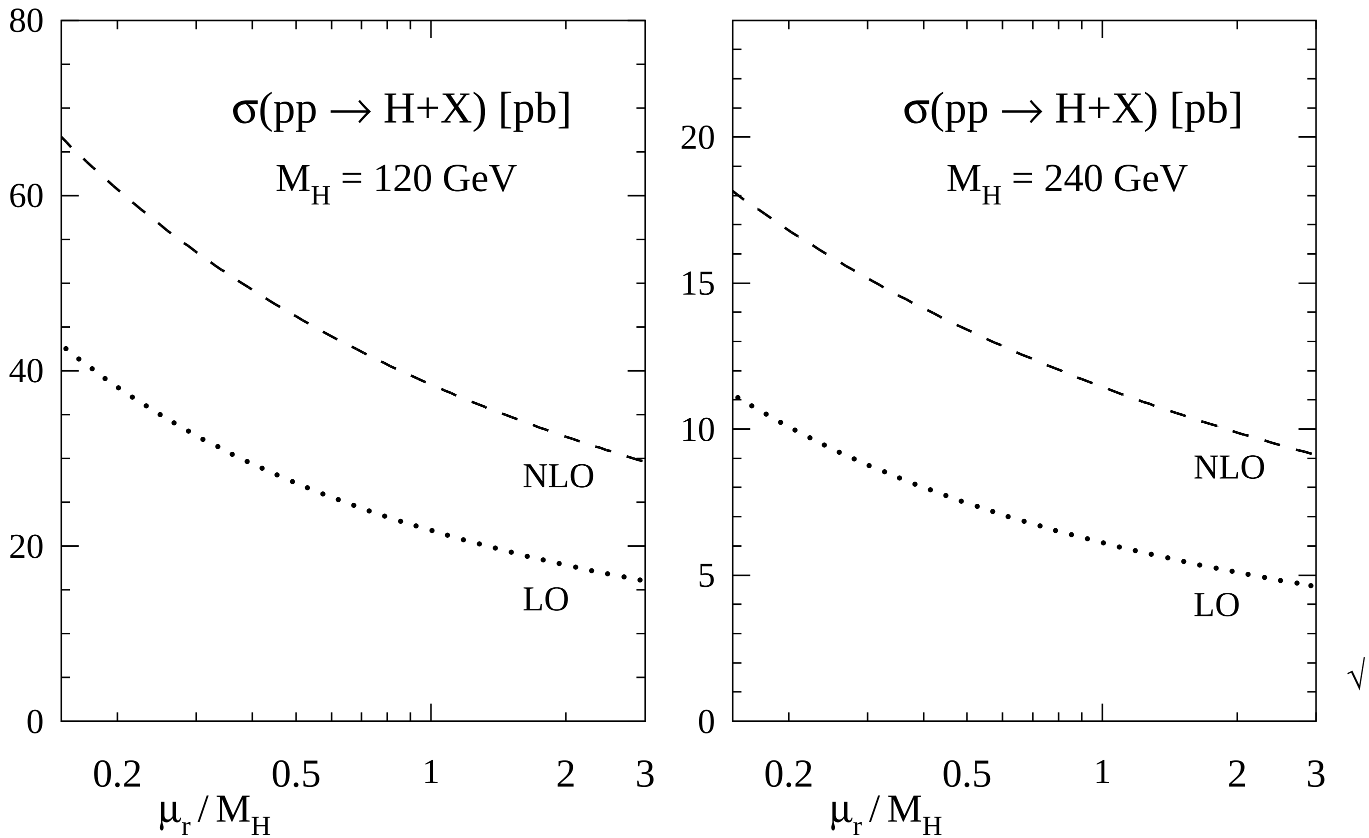
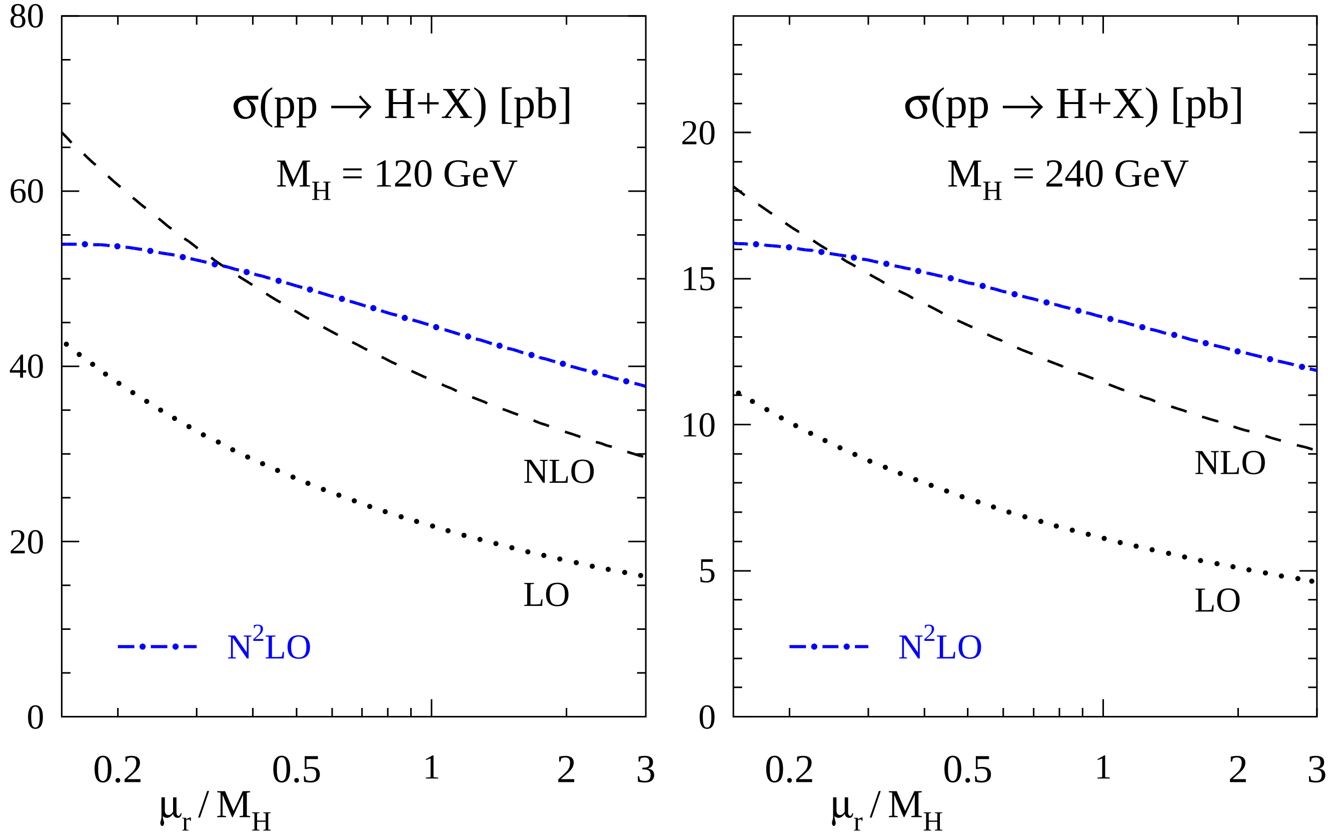
  - complete soft N^3LO corrections S.M., Vogt '05

Cross section Higgs production (cont'd)



- Variation of cross section at LHC with renormalizaion scale for different Higgs masses: $M_H = 120$GeV (left) and $M_H = 240$GeV (right)

Cross section Higgs production (cont'd)



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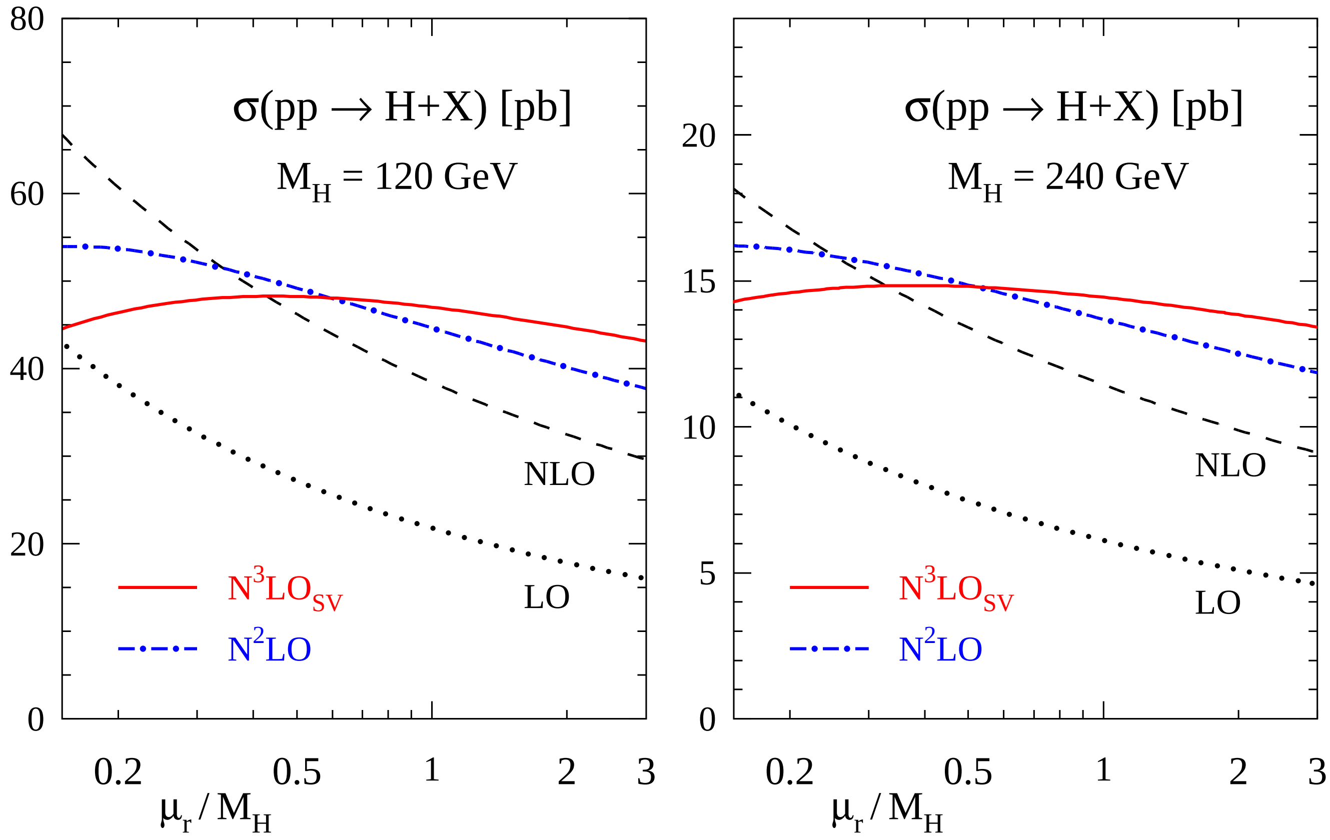

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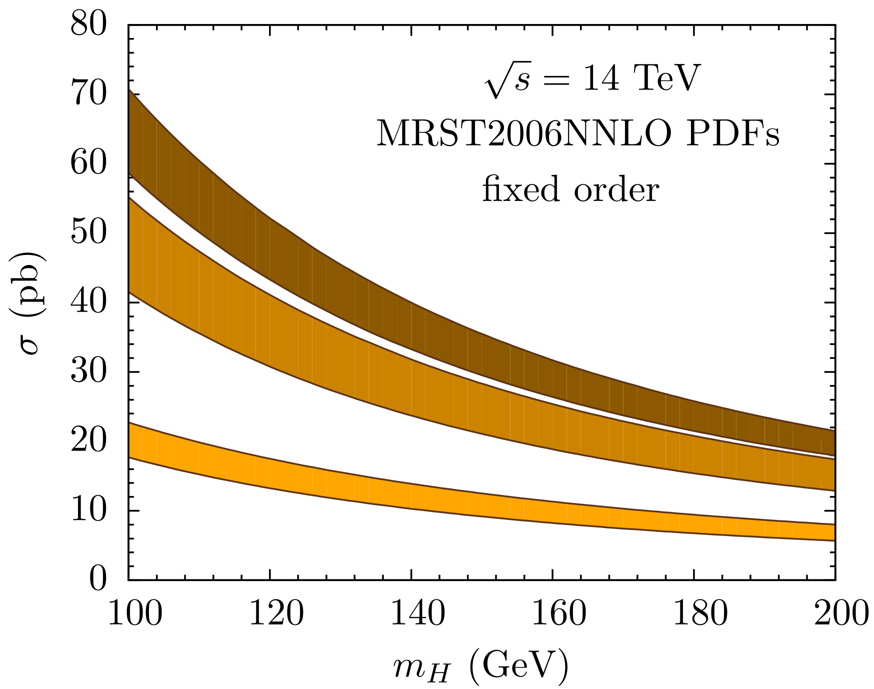
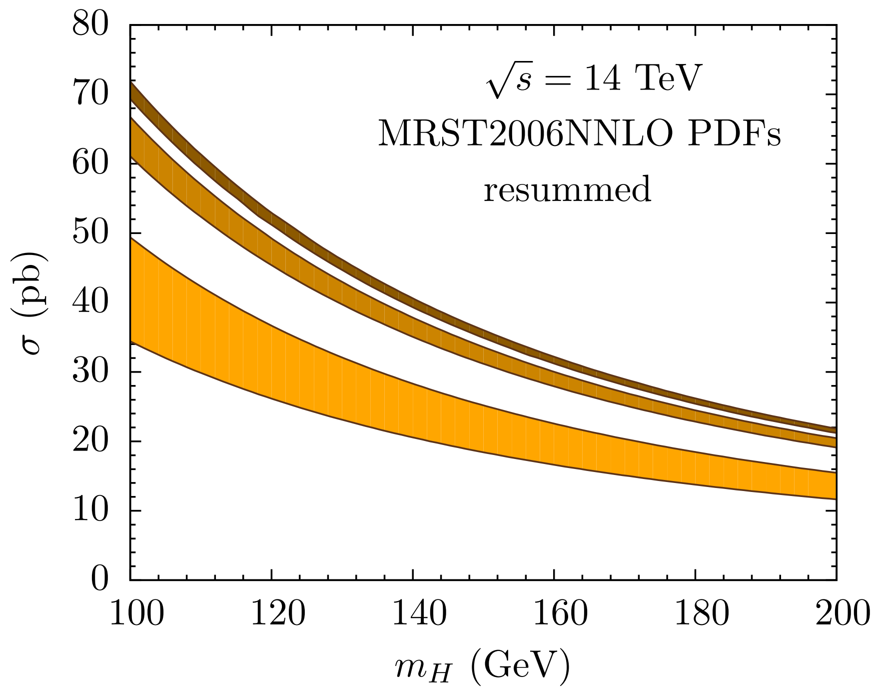
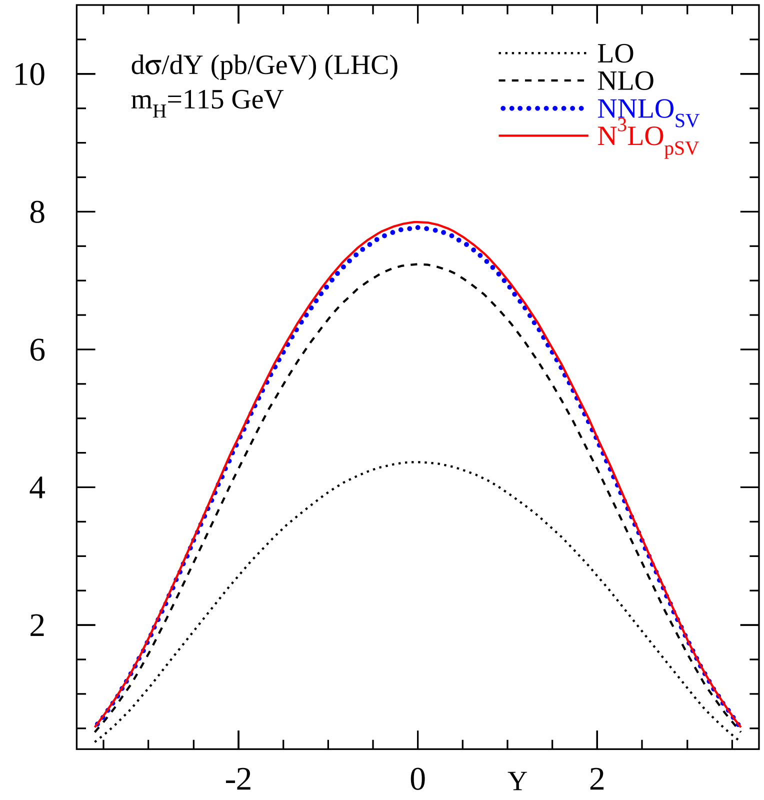
Recent phenomenological application

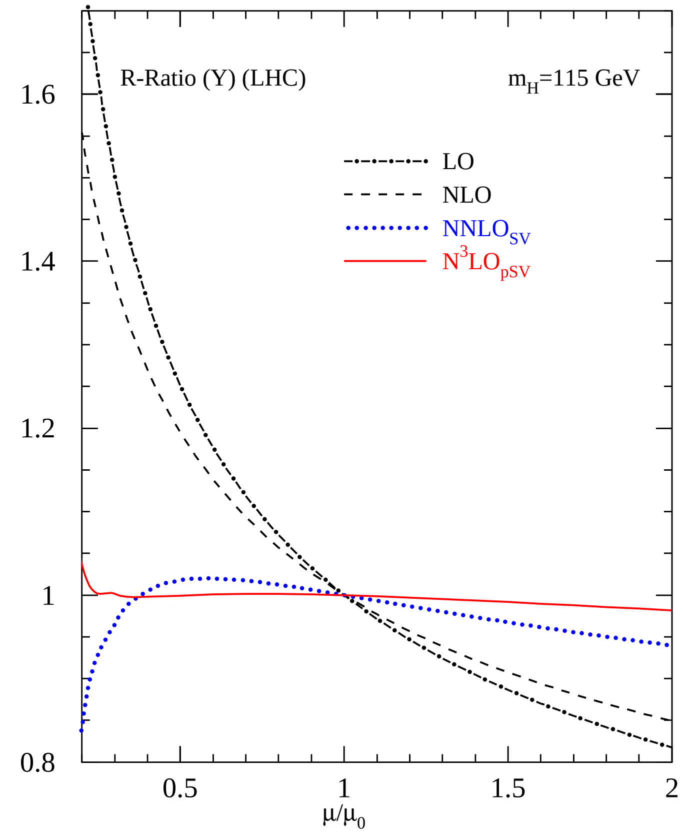




- Updated Higgs-production cross section Ahrens, Becher, Neubert, Yang ‘08
  - fixed-order perturbation theory (left)
  - resummation of soft gluons and π^2-enhanced terms (right)
- Improved convergence for resummed result

Going differential in the soft limit



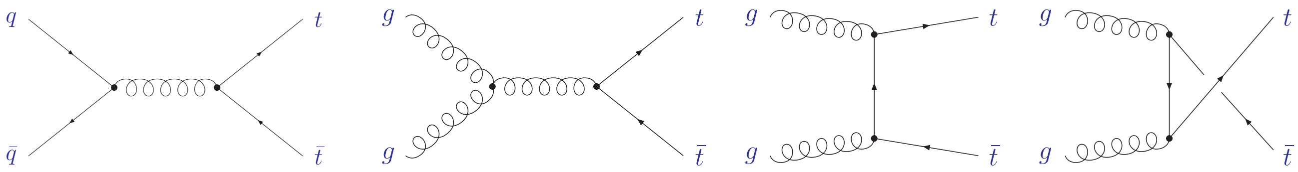


- Rapidity distribution in gluon fusion with complete soft+virtual $\mathrm{NNLO_{SV}}$ and soft $\mathrm{N^3LO_{pSV}}$ corrections Ravindran, Smith, van Neerven '06
  - $M_H = 115$GeV and $\mu = M_H$ (left)
  - renormalization scale dependence for $M_H = 115$GeV (right)

Top quark production

- Leading order Feynman diagrams

$$\mathrm{q} + \bar{\mathrm{q}} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$
$$\mathrm{g} + \mathrm{g} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$



- NLO in QCD Nason, Dawson, Ellis '88; Beenakker, Smith, van Neerven '89; Mangano, Nason, Ridolfi '92; Bernreuther, Brandenburg, Si, Uwer '04; . . .
  - accurate to $\mathcal{O}(15\%)$ at LHC
- Much activity towards higher orders in QCD
  - small-mass limit $m^2 \ll s, t, u$ for two-loop virtual corrections to $\mathrm{q}\bar{\mathrm{q}} \to \mathrm{t}\bar{\mathrm{t}}$ and $\mathrm{gg} \to \mathrm{t}\bar{\mathrm{t}}$ S.M., Czakon, Mitov '07
  - full mass dependence for two-loop virtual $\mathrm{q}\bar{\mathrm{q}} \to \mathrm{t}\bar{\mathrm{t}}$ Czakon '08
  - analytic two-loop fermionic corrections for $\mathrm{q}\bar{\mathrm{q}} \to \mathrm{t}\bar{\mathrm{t}}$ Bonciani, Ferroglia, Gehrmann, Maitre, Studerus '08
  - one-loop squared terms (NLO $\times$ NLO) Anastasiou, Mert Aybat '08; Kniehl, Merebashvili, Körner, Rogal '08

Threshold resummation

- Threshold at $s \simeq 4m^2$
  - parton cross section exhibit Sudakov-type logarithms $\ln(\beta)$ with velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$ at n$^{\mathrm{th}}$-order
- All order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
  - resummation in Mellin space (renormalization group equation)
- Resummed cross section in Mellin space

$$\frac{\hat{\sigma}^N_{ij,\,I}(m^2)}{\hat{\sigma}^{(0),N}_{ij,\,I}(m^2)} = g^0_{ij,\,I}(m^2) \cdot \exp\left(G^{N+1}_{ij,\,I}(m^2)\right) + \mathcal{O}(N^{-1}\ln^n N)$$

  - exponent in singlet-octet color basis decomposition $I = 1, 8$

$$G^N_{q\bar{q}/gg,\,I} \quad = \quad G^N_{\mathrm{DY/Higgs}} + \delta_{I,8} G^N_{Q\bar{Q}}$$

- Renormalization group equations for functions $G^N_{\mathrm{DY/Higgs}}$ and $G^N_{Q\bar{Q}}$
  - $G^N_{Q\bar{Q}}$ accounts for gluon emission from octet final state

New results

- NNLO cross section for heavy-quark hadro-production near threshold (all powers of $\ln\beta$ and Coulomb corrections) S.M., Uwer '08
  - e.g. gg-fusion for $n_f = 5$ light flavors at $\mu = m$

$$\hat{\sigma}^{(1)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}} \left\{ 96\ln^2\beta - 9.5165\ln\beta + 35.322 + 5.1698\frac{1}{\beta} \right\}$$

$$\hat{\sigma}^{(2)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}} \left\{ 4608\ln^4\beta - 1894.9\ln^3\beta + \left(-3.4811 + 496.30\frac{1}{\beta}\right)\ln^2\beta \right.$$
$$\left. + \left(3144.4 + 321.17\frac{1}{\beta}\right)\ln\beta + 68.547\frac{1}{\beta^2} - 196.93\frac{1}{\beta} + C^{(2)}_{gg} \right\}$$

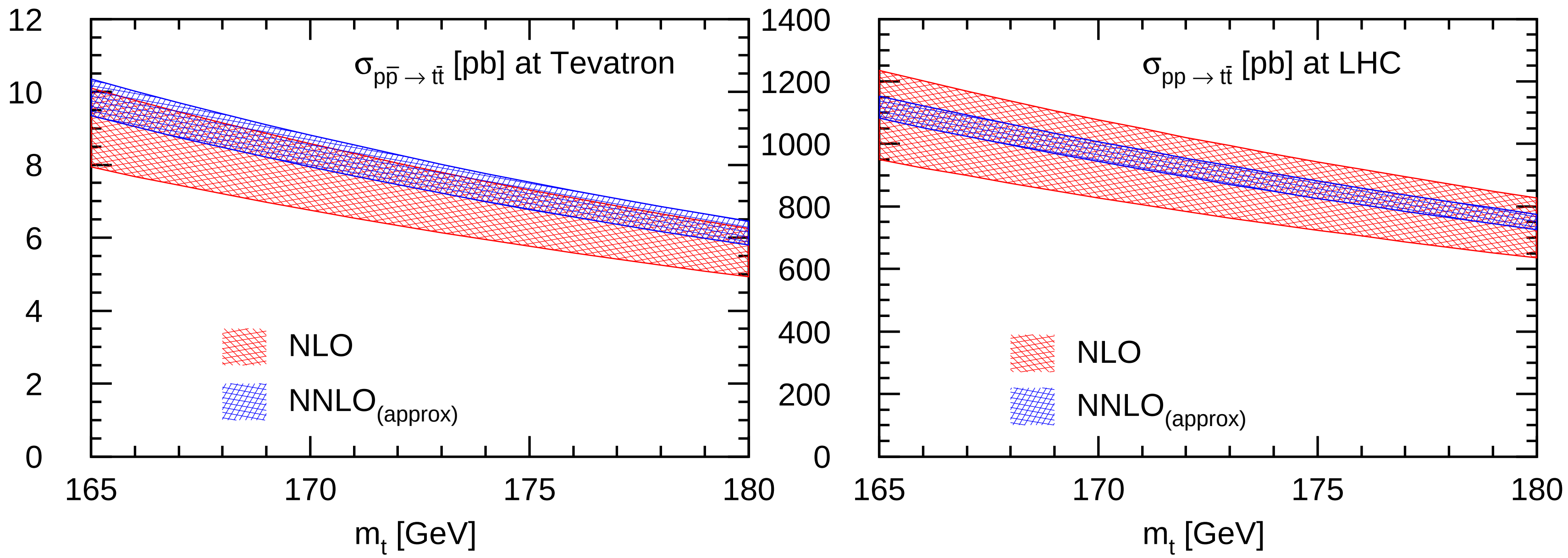
- Add all scale dependent terms
  - $\ln(\mu/m)$-terms exactly known from renormalization group methods

Upshot

- Best approximation to complete NNLO
- Similar results for new massive colored particles (4th generation quarks, squarks, gluinos, ...)
  S.M., Uwer '08; S.M., Langenfeld '09

Top-quark pair-production at NNLO

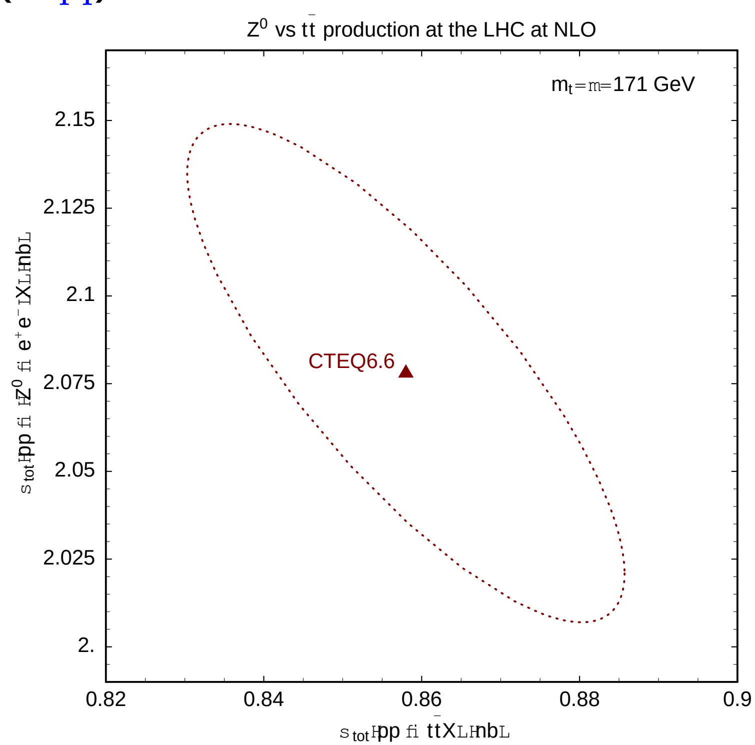
- NLO (with CTEQ6.5 PDF set)
  - scale uncertainty $\mathcal{O}(10\%) \oplus$ PDF uncertainty $\mathcal{O}(5\%)$
- NNLO$_{\mathrm{approx}}$ (with MRST2006 PDF set)
  - scale uncertainty $\mathcal{O}(3\%) \oplus$ PDF uncertainty $\mathcal{O}(2\%)$



- Theory at NNLO matches anticipated experimental precision $\mathcal{O}(10\%)$

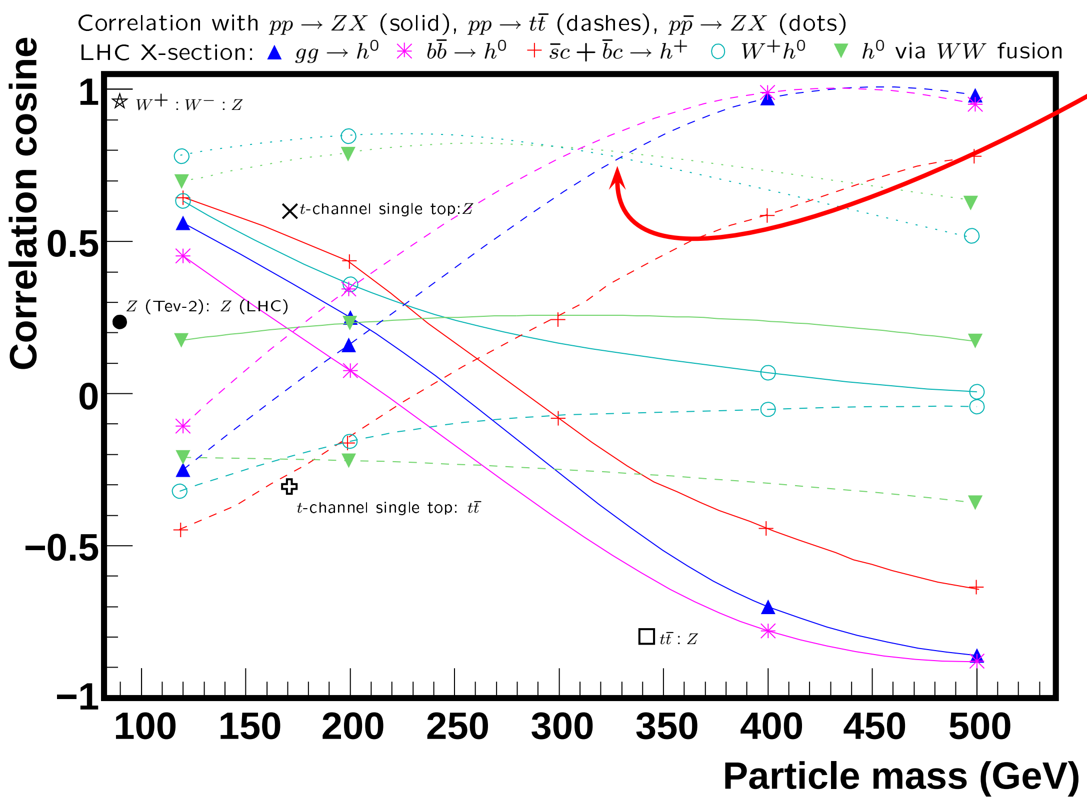
Parton luminosity

- Uncertainties in parton luminosity reduced in ratios of cross sections
  - well-known idea Dittmar, Paus, Zürcher ‘97
  - $W^{\pm}$, Z boson production “standard candle” for $L_{q\bar{q}}$ at LHC
- Drell-Yan process through $q\bar{q}$-annihilation
  - sensitive to quark PDFs at LHC ($L_{q\bar{q}}$)
- Cross section $t\bar{t}$-production at LHC
  - anti-correlated with Z boson production
  - sensitive to gluon PDFs (L_{gg}) CTEQ ‘08



Standard candles

- $\sigma_{\mathrm{pp}\to\mathrm{t\bar{t}}}$ at LHC correlated with Higgs boson production (line - - - -) espacially for larger Higgs masses



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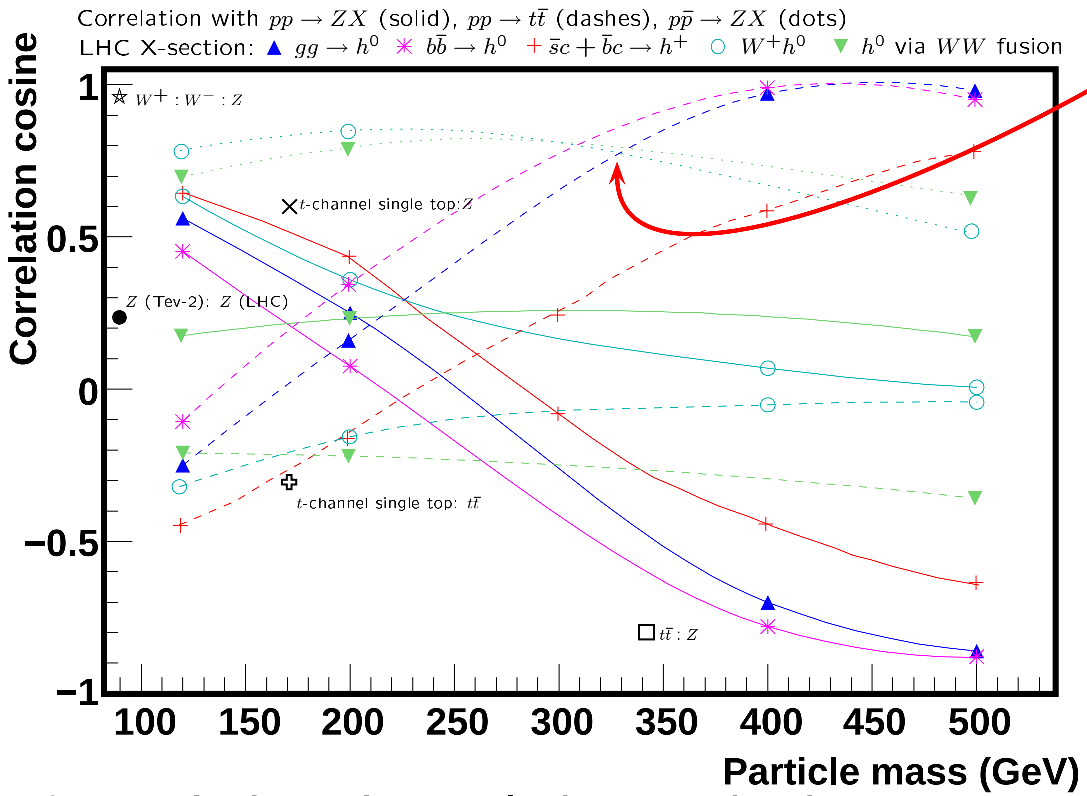

- Proposal of PDF-induced correlation method CTEQ '08
  - $\sigma_{\mathrm{pp}\to\mathrm{t\bar{t}}}$ as benchmark for all processes which are anti-correlated with Z boson production

Summary

- Soft and collinear gluons often source of large higher order corrections
- Resummation stabilizes perturbative prediction
  - apparent convergence and scale dependence
- Resummed cross section generates (yet uncalculated) higher orders
  - approximate N^{k}LO results
- Higgs production $\sigma_{\mathrm{pp}\to\mathrm{H}}$
  - N^{3}LO approximations to total rate and rapidity distribution
- Top-quark pair production $\sigma_{\mathrm{pp}\to\mathrm{t\bar{t}}}$
  - N^{2}LO approximate total cross section
  - sensitivity to gluon luminosity

Extra slides

Factorization in D-dimensions

Engineering the soft and collinear limit

- Forward Compton amplitude $\mathcal{T}_n$ in $D = 4 - 2\epsilon$-dimensions

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  - pure real-emission contributions $\mathcal{S}_n$
    (dependent on D-dimensional +-distributions $f_{k,\epsilon}$)

$$f_{k,\epsilon}(x) \;=\; \epsilon[\,(1-x)^{-1-k\epsilon}\,]_+ \;=\; -\frac{1}{k}\,\delta(1-x) + \sum_{i=0} \frac{(-k\epsilon)^i}{i\,!}\,\epsilon\left(\frac{\ln^i(1-x)}{1-x}\right)_+$$

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- Laurent-series for $\mathcal{T}_n$ at $n^{\rm th}$-order
  - mass-factorization predicts $\dfrac{1}{\epsilon^n}$
  - soft and collinear singularities in $\mathcal{F}_n$ and $\mathcal{S}_n$ behave as $\dfrac{1}{\epsilon^{2n}}$

Form factors in time-like kinematics

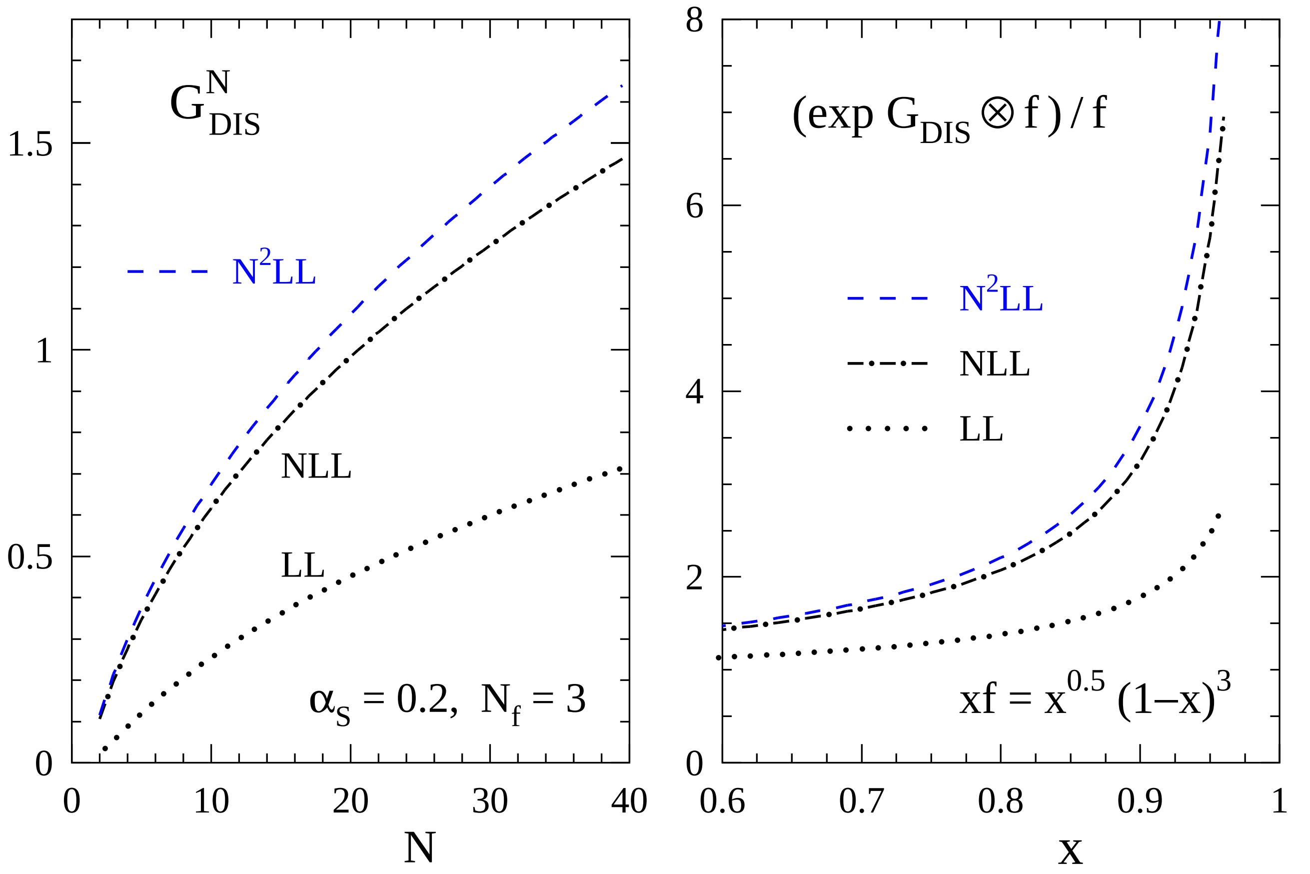
- Ratio of renormalized time-like and space-like form factors $|\mathcal{F}(q^2)/\mathcal{F}(-q^2)|$ (infrared finite)
  - analytic continuation $q^2 \to -q^2$
  - ratio known to four-loops (expansion in $a_s = \alpha_s/(4\pi)$)

$$\left|\frac{\mathcal{F}(q^2)}{\mathcal{F}(-q^2)}\right|^2 = 1 + a_s\{3\zeta_2 A_1\} + a_s^2\left\{\frac{9}{2}\zeta_2^2 A_1^2 + 3\zeta_2(\beta_0 G_1 + A_2)\right\}$$
$$+ a_s^3\left\{\frac{9}{2}\zeta_2^3 A_1^3 + 3\zeta_2^2 A_1(3\beta_0 G_1 - \beta_0^2 + 3A_2) + 3\zeta_2(A_3 + \beta_1 G_1 + 2\beta_0 G_2)\right\}$$
$$+ a_s^4\left\{\dots\right\} \ + \ \mathcal{O}(a_s^5)$$

- Numerical values for $\alpha_s(q^2)$-expansion with $n_f = 4$
  - use Padé estimate for A_4 with conservative 50% uncertainty

$$\left|\frac{\mathcal{F}(q^2)}{\mathcal{F}(-q^2)}\right|^2 = 1 + 2.094\,\alpha_s + 5.613\,\alpha_s^2 + 15.70\,\alpha_s^3 + (48.63 \pm 0.43)\,\alpha_s^4$$

DIS resummation exponent



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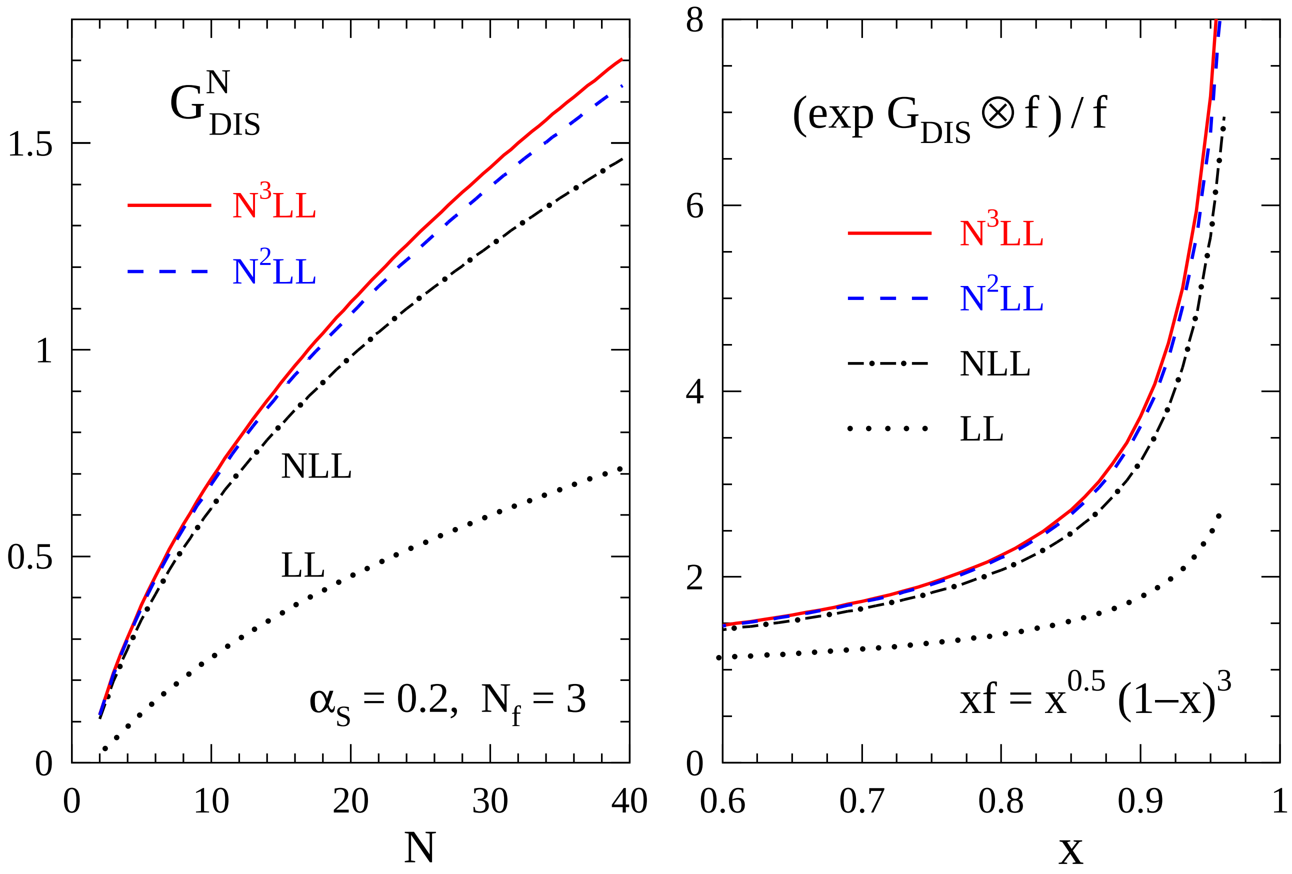

- Perturbative expansion very stable

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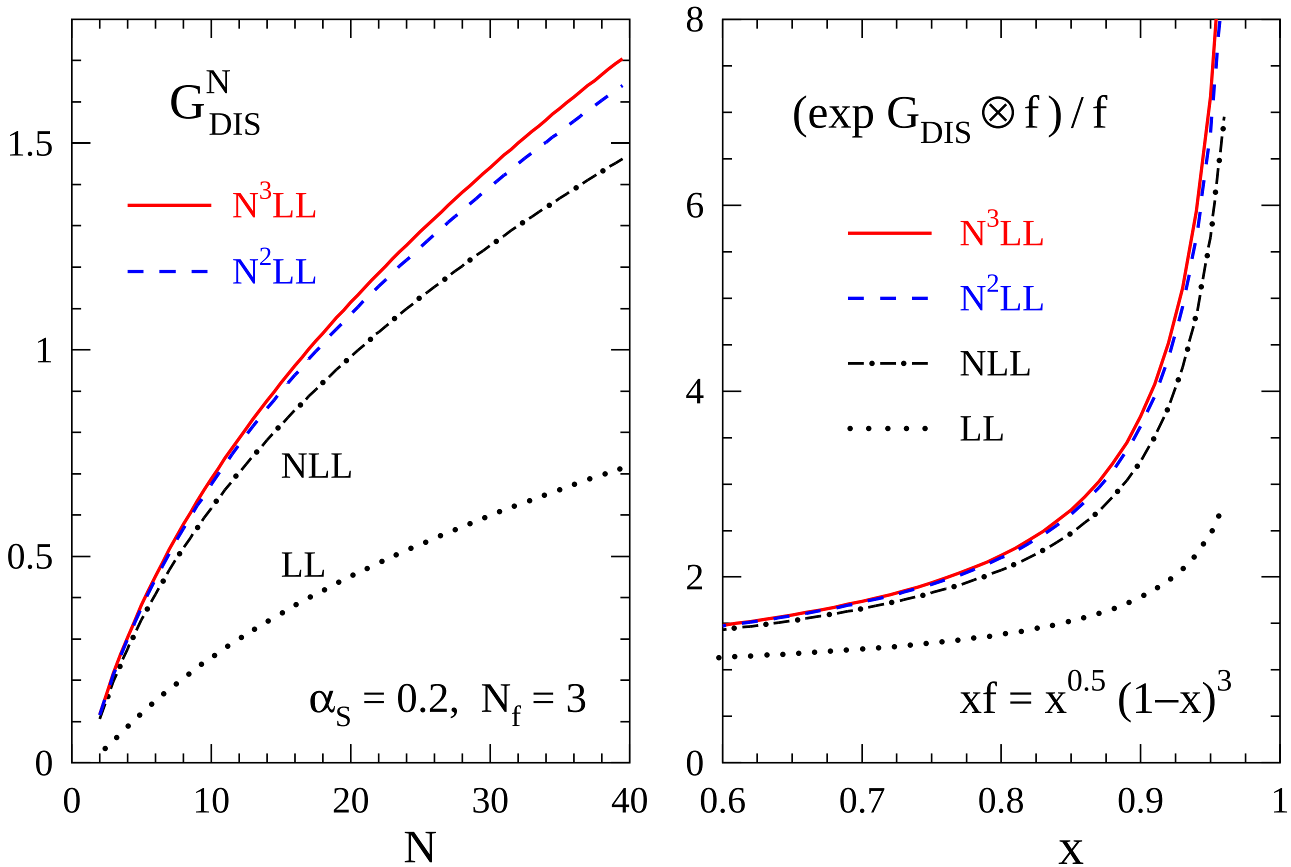

- Perturbative expansion very stable
- Resummation exponent generates perturbative expansion:
  - four-loop coeff. fct. $c^{(4)}_{2,\mathrm{q}}$ known $\left(\frac{\ln^7(1-x)}{1-x}\right)_+, \ldots, \left(\frac{\ln(1-x)}{1-x}\right)_+$