NLO + Parton Shower: POWHEG and Higgs boson production in gluon fusion

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Higgs Boson Phenomenology, Zürich
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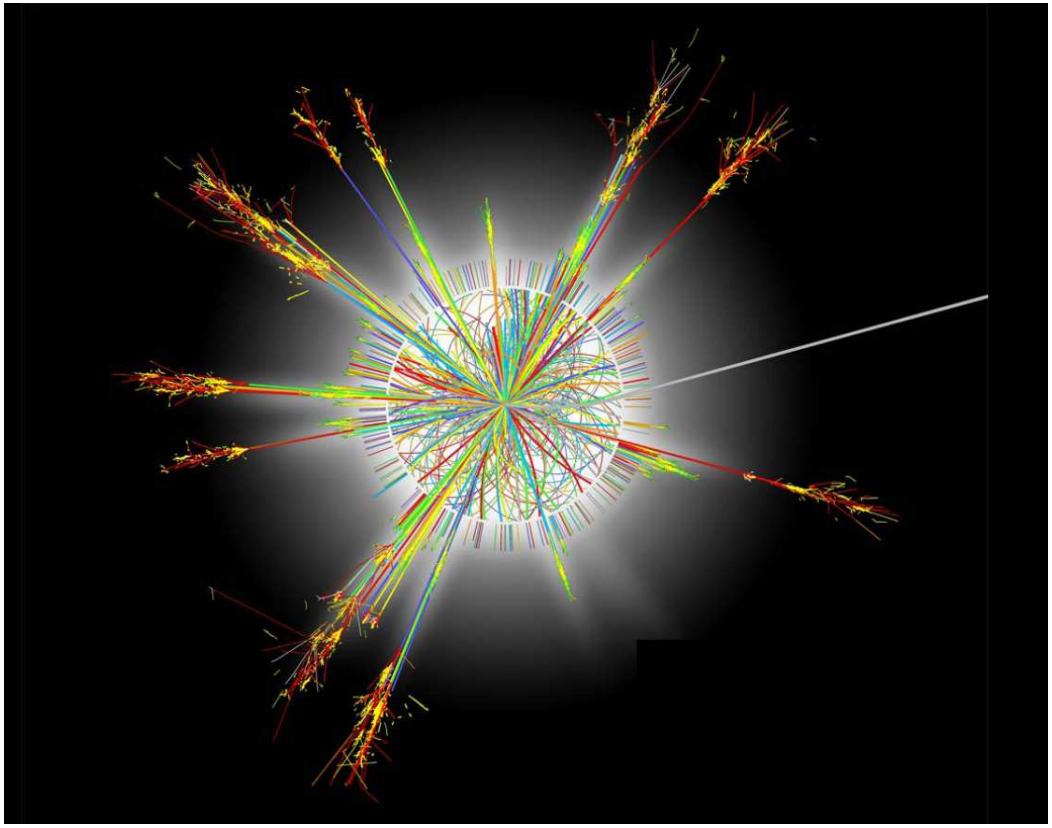
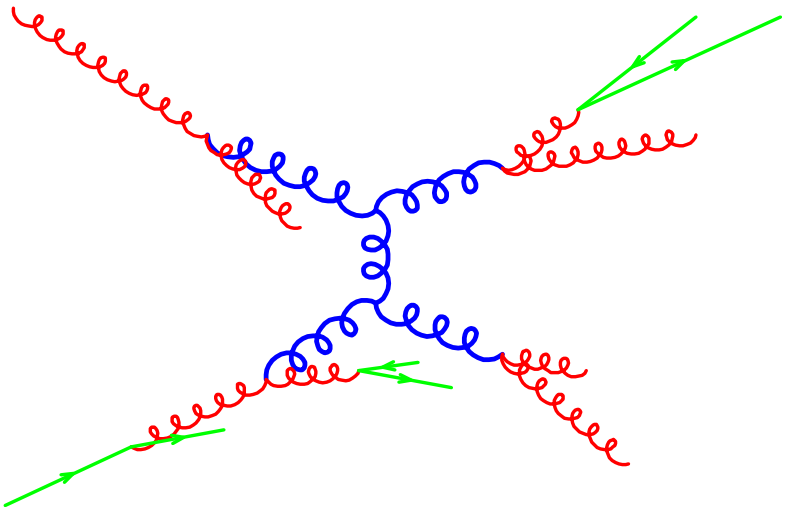
- Basics of shower Monte Carlo programs
- The POWHEG formalism
- POWHEG results
- Conclusions



Dominant corrections



Collinear-splitting processes in the initial and final state (always with transverse momenta $> \Lambda_{\rm QCD}$) are strongly enhanced. This is due to the fact that, in perturbation theory, the denominators in the propagators are small.

- The algorithms that evaluate all these enhanced contributions are called shower algorithms.
- Shower algorithms give a description of a hard collision up to distances of order $1/\Lambda_{\rm QCD}$.
- At larger distances, perturbation theory breaks down and we need to resort to non-perturbative methods (i.e. lattice calculations). However, these methods can be applied only to simple systems. The only viable alternative is to use models of hadron formation.

Hadronic final states

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS	V-X	V-Y	V-Z	V-C*T
30	NU_E	12	1	28	23	0	0	64.30	25.12	-1194.4	1196.4	0.00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
31	E+	-11	1	29	23	0	0	-22.36	6.19	-234.2	235.4	0.00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
230	PI0	111	1	155	24	0	0	0.31	0.38	0.9	1.0	0.13	4.209E-11	6.148E-11	-3.341E-11	5.192E-10
231	RHO+	213	197	155	24	317	318	-0.06	0.07	0.1	0.8	0.77	4.183E-11	6.130E-11	-3.365E-11	5.189E-10
232	P	2212	1	156	24	0	0	0.40	0.78	1.0	1.6	0.94	4.156E-11	6.029E-11	-4.205E-11	5.250E-10
233	NBAR	-2112	1	156	24	0	0	-0.13	-0.35	-0.9	1.3	0.94	4.168E-11	6.021E-11	-4.217E-11	5.249E-10
234	PI-	-211	1	157	9	0	0	0.14	0.34	286.9	286.9	0.14	4.660E-13	8.237E-12	1.748E-09	1.749E-09
235	PI+	211	1	157	9	0	0	-0.14	-0.34	624.5	624.5	0.14	4.056E-13	8.532E-12	2.462E-09	2.462E-09
236	P	2212	1	158	9	0	0	-1.23	-0.26	0.9	1.8	0.94	-4.815E-11	1.893E-11	7.520E-12	3.252E-10
237	DLTABR--	-2224	197	158	9	319	320	0.94	0.35	1.6	2.2	1.23	-4.817E-11	1.900E-11	7.482E-12	3.252E-10
238	PI0	111	1	159	9	0	0	0.74	-0.31	-27.9	27.9	0.13	-1.889E-10	9.893E-11	-2.123E-09	2.157E-09
239	RHO0	113	197	159	9	321	322	0.73	-0.88	-19.5	19.5	0.77	-1.888E-10	9.859E-11	-2.129E-09	2.163E-09
240	K+	321	1	160	9	0	0	0.58	0.02	-11.0	11.0	0.49	-1.890E-10	9.873E-11	-2.135E-09	2.169E-09
241	KL_1-	-10323	197	160	9	323	324	1.23	-1.50	-50.2	50.2	1.57	-1.890E-10	9.879E-11	-2.132E-09	2.166E-09
242	K-	-321	1	161	24	0	0	0.01	0.22	1.3	1.4	0.49	4.250E-11	6.333E-11	-2.746E-11	5.211E-10
243	PI0	111	1	161	24	0	0	0.31	0.38	0.2	0.6	0.13	4.301E-11	6.282E-11	-2.751E-11	5.210E-10

High-energy experimental physicists feed this kind of output through their detector-simulation software, and use it to determine efficiencies for signal detection, and perform background estimates.

Analysis strategies are set up using these simulated data.

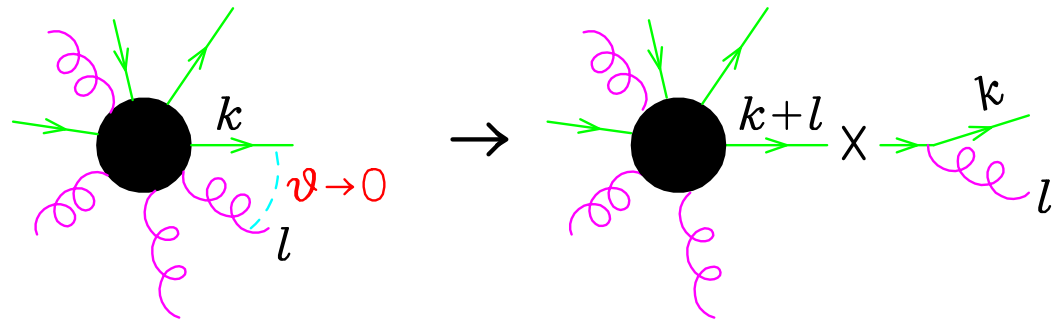
"The Monte Carlo simulation has become the major mean of visualization of not only detector performance but also of physics phenomena. So far so good. But it often happens that the physics simulations provided by the Monte Carlo generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data."

J.D. Bjorken (1992)

Shower basics: collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit



$$d\mathbf{\Phi}_{n+1} = d\mathbf{\Phi}_n \, d\Phi_r \qquad d\Phi_r \div dt\, dz\, d\varphi$$

$$|M_{n+1}|^2 d\mathbf{\Phi}_{n+1} \implies |M_n|^2 \, d\mathbf{\Phi}_n \, \frac{\alpha_s}{2\pi} \, \frac{dt}{t} P_{q,qg}(z) \, dz \, \frac{d\varphi}{2\pi} \qquad \begin{cases} \dfrac{dt}{t} \approx \dfrac{d\theta}{\theta} & \text{collinear singularity} \\ \dfrac{dz}{1-z} \approx \dfrac{dE_g}{E_g} & \text{soft singularity} \end{cases}$$

t : $(k+l)^2$, p_T^2, $E^2\theta^2 \ldots$

$z = k^0/(k^0 + l^0)$: energy (or $p_{\|}$ or p^+) fraction of quark

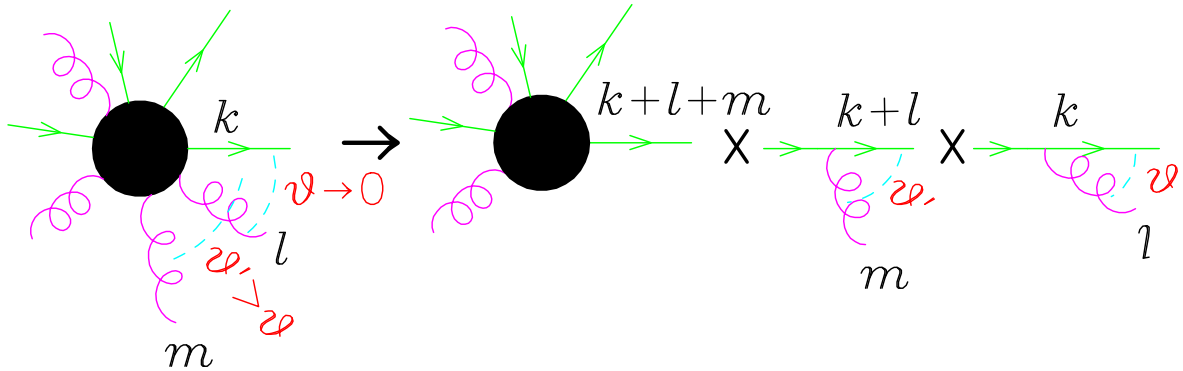
$P_{q,qg}(z) = C_F \dfrac{1+z^2}{1-z}$: Altarelli-Parisi splitting function

(ignore $z \to 1$ IR divergence for now)

Shower basics: collinear factorization

If another gluon becomes collinear, iterate the previous formula

$\theta', \theta \to 0$ with $\theta' > \theta$



$$|M_{n+1}|^2 d\mathbf{\Phi}_{n+1} \quad \Longrightarrow \quad |M_{n-1}|^2 d\mathbf{\Phi}_{n-1} \times \frac{\alpha_s}{2\pi}\,\frac{dt'}{t'}\,P_{q,qg}(z')\,dz'\,\frac{d\varphi'}{2\pi}$$
$$\times\,\frac{\alpha_s}{2\pi}\,\frac{dt}{t}\,P_{q,qg}(z)\,dz\,\frac{d\varphi}{2\pi}\,\theta(t'-t)$$

Collinear partons can be described by a factorized integral ordered in t.

Collinear factorization: multiple emissions

For n collinear emissions, the cross section goes as

$$\sigma \approx \sigma_0 \, \alpha_s^n \int_{t_0}^{Q^2} \frac{dt_1}{t_1} \frac{dt_2}{t_2} \ldots \frac{dt_n}{t_n} \, \theta\left(Q^2 > t_1 > t_2 > \ldots > t_n > t_0\right)$$

$$= \sigma_0 \, \alpha_s^n \int_{t_0}^{Q^2} \frac{dt_1}{t_1} \int_{t_0}^{t_1} \frac{dt_2}{t_2} \ldots \int_{t_0}^{t_{n-1}} \frac{dt_n}{t_n} \approx \sigma_0 \, \alpha_s^n \, \frac{1}{n!} \left(\log \frac{Q^2}{t_0}\right)^n$$

- Q^2 is an upper cutoff for the ordering variable t
- $t_0 \approx \Lambda^2 \approx \Lambda^2_{\rm QCD}$ is an infrared cutoff (quark mass, confinement scale)
- Due to the log dependence, we call it leading-log approximation.
- According to the Kinoshita-Lee-Nauenberg theorem, the virtual corrections, order by order, contribute with a comparable term, with opposite sign.
- The virtual leading-log contribution should be included in order to get sensible results!

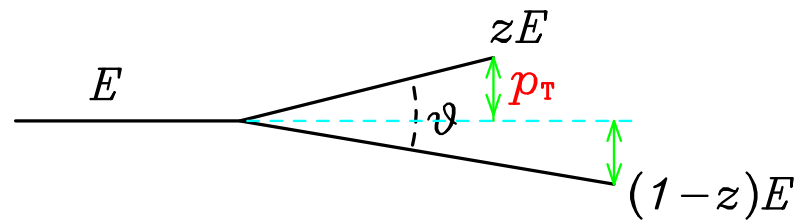
The algorithms that evaluate all these enhanced contributions are called shower Monte Carlo algorithms

Accuracy: soft divergences and double-log regions

$z \to 1$ ($z \to 0$) region problematic. In fact, for $z \to 1$, $P_{qq}, P_{gg} \div 1/(1-z)$

The choice of the ordering variable t makes a difference

$$\begin{aligned}
\text{virtuality:} \quad & t \equiv E^2 z(1-z) \overbrace{\theta^2}^{2(1-\cos\theta)} \\
p_T^2\text{:} \quad & t \equiv E^2 z^2 (1-z)^2 \theta^2 \\
\text{angle:} \quad & t \equiv E^2 \theta^2
\end{aligned}$$



$$\text{virtuality}: \; z(1-z) > t/E^2 \implies \int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z} \approx \frac{1}{4} \log^2 \frac{t}{E^2}$$

$$p_T^2: \; z^2(1-z)^2 > t/E^2 \implies \int \frac{dt}{t} \int_{t/E^2}^{1-t/E^2} \frac{dz}{1-z} \approx \frac{1}{2} \log^2 \frac{t}{E^2}$$

$$\text{angle}: \implies \int \frac{dt}{t} \int_0^1 \frac{dz}{1-z} \approx \log t \, \log \Lambda$$

Sizable difference in double-log structure!

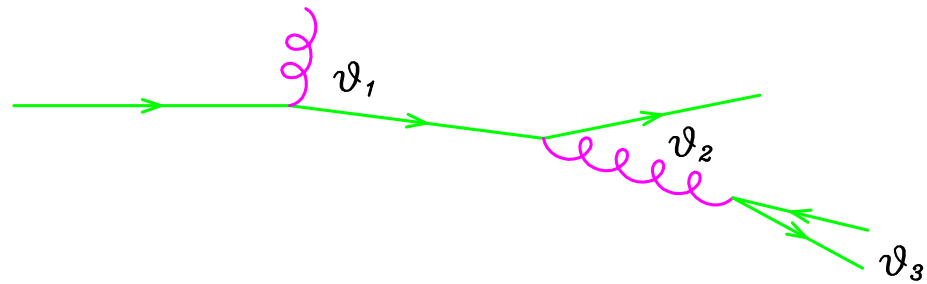
Angular ordering and color coherence

Mueller (1981) showed that angular ordering is the correct choice

$$\frac{d\theta}{\theta}\,\frac{\alpha_s\left(p_T^2\right)}{2\pi}\,P(z)\,dz$$

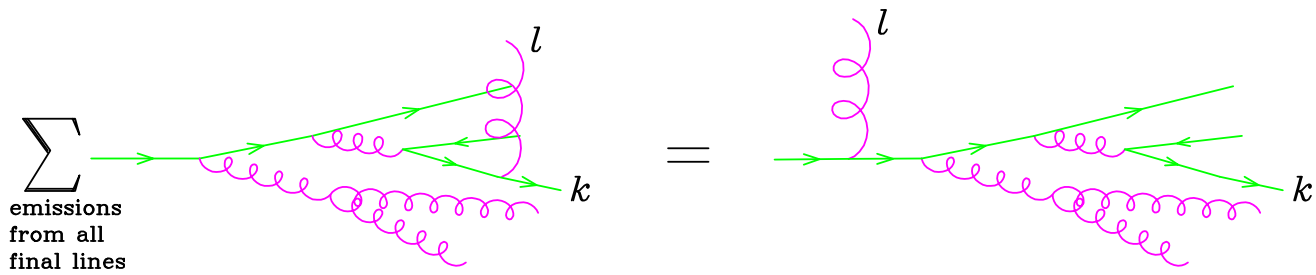
$$\theta_1 > \theta_2 > \theta_3 \ldots$$

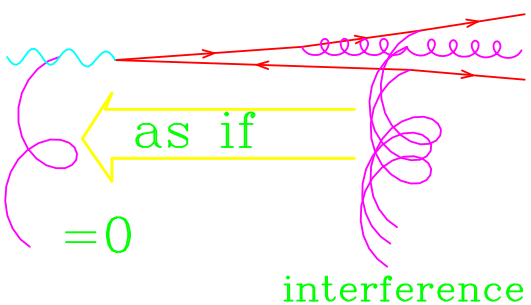
$$p_T^2 = E^2 z^2 (1-z)^2 \theta^2$$



$\alpha_s(p_T^2)$ for a correct treatment of charge renormalization in soft region

Soft gluons emitted at large angles from final-state partons add coherently





- angular ordering accounts for soft gluon interference.
- intensity for photon jets $= 0$
- intensity for gluon jets $= C_A$ instead of $2\,C_F + C_A$

POWHEG

$$d\sigma_{\rm NLO} = d\mathbf{\Phi}_n \Big\{ B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \big[R(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r) \big] \, d\Phi_r \Big\}$$

$$d\mathbf{\Phi}_{n+1} = d\mathbf{\Phi}_n \, d\Phi_r \qquad\qquad d\Phi_r \div dt \, dz \, d\varphi$$

$$V(\mathbf{\Phi}_n) = V_b(\mathbf{\Phi}_n) + \int d\Phi_r \, C(\mathbf{\Phi}_n, \Phi_r) \qquad \Longleftarrow \text{ finite}$$

$$d\sigma_{\rm SMC} = B(\mathbf{\Phi}_n) \, d\mathbf{\Phi}_n \left\{ \Delta_{t_0} + \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \Delta_t \, d\Phi_r \right\}$$

$$\Delta_t = \exp\left[- \int d\Phi'_r \frac{\alpha_s}{2\pi} P(z') \frac{1}{t'} \theta(t' - t) \right] \qquad \text{SMC Sudakov form factor}$$

$$d\sigma_{\rm POWHEG} = \overline{B}(\mathbf{\Phi}_n) \, d\mathbf{\Phi}_n \left\{ \Delta(\mathbf{\Phi}_n, p_T^{\rm min}) + \frac{R(\mathbf{\Phi}_n, \Phi_r)}{B(\mathbf{\Phi}_n)} \Delta(\mathbf{\Phi}_n, p_T) \, d\Phi_r \right\}$$

$$\overline{B}(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int d\Phi_r \big[R(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r) \big]$$

$$\Delta(\mathbf{\Phi}_n, p_T) = \exp\left[- \int d\Phi'_r \frac{R(\mathbf{\Phi}_n, \Phi'_r)}{B(\mathbf{\Phi}_n)} \theta\left(k_T(\mathbf{\Phi}_n, \Phi'_r) - p_T \right) \right] \qquad \text{POWHEG Sudakov}$$

New developments

- Interfacing Matrix Elements (ME) generators with Parton Showers
  - CKKW matching [Catani, Krauss, Küen, Webber]
  - MLM matching [Mangano]
- Interfacing NLO calculations with Parton Showers
  - MC@NLO [Frixione, Webber]
  - POWHEG [Nason]

Several other approaches have appeared

- $e^+e^- \to 3$ partons [Kramer, Mrenna, Soper]
- Shower by antenna factorization [Giele, Kosower, Skands]
- Shower by Catani-Seymour dipole factorization [Schumann, Krauss]
- Shower with quantum interference [Nagy, Soper]
- Shower by Soft Collinear Effective Theory [Bauer, Schwartz]
- Shower from the dipole formalism [Dinsdale, Ternick, Weinzierl]

Up to now, complete results for hadron colliders only from MC@NLO and POWHEG.

NLO + Parton Shower

LO-ME good for shapes. Uncertain absolute normalization

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu)\big(1 - b_0\alpha_s(\mu)\log(4)\big)^n \approx \alpha_s^n(\mu)\big(1 - n\alpha_s(\mu)\big)$$

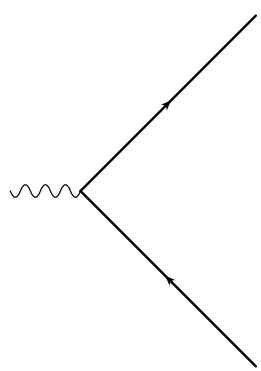
For $\mu = 100$ GeV, $\alpha_s = 0.12$, normalization uncertainty:

$W+1J$	$W+2J$	$W+3J$
$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

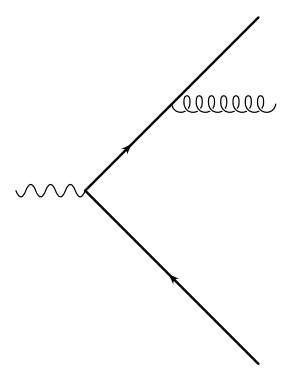
To improve on this, we need to go to NLO

The main problem in merging a NLO result and a Parton Shower is not to double-count radiation: the shower might produce some radiation already present at the NLO level.

LO:



NLO:



POsitive-Weight Hardest Emission Generator

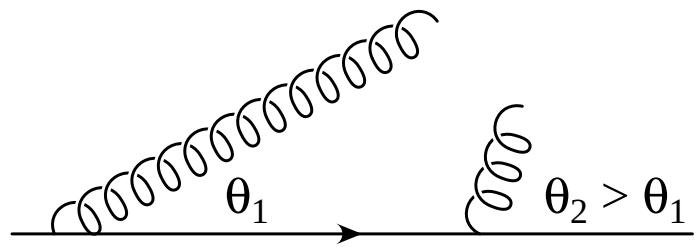
✓ it generates events with positive weights. NO negative weights to handle

✓ it is independent from parton-shower programs. Can be interfaced with PYTHIA, HERWIG, SHERPA...
It is then possible to compare the different outputs

✓ No need to implement new interfaces
Two possible ways to interface to shower Monte Carlo programs

1. Les Houches Event format. The event is written on a file that is subsequently showered by HERWIG, PYTHIA...
2. on the fly. We provide UPINIT and UPEVNT directly running in HERWIG and PYTHIA

POWHEG: truncated shower



- if the shower is ordered in p_T (for example PYTHIA), nothing else needs to be done
- if the shower is ordered in angle (for example HERWIG), we need to generate correctly soft radiation at large angle.
  - pair up the partons that are nearest in p_T
  - generate an angular-ordered shower associated with the paired parton, stopping at the angle of the paired partons (truncated shower)
  - generate all subsequent vetoed showers

This is a problem that affects all the angular-ordered shower Monte Carlo programs when the shower is initiated by a relatively complex matrix element.

Truncated shower implemented only in HERWIG++

Existing implementations

The POWHEG method has already been successfully used in

- $pp \to ZZ$ [Nason and Ridolfi, hep-ph/0606275]
- $e^+e^- \to$ hadrons [Latunde-Dada, Gieseke and Webber, hep-ph/0612281]
  $e^+e^- \to t\bar{t}$ with top decay [Latunde-Dada, arXiv:0806.4560]
- $pp \to Q\overline{Q}$ ($c\bar{c}$, $b\bar{b}$, $t\bar{t}$) with spin correlations [Frixione, Nason and Ridolfi, arXiv:0707.3088].
- $pp \to W/Z$ with spin correlations [Alioli, Nason, Oleari and Re, arXiv:0805.4802; Hamilton, Richardson and Tully, arXiv:0806.0290].
- $pp \to H$ [Alioli, Nason, Oleari and Re, arXiv:0812.0578]

All POWHEG implementations for hadronic colliders have been interfaced to both PYTHIA and HERWIG.

To appear soon

- $pp \to H$ [Hamilton, Richardson and Tully, HERWIG++ group]
- single top production [Alioli, Nason, Oleari and Re]
- $pp \to W/Z + 1$ jet [Alioli, Nason, Oleari and Re]

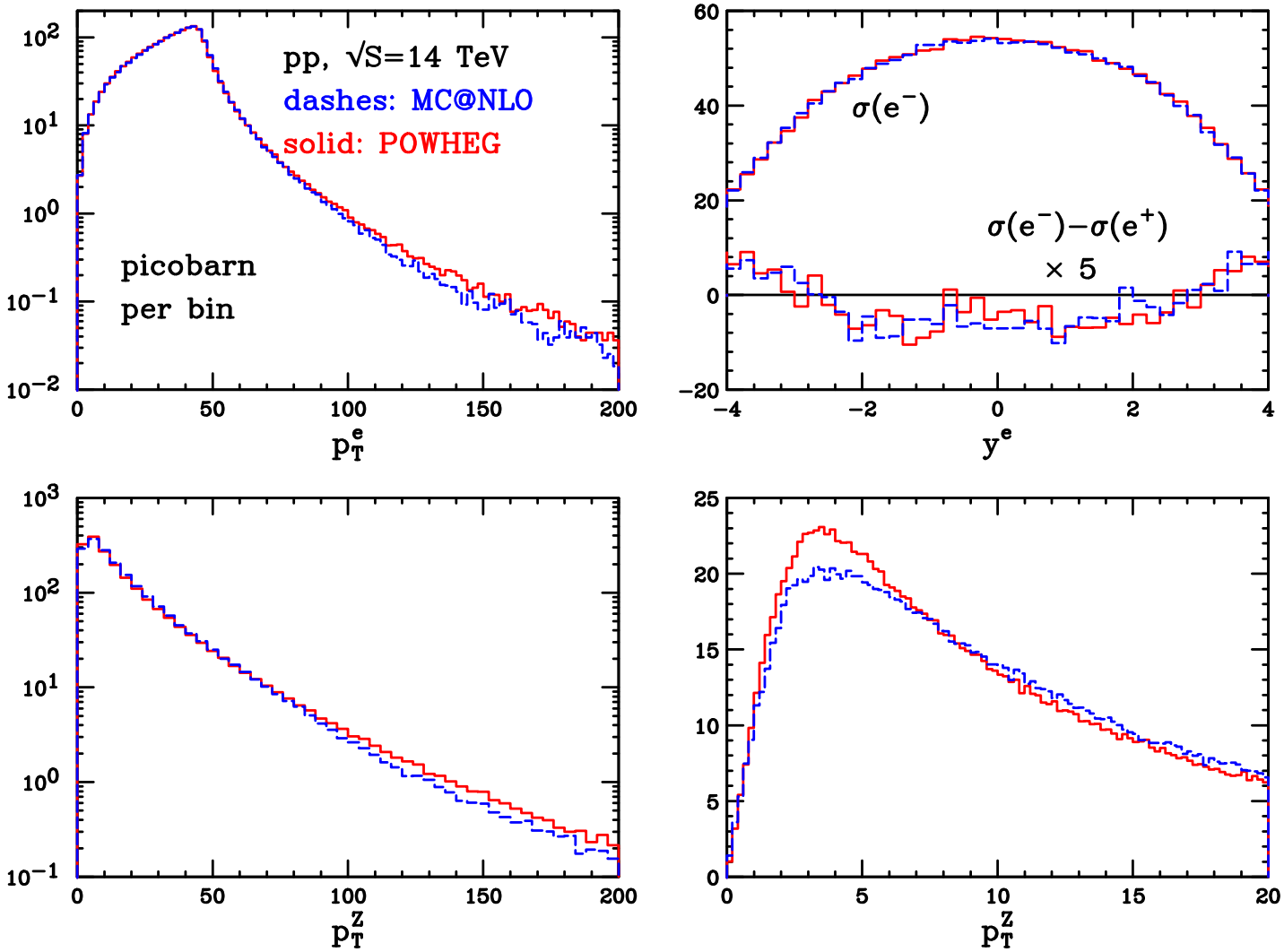
We are working now on a general framework for the implementation of any NLO process into the POWHEG formalism.

Given the Born, real and virtual amplitudes, combine them automatically to produce POWHEG events.

Truncated shower

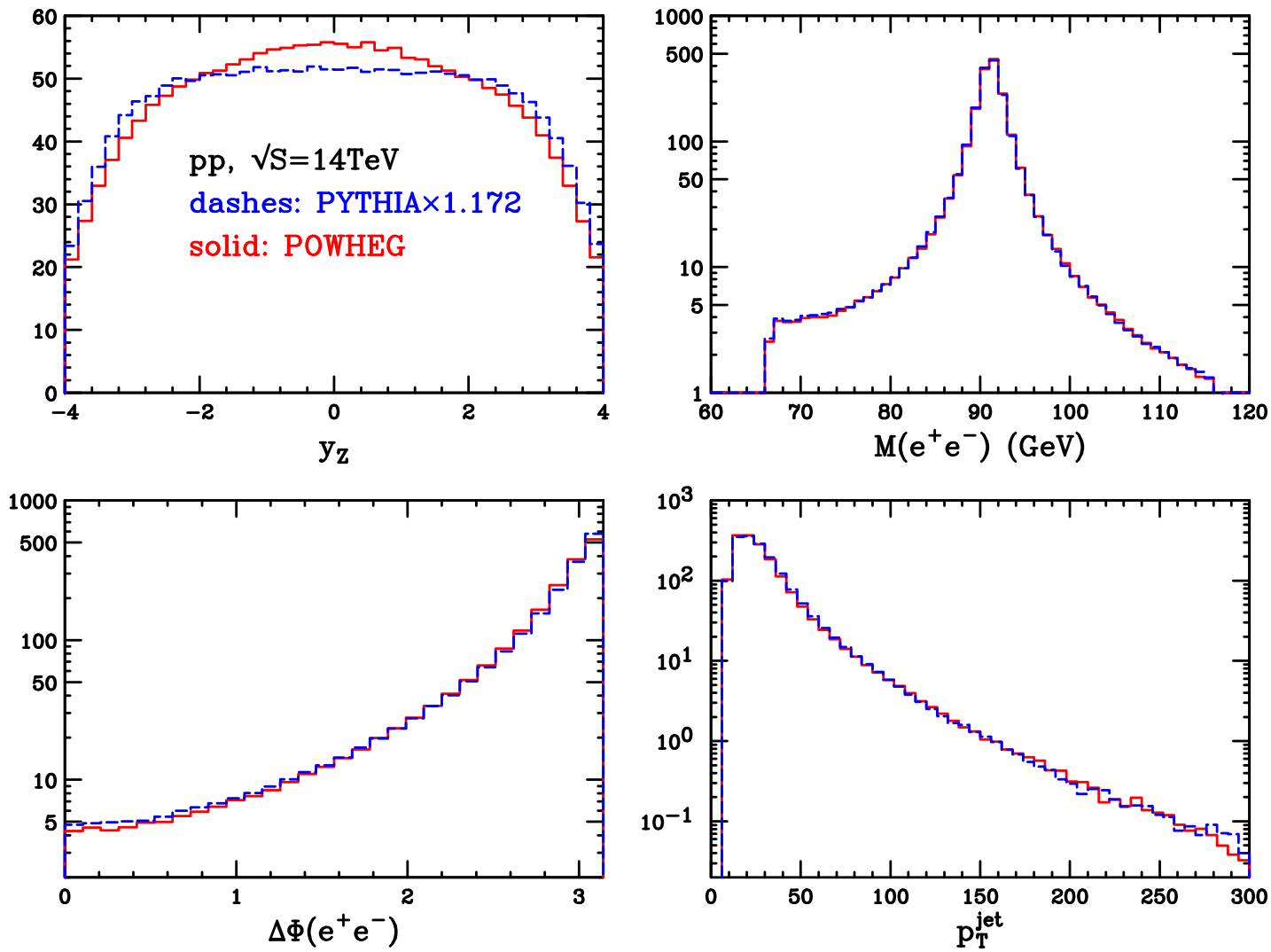
- in an approximate form, truncated shower has been studied in $e^+e^- \to$ hadrons [Latunde-Dada Gieseke and Webber, `hep-ph/0612281`]
- included in the HERWIG++ framework [Bähr, Gieseke, Gigg, Grellscheid, Hamilton, Plätzer, Richardson, Seymour and Tully, arXiv:0812.0529]

Z production: POWHEG + HERWIG vs MC@NLO



Small differences in the high- and low-p_T regions.

Z production: POWHEG + PYTHIA vs PYTHIA

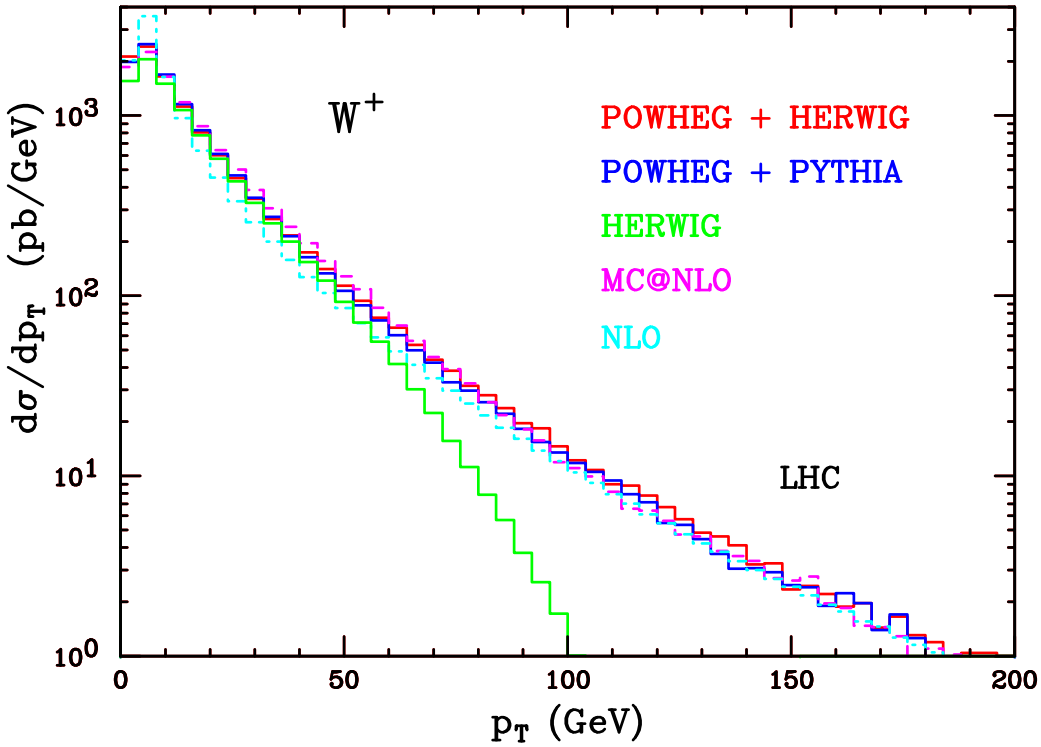


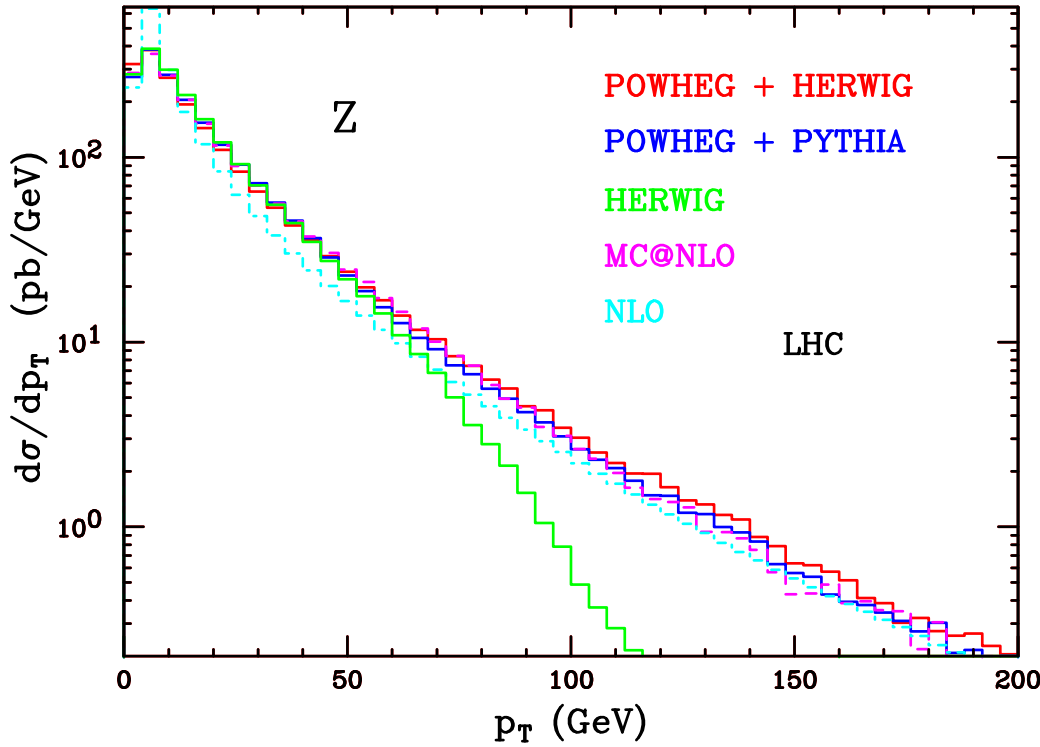
- A shower Monte Carlo is accurate in the radiation of the hardest jet only in the collinear regions.

- Only because the generation of radiation in vector-boson production in PYTHIA is very similar to the POWHEG one, we can make comparisons of $p_T^{\rm jet}$ and p_T^Z distributions.

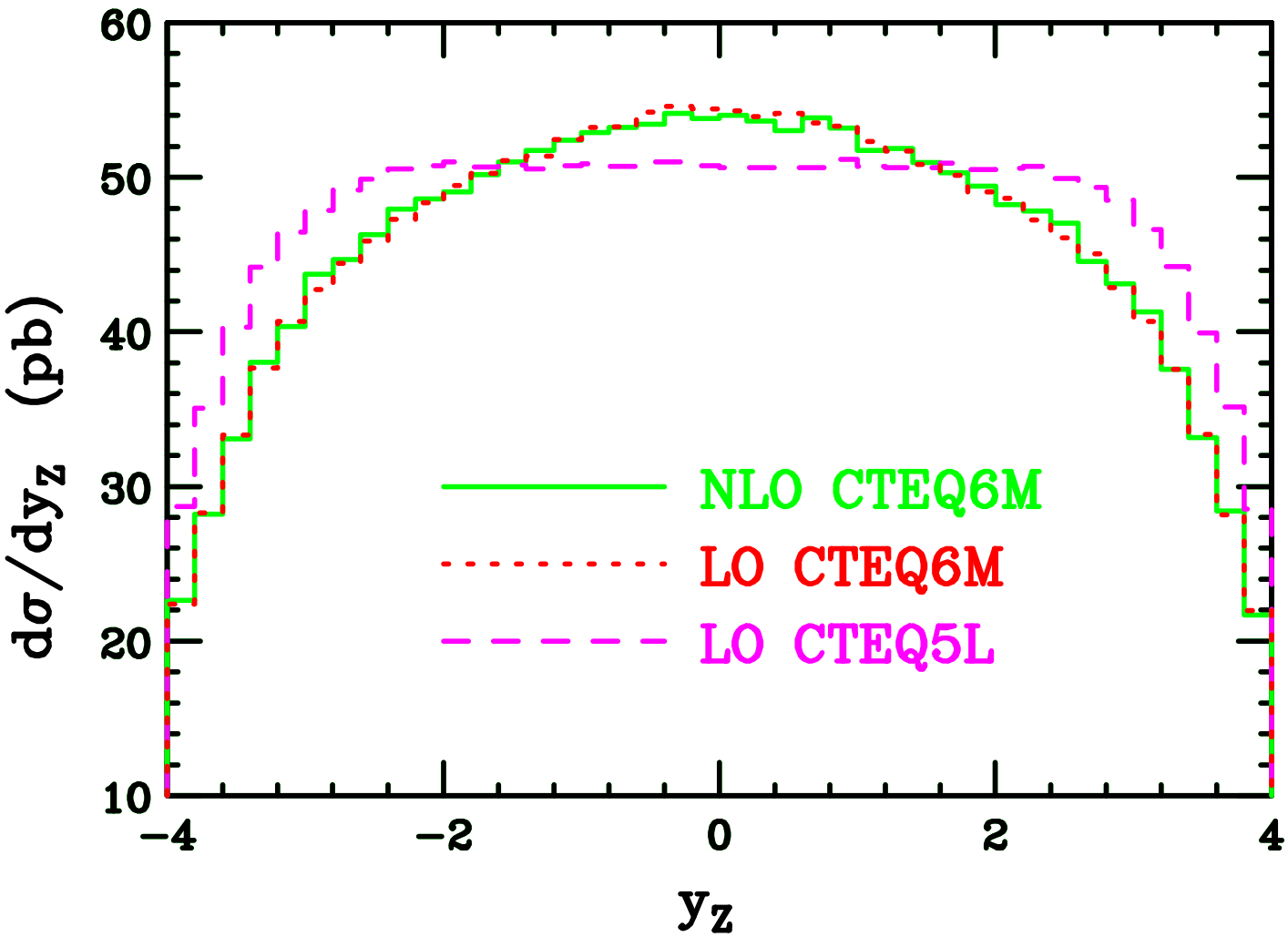
Differences in y_Z ascribed to the use of LO parton densities.

W/Z production



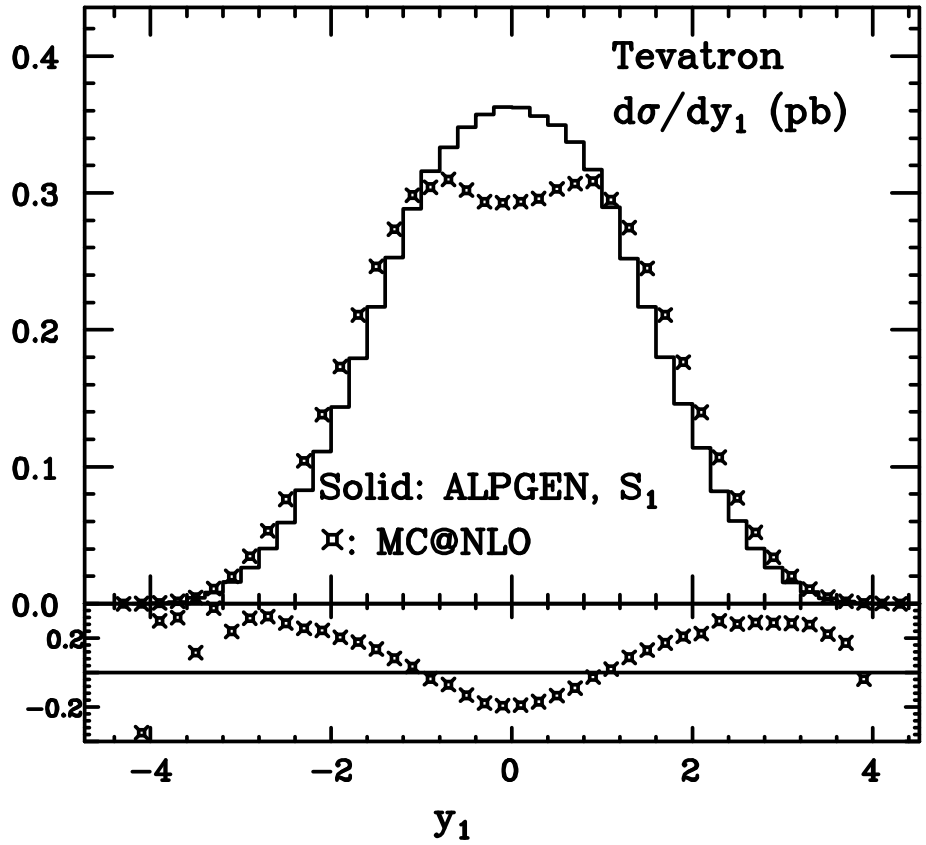


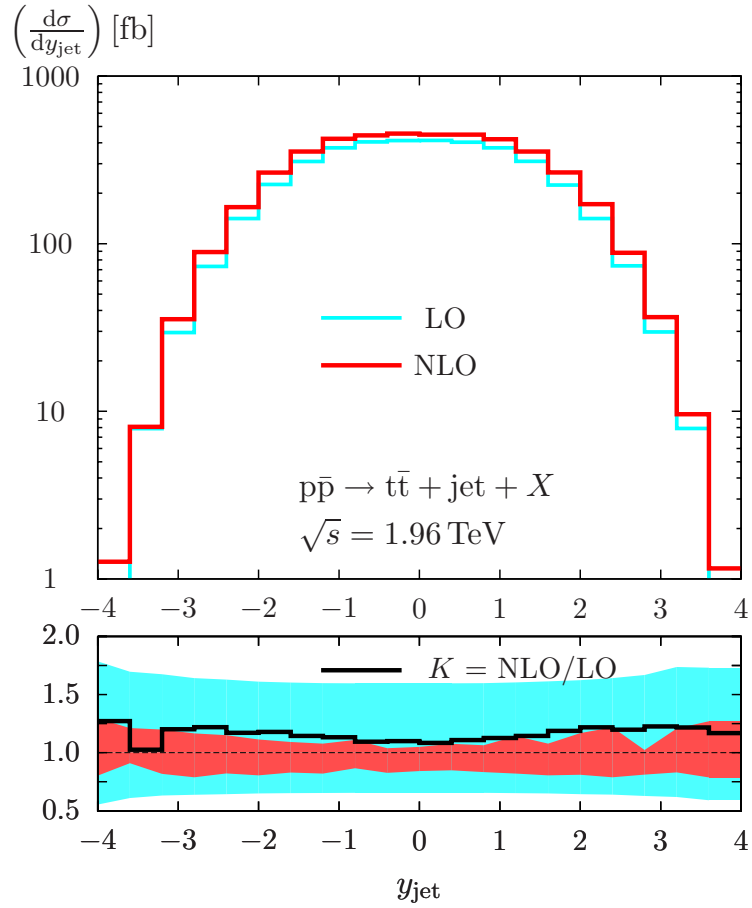
Z production: fixed order



Plots normalized to the NLO total cross section.

ALPGEN and NLO vs MC@NLO: $t\bar{t}$ + 1 jet



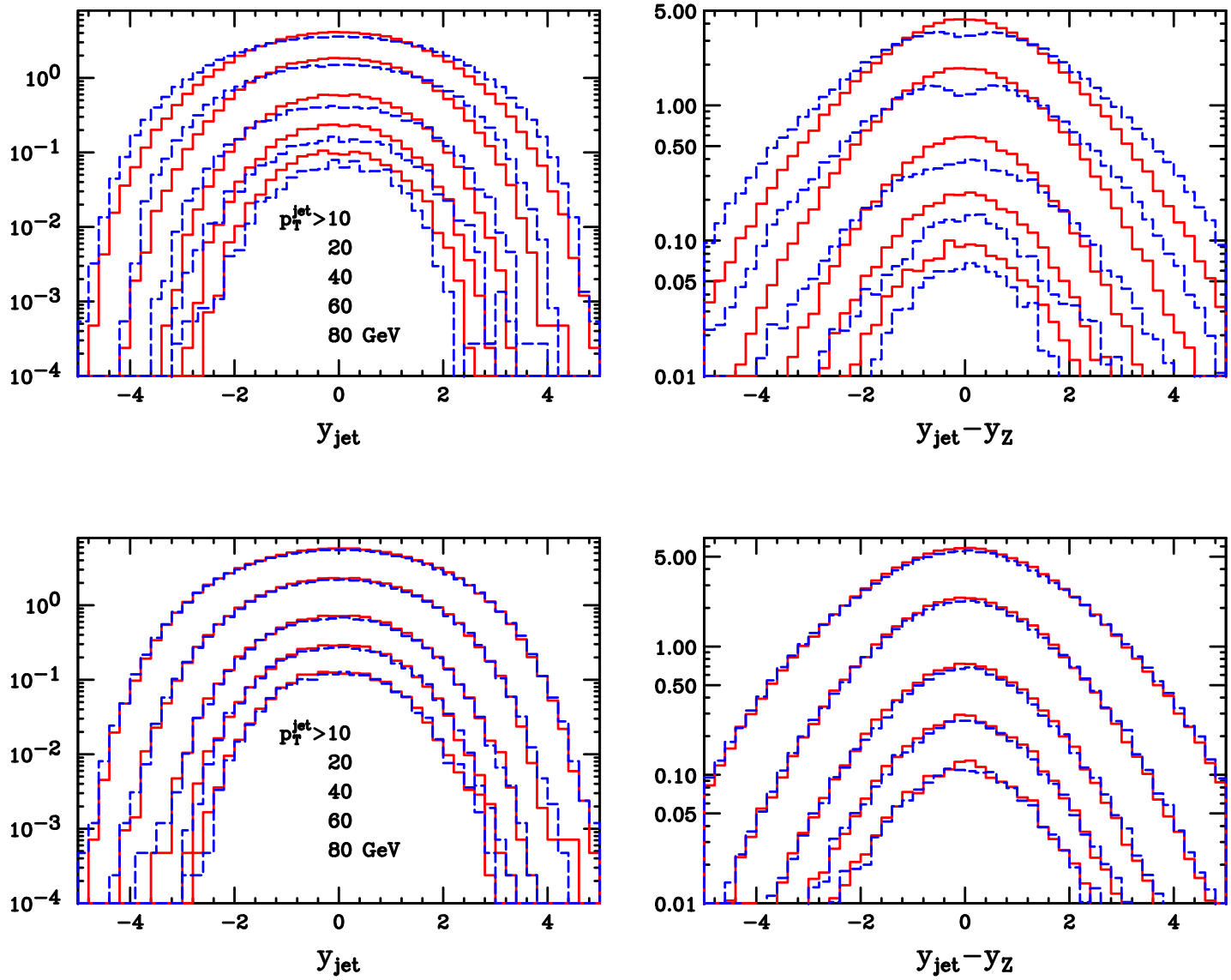


Rapidity y_1 of the leading jet (highest p_T).

Comparison ALPGEN vs MC@NLO [Mangano, Moretti, Piccinini & Treccani, `hep-ph/0611129`]

$pp \to t\bar{t}$ + jet at NLO shows no dip too [Dittmaier, Uwer, Weinzierl, arXiv:0810.0452]

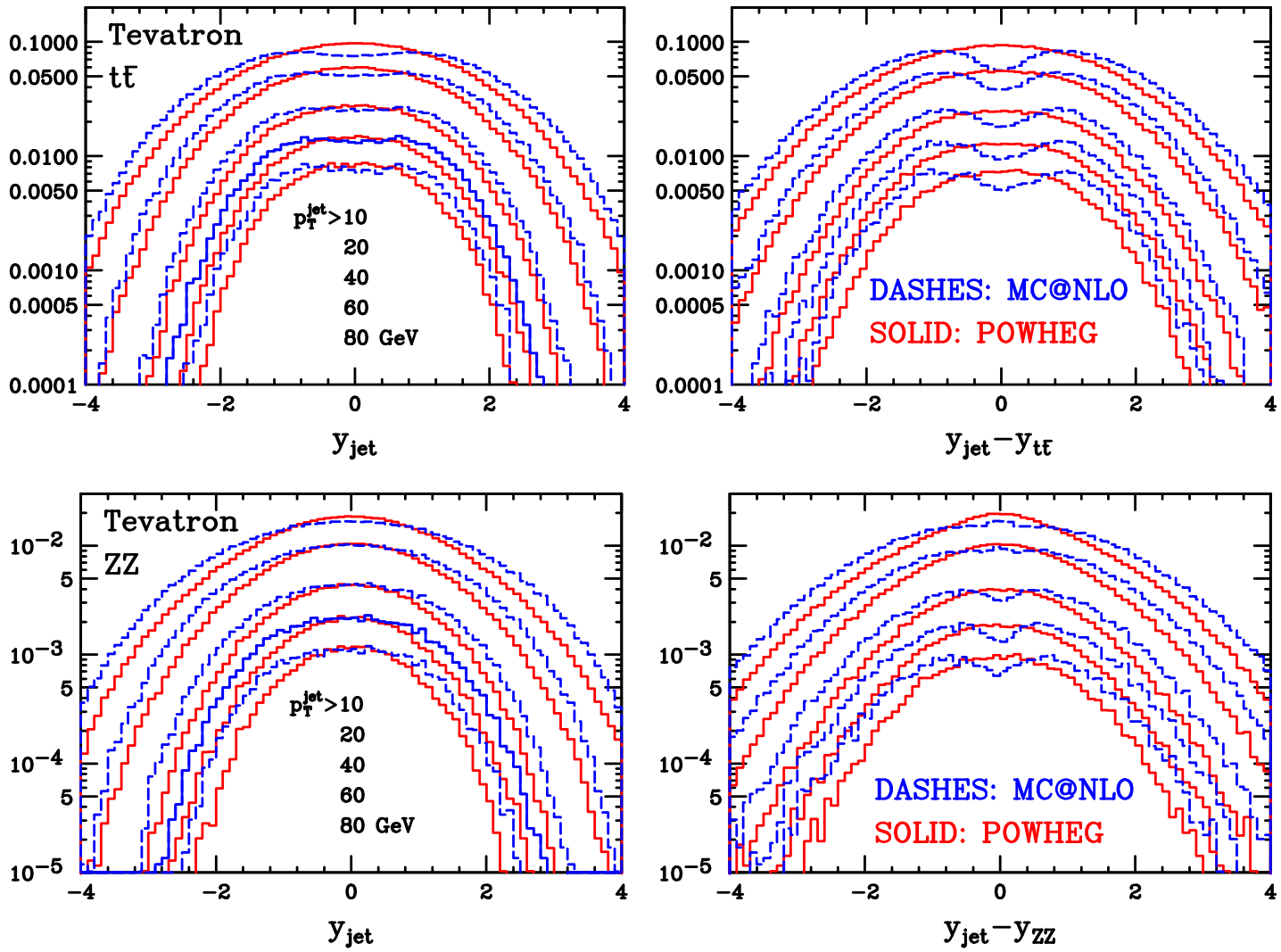
Rapidity distribution of hardest jet at Tevatron



POWHEG+HERWIG
MC@NLO

POWHEG+PYTHIA
PYTHIA

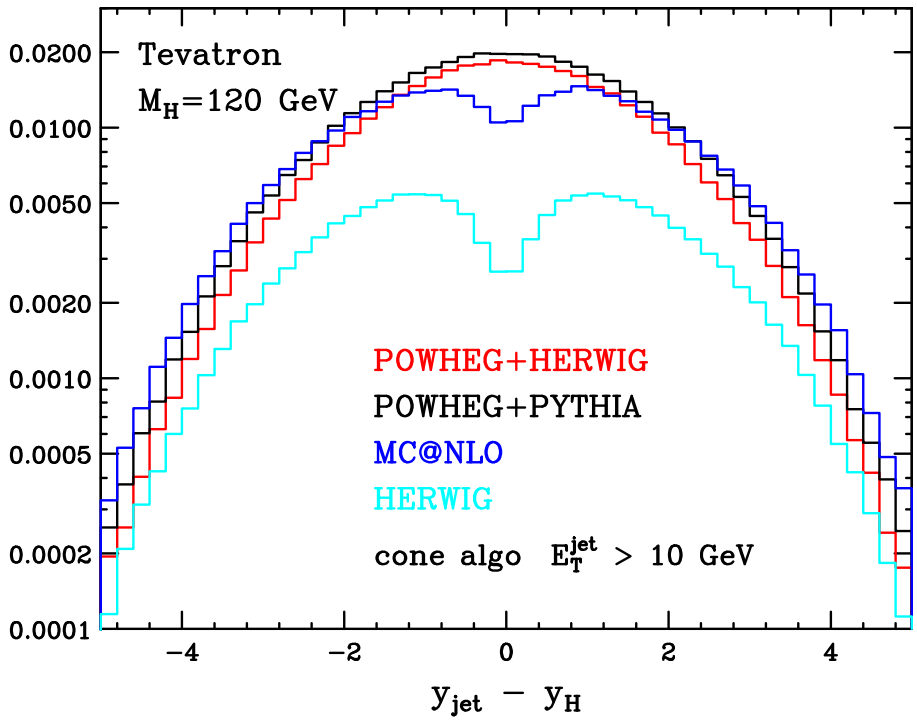
Rapidity distribution of hardest jet at Tevatron

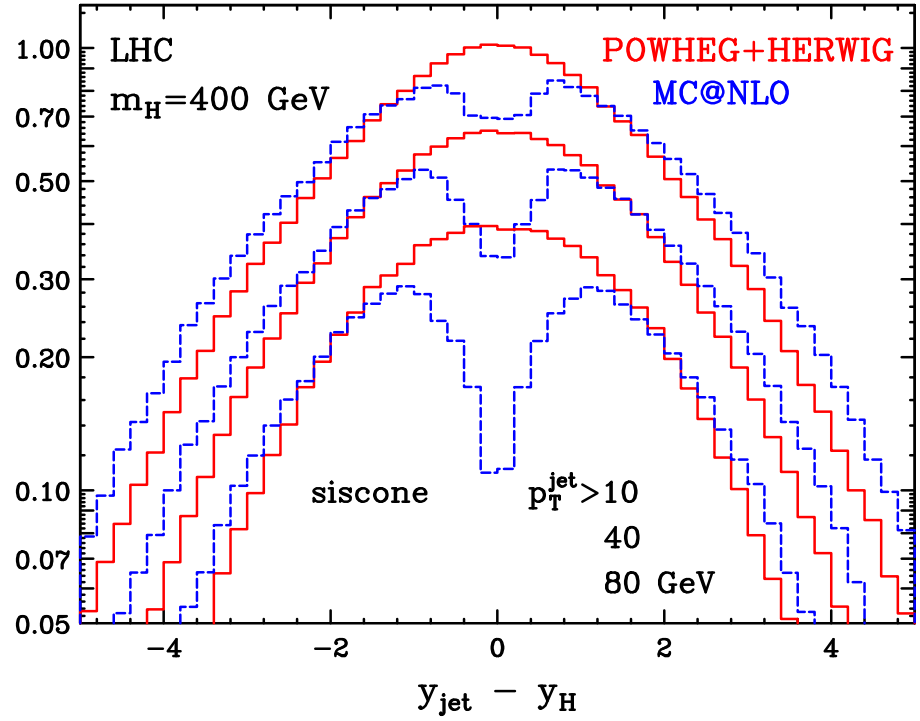


POWHEG+HERWIG
MC@NLO

POWHEG+HERWIG
MC@NLO

Higgs boson rapidity distribution at Tevatron and LHC

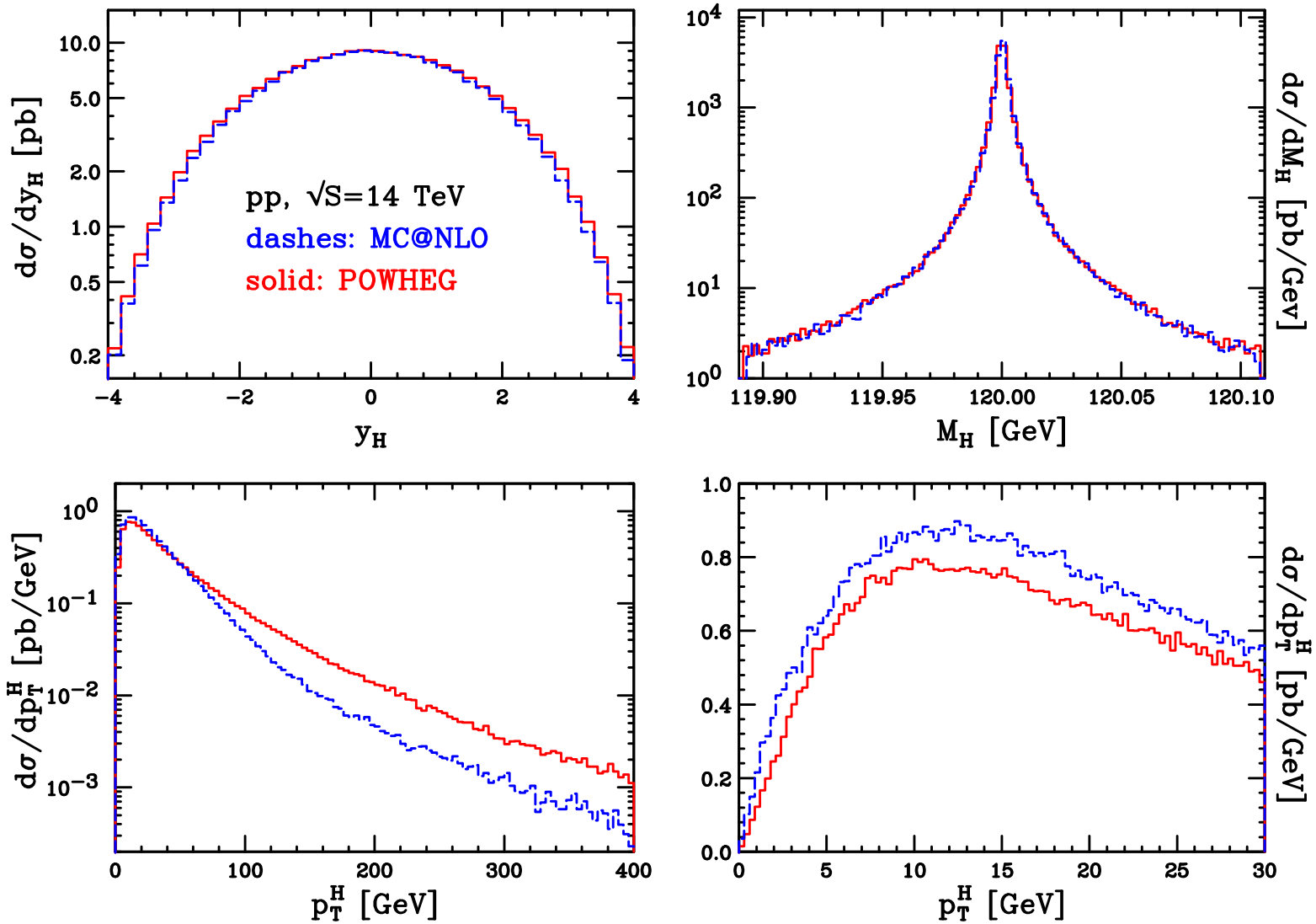




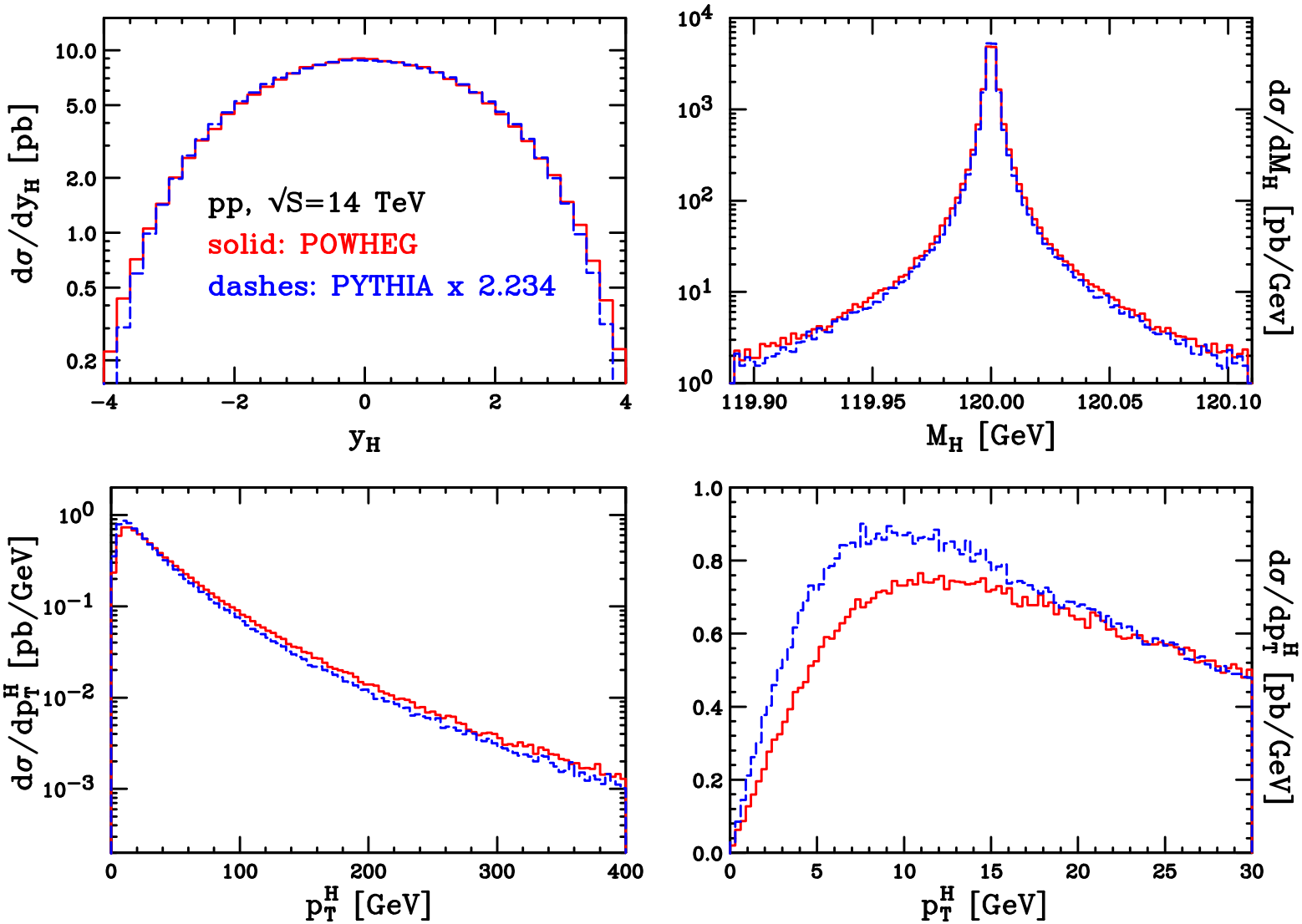
Dip inherited from the even-deeper dip of HERWIG. MC@NLO fills partially the dip.

The dip in the MC@NLO result is compatible with an effect beyond NLO.

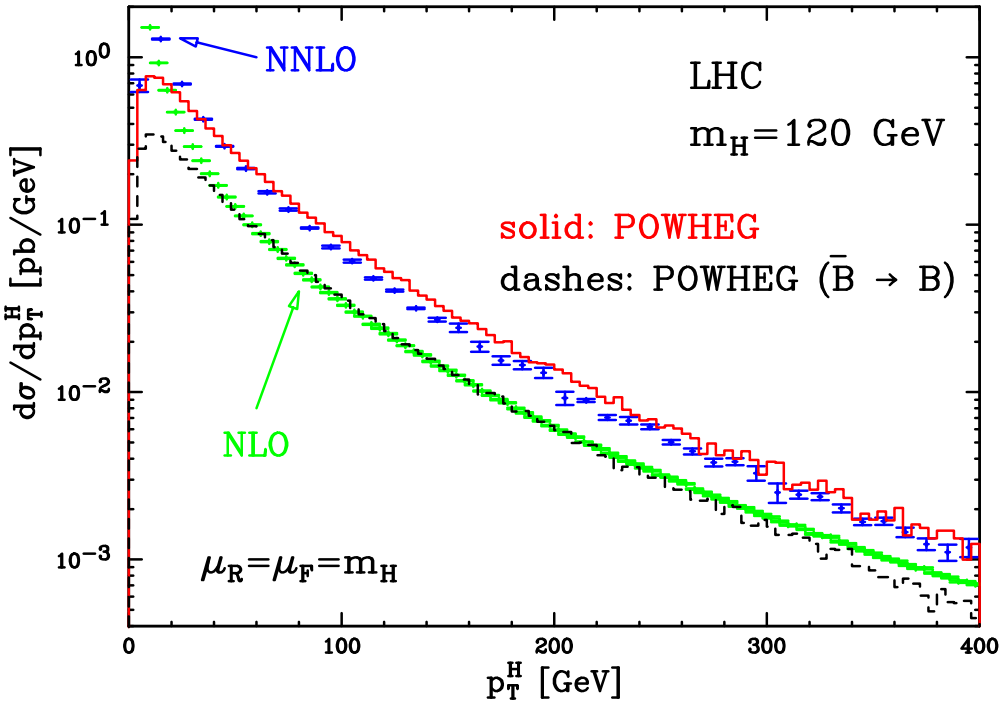
Higgs boson production at the LHC

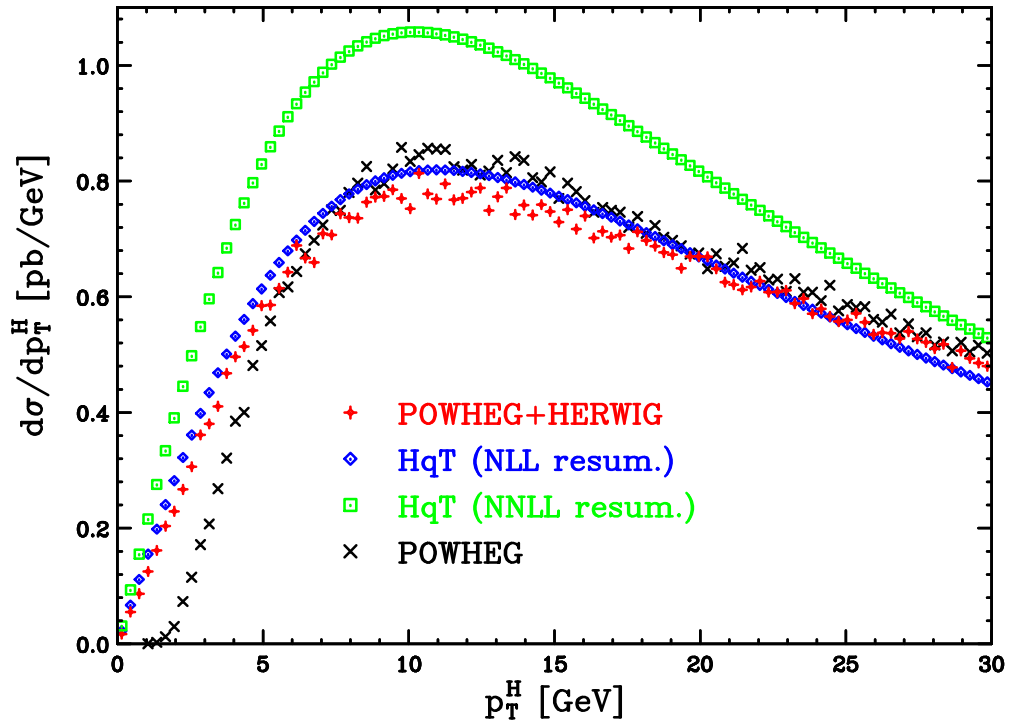


Higgs boson production at the LHC



Higgs boson production at the LHC



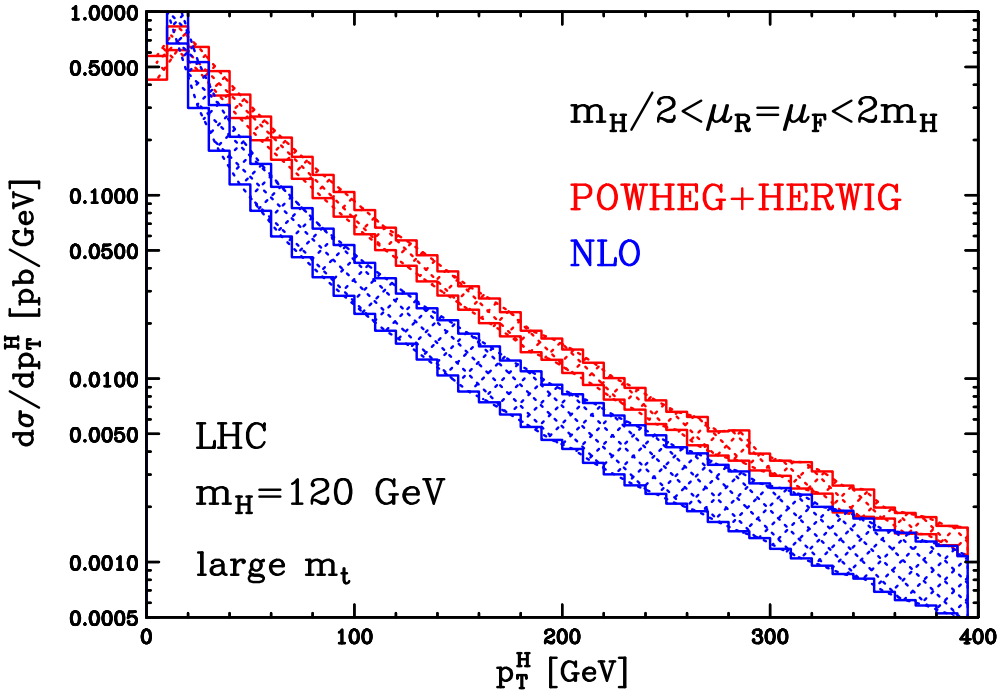


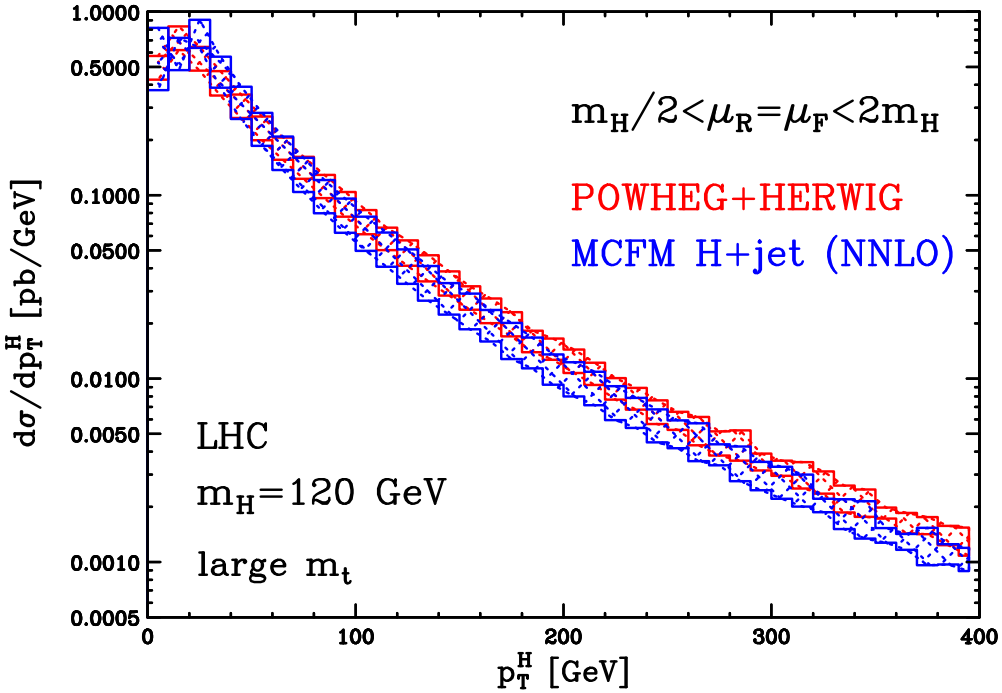
$$\overline{B}(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int d\Phi_r \left[R(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r)\right]$$

$$d\sigma = \overline{B}(\mathbf{\Phi}_n)\, d\mathbf{\Phi}_n \left\{ \Delta(\mathbf{\Phi}_n, p_T^{\min}) + \Delta(\mathbf{\Phi}_n, p_T)\, \frac{R(\mathbf{\Phi}_n, \Phi_r)}{B(\mathbf{\Phi}_n)}\, d\Phi_r \right\}$$

$$d\sigma_{\rm rad} \approx \frac{\overline{B}(\mathbf{\Phi}_n)}{B(\mathbf{\Phi}_n)}\, R(\mathbf{\Phi}_{n+1})\, d\mathbf{\Phi}_{n+1} = \left\{1 + \mathcal{O}(\alpha_s)\right\} R(\mathbf{\Phi}_{n+1})\, d\mathbf{\Phi}_{n+1}$$

Higgs boson production at the LHC





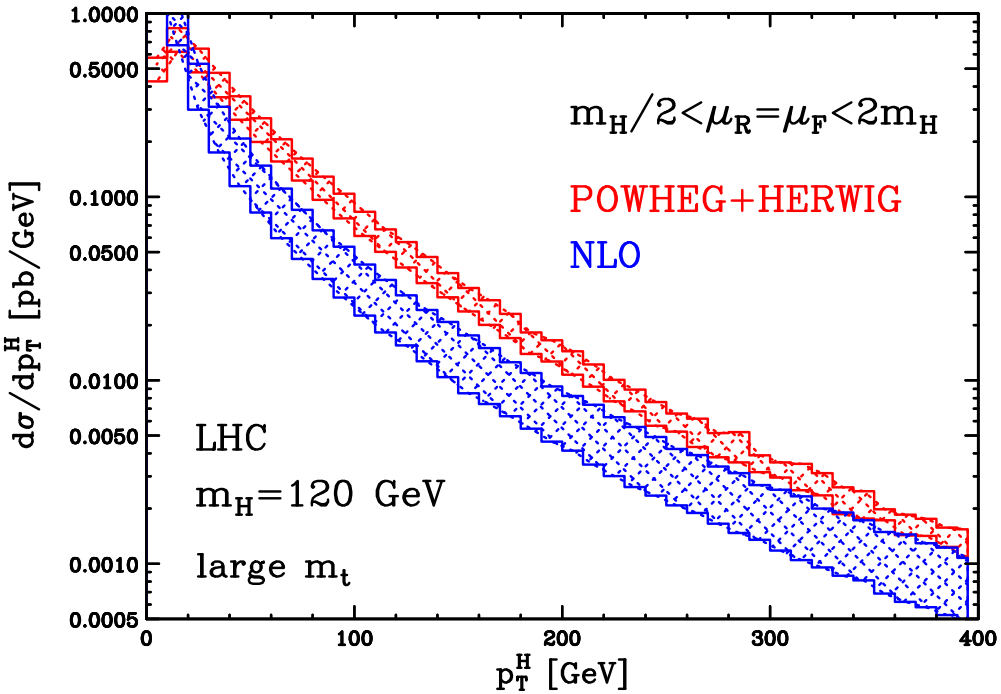
The NLO result is in reality a LO one $\Longrightarrow$ it depends upon $\alpha_s^3(\mu_R)$

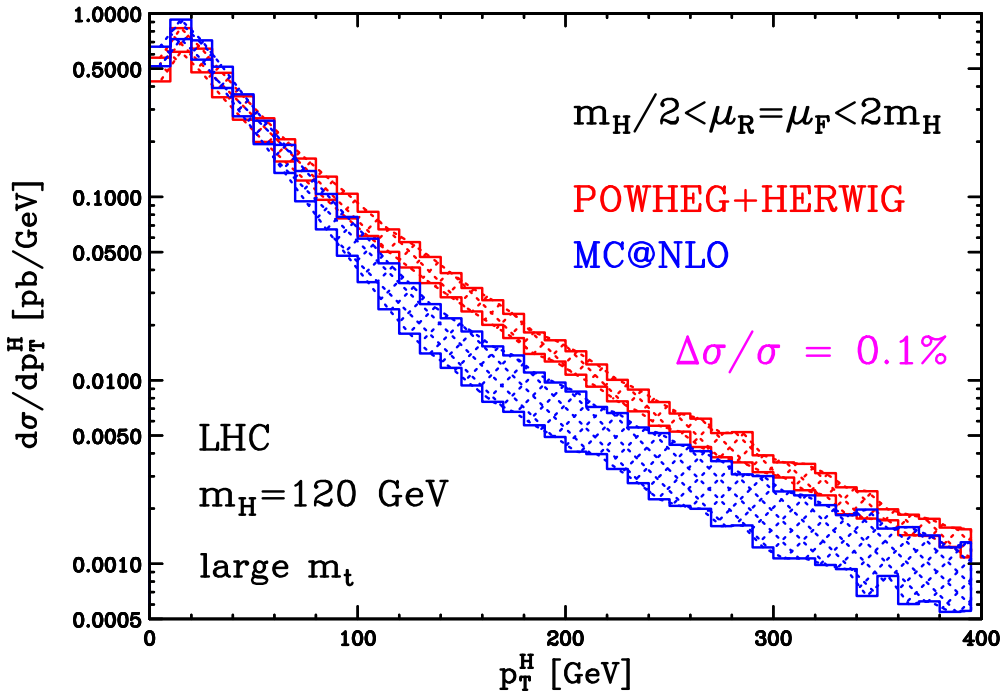
$$\overline{B}(\mathbf{\Phi}_n, \mu_R) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n, \alpha_s(\mu_R)) + \int d\Phi_r \left[R(\mathbf{\Phi}_n, \Phi_r, \alpha_s(\mu_R)) - C(\mathbf{\Phi}_n, \Phi_r)\right]$$

$$d\sigma = \overline{B}(\mathbf{\Phi}_n, \mu_R)\, d\mathbf{\Phi}_n \left\{\Delta(\mathbf{\Phi}_n, p_T^{\min}) + \Delta(\mathbf{\Phi}_n, p_T)\, \frac{R(\mathbf{\Phi}_n, \Phi_r)}{B(\mathbf{\Phi}_n)}\, d\Phi_r\right\}$$

$$\Delta(\mathbf{\Phi}_n, p_T) = \exp\left[-\int d\Phi_r' \frac{R(\mathbf{\Phi}_n, \Phi_r', \alpha_s(k_T))}{B(\mathbf{\Phi}_n)}\, \theta\big(k_T(\mathbf{\Phi}_n, \Phi_r') - p_T\big)\right]$$

Higgs boson production at the LHC





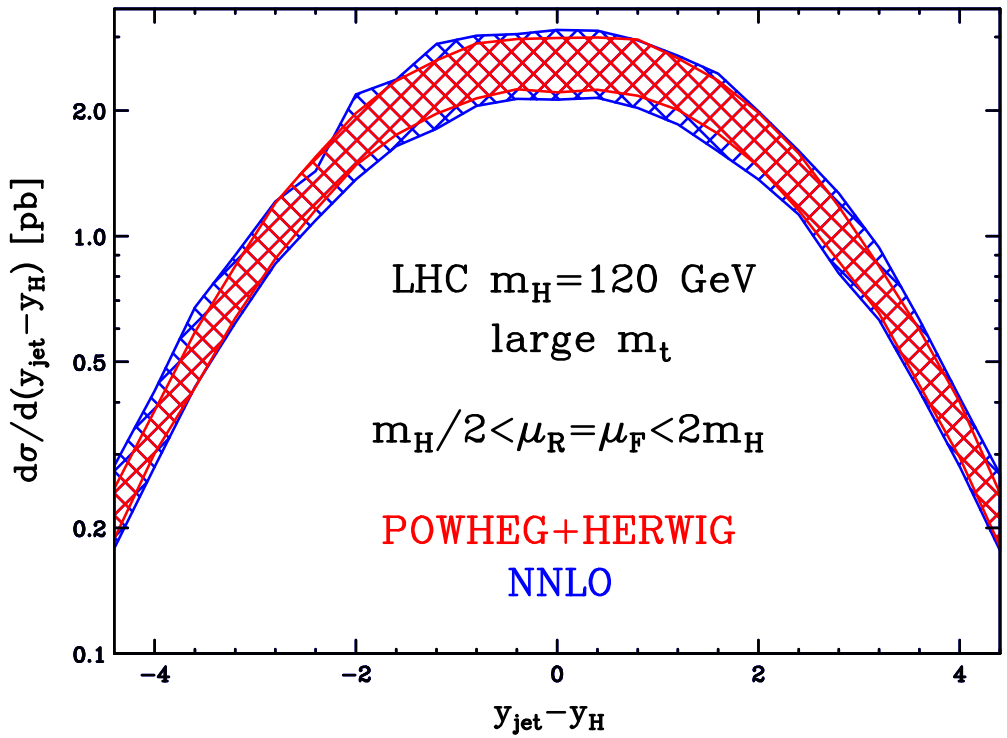
$$R = R \times F + R \times (1-F) = R_{\bar{B}} + R_{\rm reg} \qquad F < 1 \qquad F \to 1 \text{ when } p_T \to 0$$

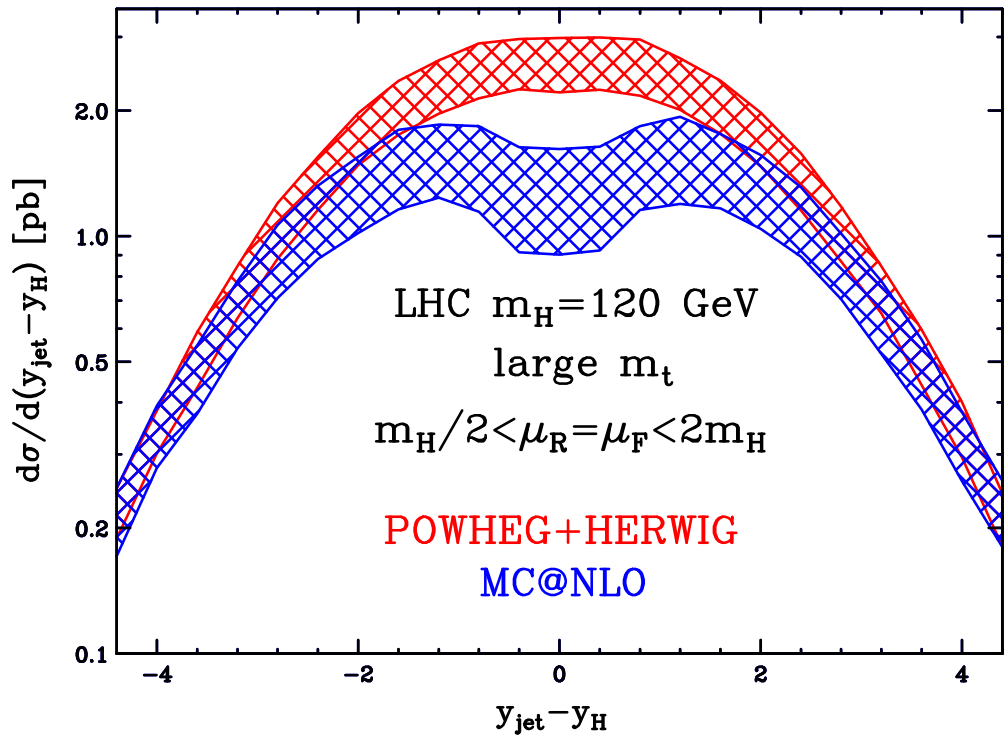
$$\sigma = \sigma_{\bar{B}} + \sigma_{\rm reg}$$

$$\sigma_{\bar{B}} = \int d\mathbf{\Phi}_n \, \overline{B}(\mathbf{\Phi}_n) = \int d\mathbf{\Phi}_n \left\{ B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int d\Phi_r \left[R_{\bar{B}}(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r) \right] \right\}$$

$$\sigma_{\rm reg} = \int d\mathbf{\Phi}_n \, d\Phi_r \, R_{\rm reg}$$

Higgs boson production at the LHC





NNLO result obtained with HNNLO by Catani & Grazzini

From NLO to POWHEG

POWHEG is a **method**, NOT (only) a set of programs!

POWHEG is fully general and can be applied to any NLO subtraction framework.

We have provided any user with all the formulae and ingredients to implement an existing NLO calculation in the POWHEG formalism [Frixione, Nason and Oleari, arXiv:0709.2092].

We have looked in detail at POWHEG in two subtraction schemes:

- the Frixione, Kunszt and Signer scheme
- the Catani and Seymour scheme.

We have discussed, in a pedagogical way, two examples:

- $e^+e^- \to q\bar{q}$
- $q\bar{q} \to V$

The `fortran` implementation of the POWHEG code for these two processes (and all the others) can be found at

`http://moby.mib.infn.it/~nason/POWHEG`

Strategy and conclusions

✓ Shower Monte Carlo programs to do the final shower already exist

✓ Most of them implement a p_T veto

✓ Most of them comply with a standard interface to hard processes, the so called Les Houches Interface (LHI)

SO...

- construct a POWHEG for a NLO process. Output on LHI
- if needed, construct a generator capable to add truncated showers to events from the LHI. Output again on LHI
- use standard shower Monte Carlo programs to perform the p_T-vetoed final shower from the event on LHI.