Practical Statistics Problems, DESY, Feb 2016

Here are just 2 problems. The first is designed to show you that it can be easy to calculate the best value and the range for a parameter by the likelihood method. You can also use the information from my Likelihood lecture to check that the best value is correct, and that the range is reasonable.

The second is to help you understand the concept of coverage, and why there are discontinuities in a plot of coverage as a function of μ for Poisson data.

The first problem requires you to write a short computer programme; the second can be done with just a simple calculator.

Please do these problems, as they will improve your understanding of these topics.

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1) An experiment is determining the decay rate λ for a new particle X, whose probability density for decay at time t is proportional to exp(-λt). A total of nine decays are observed at decay times 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 picoseconds. Calculate the likelihood function L(λ) at suitable values of λ (most easily done by a simple computer program), and draw a graph of the results. Find the best estimate of λ from the maximum of the likelihood curve, and a "±σ" range for λ by finding the values of λ where the logarithm to the base e of the likelihood function decreases by 0.5 units from its maximum value.

2) The coverage C(μ) is a property of a statistical technique for estimating a range for a parameter μ at a confidence level α (e.g. 68%, 90% or whatever). It is the fraction of times that, in repetitions of the procedure with different data each with its own statistical fluctuations, the estimated range contains the true value μ.
In a Poisson counting experiment with n observed events, one method of estimating a range for the Poisson parameter μ uses the estimate n +- sqrt(n) i.e. from n - sqrt(n) to n + sqrt(n). This is supposed to have 68% coverage. Determine the actual coverage C(μ) at μ = 3.41 and 3.42 as follows:
Determine for which measured values of n the nominal range from the "n+-sqrt(n)" procedure

includes the specified true value, and then add up the Poisson probabilities for obtaining these measured values, again assuming the specified value of the Poisson parameter.
Explain why a plot of the coverage $C(\mu)$ as a function of the Poisson parameter value μ has discontinuities.