Non-locality in General Relativity and Quantum Field Theory

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BE STILL AND KNOW

Introduction

- We have good reasons to think that length scales smaller than the Planck scale are not observables due to the formation of small Planckian black holes: there is a minimal length i.e. a form of non-locality.

- In that sense space-time singularities should be smeared by quantum effects and effective theories might be all we need to study a unification of quantum mechanics and general relativity.

- Can we see this non-locality appearing using effective field theory techniques?

A minimal length from QM and GR

Claim: GR and QM imply that no operational procedure exists which can measure a distance less than the Planck length.

Assumptions:

- Hoop Conjecture (GR): if an amount of energy E is confined to a ball of size R, where R < E, then that region will eventually evolve into a black hole.
- Quantum Mechanics: uncertainty relation.

Minimal Ball of uncertainty:

Consider a particle of Energy E which is not already a black hole.
Its size r must satisfy:

$$r \gtrsim \max\,[1/E\,,\;E\,]$$

where 1/E is the Compton wavelength and E comes from the Hoop Conjecture. We find:

$$r \sim l_P$$

Can we identify this minimal length in GR and QFT? It is a form of non-locality.

Effective action for GR coupled to known matter

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Planck scale $(M^2 + \xi v^2) = M_P^2$ $M_P = 2.4335 \times 10^{18}$ GeV
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1 = 5 \times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6 \times 10^{15}$ [xc & Atkins, 2013]

    Higgs inflation requires $\xi \sim 10^4$.

Unitarity in linearized GR

Let us consider gravitational scattering of the particles included in our model (s-channel, we impose different in and out states)

(Han & Willenbrock 2004, xc & Atkins 2011)

$$\mathcal{A} = 16\pi \sum_J (2J+1) a_J d^J_{\mu,\mu'}$$

$$|\mathrm{Re}\ a_J| \leq 1/2$$

Let us look at J=2 partial wave

$$a_2 = -\frac{1}{320\pi} \frac{s}{\bar{M}_P^2} N$$

One gets the bound:

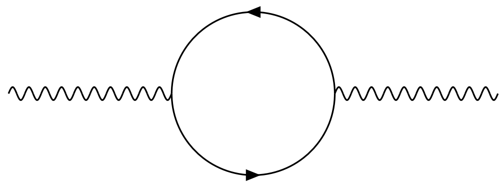
$$N = 1/3 N_s + N_\psi + 4 N_V$$

$$E^*_{\mathrm{CM}} \leq \bar{M}_P \sqrt{\frac{160\pi}{N}}$$

For large *N*, unitarity can be violated well below the Planck mass.

Self-healing of unitarity

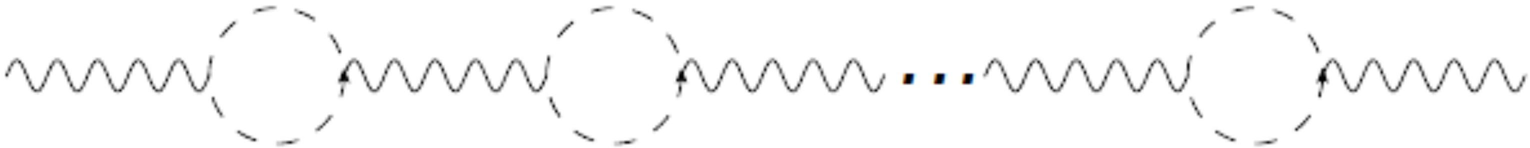
- Aydemir, Anber & Donoghue argued that the effective theory heals itself.
- First let's calculate the leading quantum corrections to the previous amplitude (still working in linearized GR in flat space-time)



- Insert any matter in your model in that loop.
- Typically there is more matter than gravitational degrees of freedom, we can thus ignore gravitons in that loops for energies below the Planck mass.
- Honest calculation: regularized using dim-reg and absorb divergencies in R^2 etc.
- Obviously the theory is still not renormalizable, but that's not an issue for an effective field theory.

Self-healing of unitarity

- In the case of linearized gravity coupled to QFT, resum:



- in the large N limit, keeping NG_N small, one obtains a resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

$$N = N_s + 3N_f + 12N_V$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu/q^2$$

- One can check explicitly

$$|A_{dressed}|^2 = \mathrm{Im}(A_{dressed})$$

Poles in the resummed graviton propator

- In linearized GR, the effective theory self-heals itself.
- However, the resummed graviton propagator has poles: sign of strong interaction.
- The positions of these poles depend on the number of fields
- One finds

$$
\begin{aligned}
q_1^2 &= 0, \\
q_2^2 &= \frac{1}{G_N N} \frac{120\pi}{W\left(\frac{-120\pi M_P^2}{\mu^2 N}\right)}, \\
q_3^2 &= (q_2^2)^*,
\end{aligned}
$$

Poles as Planckian Black Holes?

- It is tempting to interpret these poles as black hole precursors.
- In the SM

$$N_s = 4,\ N_f = 45,\ \text{and}\ N_V = 12$$

- We thus find

$$(7 - 3i) \times 10^{18}\ \text{GeV}\ \text{and}\ (7 + 3i) \times 10^{18}\ \text{GeV}$$

- The first one corresponds to a state with mass $p_0^2 = (m - i\Gamma/2)^2$

$$7 \times 10^{18}\ \text{GeV}$$

- and width

$$6 \times 10^{18}\ \text{GeV}$$

Back to the effective field theory

- Note that the 2nd pole has the wrong sign for a particle: it is a ghost

$$(7+3i)\times 10^{18}\ \mathrm{GeV}$$

- In terms of EFT, the poles correspond to the non local effects in gravity parameterized by

$$S=\int d^4x\sqrt{g}\left[R\log\left(\frac{\Box}{\mu^2}\right)R\right]$$

- Can these effects soften singularities?

- The effective theory breaks down at the energy scale $M_\star$ corresponding to these poles.

Gravity leads to non-local effects in Matter

XC, Croon & Fritz (2015)

- Using the resummed graviton propagator we can now calculate the dressed amplitude for the gravitational scattering two identical scalar fields (s, t & u channels):

$$A_{dressed} = A_{tree} + A^{(1)} + \dots$$

- The tree-level amplitude has been known for a long time. The nonlocal correction is given by

$$\begin{aligned} A^{(1)} = & \frac{2}{15} G_N^2 N \left(m^4 \left(\log \left(-\frac{stu}{\mu^6} \right) \right) \right. \\ & + \log \left(-\frac{s}{\mu^2} \right) (2m^2 + t)(2m^2 + u) + \log \left(-\frac{t}{\mu^2} \right) (2m^2 + s)(2m^2 + u) \\ & \left. + \log \left(-\frac{u}{\mu^2} \right) (2m^2 + s)(2m^2 + t) \right) . \end{aligned}$$

Gravity leads to non-local effects in Matter

- It is easy to see that $A^{(1)}$ can be obtained from this effective operator:

$$O_8 \;=\; \frac{2}{15}G_N^2 N\left(\partial_\mu\phi(x)\partial^\mu\phi(x) - m^2\phi(x)^2\right)\log\left(-\frac{\Box}{\mu^2}\right)\left(\partial_\nu\phi(x)\partial^\nu\phi(x) - m^2\phi(x)^2\right)$$

- This is a non-local operator, we need to make sense of the log term to obtain a causal theory (Espiru et al. (2005), Donoghue &El-Menoufi (2014) and Barvinsky et al in the 80's.)

$$L \;\supset\; \sqrt{-g(x)}\frac{2}{15}G_N^2 N \\ \left(\left(\partial_\mu\phi(x)\partial^\mu\phi(x) + m^2\phi(x)^2\right)\int d^4y\sqrt{-g(y)}\langle x|\log\left(-\frac{\Box}{\mu^2}\right)|y\rangle\left(\partial_\nu\phi(y)\partial^\nu\phi(y) - m^2\phi(y)^2\right)\right)$$

- with the definition:

$$\mathcal{L}(x,y) = \langle x|\log\left(-\frac{\Box}{\mu^2}\right)|y\rangle$$

It can be calculated explicitly when needed.

Non-local effects in QFT

- We have seen that the non-local effects observed in gravity feedback into matter.
- This is compatible with our interpretation of the poles of the resummed propagators as quantum black holes (black hole precursors) which are extended objects.
- The new higher dimensional operators have an approximate shift symmetry

$$\phi \to \phi + c, \text{ where } c \text{ is a constant}$$

- which is broken explicitly by the mass of the scalar field.
- This is interesting for models of inflation.
- Very small non-Gaussianities even for a single scalar inflation model.

Conclusions

- We have discussed a conservative effective action for quantum gravity within usual QFTs such as the standard model.
- Thought experiments point towards the existence of a minimal length in nature when combining GR and QM.
- We can identify these effects in QFT coupled to GR.
- We have found the sign of strong dynamics in the resummed graviton propagator.
- These poles can be interpreted as black hole precursors and their masses and width can be calculated.
- These quantum black holes lead to non-local effects in QFT.

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Thanks for your attention!