Recent Developments in Superconformal Field Theories

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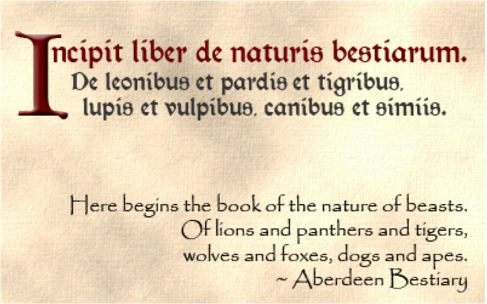
Bad Honnef, Feb 2016

Outline

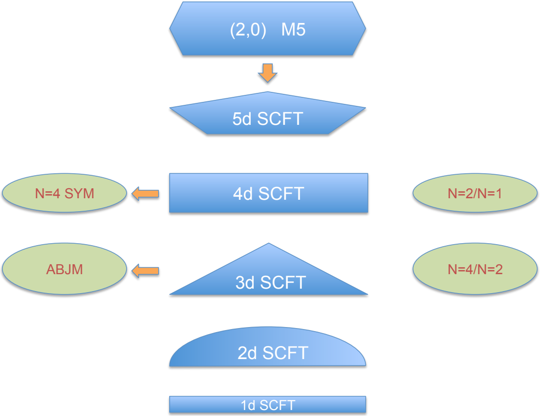
- **Part I: Zoology**
- **Part II: Euclidean SCFT in Finite Volume**
- **Part III: Low Dimensions**

PART I : ZOOLOGY



Bestiary from String Theory

A deeply interconnected web of superconformal theories arising from branes, most often strongly coupled, related by various types of dualities.



SCFT and Nahm's classification

Conformal field theories

- $d > 2$: Poincarè $(P_\mu, M_{\mu\nu})+$ Scaling $(D)+$ Special Conformal (K_μ)

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Nahm's classification of unitary SCFT back in 1977

- maximally supersymmetric (32 real charges): $d = 2, 3, 4, 6$
- (16 real charges): $d = 2, 3, 4, 5, 6$
- (8 real charges): $d = 2, 3, 4$

Four dimensional gauge theories

Many examples in 4d

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- a superconformal zoo of $N = 1$ SCFT (Leigh-Strassler argument)

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And yet a new beast: people claims the existence of $N = 3$ SCFT (24 supercharges) [Aharony,Evtikhiev; $\cdots$]

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And yet a new beast: people claims the existence of $N = 3$ SCFT (24 supercharges) [Aharony,Evtikhiev; $\cdots$]

- Perturbatively $N = 3$ is the same as $N = 4$
- Isolated fixed point at strong coupling

Three dimensional gauge theories

It is easy to find SCFT in 3d

The gauge coupling is dimensionful and gauge theories become strongly coupled in the IR.

$$\frac{1}{g^2} \sim [M]^{-1}$$

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- large number of proposals for $N = 2$ SCFT

Five and Six dimensional gauge theories

Is this possible? 5d and 6d gauge theories are not renormalizable and IR free.

The gauge coupling is again dimensionful and grows in the UV. UV fixed point?

$$\mathcal{L}_{CFT}\big|_{g=\infty} + \frac{1}{g^2} F_{\mu\nu}^2 \qquad \text{relevant operator}$$

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- 6d theories with tensors and without a known Lagrangian

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Back in 96, recent revival...

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Original examples (1996) [seiberg,morrison]

$SU(2)$ gauge theory with N_f hypermultiplets

with ehnancement of global symmetry from $SO(2N_f) \times U(1)$ to E_{N_f+1}.

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The effective gauge coupling

$$\frac{1}{g_{eff}(\phi)^2} = \frac{1}{g_0^2} + c|\phi|$$

For $g_0 \to \infty$, $g_{eff}(\phi = 0) = 0$ and you find a strongly coupled fixed point at the origin.

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Recent revival, many examples from branes, pq-web, etc...

[Kim,Kim,Lee; Bergman,Rodrigez-Gomes,Zafrir; Bao, Mitev,Pomoni,Yagi; $\cdots$]

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- $(1,1)$ supersymmetry has vector multiplets: $A_\mu, \lambda^+, \lambda^-$ and 4 scalars. D5 branes in IIB.
- $(2,0)$ supersymmetry has tensor multiplets: $B_{\mu\nu}, \lambda^+, \lambda^+$ and 5 scalars. M5 branes in M theory. Theory of (non-abelian) self-dual tensor multiplets:

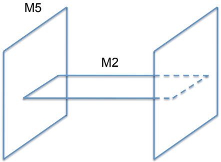
$$H = dB \qquad H = *H$$

Only $(2,0)$ is superconformal.

Six dimensional SCFT

The mysterious $(2,0)$ 6d theory is realized in string theory (1995)

- Worldvolume theory of M5 branes.
- Tensionless string theory.
- Theory of (non-abelian) self-dual tensor multiplets with no Lagrangian



$$H = dB \qquad H = *H$$

Six dimensional SCFT

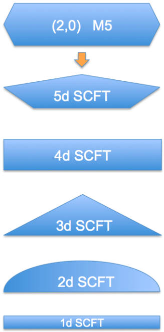
Most of brane constructions originate from the mysterious (2, 0) 6d theory

- Gauge fields upon compactification

$$6d \to 5d : A_\mu = B_{\mu 6}$$

- Lagrangian and non Lagrangian theories in $d \leq 6$

  [Gaiotto,AGT,$\cdots$ (2009)]



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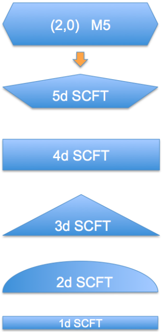

Also $(1,0)$ SCFT theories in 6 d (97, recent revival)

[97: seiberg; intriligator, morrison; hanany,AZ; blum,karch,$\cdots$]

[recent: vafa, morrison, heckman, tomasiello,$\cdots$]

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Maximal SYM has no 5d fixed points. What's about a different UV completion?
[Douglas; Lambert, Papageorgakis, Schmidt-Sommerfeld]

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6d $N = (2,0)$ SCFT on $S^1 \longrightarrow$ 5d max SYM

$m_{KK} = 1/R = 1/g^2$ instantons

Tools for SCFT

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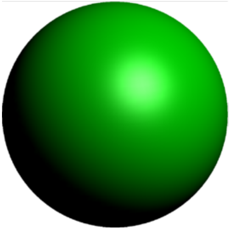
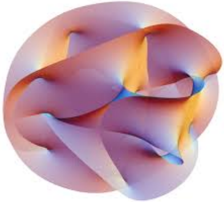
- Computing exact quantities: partition functions through localization.
- Holographically: find a gravity dual at large N.
- Constructively: bootstrap methods

PART II : Euclidean Theories in Finite Volume





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To regularize IR physics we can put the theory on a Euclidean compact manifold.
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Topological Field Theories [Witten, 1988]

- $N = 2$ theories in 4d with $SU(2)_R$ R-symmetry can be consistently defined on any M_4 euclidean manifold
- The resulting theory is *topological*: independent of metric.

$$T_{\mu\nu} = \{Q, \cdots\}$$

Euclidean SCFT in finite volume

To regularize IR physics we can put the theory on a Euclidean compact manifold. Can we preserve supersymmetry?

SCFT on Spheres [Pestun 2007]

- Just use conformal map from $\mathbb{R}^n$ to S^n

Euclidean SCFT in finite volume

To regularize IR physics we can put the theory on a Euclidean compact manifold. Can we preserve supersymmetry?

- String theory strongly relies on the existence of extra-dimensions. The requirement that a vacuum is supersymmetric selects particular type of internal geometries (Calabi-Yau, Generalized Geometries, ...) which have been objects of intense investigation in the last thirty years.
- The recent QFT interest is in an even simpler question: when can we define a supersymmetric quantum field theory on a nontrivial manifold M ?

Supersymmetric theories

Supersymmetric theories are usually formulated on Minkowski space-time $\mathbb{R}^{3,1}$. At the classical level, we have an action for bosonic and fermionic fields

$$S_{\rm SUSY}(\phi(x), \psi(x), A_\mu(x), ...)$$

invariant under transformations that send bosons into fermions and viceversa

$$\delta\phi(x) = \epsilon\psi(x)\,, \qquad \delta\psi = \partial_\mu\phi\gamma^\mu\epsilon + \cdots$$

where ϵ is a constant spinor.

The symmetry group of the theory contains translations, Lorentz transformations $SO(3,1)$ and the fermonic symmetries with the corresponding fermionic Noether charges Q. The theory can be also formulated on Euclidean space $\mathbb{R}^4$.

Can we define the theory on a general manifold M preserving supersymmetry?

Supersymmetric theories on curved spaces

The general strategy is to promote the metric to a dynamical field [Festuccia,Seiberg] .

This is done by coupling the rigid theory to the multiplet of supergravity $(g_{\mu\nu}, \psi_\mu, ...)$

$$S_{\mathrm{SUGRA}}(\phi(x), \psi(x), g_{\mu\nu}(x), \psi_\mu(x), ...)$$

which is invariant under local transformations

$$\delta\phi(x) = \epsilon(x)\psi(x)\,, \qquad \delta e^a_\mu(x) = \bar{\epsilon}(x)\gamma^a\psi_\mu(x) + \cdots$$

We are gauging the original symmetries of the theory. At linear level this is just the Noether coupling

$$-\frac{1}{2}g_{mn}T^{mn} + \bar{\psi}_m\mathcal{J}^m$$

Supersymmetric theories on curved spaces

The rigid theory is obtained by freezing the fields of the metric multiplet to background values

$$g_{\mu\nu} = g^M_{\mu\nu}\,, \quad \psi_\mu = 0$$

The resulting theory will be supersymmetric if the variation of supersymmetry vanish

$$\begin{aligned} \delta e^a_\mu(x) &= \bar\epsilon(x)\gamma^a\psi_\mu(x) + \cdots \equiv 0 \\ \delta\psi_\mu(x) &= \nabla_\mu\epsilon + \cdots \equiv 0 \end{aligned}$$

The graviton variation gives a differential equation for $\epsilon(x)$ which need to be solved in order to have supersymmetry and gives constraints on M.

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Consider for example, $N = 1$ SCFTs.

The group of symmetries of a $N = 1$ SCFT is enlarged to the algebra $SU(2, 2|1)$

- translations + Lorentz $SO(3, 1) \rightarrow$ conformal group $SO(4, 2)$
- supersymmetry Q is doubled: (Q, S)
- extra bosonic global symmetries rotating (Q, S) (R-symmetries)

$$U(1) : \qquad Q \rightarrow e^{i\alpha} Q$$

Superconformal theories on curved spaces

The strategy here is to couple the CFT to conformal supergravity.
Consider for example, $N = 1$ SCFTs.

The $N = 1$ conformal supergravity multiplet $(g_{\mu\nu}, \psi_\mu, A_\mu)$ contains gauge fields for the superconformal symmetries

$$-\frac{1}{2} g_{mn} T^{mn} + A_m J^m + \bar{\psi}_m \mathcal{J}^m$$

We freeze $(g_{\mu\nu}, A_\mu)$ to background values and set $\psi_\mu = 0$. In order to preserve some supersymmetry, the gravitino variation must vanish.

$$(\nabla_a - iA_a)\epsilon_+ + \gamma_a \epsilon_- = 0$$

$\epsilon_\pm$ parameters for the supersymmetries and the superconformal transformations.

Superconformal theories on curved spaces

The variation of the gravitino can be written as a (twisted) conformal Killing equation

$$(\nabla_a - iA_a)\epsilon_+ + \gamma_a \epsilon_- = 0 \qquad \Longrightarrow \qquad \nabla_a^A \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+$$

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- for theories with more supercharges and in different dimensions, A_a is promoted to a non-abelian gauge field, there can be other backgrounds tensor fields and extra conditions.

Superconformal theories on curved spaces

A possible solution is to have covariantly constant spinors

$$\nabla_a^A \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+ \qquad \text{solved by} \qquad \nabla_a^A \epsilon_+ = 0$$

Realized for Topological Field Theories. We can turn on a $SU(2)_R$ background gauge field for the $N = 2$ R-Symmetry in order to cancel the $SU(2)$ spin connection

$$\nabla_a^A \epsilon_+ = \partial_a \epsilon_+ + \frac{1}{4}\omega_a^{\alpha\beta}\epsilon_+ + A_a \epsilon_+ = \partial_a \epsilon_+$$

ϵ_+ transform in the $(\mathbf{2}, \mathbf{0})$ representation of the local Euclidean group $SO(4) = SU(2) \times SU(2)$

which can be solved by $\epsilon_+ =$ constant on any manifold M_4.

Superconformal theories on curved spaces

A possible solution is to have Killing spinors

$$\nabla_a^A \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+ \qquad \text{solved by} \qquad \nabla_a^A \epsilon_+ = \gamma_a \epsilon_+$$

Realized for Theories on Spheres, with zero background field $A = 0$.

Superconformal theories on curved spaces

Another possible solution is to have Killing spinors on S^d and holonomies on S^1

$$\nabla_a^A \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+ \qquad \text{preserves susy on} \qquad S^d \times S^1$$

The path integral on $S^d \times S^1$ can be written as a trace over a Hilbert space

$$Z_{S^d \times S^1} = \mathrm{Tr}(-1)^F e^{-\beta\{Q,S\}+\sum \Delta_a J_a}$$

This is the superconformal index

[Romelsberger; Kinney,Maldacena,Minwalla,Rahu]

computed for large class of 4d SCFT [Gadde,Rastelli,Razamat,Yan]

Superconformal theories on curved spaces

In between a pletora of possibilities. Backgrounds with (twisted) conformal Killing spinors have been classified in various dimensions and with various amount of supersymmetry. For example,

- In $N = 1$ in 4d, the existence of a CKS is (locally) equivalent to the existence of a complex structure.
- In $N = 2$ in 3d, the existence of a CKS is (locally) equivalent to the existence of a holomorphic transverse foliation.

[klare,tomasiello,A.Z.;dumitruescu,festuccia,seiberg]

Localization

Exact quantities in supersymmetric theories with a charge $Q^2 = 0$ can be obtained by a saddle point approximation

$$Z = \int e^{-S} = \int e^{-S+t\{Q,V\}} \underset{t\gg 1}{=} e^{-\bar{S}|_{class}} \times \frac{\det_{fermions}}{\det_{bosons}}$$

$$\partial_t Z = \int \{Q, V\} e^{-S+t\{Q,V\}} = 0$$

Very old idea that has become very concrete recently, with the computation of partition functions on spheres and other manifolds supporting supersymmetry.

Localization

Localization ideas apply to path integral of Euclidean supersymmetric theories

- Compact space provides IR cut-off, making path integral well defined
- Localization reduces it to a finite dimensional integral, a matrix model

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$$\int \prod_{i=1}^{N_1} du_i \prod_{j=1}^{N_2} dv_j \frac{\prod_{i<j} \sinh^2 \frac{u_i - u_j}{2} \sinh^2 \frac{v_i - v_j}{2}}{\prod_{i<j} \cosh^2 \frac{u_i - v_j}{2}} e^{\frac{ik}{4\pi}\left(\sum u_i^2 - \sum v_j^2\right)}$$

ABJM, 3d Chern-Simon theories, [Kapustin,Willet,Yakoov;Drukker,Marino,Putrov]

Localization

Carried out recently in many cases

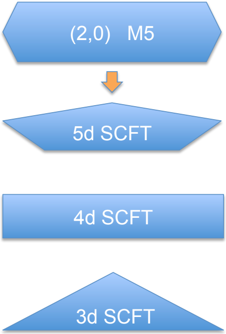
- many papers on topological theories
- S^2, T^2
- S^3, $S^3/\mathbb{Z}_k$, $S^2 \times S^1$, Seifert manifolds
- S^4, $S^4/\mathbb{Z}_k$, $S^3 \times S^1$, ellipsoids
- S^5, $S^4 \times S^1$, Sasaki-Einstein manifolds

with addition of boundaries, codimension-2 operators, ...

Pestun 07; Kapustin,Willet,Yakoov; Kim; Jafferis; Hama,Hosomichi,Lee, too many to count them all ⋯

Free energy on S^d

Successfully testing holographic prediction at large N: large N matrix model techniques



$$AdS_7 \quad \rightarrow \quad \mathcal{F} \sim N^3$$

$$AdS_6 \quad \rightarrow \quad \mathcal{F} \sim N^{5/2}$$

$$AdS_5 \quad \rightarrow \quad \mathcal{F} \sim N^2$$

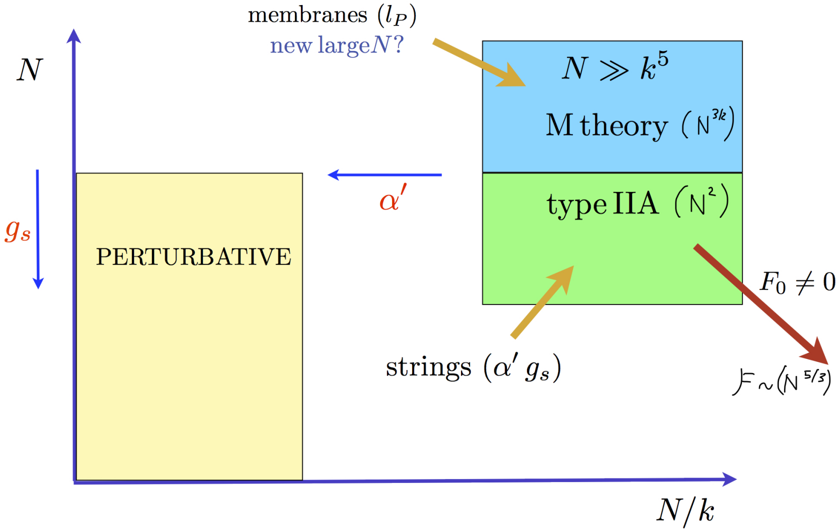
$$AdS_4 \quad \rightarrow \quad \mathcal{F} \sim N^{3/2}, \quad N^{5/3} \ldots$$

Free energy on S^3

A particularly interesting case is the sphere partition function on S^3

- the free energy $F = -\log Z_{S^3}$ counts d.o.f, like a central charge
- F decreases along the RG [Jafferis,Klebanov,Pufu,Safdi;Casini,Huerta]
- the exact superconformal symmetry maximizes F [jafferis]
- variety of behaviours at large N

3d Phase diagram



5d amusement

- $Z_{S^4\times S^1}$ = Superconformal index can see enhancement of global symmetry

$$SU(2)\,\text{with}\,N_f\,\text{flavors}\ :\ SO(2N_f)\to E_{N_f+1}$$

[Kim,Kim,Lee; Bergman,Rodrigez-Gomes,Zafrir; Bao, Mitev,Pomoni,Yagi; ⋯]

- maximally supersymmetric 5d SYM: $logZ\sim N^3$

$$Z_{S^5}(SYM_{5d})=Z_{S^5\times S^1}((2,0)_{6d})$$

[Kim^3, Kallen,Zabzine,Mihanan, Nedelin; Hosomichi,Seong,Terashima; ⋯]

PART I : LOW DIMENSIONS

Two dimensions

Surprises in 2d? CFT and SCFT well understood. New techniques nevertheless for computing

- Partition function on S^2. Kähler potential of gauged linear sigma models. [Benini,Cremonesi; Doroud,Gomis,LeFloch]
- Topologically twisted partition function on S^2. Amplitudes of gauged linear sigma models. [Benini,AZ; Cremonesi, Closset,Park]
- Partition function on T^2. Elliptic genus. [Benini,Eager,Hori,Tachikawa]

One dimension

Almost totally unexplored. Conformal algebra in 1d involves $SL(2, R)$

$$H\,, \qquad D, \qquad K$$

with various superconformal extensions.

- Basic tools (state/op correspondence) lacking in 1d
- AdS_2/CFT_1 is not understood at all

One dimension

However supersymmetric black holes have horizon $\mathrm{AdS}_2 \times S^2$

- CFT_1 living at the horizon of black holes
- density of states = BH entropy

One dimension

One can use localization to evaluate the Witten index of QM with two supercharges [Hori,Kim,Yi]

$$\mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ_F A^F} e^{-\beta H}\right)$$

$Q^2 = H - \sigma^F J_F$

holomorphic in $A^F + i\sigma^F$

where J_F is the generator of the global symmetry.

- Unfortunately a SCQM has no mass gap and the index is ill-defined.

Application: AdS_4 black holes

A nice application where many of the previous ingredients come together is given by the counting of microstates of asymptotically AdS_4 BPS black holes [Benini,Hristov, AZ]

AdS_4 black holes

There is a large class of 1/4 BPS asymptotically AdS_4 static black holes

[Cacciatori, Klemm; Gnecchi, Dall'agata; Hristov, Vandoren];

Holography tells us that they corresponds to states in some dual 3d SCFT theory.

AdS_4 black holes

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[Cacciatori, Klemm; Gnecchi, Dall'agata; Hristov, Vandoren];

They are characterized by

- characterized by a collection of magnetic charges $\int_{S^2} F$
- preserving supersymmetry via a twist

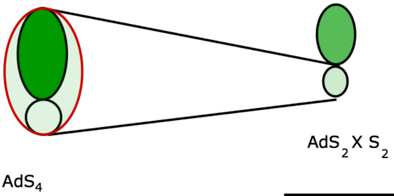
$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu \epsilon \qquad \Longrightarrow \qquad \epsilon = \text{cost}$$

They correspond to 3d topologically twisted SCFT in the presence of magnetic charges for the global symmetries.

AdS_4 black holes

Asymptotically AdS backgrounds describe CFTs on curved space-times

$$ds^2_{d+1} = \frac{dr^2}{r^2} + (r^2 ds^2_{M_d} + O(r)) \qquad A = A_{M_d} + O(1/r)$$



Entropy of black holes

Counting of microstates

Partition function of twisted

3d CFT on S_2 X S_1

QM fixed point

AdS_4 black holes

The BH entropy is a ugly function of the magnetic charges

- For an M theory black hole in $AdS_4 \times S^7$ depending on one magnetic charge

$$S_{BH} = \sqrt{-1 + 6\mathfrak{n}_1 - 6\mathfrak{n}_1^2 + (-1 + 2\mathfrak{n}_1)^{3/2}\sqrt{-1 + 6\mathfrak{n}_1}}$$

AdS_4 black holes

Can we extract the BH entropy for field theory?

- Dual CFT is a 3d gauge theory on $S^2 \times S^1$, topologically twisted on S^2.
- A QM is obtained from compactification $S^2 \times S^1 \to S^1$

The partition function depends on a set of magnetic charges q and chemical potentials A

$$Z^{\text{twisted}}_{S^2\times S^1}(q, A) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ_F A^F} e^{-\beta H}\right)$$

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When extremized with respect to the A reproduces the Black Hole entropy

$$Z^{\text{twisted}}_{S^2 \times S^1}(q, \bar{A}) \equiv S_{BH}(q)$$

For a class of BH asymptotic to $AdS_4 \times S^7$ (dual theory = ABJM)

[Benini,Hristov, AZ]

AdS_4 black holes

Can we extract the BH entropy for field theory?

- Dual CFT is a 3d gauge theory on $S^2 \times S^1$, topologically twisted on S^2.
- A QM is obtained from compactification $S^2 \times S^1 \to S^1$

Where extremization has a nice interpretation in terms of selection of the exact R-symmetry of the QM.

[Benini,Hristov, AZ]

More details

The ABJM twisted index is

$$Z = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\tilde{\mathfrak{m}} \in \mathbb{Z}^N} \int \prod_{i=1}^{N} \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} x_i^{k\mathfrak{m}_i} \tilde{x}_i^{-k\tilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^{N} \left(1 - \frac{x_i}{x_j}\right)\left(1 - \frac{\tilde{x}_i}{\tilde{x}_j}\right) \times$$

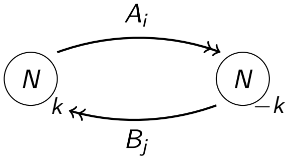
$$\times \prod_{i,j=1}^{N} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_1}}{1 - \frac{x_i}{\tilde{x}_j} y_1}\right)^{\mathfrak{m}_i - \tilde{\mathfrak{m}}_j - \mathfrak{n}_1 + 1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_2}}{1 - \frac{x_i}{\tilde{x}_j} y_2}\right)^{\mathfrak{m}_i - \tilde{\mathfrak{m}}_j - \mathfrak{n}_2 + 1}$$

$$\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_3}}{1 - \frac{\tilde{x}_j}{x_i} y_3}\right)^{\tilde{\mathfrak{m}}_j - \mathfrak{m}_i - \mathfrak{n}_3 + 1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_4}}{1 - \frac{\tilde{x}_j}{x_i} y_4}\right)^{\tilde{\mathfrak{m}}_j - \mathfrak{m}_i - \mathfrak{n}_4 + 1}$$

where $\mathfrak{m}, \tilde{\mathfrak{m}}$ are the gauge magnetic fluxes and y_i are fugacities for the three independent $U(1)$ global symmetries ($\prod_i y_i = 1$)

More details

Black hole supported by magnetic charges: is the ABJM theory with gauge group $U(N) \times U(N)$



with quartic superpotential

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

defined on twisted $S^2 \times \mathbb{R}$ with magnetic fluxes $\mathfrak{n}_i$ for the R/global symmetries

$$SU(2)_A \times SU(2)_B \times U(1)_B \times U(1)_R \subset SO(8)$$

.

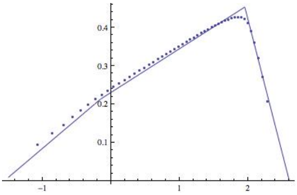
The large N limit

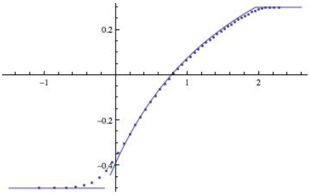
Step 1: solve the large N Limit of algebraic equations giving the positions of poles

$$1 = x_i^k \prod_{j=1}^{N} \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})}$$

with an ansatz

$$\log x_i = i\sqrt{N} t_i + v_i \,, \qquad \log \tilde{x}_i = i\sqrt{N} t_i + \tilde{v}_i$$





The large N limit

Step 1: we dubbed this set of equations

$$e^{iB_i} = e^{i\tilde{B}_i} = 1$$

Bethe Ansatz Equations in analogy with similar expressions in integrability business [Nekrasov-Shatashvili]. They can be derived by a BA potential $\mathcal{F}_{BA}$

The large N limit

Step 2: plug into the partition function. The final result is surprisingly simple

$$\log Z = -\frac{1}{3} N^{3/2} \sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4} \sum_a \frac{\mathfrak{n}_a}{\Delta_a} \qquad\qquad y_i = e^{i\Delta_i}$$

The main result

The index is:

$$\log Z = -\frac{1}{3} N^{3/2} \sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4} \sum_a \frac{\mathfrak{n}_a}{\Delta_a} \qquad\qquad y_i = e^{i\Delta_i}$$

This function can be extremized with respect to the Δ_i and

$$\boxed{\log Z|_{crit}(\mathfrak{n}_i) = \mathrm{BH\,Entropy}(\mathfrak{n}_i)}$$

$$\boxed{\Delta_i|_{crit} \sim X^i(r_h)}$$

The main result

Compare the field theory formula

$$\log Z = -\frac{1}{3} N^{3/2} \sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4} \sum\nolimits_a \frac{\mathfrak{n}_a}{\Delta_a}$$

with the gravity one

$$S = -\frac{\pi}{G_4} \sqrt{X_1(r_h)X_2(r_h)X_3(r_h)X_4(r_h)} \sum_a \frac{\mathfrak{n}_a}{X_a(r_h)}$$

Conclusions and Omissis

The world of SCFT in $1 \leq d \leq 6$ is a lot of fun.

Many unexpected progresses recently, many expected in the future.

A long series of omissis, with recent progresses

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- Classification of 6d (1,0) SCFTs

Thank you for the attention !