Worldsheet string theory in AdS/CFT: beyond perturbation theory

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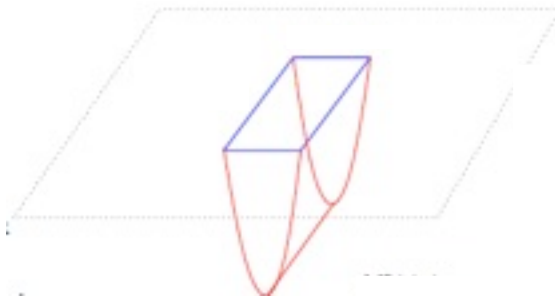
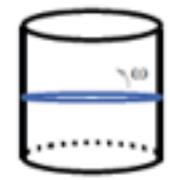
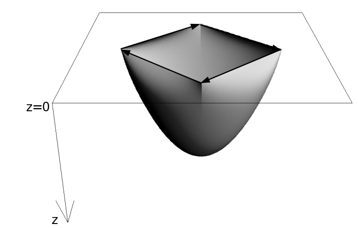
Motivation

Beautiful recent progress in AdS/CFT: for some gauge theory ``observables''







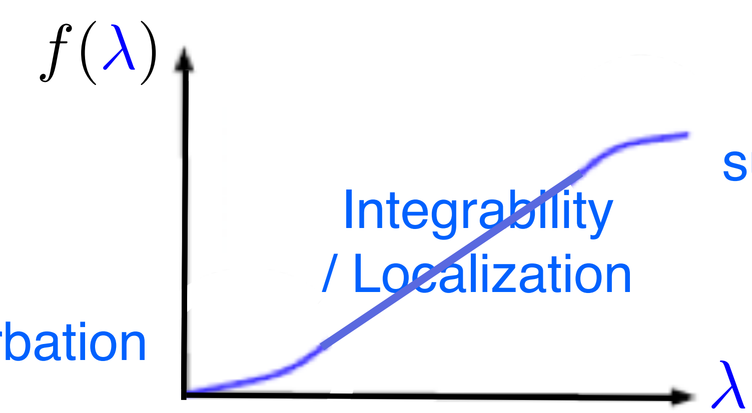
(particular classes of Wilson loops, dimensions and amplitudes, or equivalently string minimal surfaces and energies of string states) exact results can be obtained

> from integrability
> from supersymmetric localization

$$\frac{R^2}{\alpha'} \equiv \sqrt{g_{\rm YM}^2 N} = \sqrt{\lambda}$$

$$f(\lambda) = a\,\lambda + b\,\lambda^2 + \cdots$$

Gauge perturbation theory



Quantum superstrings

$$f(\lambda) = c\sqrt{\lambda} + d + e\,\frac{1}{\sqrt{\lambda}} + \cdots$$

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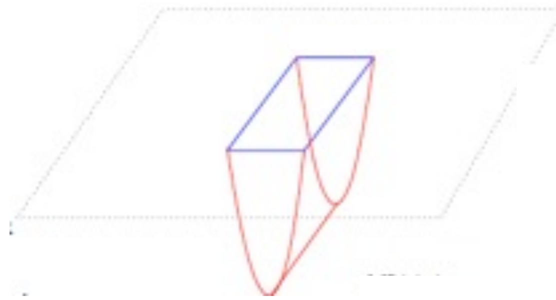
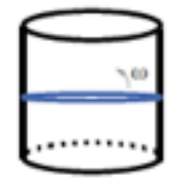
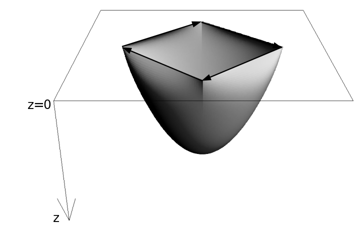
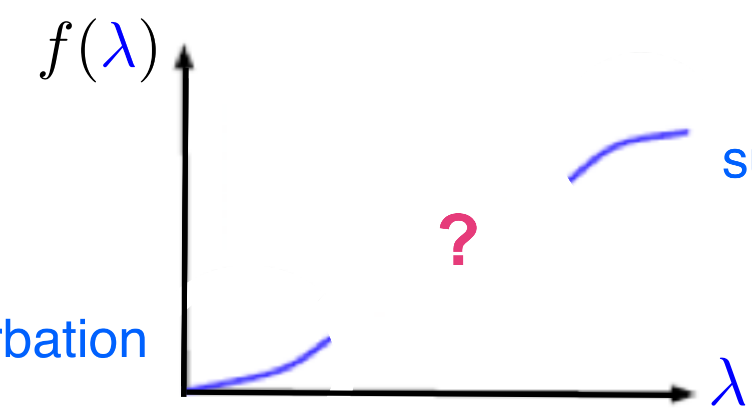




(particular classes of Wilson loops, dimensions and amplitudes, or equivalently string minimal surfaces and energies of string states) exact results can be obtained

> from integrability (assumed, just classical string theory)
> from supersymmetric localization (BPS quantities, not in string theory)

$$\frac{R^2}{\alpha'} \equiv \sqrt{g_{\mathrm{YM}}^2 N} = \sqrt{\lambda}$$

$$f(\lambda) = a\,\lambda + b\,\lambda^2 + \cdots$$



Quantum superstrings

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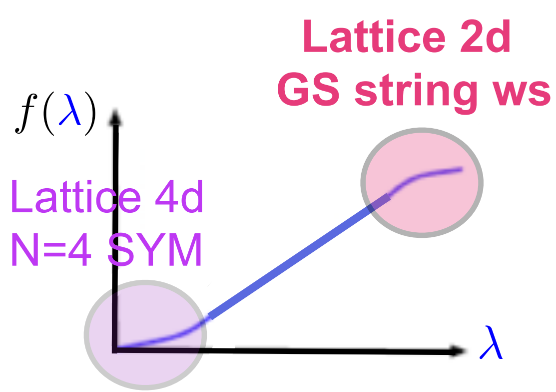
Gauge perturbation theory

Question: can genuine 2d QFT cover the finite-coupling region?

Motivation

Lattice study of Green-Schwarz string worldsheet σ-model in AdS_5xS^5
assumptions-free, potentially powerful tool to test integrability, localization, AdS/CFT.

[McEowan Roiban 13]



Lattice and AdS/CFT: existing program for gauge theory, main issue is susy - good results at weak coupling.

[Catterall et al.]

> 2d: computationally cheap

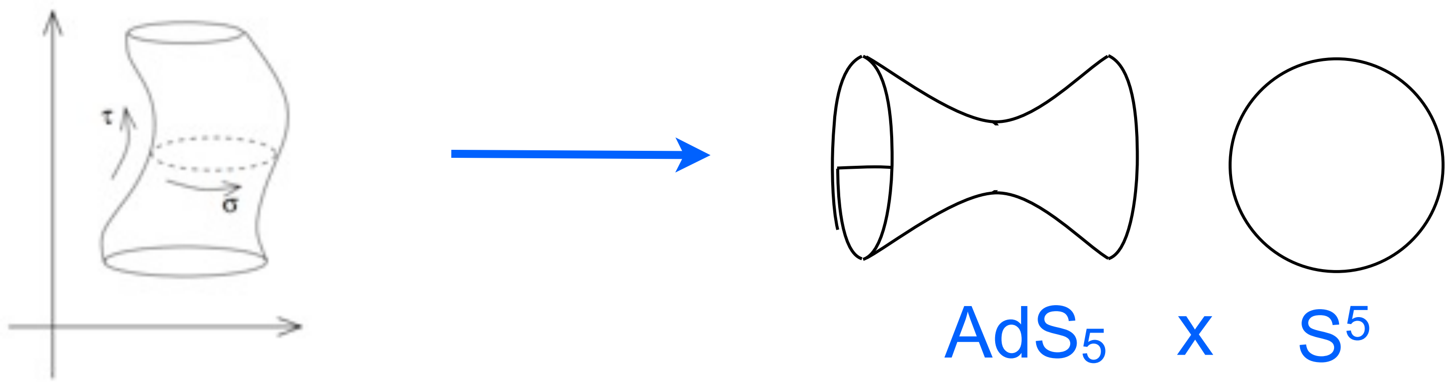
> no world-sheet susy (Green-Schwarz), local symmetries are fixed.

> no gauge fields, only scalars (anticommuting)

> "strong coupling" analytically known (perturbative $\mathcal{N}=4$ SYM theory)

The model in perturbation theory

Green-Schwarz string in $AdS_5 x S^5$ + RR flux



[Metsaev Tseytlin 1998]

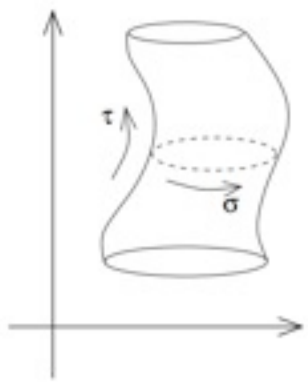
Sigma-model on $G/H = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$, $J_\alpha = g^{-1}\partial_\alpha g = J_{\alpha 0} + J_{\alpha 1} + J_{\alpha 2} + J_{\alpha 3}$

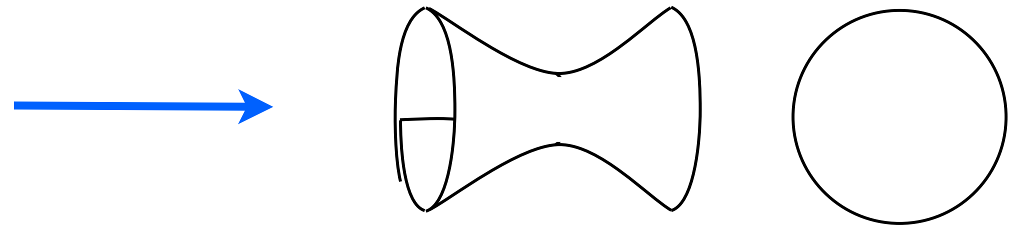
$$S = \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \, \mathrm{Str}[g^{\alpha\beta} J_{\alpha 2} J_{\beta 2} + i\, \epsilon^{\alpha\beta} J_{\alpha 1} J_{\alpha 3} \,]$$

Symmetries: **global** $PSU(2,2|4)$, **local** bosonic (diffeomorphism) and fermionic (κ-)

hidden integrability.

Green-Schwarz string in $AdS_5 x S^5$ + RR flux perturbatively





AdS_5 x S^5 [Metsaev Tseytlin 1998]

Sigma-model on $G/H = \dfrac{PSU(2,2|4)}{SO(1,4)\times SO(5)}$, $J_\alpha = g^{-1}\partial_\alpha g = J_{\alpha 0} + J_{\alpha 1} + J_{\alpha 2} + J_{\alpha 3}$

$$S = \frac{\sqrt{\lambda}}{4\pi}\int d\tau d\sigma\, \mathrm{Str}[g^{\alpha\beta} J_{\alpha 2} J_{\beta 2} + i\,\epsilon^{\alpha\beta} J_{\alpha 1} J_{\alpha 3}\ ]$$

$$= \frac{\sqrt{\lambda}}{4\pi}\int d\tau d\sigma\,[\,\partial_a X^\mu \partial^a X^\nu G_{\mu\nu} + \bar{\theta}\,\Gamma(D + F_5)\theta\,\partial X + \bar{\theta}\theta\bar{\theta}\theta\,\partial_a X \partial^a X + ...]$$

Highly non-linear, to quantize it use **semiclassical methods**.

$$X = X_{\rm cl} + \tilde{X} \longrightarrow E = g\left[E_0 + \frac{E_1}{g} + \frac{E_2}{g^2} + \cdots\right] \qquad g = \frac{\sqrt{\lambda}}{4\pi} = \frac{R^2}{4\pi\alpha'}$$

classical, 1 loop, 2 loops...

Green-Schwarz string in $AdS_5 x S^5$ + RR flux perturbatively

Highly non-linear, to quantize it use **semiclassical methods.**

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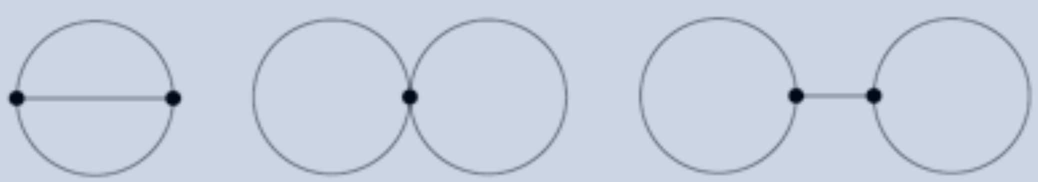
> **General** description of fluctuations in terms of background **geometry**

[Drukker Gross Tseytlin] [Buchbinder Tseytlin 14] [**VF** Giangreco Griguolo Seminara Vescovi 15]

> Only for **restricted** class an explicit, analytic form of one-loop partition function (for BPS cases - e.g. dual to circular Wilson loop - **discrepancy** with known result).

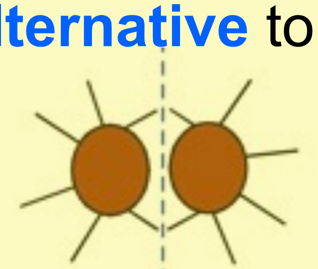
[**VF** w/ Beccaria Dunne Tseytlin, Drukker, Giangreco Ohlson Sax Vescovi ....]
[Kruczenski Tirziu 08] [**VF** Giangreco Griguolo Seminara Vescovi, 15] [Pando-Zayas Trancanelli et al.16]

> **2 loops** is the **current limit**: ``homogenous'' configurations, AdS lc gauge-fixing.



[Giombi Ricci Roiban Tseytlin 09]   [Bres Bianchi[2] **VF** Vescovi 14]

Efficient **alternative** to Feynman diagrams: **unitarity cuts in d=2**



for **on-shell** objects (worldsheet S-matrix):

[Bianchi **VF** Hoare 13]   [Engelund Roiban 13] [Bianchi Hoare 14]

Beyond perturbation theory

Emmy Noether group (L. Bianchi, VF, E. Vescovi), M. S. Bianchi + B. Leder

arXiv:1601.04670   arXiv:1602.xxxxx

Test observable: cusp anomaly of N=4 SYM

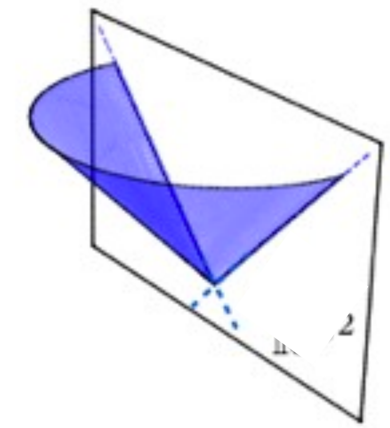
Completely solved via integrability [Beisert Eden Staudacher 2006].

Expectation value of a light-like cusped Wilson loop

$$\langle W[C_{\rm cusp}]\rangle \sim e^{-f(g)\,\phi\,\ln\frac{L_{\rm IR}}{\epsilon_{\rm UV}}}$$

AdS/CFT

$$Z_{\rm cusp} = \int [D\delta X][D\delta\theta]\, e^{-S_{\rm IIB}(X_{\rm cusp}+\delta X,\delta\theta)} = e^{-\Gamma_{\rm eff}}$$



String partition function with ``cusp'' boundary conditions

In Poincaré patch (boundary at z=0)

$$ds^2_{AdS_5} = \frac{dz^2 + dx^+\,dx^- + dx^*\,dx}{z^2} \qquad x^\pm = x^3 \pm x^0 \qquad x = x^1 \pm i\,x^2$$

the "cusp" ($0 = z^2 = -2x^+x^-$) classical solution is $z = \sqrt{\frac{\tau}{\sigma}},\quad x^+ = \tau,\quad x- = -\frac{1}{2\sigma}$

[Giombi Ricci Roiban Tseytlin 2009]

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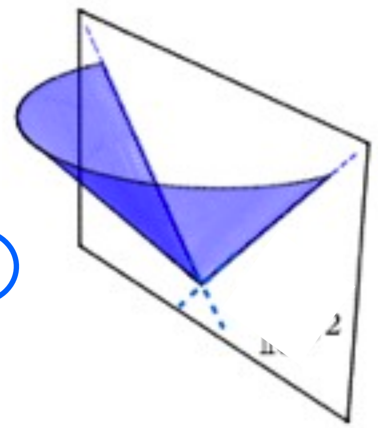


String partition function with ``cusp'' boundary conditions

Evaluated perturbatively

$$\Gamma_{\rm eff} = \Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)} + \dots$$

$$= V\, g\,(a_0 + \frac{a_1}{g} + \frac{a_2}{g^2} + \dots) \equiv V\, f(g)$$

$$V = \int_0^\infty dt \int_0^\infty ds$$

Test observable: cusp anomaly of N=4 SYM

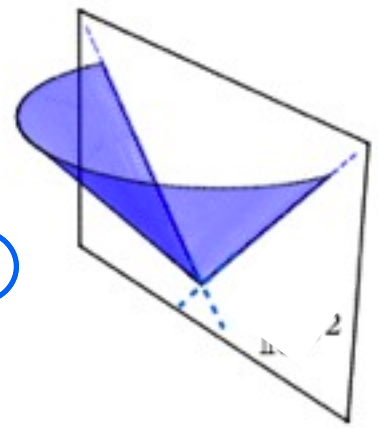
Completely solved via integrability [Beisert Eden Staudacher 2006].

Expectation value of a light-like cusped Wilson loop

AdS/CFT

$$\langle W[C_{\text{cusp}}]\rangle \sim e^{-f(g)\,\phi \ln \frac{L_{\text{IR}}}{\epsilon_{\text{UV}}}}$$

$$Z_{\text{cusp}} = \int [D\delta X][D\delta\theta]\, e^{-S_{\text{IIB}}(X_{\text{cusp}}+\delta X, \delta\theta)} = e^{-\Gamma_{\text{eff}}} = e^{-f(g)\,V}$$



Perturbation theory:

$$\Gamma_{\text{eff}} = \Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)} + \dots$$

$$= V\, g\, (a_0 + \frac{a_1}{g} + \frac{a_2}{g^2} + \dots) \equiv V\, f(g)$$

$$V = \int_0^\infty dt \int_0^\infty ds$$

A lattice approach prefers **expectation values**

$$\langle S\rangle = \frac{\int [D\delta X][D\delta\theta]\, S\, e^{-S}}{\int [D\delta X][D\delta\theta]\, e^{-S}} = -g\frac{d\ln Z}{dg} = V\, g\, \frac{df}{dg}$$

Our simulated observable

Simulations in lattice QFT

Goal: expectation value of some functional of field variables

$$\langle A \rangle = \frac{1}{Z} \int [\mathcal{D}\phi]\, A[\phi]\, e^{-S[\phi]} \qquad Z = \int [\mathcal{D}\phi]\, e^{-S[\phi]}$$

Spacetime grid with lattice spacing $a = \mathrm{const.}$

$$\Lambda = a\,\mathbf{Z}^2 = \{\xi \,|\, \xi^\alpha / a \in \mathbf{Z}\} \quad \text{so that} \quad \xi^\alpha = (\tau, \sigma) \equiv (a\; n_0, a\; n_1) \equiv a\; n$$

- **Natural regularization**: momenta in the first Brillouin zone $\mathcal{B} = \{-\frac{\pi}{a} < p_\alpha \leq \frac{\pi}{a}\}$

- **Definition of the PI measure:** fields are defined on sites $\phi \equiv \phi_n$

  and the PI becomes a multidimensional integral $[\mathcal{D}\,\phi] = \prod_n d\phi_n$

  If $0 \leq n_0 \leq N-1$, $0 \leq n_1 \leq N-1$ so that lattice size in each direction is $N a = L$

$$\partial_\mu \phi \longrightarrow \frac{1}{a}\Big[f(\xi + a\vec{\mu}) - f(\xi)\Big]$$

Then $\int \prod_n d\phi_n\, e^{-S_{\mathrm{discr}}} \sim$ Statistical system with N^2 dof and Hamiltonian S that one can study with **Montecarlo simulation**.

Simulations in lattice QFT

Montecarlo approach: generate a number (*K*) of **field configurations**
or **ensemble** $\{\Phi_1, ..., \Phi_K\}$ each weighted with a **probability** $P[\Phi_i] = \frac{e^{-S_E[\Phi_i]}}{Z}$

Ensemble average:

$$\langle A \rangle = \int [D\Phi]\, P[\Phi]\, A[\Phi] = \frac{1}{K}\sum_{i=1}^{K} A[\Phi_i] + \mathcal{O}\Big(\frac{1}{\sqrt{K}}\Big)$$

Simulations in lattice QFT

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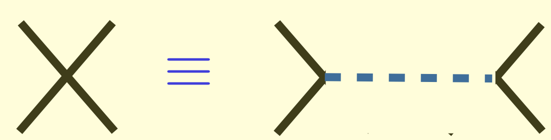
Graßmann-odd fields are formally integrated out,

their determinant becomes part of the weight:

$$P[\Phi_i] = \frac{e^{-S_E[\Phi_i]}}{Z}\ \det O_F$$

> action must be **quadratic** in fermions (linearization via auxiliary fields)

Here:  Introduce **auxiliary** fields (7 complex bosons)

> their determinant must be **positive**

$$\det O_F \to \sqrt{\det(\mathcal{O}_F \mathcal{O}_F^\dagger)} = \int D\zeta D\bar{\zeta}\, e^{-\int d^2\xi\, \bar{\zeta} (\mathcal{O}_F \mathcal{O}_F^\dagger)^{-1/2} \zeta}$$

Potential ambiguity!

The simulation: final lagrangean

GS string in AdS5xS5 cusp background (AdS light-cone gauge, after linearization)

[Metsaev Tseytlin 00, Metsaev Thorn Tseytlin 00] [Giombi Ricci Tseytlin 09] [Roiban McEowan 13]

$$\mathcal{L} = |\partial_t \tilde{x} + \frac{1}{2}\tilde{x}|^2 + \frac{1}{\tilde{z}^4}|\partial_s \tilde{x} - \frac{1}{2}\tilde{x}|^2 + (\partial_t \tilde{z}^M + \frac{1}{2}\tilde{z}^M)^2 + \frac{1}{\tilde{z}^4}(\partial_s \tilde{z}^M - \frac{1}{2}\tilde{z}^M)^2$$
$$+ \frac{1}{2}\tilde{\phi}^2 + \frac{1}{2}(\tilde{\phi}_M)^2 + \underline{\psi^T M \psi}$$

with $\psi \equiv (\tilde{\theta}^i, \tilde{\theta}_i, \tilde{\eta}^i, \tilde{\eta}_i)$ $i = 1, \cdots, 4$, it is $\theta^i = (\theta_i)^\dagger,\ \eta^i = (\eta_i)^\dagger$ and

$$M = \begin{pmatrix} 0 & i\partial_t & -\mathrm{i}\rho^M\left(\partial_s + \frac{1}{2}\right)\frac{\tilde{z}^M}{\tilde{z}^3} & 0 \\ \mathrm{i}\partial_t & 0 & 0 & -\mathrm{i}\rho^\dagger_M\left(\partial_s + \frac{1}{2}\right)\frac{\tilde{z}^M}{\tilde{z}^3} \\ \mathrm{i}\frac{\tilde{z}^M}{\tilde{z}^3}\rho^M\left(\partial_s - \frac{1}{2}\right) & 0 & 2\frac{\tilde{z}^M}{\tilde{z}^4}\rho^M\left(\partial_s\tilde{x} - \frac{\tilde{x}}{2}\right) & i\partial_t - A^\dagger \\ 0 & \mathrm{i}\frac{\tilde{z}^M}{\tilde{z}^3}\rho^\dagger_M\left(\partial_s - \frac{1}{2}\right) & \mathrm{i}\partial_t + A & -2\frac{\tilde{z}^M}{\tilde{z}^4}\rho^\dagger_M\left(\partial_s\tilde{x}^* - \frac{\tilde{x}^*}{2}\right) \end{pmatrix}$$

$$A^i{}_j = \frac{1}{\sqrt{2}\tilde{z}^2}\tilde{\phi}_M \rho^{MNi}{}_j \tilde{z}_N - \frac{1}{\sqrt{2}\tilde{z}}\tilde{\phi}\,\delta^i{}_j + \mathrm{i}\,\frac{\tilde{z}_N}{\tilde{z}^2}\rho^{MNi}{}_j\,\partial_t \tilde{z}^M$$

where $(\rho^M)_{ij}$ are off-diagonal blocks of SO(6) Dirac matrices $\gamma^M \equiv \begin{pmatrix} 0 & \rho^\dagger_M \\ \rho^M & 0 \end{pmatrix}$

The simulation: final lagrangean

> Keep track of dimensionful parameters (subject to renormalization): $m \sim P_+$

> A naive regularization leads to “fermion doublers”: add “Wilson term” to the action.

Explicit SO(6) symmetry breaking: we study SO(6) singlets, and this might only affect the way the continuum limit is taken

m

$$\mathcal{L} = |\partial_t \tilde{x} + \frac{1}{2}\tilde{x}|^2 + \frac{1}{\tilde{z}^4}|\partial_s \tilde{x} - \frac{1}{2}\tilde{x}|^2 + (\partial_t \tilde{z}^M + \frac{1}{2}\tilde{z}^M)^2 + \frac{1}{\tilde{z}^4}(\partial_s \tilde{z}^M - \frac{1}{2}\tilde{z}^M)^2 + \frac{1}{2}\tilde{\phi}^2 + \frac{1}{2}(\tilde{\phi}_M)^2 + \underline{\psi^T M \psi}$$

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The simulation: parameter space

- In the continuum model there are two parameters, $g = \frac{\sqrt{\lambda}}{4\pi}$ and $m \sim P_+$.
  In perturbation theory divergences cancel, dimensionless quantities are pure functions of the (bare) coupling

$$F = F(g) \ .$$

- Our discretization cancels (1-loop) divergences, and reproduces the 1-loop cusp anomaly
  Assume it is true nonperturbatively for lattice regularization.
  Only additional scale: lattice spacing a (box size $L^2 = (N\,a)^2 = V$)
  Three dimensionless (input) parameters:

$$g\,, \qquad N \equiv \frac{L}{a}\,, \qquad M \equiv a\,m$$

  Therefore

$$F_{\rm LAT} = F_{\rm LAT}(g, N, M)$$

The simulation: continuum limit $(a \to 0)$

In the continuum, "effective" masses of field excitations undergo a *finite* renormalization

E.g. $$m_x^2(g) = \frac{m^2}{2}\left(1 - \frac{1}{8\,g} + \mathcal{O}(g^{-2})\right) \quad *$$

Dimensionless physical quantities natural to keep **constant** when $a \to 0$:

$$L^2 m_x^2 = \text{const} \quad \longrightarrow \quad L^2 m^2 \equiv (NM)^2 = \text{const}$$

If * true in the discretized model,and fixing *g* (assume it not renormalized)

The simulation: continuum limit ($a \to 0$)

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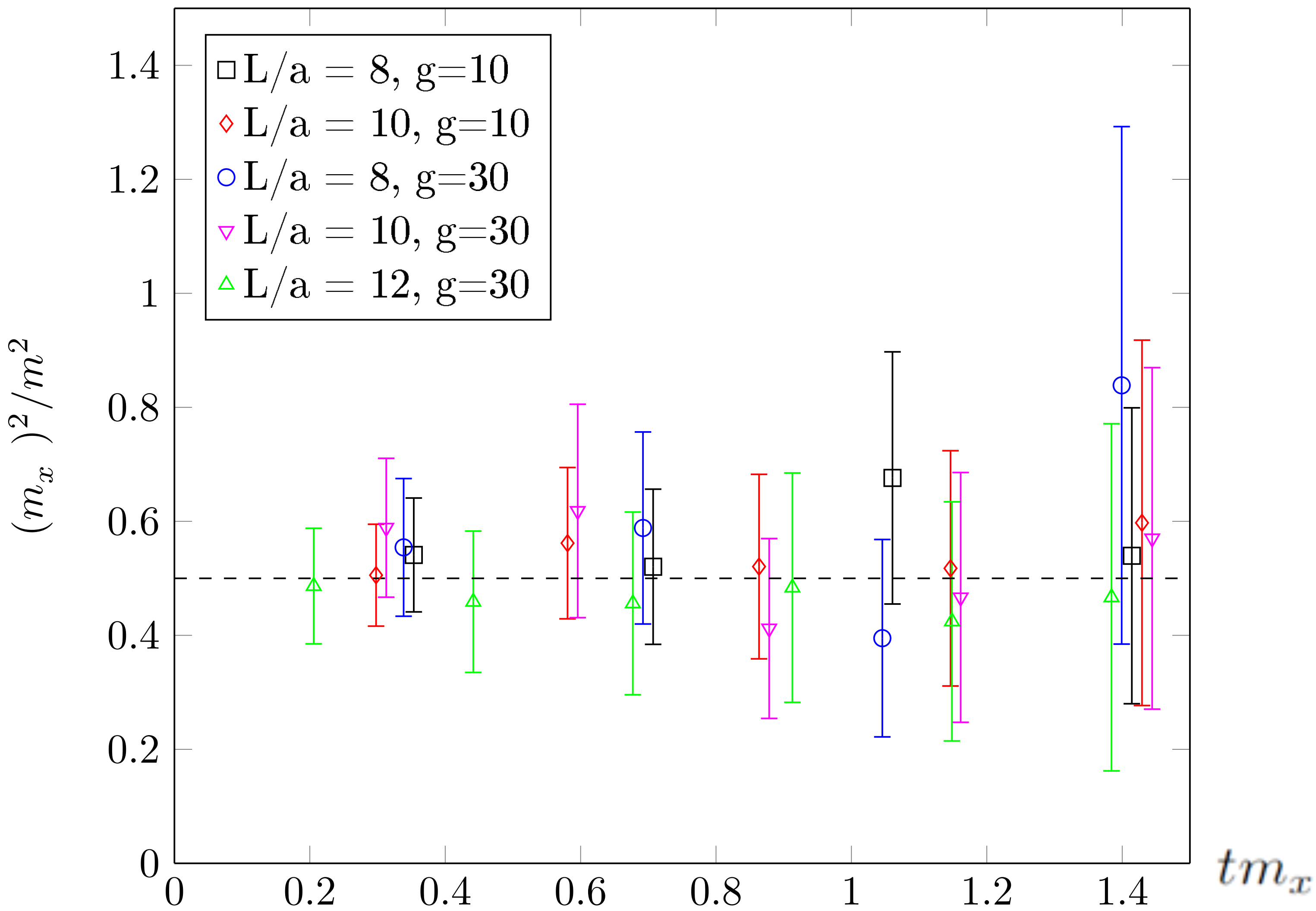


The simulation: continuum limit ($a \to 0$)

Remove the cutoff and compare to other results (here: integrability) or other regularizations.

If there are no divergences (i.e. no terms proportional to 1/a)

$$F_{\mathrm{LAT}}(g, N, M) = F(g) + \mathcal{O}\left(\frac{1}{N}\right) + \mathcal{O}(M) + \mathcal{O}(e^{-MN})$$

finite lattice spacing
(~a) effects

finite volume
(~ m L) effects

Recipe:

> fix g

> fix $MN = mL$, large enough so that finite volume effects are small

> compute F_{LAT} for $N = 6, 8, 10, 12, 16, \ldots$

> extrapolate to $1/N \to 0$

The simulation: the observable

$$\langle S \rangle = -g \frac{d \ln Z_{\rm cont}}{dg} \equiv g \frac{V_2}{8} f'(g)$$

The partition function on the lattice is **modified** (auxiliary fields + pseudofermions ⟶ Jacobians)

$Z_{\rm LAT} \sim J(g)\, Z_{\rm cont}$ so that thee relation of $\langle S_{\rm LAT}\rangle$ to $f(g)$ picks a **constant** (in g) factor

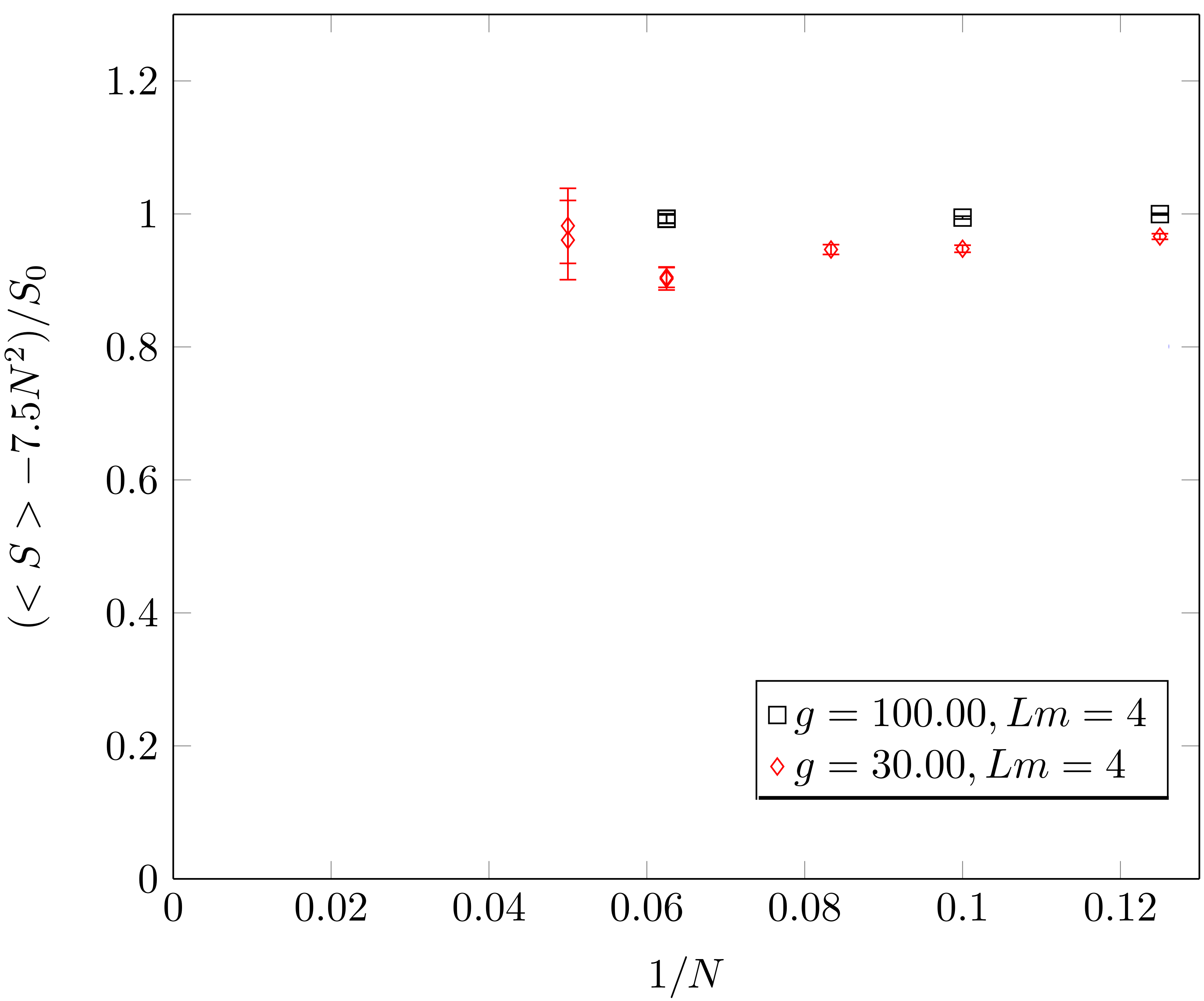
$$\langle S_{\rm LAT}\rangle = -g \frac{d \ln Z_{\rm LAT}}{dg} = g \frac{d \ln J(g)_{\rm tot}}{dg} - g \frac{d \ln Z_{\rm cont}}{dg}$$

$$\langle S \rangle_{\rm LAT} = \frac{15}{2} N^2 + \frac{1}{8} m^2 V\, g\, f'(g) \qquad m^2 = \frac{M^2}{a^2} \qquad V = a^2 N^2$$

1. Fit $\frac{\langle S \rangle_{\rm LAT}}{N^2} = \frac{c}{2} + \frac{1}{2} M^2 g$ to find c, having in mind $f(g) = 4\, g\,, \ g \gg 1$ ✓ $c = 7.5(1)$
2. Compute the continuum limit of $\frac{\langle S \rangle_{\rm LAT} - c N^2}{\frac{1}{2} M^2 N^2 g} = \frac{1}{4} f'(g)$

The simulation: the observable

Continuum limit ($N \to \infty$), at very large *g* good agreement with prediction $f(g) = 4\,g\,,\;\; g \gg 1$

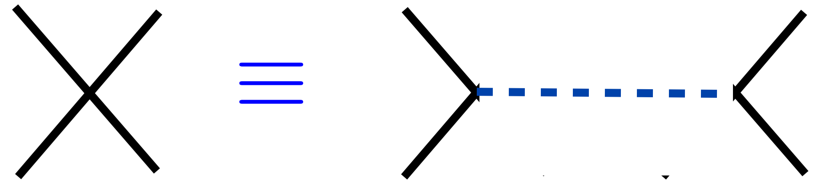


$$\underbrace{\frac{\langle S\rangle - cN^2}{\frac{1}{2}M^2N^2g}}_{S_0} = \frac{1}{4}f'(g)$$

At smaller *g* a SEVERE phase problem appears!

Phase problem

In fact, our Hubbard-Stratonovich transformation "encoded" a phase



$$e^{-S_E^{\rm ferm4}} = e^{-\int (i\eta\rho\eta)^2} = e^{-\frac{b^2}{4a}} \equiv \int_{-\infty}^{+\infty} dx\, e^{-ax^2+ibx}$$

fermionic quadratic part
(entering Boltzmann weight!)

Namely, the weight in the probability

$$(\det M)^{1/2} = (\det MM^\dagger)^{1/4}$$

Phase problem

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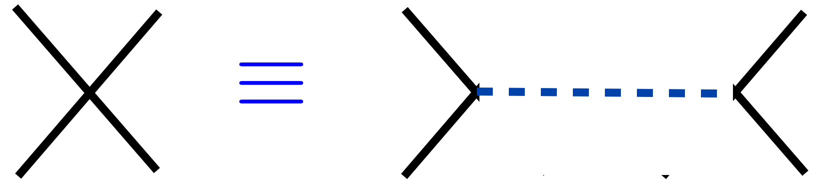


$$e^{-S_E^{\mathrm{ferm4}}} = e^{-\int (i\eta\rho\eta)^2} = e^{-\frac{b^2}{4a}} \equiv \int_{-\infty}^{+\infty} dx\, e^{-ax^2+ibx}$$

fermionic quadratic part
(entering Boltzmann weight!)

Namely, the weight in the probability is **not** definite positive.

$$(\det M)^{1/2} = (\det MM^\dagger)^{1/4} \exp(i\theta) = \sqrt{|\det M|}\, \exp(i\theta)$$

Standard **reweighting**: non positive part of the weight incorporated into the observable.

$$P[\Phi_i] = \rho[\Phi_i] \exp(i\theta[\Phi_i])$$

positive weight

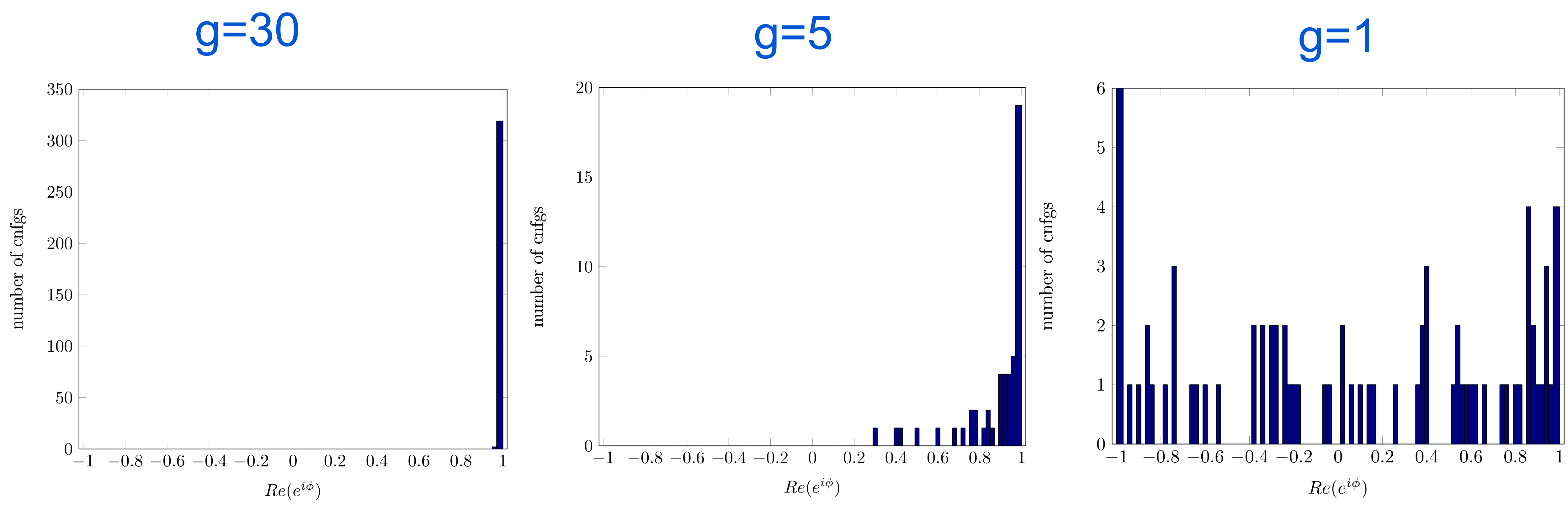
and

$$\langle A \rangle_\rho = \frac{\langle A \exp(i\theta) \rangle_\rho}{\langle \exp(i\theta) \rangle_\rho}$$

It gives meaningful results **as long as the phase does not average to zero.**

Phase problem

In the interesting (*g=1*) region the phase has a **flat** distribution: **"reweighting" not meaningful!**



Alternative algorithms: active field of study, no general proof of convergence.

...but here we know the result from integrability.

Phase problem as opportunity to see whether/which algorithm works.

Alternative linearization and auxiliary fields set : in progress.

Conclusions

Solving a 4d qft is **hard** ⟶ Reduce the problem via AdS/CFT, and "solve a (non-trivial) 2d qft": Green-Schwarz string sigma model in $AdS_5 x S^5$.

String worldsheet model on the lattice:

✓ good discretization, good control on "weak coupling" region

✓ good (Fortran, Matlab) implementations, internal consistency checks

X phase problem occurring, continuum limit problematic

Next steps

> alternative linearization to eliminate the phase: in progress

> correlation functions for all fields

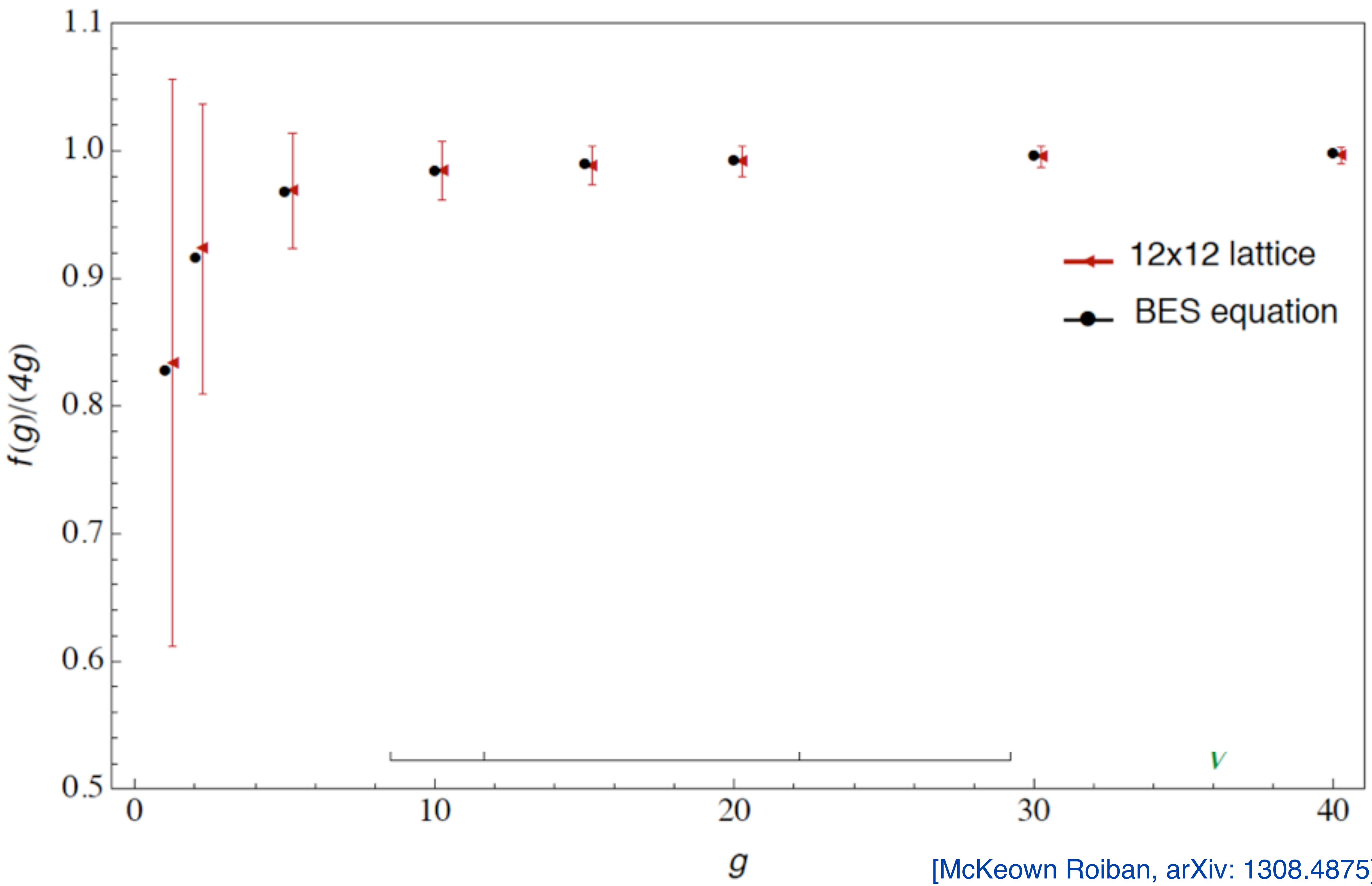
> cusp anomaly of AdS_4/CFT_3

> correlators of string vertex operators (three-point functions in gauge theory)

More **general** analysis (**not** limited to this background/gauge-fixing) **should** give a useful device in numerical holography.

Extra slides

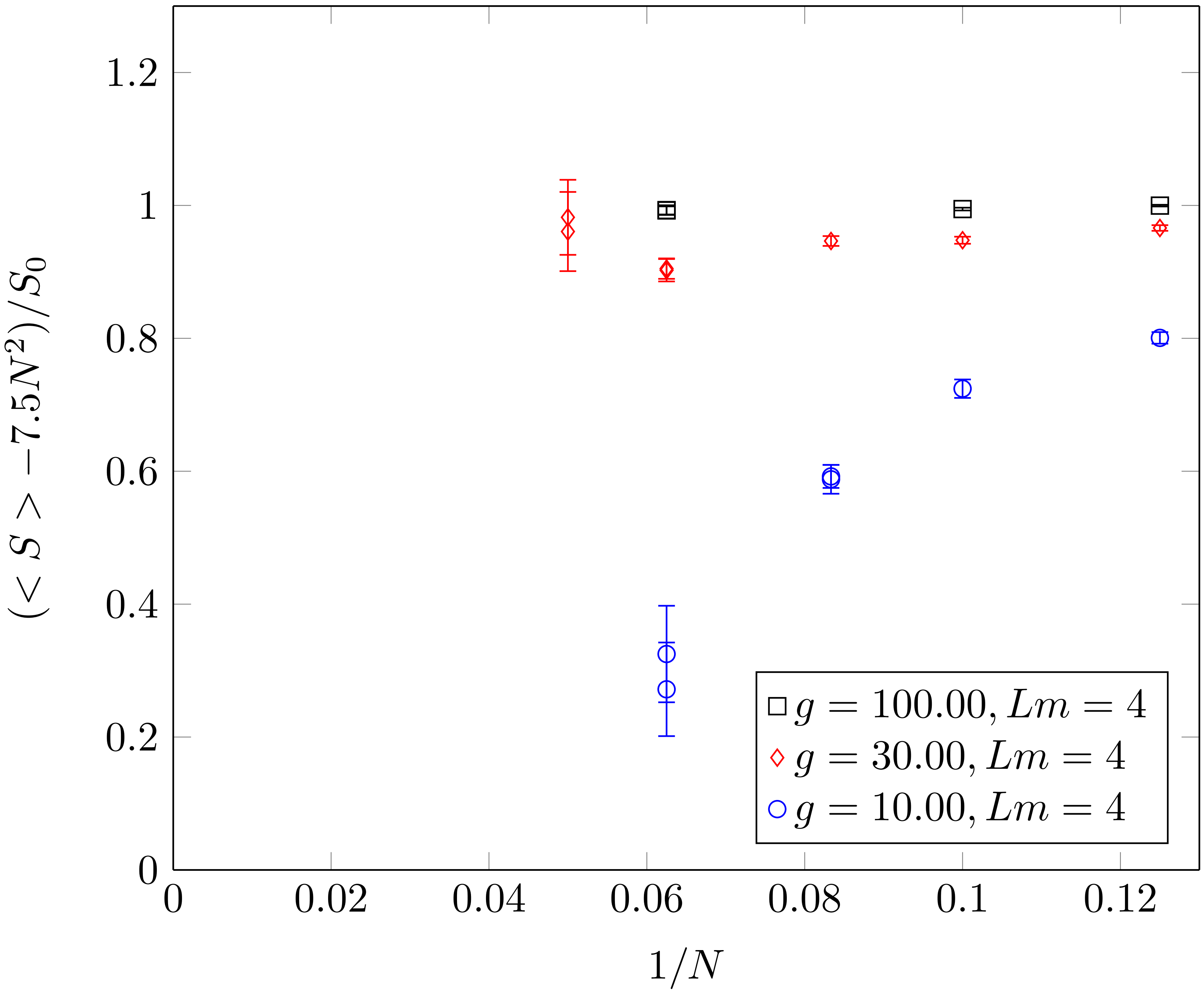
Roiban McKeown 2013



[McKeown Roiban, arXiv: 1308.4875]

The simulation: the observable

Continuum limit ($N \to \infty$), at very large *g* good agreement with prediction $f(g) = 4\,g\,,\ \ g \gg 1$



$$\underbrace{\frac{\langle S \rangle - cN^2}{\frac{1}{2}M^2N^2g}}_{S_0} = \frac{1}{4}f'(g)$$

Strong deviation:
can be "reabsorbed" with
alternative interpretation
of the "constant" *c*
(in progress)

Cusp anomaly at weak and at strong coupling

$$f(g)|_{g\to 0} = 8g^2\left[1 - \frac{\pi^2}{3}g^2 + \frac{11\,\pi^4}{45}g^4 - \left(\frac{73}{315} + 8\,\zeta_3\right)g^6 + ...\right]$$

$$f(g)|_{g\to\infty} = 4g\left[1 - \frac{3\ln 2}{4\pi}\frac{1}{g} - \frac{K}{16\pi^2}\frac{1}{g^2} + ...\right]$$

$$-\frac{3\ln 2}{\pi} = -0.661907$$

$$-\frac{K}{4\pi^2} = -0.0232017$$

“Wilson term” solving the doubling problem

$$K_f = \begin{pmatrix} 0 & i\,p_0\mathbb{1} & -(i\,p_1 + \frac{1}{2})\rho^6 & 0 \\ i\,p_0\mathbb{1} & 0 & 0 & -(i\,p_1 + \frac{1}{2})\rho_6^\dagger \\ (i\,p_1 - \frac{1}{2})\rho^6 & 0 & 0 & i\,p_0\mathbb{1} \\ 0 & (i\,p_1 - \frac{1}{2})\rho_6^\dagger & i\,p_0\mathbb{1} & 0 \end{pmatrix}$$

$$p \to \mathring{p} = \frac{1}{a}\sin ap$$

Jacobians and the intercept in the simulated <S>

The action simulated on the lattice is **modified** (auxiliary fields + pseudofermions $\longrightarrow$ jacobians)

$$\langle S_{LAT}\rangle = \frac{\int D\phi_{\rm phys} D\phi_{\rm aux} D\zeta_{\rm ps.ferm.}\, S[\phi_{\rm phys},\phi_{\rm aux},\zeta_{\rm ps.ferm.}]\, e^{-S[\phi_{\rm phys},\phi_{\rm aux},\zeta_{\rm ps.ferm.}]}}{\int D\phi_{\rm phys} D\phi_{\rm aux} D\zeta_{\rm ps.ferm.}\, e^{-S[\phi_{\rm phys},\phi_{\rm aux},\zeta_{\rm ps.ferm.}]}} = -g\frac{d\ln Z_{\rm LAT}}{dg}$$

where

$$Z_{\rm LAT} = \int D\phi_{\rm phys} D\phi_{\rm aux} D\zeta_{\rm ps.ferm.}\, e^{-S[\phi_{\rm phys},\phi_{\rm aux}]} e^{-\int dtds\, \zeta^\dagger \left(g\, \mathcal{O}_F \mathcal{O}_F^\dagger\right)^{-\frac{1}{4}} \zeta}$$

$$\sim J_{HS}^{-1}(g)\, (\sqrt{g})^{-\frac{16}{2}} \int D\phi_{\rm phys} D\psi\; e^{-S[\phi_{\rm phys}]}\; e^{-g\int dtds\, \psi^T\, O_F\, \psi} \;\sim\; J_{\rm tot}(g)\, Z_{\rm cont}$$