A Slippery Slope in the Inflationary Landscape

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based on work with Roberto Gobetti, Enrico Pajer, and I-Sheng Yang

String Theory is a beautiful, consistent theory of quantum gravity with no free parameters.

Appears to have a large *landscape* of solutions that look approximately like our universe: 4 large dimensions.

Data about compactification (e.g. size and shape of compact manifold) $\rightarrow$
Large number of fields in 4D description.

Guess based on current theoretical/observational data:

- Landscape contains many different regions where inflation can occur.
- Inflation occurs in a high-dimensional field space.

Many choices for inflaton potential, each one very complicated.

How to make predictions in this context?

Idea: Perhaps analysis simplifies with a large number of fields.

Central limit theorem analogy: Suppose $x = \sum_{i=1}^{N} x_i$.
If N large, statistics of individual x_i unimportant; x Gaussian.

In our case, many contributions to potential- perhaps potential becomes "Gaussian."

What does such a potential look like?

Claim of this talk:

- This idea is predictive: universal form for potential.
- Inflation can occur in such potentials.
- The predictions disagree with observation.

(I will not prove this.)

Physics issue: Inflation requires a bit of fine-tuning of the potential to satisfy slow-roll conditions for 60 e-foldings: V' and V'' small.

With many fields, more difficult to tune.

Eigenvalue repulsion pushes most tachyonic direction to be steep.

Even if begin inflation in a region with small derivatives, they evolve as we move through field space.
Potential tends to steepen: "slippery slope."

Possible observational issues:

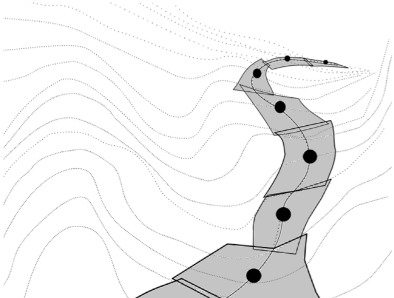
- Predict spatial curvature too large?
- Predict spectrum too far from scale invariance?

Concrete realization [Marsh McAllister Pajer Wrase 13] :
"Charting an Inflationary Landscape with Random Matrix Theory"

N scalar fields ϕ^i, with $i = 1 \ldots N$.
Locally, approximate potential to quadratic order. Near $\phi^i = 0$,

$$V(\phi^i) = V(0) + \partial_i V(0)\phi^i + \frac{1}{2}\partial_i\partial_j V(0)\phi^i\phi^j$$

- Probability distribution for parameters of potential $V(0), \partial_i V(0), \partial_i\partial_j V(0)$.
- Rule for how parameters change as move through field space.



Parameters

Near any point in field space,

$$V = \Lambda_v^4 \sqrt{N} \left[v_0 + v_a \frac{\phi^a}{\Lambda_h} + \frac{1}{2} v_{ab} \frac{\phi^a \phi^b}{\Lambda_h^2} + \dots \right] ,$$

v_0, v_a, and v_{ab} independent Gaussian random variables with mean zero and variance $1/\sqrt{N}$.

Small number of parameters:

- Characteristic scale of variation in field space: $\Delta\phi = \Lambda_h$
- Typical energy scale of potential Λ_v
- Number of fields N.

Evolution

As move through field space, parameters of potential evolve according to:

$$\delta v_0 = v_a \frac{\delta\phi^a}{\Lambda_h} , \tag{1}$$

$$\delta v_a = v_{ab} \frac{\delta\phi^b}{\Lambda_h} . \tag{2}$$

To avoid an infinite set of equations, MMPW assume that the matrix of second derivatives evolves stochastically.
δv_{ab} is Gaussian random variable with

$$\begin{aligned} \langle \delta v_{ab} \rangle &= -v_{ab} \, \frac{||\delta\boldsymbol{\phi}||}{\Lambda_h} , \\ \langle (\delta v_{ab})^2 \rangle &= \frac{1}{N} \frac{(1+\delta_{ab})}{2} \, \frac{||\delta\boldsymbol{\phi}||}{\Lambda_h} , \end{aligned} \tag{3}$$

Hessian evolves by Dyson Brownian motion.
Simple, tractable model with only a few parameters.

Inflation

Inflation can happen because

- Parameters are such that inflation happens at generic places in potential.
- Inflation occurs at special places where potential is accidentally flat.

We focus on the last possibility.

Assume $\Lambda_h < M_P$.

Statistics

Basic effect: eigenvalue repulsion. Independent entries of Hessian v_{ab} does not lead to statistically independent eigenvalues. Matrices with nearly degenerate eigenvalues are rare.

Dyson description: Eigenvalues are charged particles that repel each other, confined by a harmonic potential.

Eigenvalue distribution at large N: "Wigner semicircle."

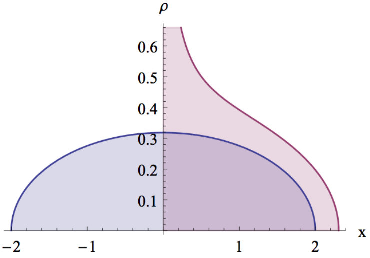


Figure: Blue curve shows density of eigenvalues.

To allow inflation, need to avoid significantly tachyonic eigenvalues.

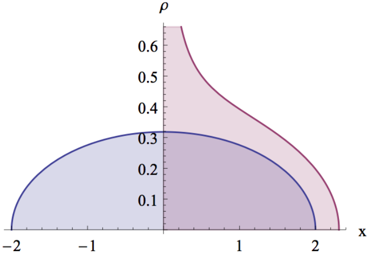


Figure: Inflation can only occur in patches with an unusual fluctuation in the eigenvalue distribution (red), rather than the typical distribution (blue)

As inflation proceeds, most negative eigenvalue becomes more negative due to eigenvalue repulsion.

Dyson description can be extended to dynamics:
Eigenvalues evolve according to harmonic potential, electrical repulsion, random force (Dyson Brownian Motion).

Simply due to

$$\langle(\delta v_{ab})^2\rangle = \frac{1}{N}\frac{(1+\delta_{ab})}{2}\,\frac{||\delta\phi||}{\Lambda_h}\,, \tag{4}$$

Same math as calculating perturbed energy eigenvalues in quantum mechanics.

$$v_{ab}(\phi^a) = v_{ab}(0) + \delta v_{ab}$$

We improve somewhat on MMPW and find

$$\lambda_{\min}(\phi) = \lambda_{\min}(0) - \left(\frac{||\phi||}{\Lambda_h}\right)^{2/3}$$

For the mass this is

$$m^2_{\min}(\phi) = m^2_{\min}(0) - \sqrt{N}\Lambda_v^4 \left(\frac{||\phi||}{\Lambda_h}\right)^{2/3}$$

We now want to use these results to calculate observational quantities.

Many open questions because inflation is generally multi-field.

We find some problems which will probably not be solved by a better accounting of multi-field effects.

MMPW: Background evolution well approximated by effective single-field model,

$$V(\phi) = V_0 - V_1\phi - \frac{1}{2}m^2(\phi)\phi^2 \ .$$

Putting in our result for $m^2(\phi)$ gives

$$V(\phi) = V_0 - V_1\phi - \frac{1}{2}m_0^2\phi^2 - c\phi^{8/3}$$

where $c = \sqrt{N}\Lambda_v^4/\Lambda_h^{8/3}$.

First issue: statistics for the slope.

Begin inflation from arbitrary point in landscape.
Mostly, do not inflate.
When we do inflate, number of efolds given by

$$N_e = \frac{H^3}{V_1}$$

neglecting m_0^2.

V_1 is magnitude of gradient,

$$V_1 = \sqrt{N}\frac{\Lambda_v^4}{\Lambda_h}\sqrt{v_a v^a}$$

Magnitude of a random vector in N dimensions, so

$$p(v_1) \sim v_1^{N-1}$$

Convert to probability distribution for the number of efoldings:

$$p(N_e) \sim N_e^{-5N/2}$$

More efoldings strongly disfavored.

Not clear if it is a problem that 20 efoldings is much more likely than 60: selection effect (anthropic principle).

But no anthropic problem with $\Omega_k \sim .5$ today.

Probability for curvature to agree with observation, conditioning on $\Omega_k \lesssim .5$:

$$\frac{P(N_e > 60)}{P(N_e > 58)} = \left(\frac{58}{60}\right)^{5N/2}$$

This is small for large N. (1% for $N = 50$)

Might think that dynamics help with this problem: first roll down to region of small gradient, then inflate.

Assuming this happens, set gradient to zero $\rightarrow$ inflation near a critical point

$$V(\phi) = V_0 - \frac{1}{2}m_0^2\phi^2 - c\phi^{8/3}$$

No obvious problem at level of background, at least for some choices of parameters.
Want to consider perturbations.

We do the only tractable thing: **assume** the single-field model also is correct for the perturbations.

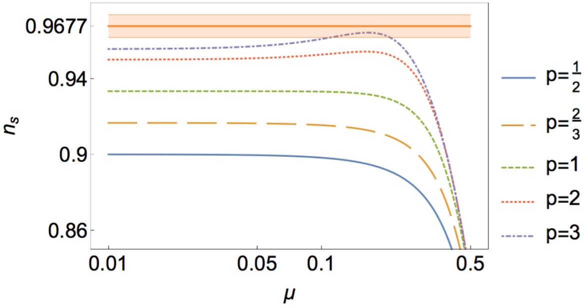
Calculate spectral index n_s for the potential

$$V(\phi) = V(\phi) = V_0 - \frac{1}{2} m_0^2 \phi^2 - c\phi^{p+2}$$

Find

$$n_s - 1 \lesssim -\frac{2\mu^2}{3} \frac{\exp(\mu^2 p N_e/3) + p}{\exp(\mu^2 p N_e/3) - 1}$$

where $\mu = m_0/H$.
Assumption (valid in our model): Inflation ends in the region where ϕ^p dominates.



Find n_s too small for reasonable values of p, regardless of choice of μ.

No anthropic argument for n_s. Simply not our universe.

Bold Conclusion: Our universe not well described by large N random landscape.

Other hints in this direction:

- Wasteland of random supergravities: critical points are *very* unlikely to be minima at large N due to eigenvalue repulsion [Marsh McAllister Wrase].
- If find a minimum, it tends to have a fast nonperturbative instability at large N [Greene Kagan Masoumi Mehta Weinberg Xiao 13][Dine Paban 15][Masoumi Vilenkin 16] . . . .

Alternatively, perhaps the model or our analysis should be improved.

Issues with the model: Evolution of Hessian

Behavior of the Hessian for small field displacements is unphysical. Assuming only that 3rd derivative of the potential exists,

$$\delta v_{ab} = v_{abc}\left(\frac{\delta\phi^c}{\Lambda_h}\right) \sim \frac{1}{\sqrt{N}}\frac{||\delta\phi||}{\Lambda_h}$$

But model gives

$$\langle(\delta v_{ab})^2\rangle = \frac{1}{N}\frac{(1+\delta_{ab})}{2}\frac{||\delta\boldsymbol{\phi}||}{\Lambda_h}$$

so

$$\delta v_{ab} \sim \frac{1}{\sqrt{N}}\sqrt{\frac{||\delta\phi||}{\Lambda_h}}$$

Fixing this problem will only change the exponent p, but n_s will remain too small.

Issues with the model: Probability distribution for v_0, v_a.

Consistency check: statistics for the parameters in one patch should be invariant under evolution in field space.

Otherwise, these statistics would have to depend on location in field space.

This works well for Hessian.

But variance of v_a and v_0 increase indefinitely under evolution.

$$\delta v_a = v_{ab} \delta \phi^b$$

Conclusions

Plausible conclusion from string landscape: inflation occurs in high-dimensional field space.

One well-motivated, tractable approach: random potentials.

Potential simplifies for large number of fields.

I have argued that the resulting potential disagrees with observation.

I encourage you to either

- Prove me wrong by improving on our analysis, model.
- Look for inflation in a context with more structure: symmetries, supersymmetric stabilization of most fields, etc.