On the physical realization of Seiberg dual phases in branes at singularities

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Based on: Ongoing work with Iñaki García-Etxebarria and Ben Heidenreich

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Introduction

- Seiberg duality is an important aspect of $\mathcal{N} = 1$ supersymmetric gauge theory.
- In string theory, Seiberg duality is often realized by supersymmetric deformations of systems of branes that induce irrelevant deformations of the low energy EFT.
- In the context of chiral quiver theories, the algebraic (chiral ring) content of Seiberg duality can be understood at the level of topological string theory.[Berenstein, Douglas, '02] An interesting question is finding to what extend the same occurs in the full string theory, once we take into account BPS conditions for the brane system.
- In particular, we will be interested in the case of D-branes sitting on a singularity inside a Calabi-Yau manifold.
- The quiver gauge theory will be physically realized when the periods characterizing the central charges for the associated fractional branes are aligned.[e.g. Aspinwall, Melnikov, '04]

Warm up: $\mathbb{CP}^2$

Let us consider $\mathbb{CP}^2$ [García-Etxebarria, Heidenreich, Wrase, '13].

There is a phase (phase I) with $B_2 = 0$ which corresponds to the "orbifold" phase.

The Seiberg dual phase (phase II) corresponds to $B_2 = 1/2$.

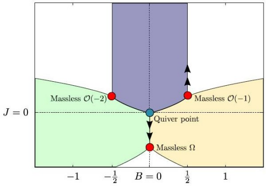

Figure from 1307.1701

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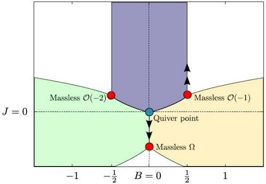

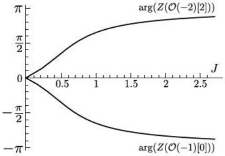

Figures from 1307.1701

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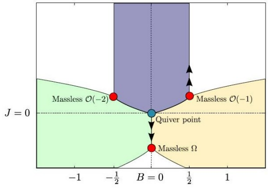
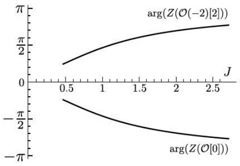
Phase I is supersymmetric at the orbifold point.

Phase II is not supersymmetric.

Therefore, only phase I is physically realized.





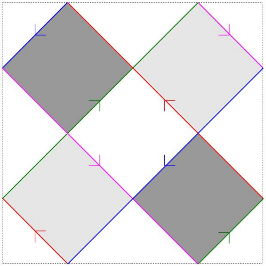
Figures from 1307.1701

D-branes in toric singularities

- D-branes in toric singularities have been extensively researched:
  - Hanany, Zaffaroni, '98; Hanany, Uranga, '98; Hori, Iqbal, Vafa, '00; Cachazo, Fiol, Intriligator, Katz, Vafa, '01; Feng, Hanany, He, '01, '02; Feng, Hanany, He, Uranga, '01; Feng, Franco, Hanany, He, '02; Berenstein, Douglas, '02; Aspinwall, Melnikov, '04; Feng, He, Lam, '04; Franco, Hanany, Kennaway, Vegh, Wecht, '05; Franco, Hanany, Martelli, Sparks, Vegh, Wecht, '05; García-Etxebarria, Saad, Uranga, '06; Ueno, Yamazaki, '07; Yamazaki, '08'; ... And many more.

D-branes in toric singularities. Strong string coupling

- We use the brane tiling description obtained after two T-dualities[Hanany, Zaffaroni, '98; Hanany, Uranga, '98].
- NS5-branes wrap straight 1-cycles on a T^2 while D5-branes extend between them.
- The BPS condition implies that the total NS5 charge on the T^2 has to vanish.
- As shown in [Ueda, Yamazaki, '07], for the toric Fano cases, we only have one quiver diagram in the strong string coupling limit.



D-branes in toric singularities. Weak string coupling

- We will use the mirror manifold that is obtained after three T-dualities.
- The mirror manifold is given by a hypersurface [Hori, Iqbal, Vafa, '00]

$$uv = P(x,y), \quad u, c \in \mathbb{C}, \quad x, y \in \mathbb{C}^*$$

- It is useful to write it as a double fibration over a complex plane

$$\begin{aligned} W &= uv \\ W &= P(x,y) \end{aligned}$$

- D6-branes extend between the origin and the points where $P(x,y)$ becomes singular.
- The BPS conditions restrict the possible values of the coefficients of $P(x,y)$

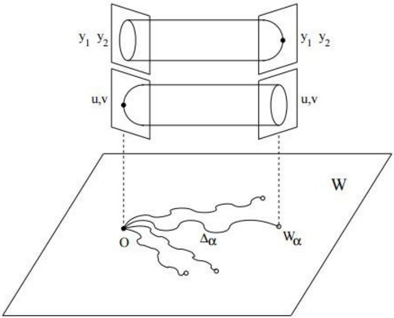
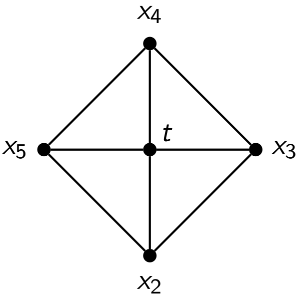


Figure taken from hep-th/0110028.

Complex cone over $\mathbb{F}_0$

Toric diagram:



Newton polynomial:

$$P(x,y) = \frac{b}{y} + cx + dy + \frac{e}{x} + t$$

Mori cone:

	x_2	x_3	x_4	x_5	t
$\mathcal{C}_1$	0	1	0	1	-2
$\mathcal{C}_2$	1	0	1	0	-2

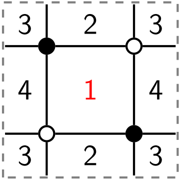
$$z_1 = \frac{ce}{t^2}, \qquad z_2 = \frac{bd}{t^2}$$

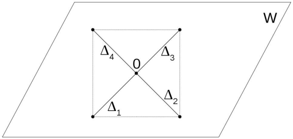
$\mathbb{F}_0$: Toric phases, Seiberg duality and mirror picture

[Feng, Hanany, He, '01]

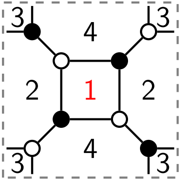
[Cachazo, Fiol, Intriligator, Katz, Vafa, '01.]

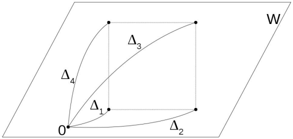
Phase I



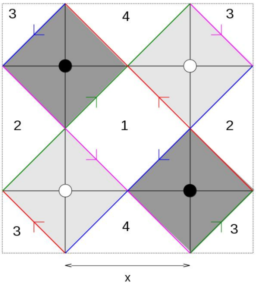


Phase II





$\mathbb{F}_0$. Strong string coupling.



- Only the brane tiling for phase I can be constructed.
- After imposing the BPS conditions only one parameter remains.

$\mathbb{F}_0$. Weak string coupling: Periods and locus quiver

For the Mori cone

	x_2	x_3	x_4	x_5	t
$\mathcal{C}_1$	0	1	0	1	-2
$\mathcal{C}_2$	1	0	1	0	-2

the Picard-Fuchs equations we need to solve are

$$\mathcal{L}_1\Phi(z_1,z_2) = (\theta_1^2 - z_1(2\theta_1+2\theta_2)(2\theta_1+2\theta_2+1))\Phi(z_1,z_2) = 0,$$

$$\mathcal{L}_2\Phi(z_1,z_2) = (\theta_2^2 - z_2(2\theta_1+2\theta_2)(2\theta_1+2\theta_2+1))\Phi(z_1,z_2) = 0,$$

where

$$\theta_i = z_i\frac{\partial}{\partial z_i}, \quad z_1 = \frac{ce}{t^2}, \quad z_2 = \frac{bd}{t^2}.$$

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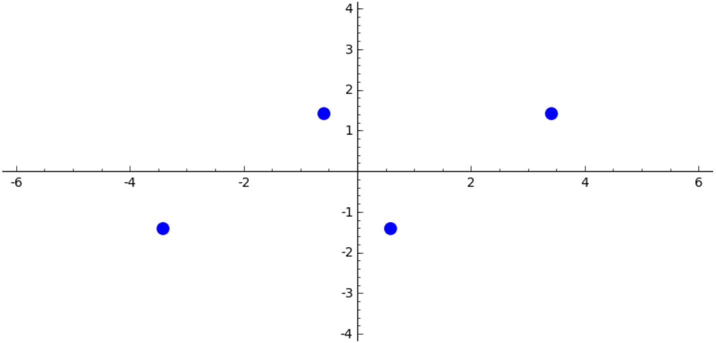
Quiver locus

$$z_1, z_2 \to \infty \quad \text{and} \quad \frac{z_1}{z_2} = e^{2\pi i\theta}$$

[Aspinwall, Melnikov, '04]

$\mathbb{F}_0$: Physical realization of the phases. Phase I

Phase I is physically realized.
Take e.g. $c = d = e = 1$, $b = i$ and $t \to 0$ in the Newton polynomial. The mirror is



$\mathbb{F}_0$: Physical realization of the phases. Phase II.

For the phase II, the situation is different.

We can obtain the phase II if one of the singularities in the mirror goes through the origin.

We need to know the intersection between the quiver locus and the conifold locus.

$\mathbb{F}_0$: Physical realization of the phases. Phase II.

The singularities of

$$W = \frac{e}{x} + \frac{b}{y} + cx + dy + t$$

are given by

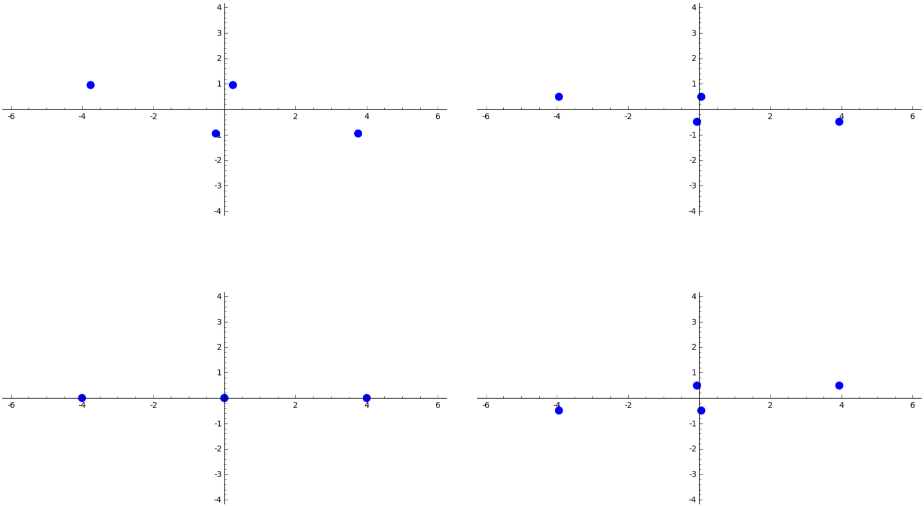
$$w = 4(z_1 + z_2) \pm 8\sqrt{z_1 z_2}, \qquad w = \left(1 - \frac{W}{t}\right)^2$$

For $z_1 = z_2$ there is a double root at $W = t$.

In the $t \to 0$ limit $(z_1, z_2 \to \infty)$, there is a double root at $W = 0$, so there are precisely *two* branes becomming massless simultaneously.

Therefore, the phase II will never be physically realized.

$\mathbb{F}_0$: Physical realization of the phases. Phase II.



Conclusions and future directions

Conclusions

- We have studied the physical realization of Seiberg dual theories in branes at singularities.
- In the $\mathbb{F}_0$ case, only phase I can be physically realized at weak gauge coupling.

Future directions

- Extend the analysis to the case of the complex cone over dP_2 (on the way) and dP_3.

Thank you!