Towards a world-sheet description of DFT

Erik Plauschinn

LMU Munich

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based on ...

This talk is based on ::

- *T-duality revisited* [arXiv:1310.4194]
- *On T-duality transformations for the three-sphere* [arXiv:1408.1715]
- and work in progress with I. Bakas & D. Lüst [arXiv:1602.xxxxx]

outline

1. motivation
2. t-duality
3. example

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motivation :: dualities

Duality :: two different theories describe the same physics.

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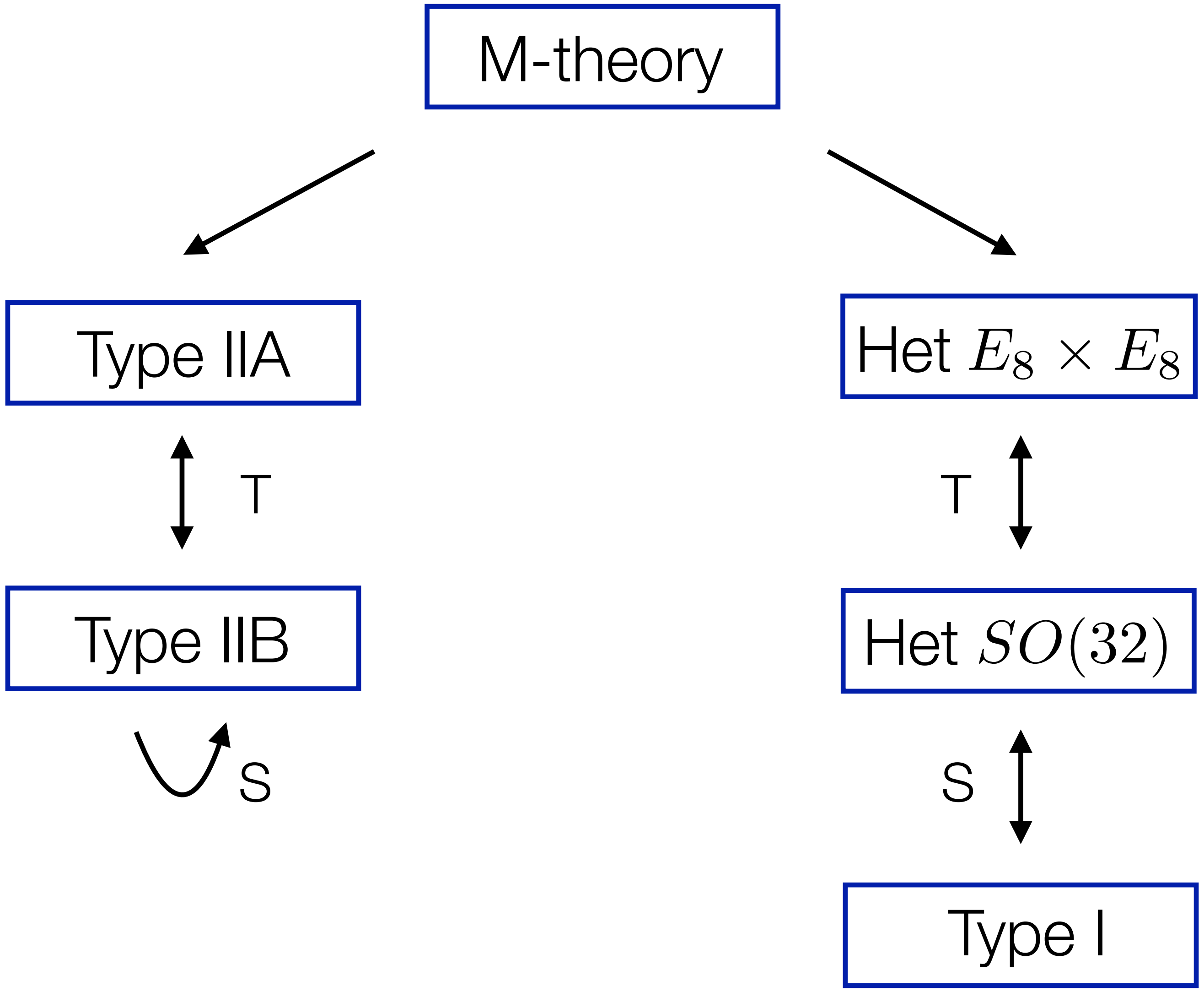
They play an important role in understanding string theory.



motivation :: main idea

Duality ::

Idea ::

Duality :: is a symmetry of the equations of motion.

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Duality :: is a symmetry of the equations of motion.

Idea :: make duality into a symmetry of an action.

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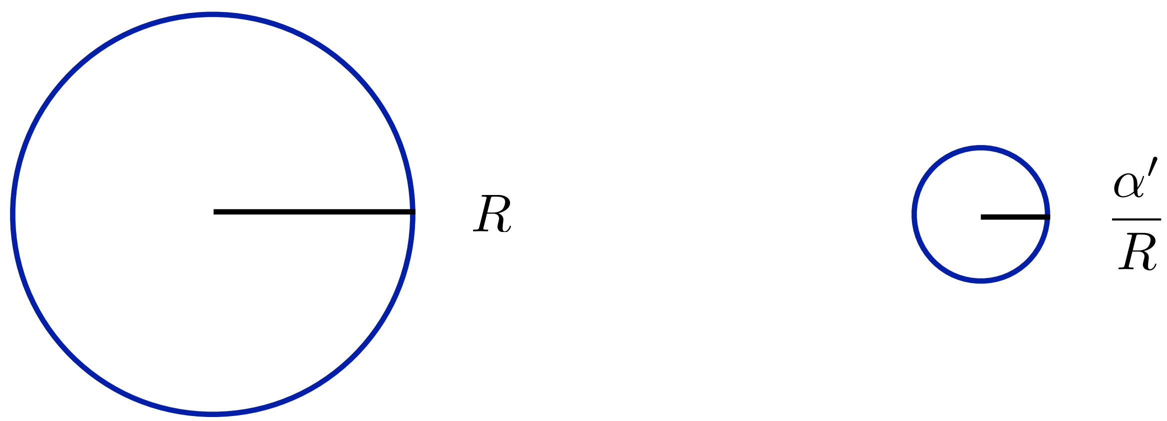
3. example

t-duality :: generalities

T-duality :: the physics of string theory compactified on two circles

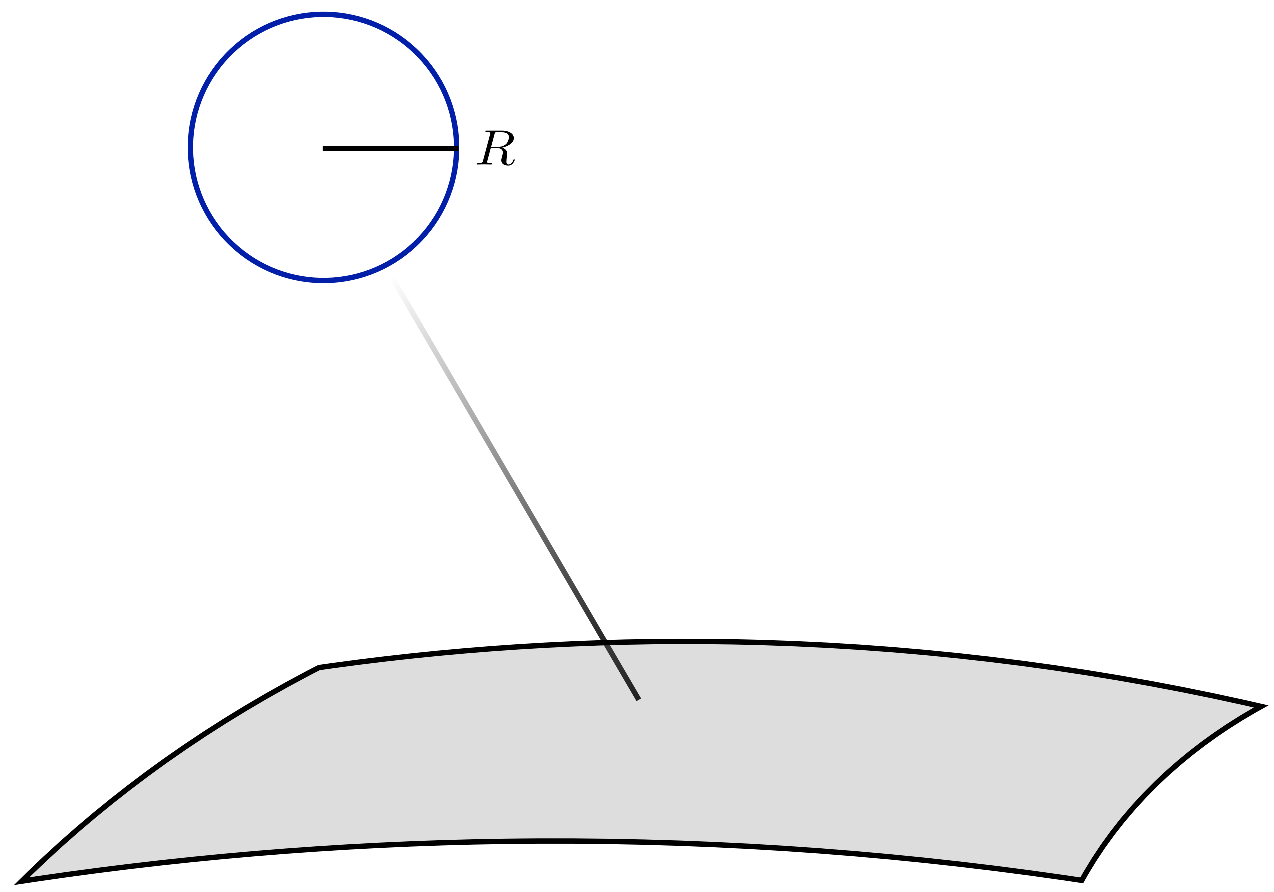


is indistinguishable.

t-duality :: doubled geometry and dft

circle fibration

base manifold



Bouwknegt, Evslin, Mathai - 2003
Hull - 2004
Hull, Zwiebach - 2009

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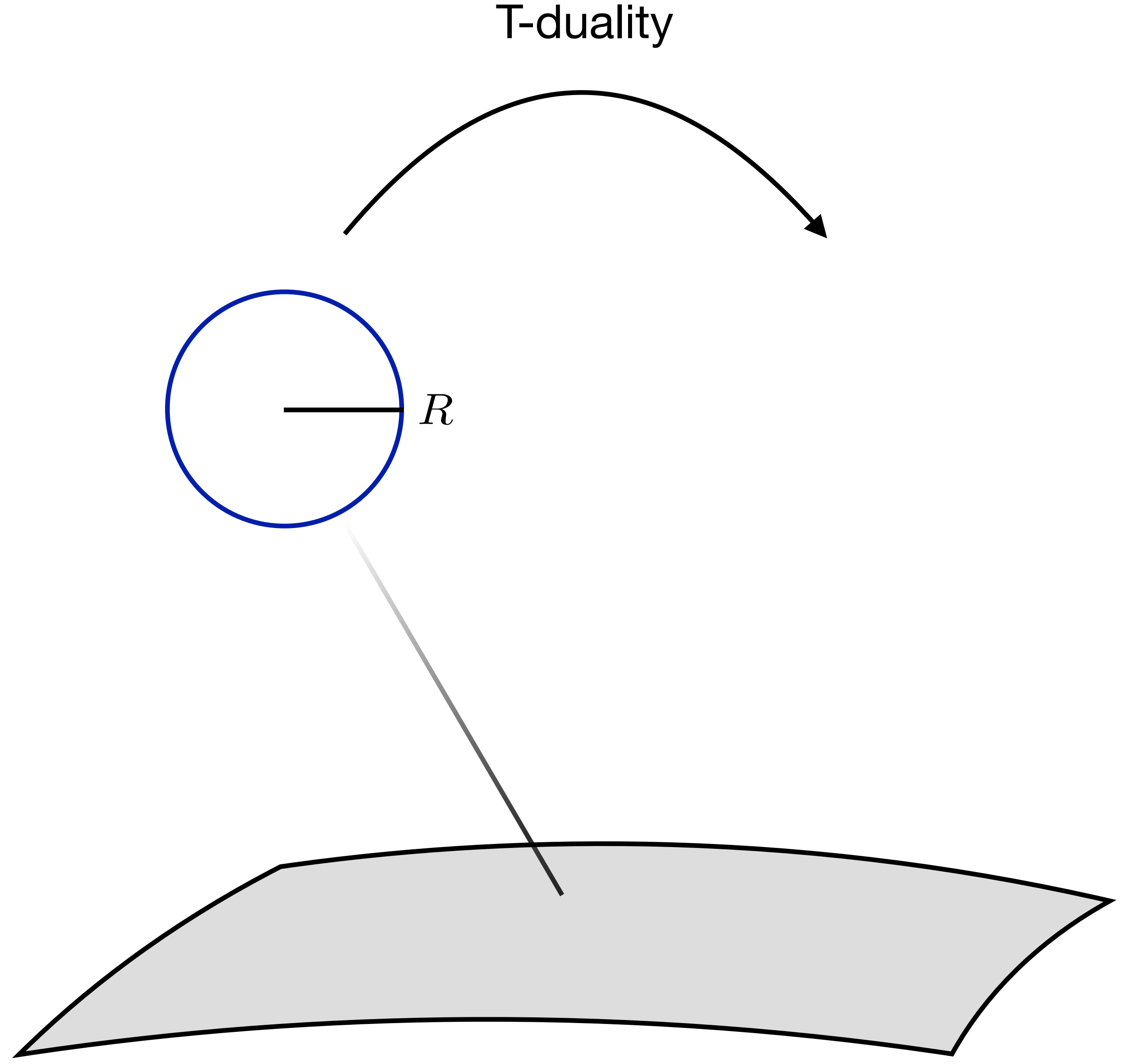

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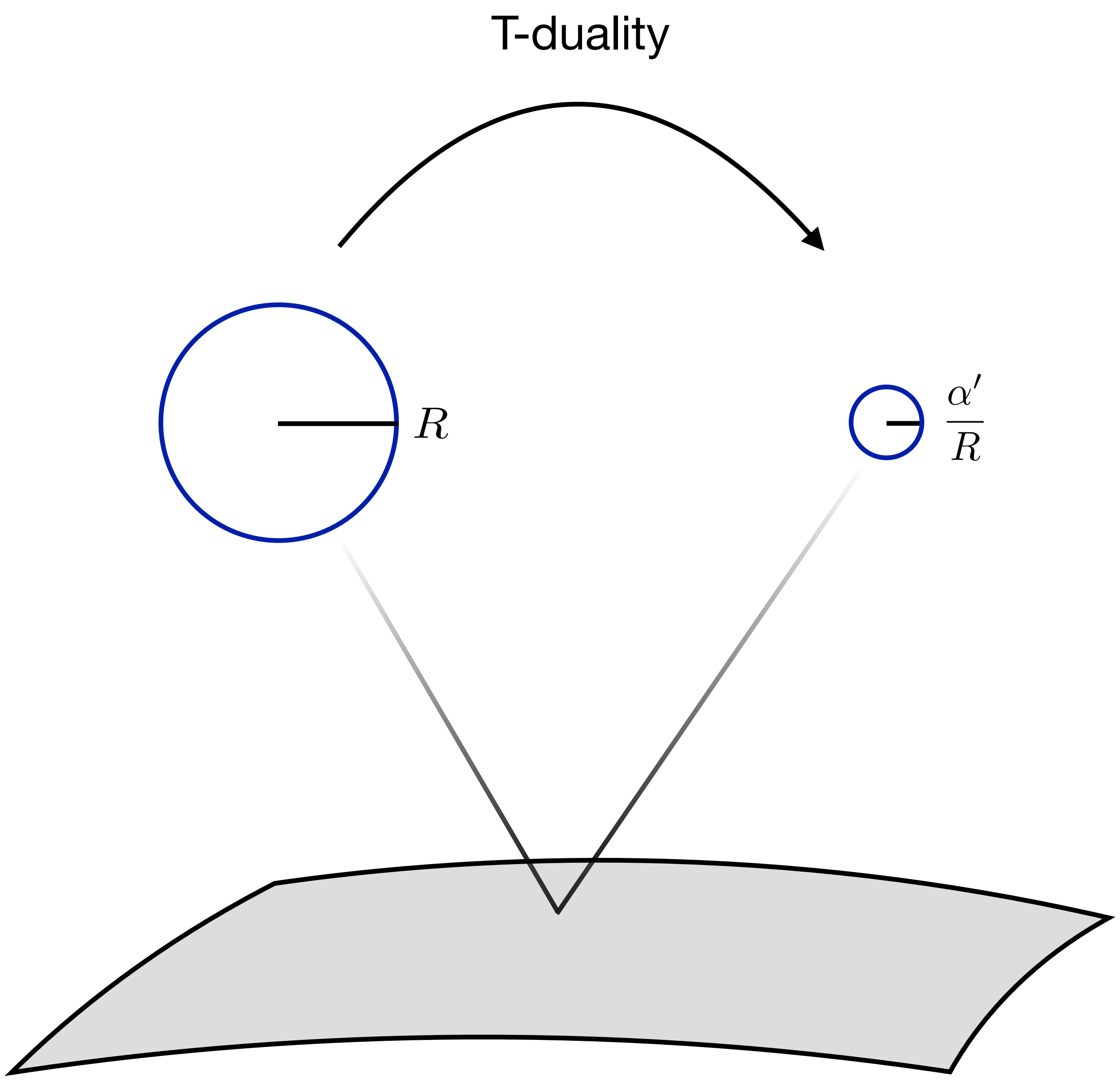

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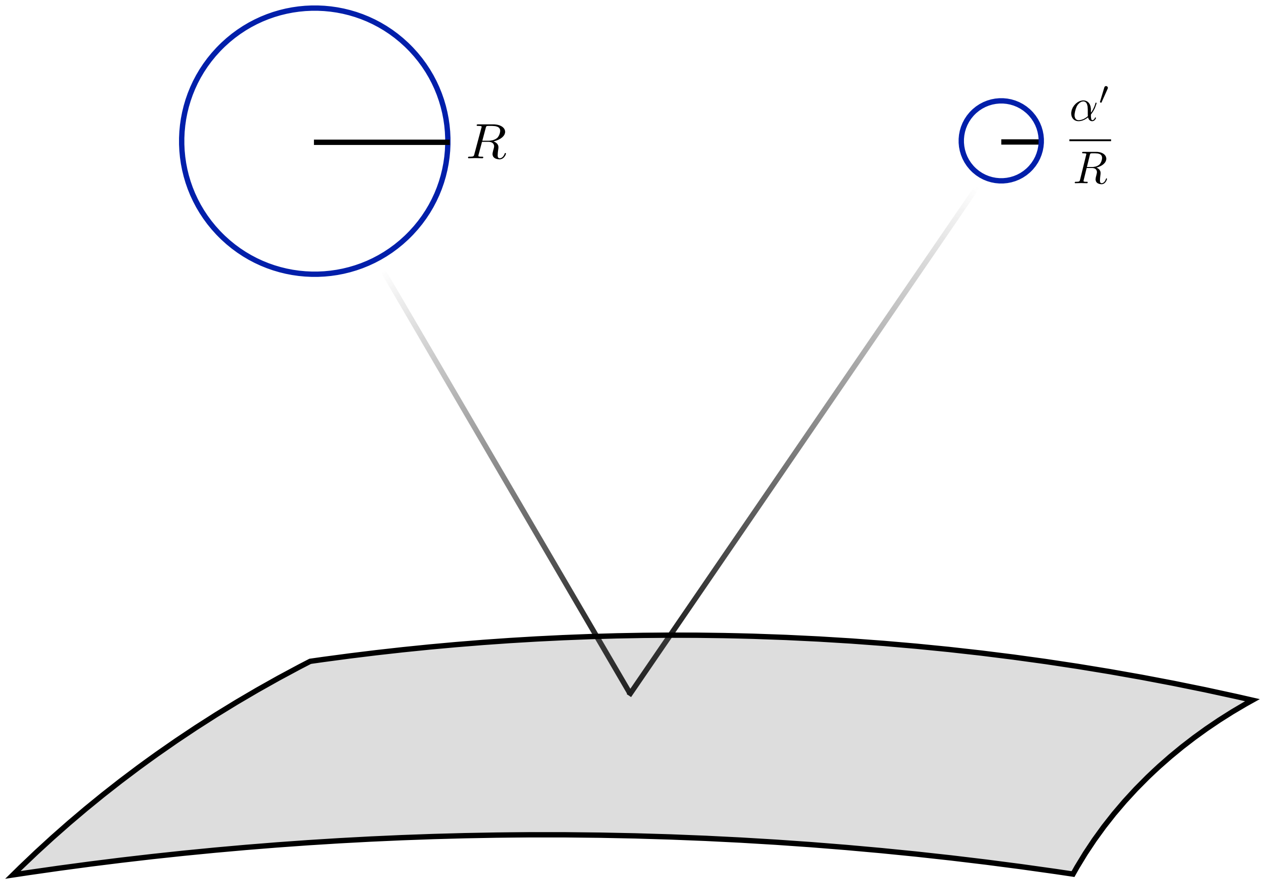

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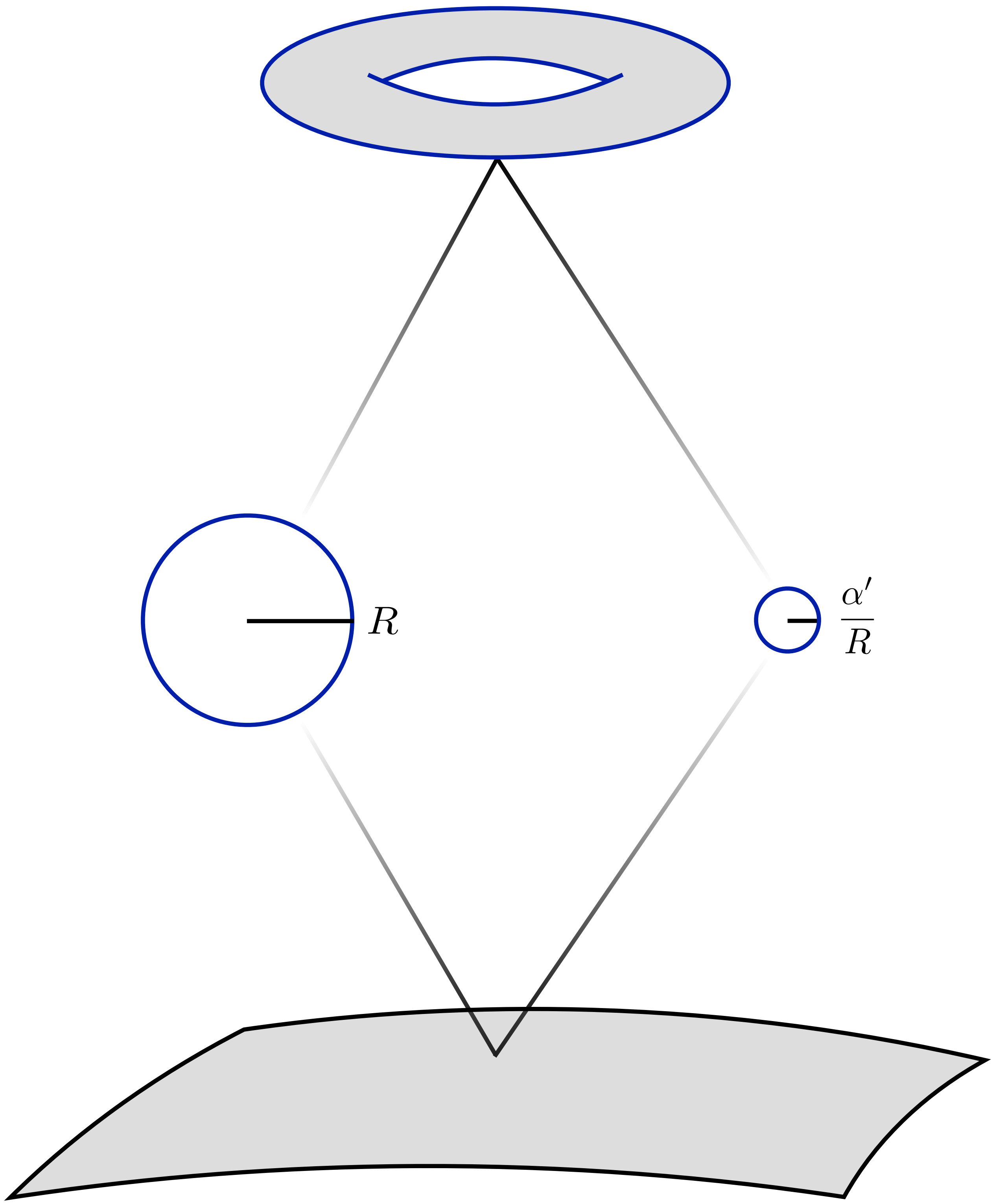
t-duality :: doubled geometry and dft

doubled geometry

circle fibration

base manifold



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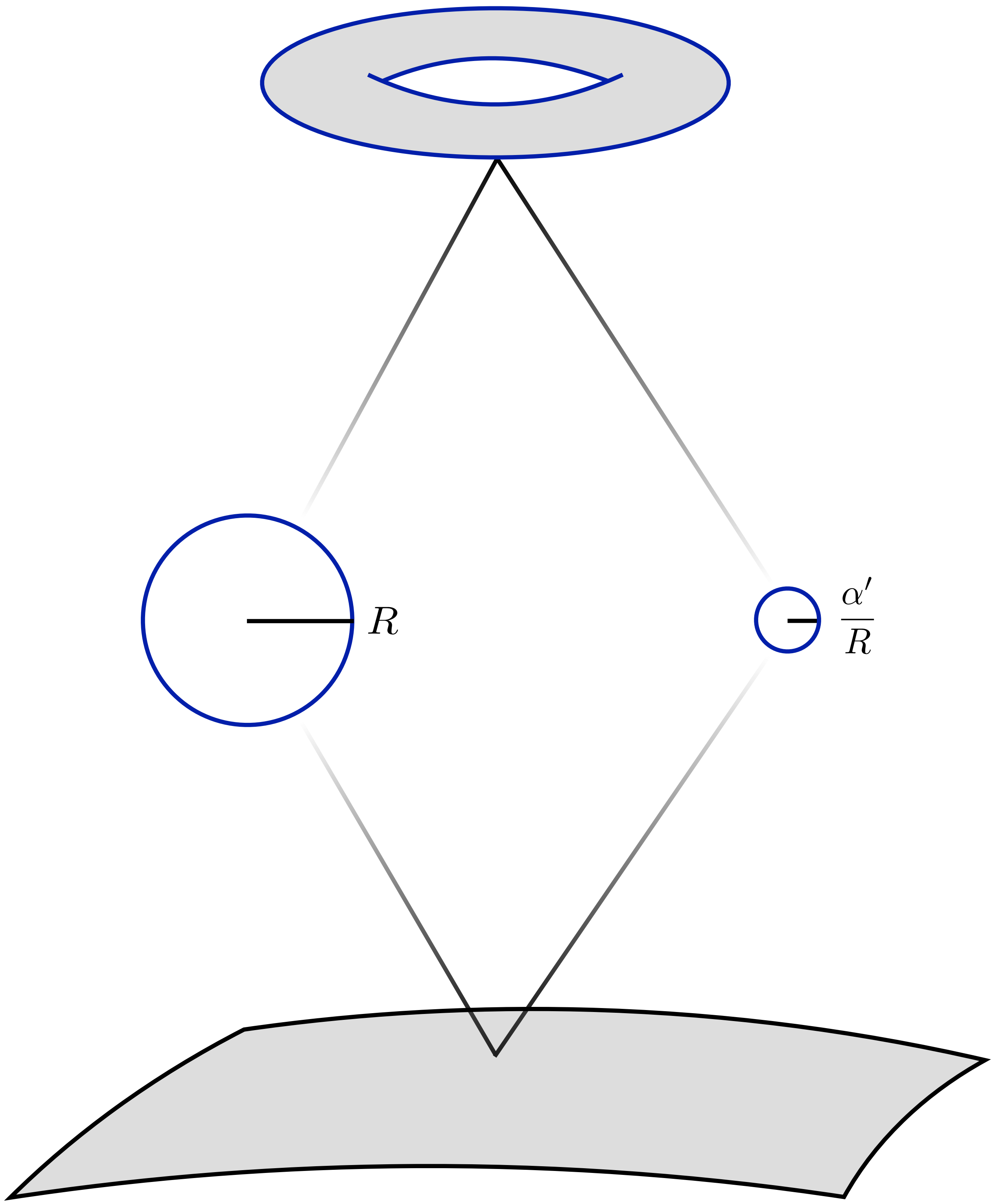
t-duality :: doubled geometry and dft

doubled geometry — symmetry

circle fibration — duality

base manifold



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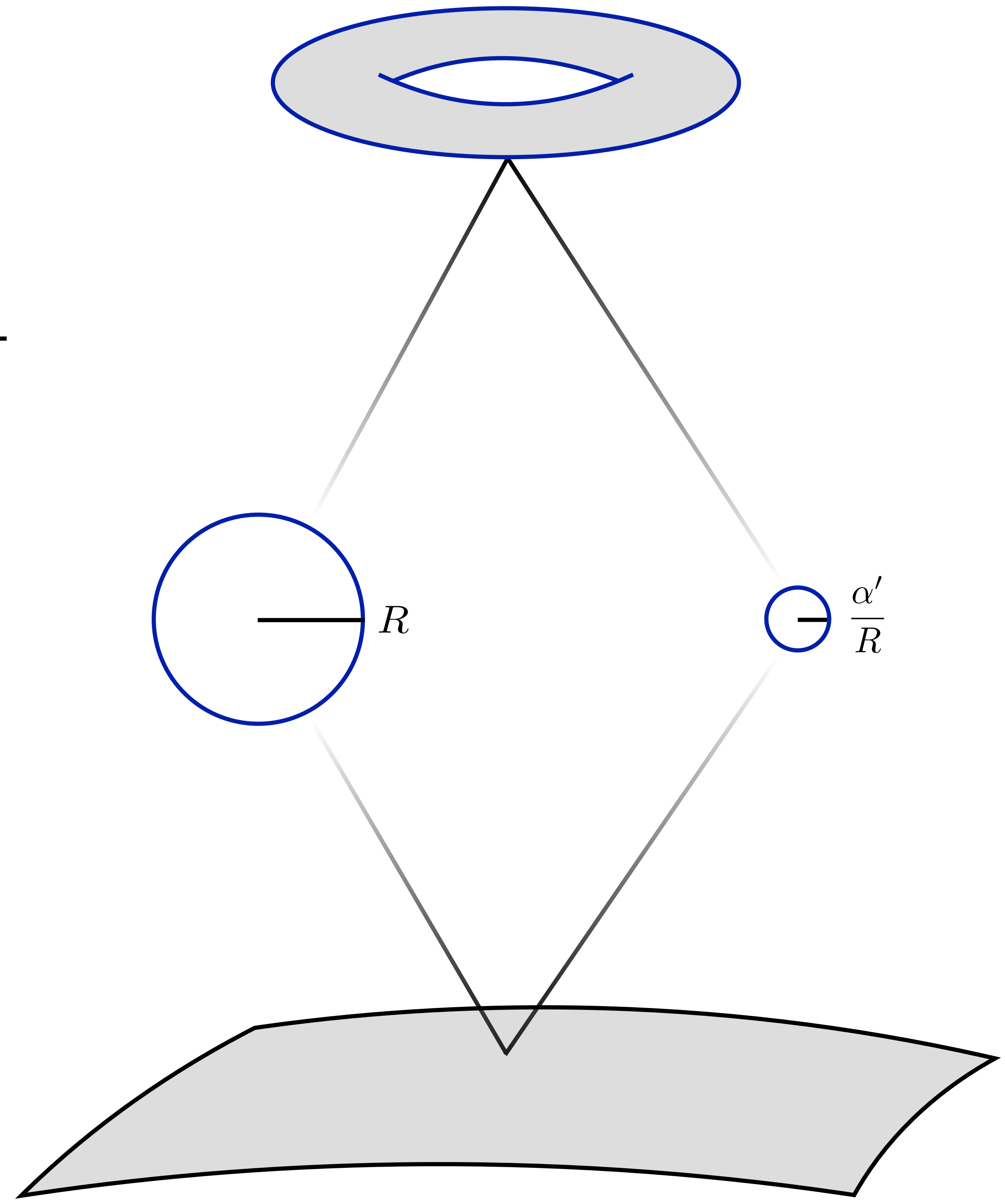
t-duality :: doubled geometry and dft

doubled geometry	symmetry	
circle fibration	duality	
base manifold		



Bouwknegt, Evslin, Mathai - 2003
Hull - 2004
Hull, Zwiebach - 2009

t-duality :: reduction via gauging

The reduction

doubled geometry

↓

physical geometry

can be realized via a gauging procedure.

Main idea ::

1. Identify a global symmetry (circle) in the action.
2. Gauge this symmetry.
3. Integrate-out the gauge field.

Consider the following sigma-model action for the doubled geometry

$$\mathcal{S} = -\frac{1}{4\pi\alpha'}\int_{\partial\Sigma}\Big[G_{ij}\,dX^i\wedge\star dX^j + \alpha' R\,\Phi\star 1\Big] - \frac{i}{2\pi\alpha'}\int_{\Sigma}\tfrac{1}{3!}\,H_{ijk}\,dX^i\wedge dX^j\wedge dX^k\,.$$

This action is invariant under global transformations $\delta_\epsilon X^i = \epsilon\,k^i(X)$ if

$$\mathcal{L}_k G = 0\,, \qquad \iota_k H = d\mathsf{v}\,, \qquad \mathcal{L}_k\Phi = 0\,.$$

$\mathcal{L}_k G = 0$ ↓ isometry (circle)

$\iota_k H = d\mathsf{v}$ ↓ one-form v

t-duality :: reduction — step II

The gauged action takes the following form

$$\widehat{\mathcal{S}} = -\frac{1}{4\pi\alpha'}\int_{\partial\Sigma}\Big[G_{ij}(dX^i + k^i A)\wedge\star(dX^j + k^j A) + 2i\,\mathsf{v}\wedge A + \alpha' R\,\Phi\star 1\Big]$$
$$-\frac{i}{2\pi\alpha'}\int_{\Sigma}\tfrac{1}{3!}\,H_{ijk}\,dX^i\wedge dX^j\wedge dX^k\,.$$

This action is invariant under local symmetry transformations provided that

$$\iota_k\mathsf{v} = k^m\mathsf{v}_m = 0\,.$$

Hull, Spence - 1989 & 1991

For the gauged and ungauged theories to be equivalent, impose the constraint

$$0 = F = dA\,.$$

t-duality :: reduction — step III

The equation of motion for the gauge field has the solution

$$|k|^2 A = -k^i G_{ij}\, dX^j - i \star \mathsf{v}\,.$$

Integrating-out the gauge field results in an action specified by

$$\check{G} = G - \frac{1}{|k|^2}\,\mathsf{k}\wedge\star\mathsf{k} + \frac{1}{|k|^2}\,\mathsf{v}\wedge\star\mathsf{v}\,, \qquad \check{H} = H + d\left(\frac{1}{|k|^2}\,\mathsf{k}\wedge\mathsf{v}\right)\,.$$

Note that k is a null-eigenvector for $\check{G}$ and $\check{H}$ ::

$\iota_k \check{G} = 0$, $\iota_k \check{H} = 0$ ⟶ change of coordinates ⟶ dimensions reduced

t-duality :: reduction — step III

The equation of motion for the gauge field has the solution

$$\mathcal{T}^i{}_j = \left(\begin{array}{c:c} k^1 & 0 \\ \hdashline k^2 & \\ \vdots & \mathbb{1} \\ k^D & \end{array}\right) \qquad \check{\mathcal{G}}_{ij} = \left(\mathcal{T}^T \check{G}\, \mathcal{T}\right)_{ij} = \left(\begin{array}{c:c} 0 & 0 \\ \hdashline 0 & \check{G}_{ab} \end{array}\right)$$

$$\check{\mathcal{H}}_{ijk} = \check{H}_{lmn} \mathcal{T}^l{}_i \mathcal{T}^m{}_j \mathcal{T}^n{}_k$$

$$\check{\mathcal{H}}_{1jk} = 0$$

Integrating

Note that k

$\iota_k \check{G} = 0$, $\iota_k \check{H} = 0$ → change of coordinates → dimensions reduced

t-duality :: reduction — step III

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Summary ::

- through a gauging procedure on the world-sheet,
- the target-space dimensions can be reduced.

Constraints ::

- global symmetry $\quad \mathcal{L}_k G = 0\,, \qquad \iota_k H = d\mathsf{v}\,, \qquad \mathcal{L}_k \Phi = 0\,,$
- gauging $\quad \iota_k \mathsf{v} = 0 = \iota_v \mathsf{k}\,,$
- vanishing F $\quad \mathcal{L}_v G = 0\,, \qquad \iota_v H = d\mathsf{k}\,, \qquad \mathcal{L}_v \Phi = 0\,.$

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Outlook ::

- T-duality corresponds to interchanging $k \longleftrightarrow v$.

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example :: su(2) wzw

Consider the SU(2) WZW model (three-sphere with H-flux) ::

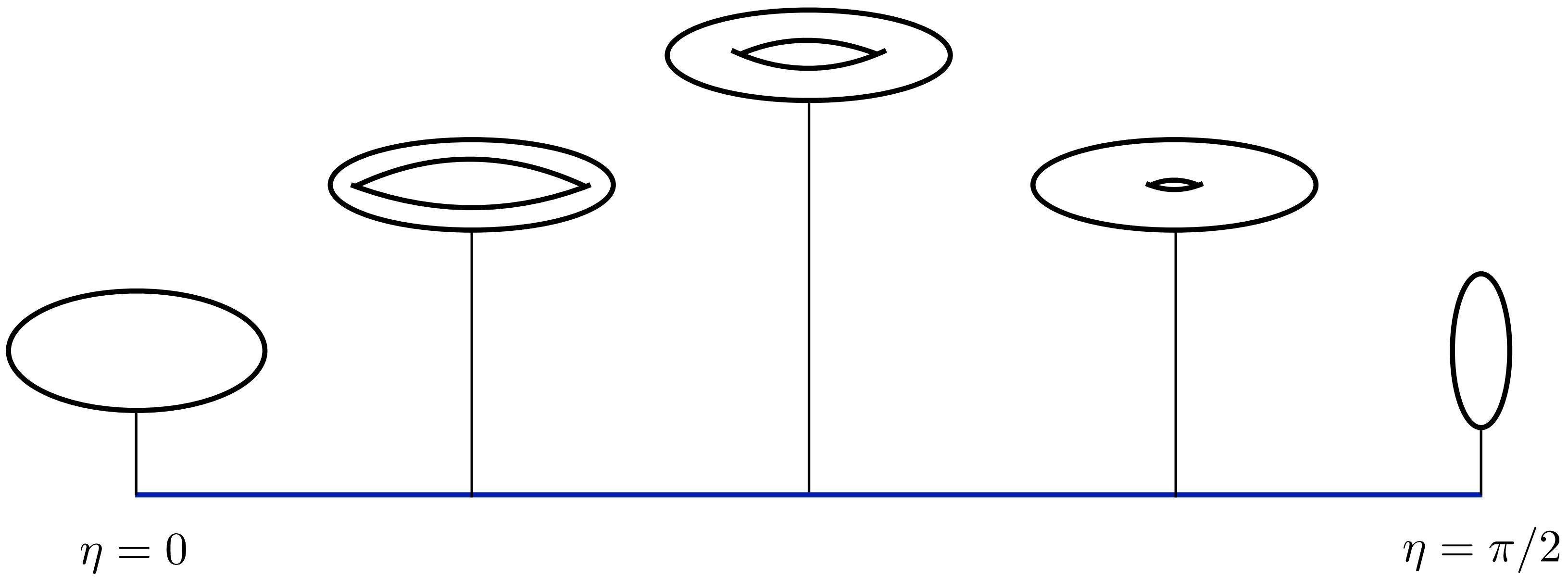
$$ds^2 = R^2\left(\sin^2\eta\, d\zeta_1^2 + \cos^2\eta\, d\zeta_2^2 + d\eta^2\right),$$

$$H = 2\,R^2 \sin\eta\cos\eta\, d\zeta_1\wedge d\zeta_2\wedge d\eta\,,$$

$$\Phi = \text{const.}$$

$$\zeta_{1,2}\in[0,2\pi)\,,$$

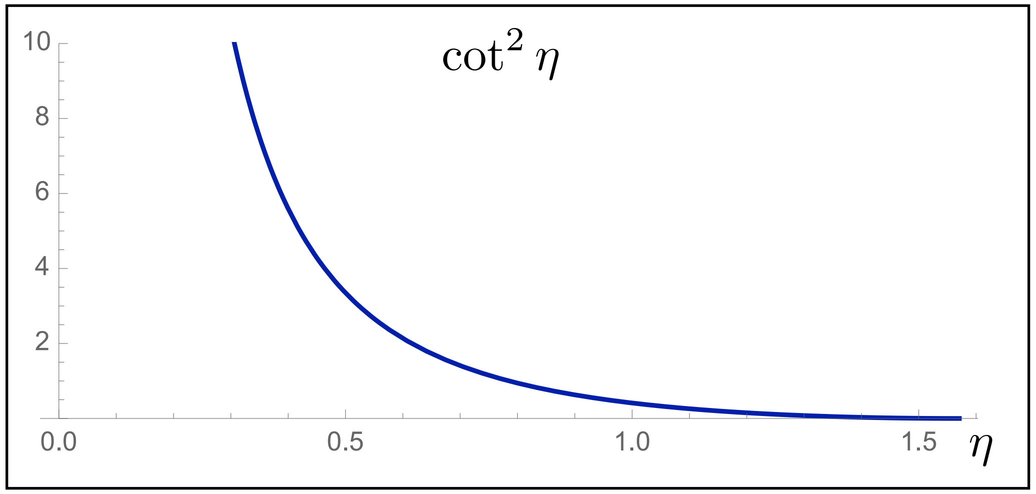
$$\eta\in[0,\pi/2]\,.$$

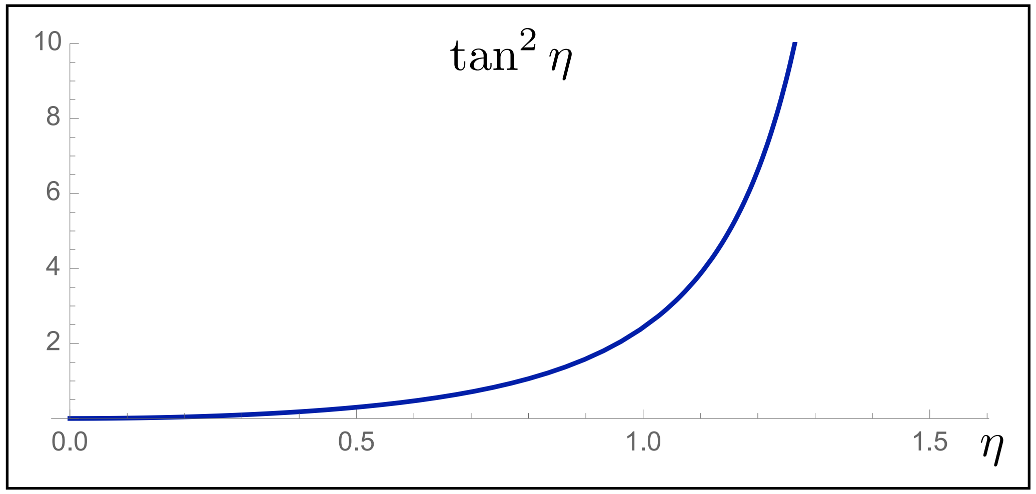


This geometry has two directions of isometry.

The reduction constraints are satisfied by precisely two Killing vectors ::

choice 1 — $k = \partial_{\zeta_1}$	choice 2 — $k = \partial_{\zeta_2}$
$\check{ds}^2 = R^2\left(\cot^2\eta\, d\zeta^2 + d\eta^2\right)\,,$	$\check{ds}^2 = R^2\left(\tan^2\eta\, d\zeta^2 + d\eta^2\right)\,,$
$\check{H} = 0\,,$	$\check{H} = 0\,,$
$\check{\Phi} = \Phi - \log(\sin\eta)\,,$	$\check{\Phi} = \Phi - \log(\cos\eta)\,.$





example :: reduced theories

The reduction constraints are satisfied by precisely two Killing vectors ::

choice 1 — $k = \partial_{\zeta_1}$

$$\check{ds}^2 = R^2 \left(\cot^2\eta \, d\zeta^2 + d\eta^2\right) ,$$

$$\check{H} = 0 ,$$

$$\check{\Phi} = \Phi - \log(\sin\eta) ,$$

choice 2 — $k = \partial_{\zeta_2}$

$$\check{ds}^2 = R^2 \left(\tan^2\eta \, d\zeta^2 + d\eta^2\right) ,$$

$$\check{H} = 0 ,$$

$$\check{\Phi} = \Phi - \log(\cos\eta) .$$

The reduced theories are conformal, and T-dual to each other.

Summary ::

- the SU(2) WZW model (D=3) allows for two reductions,
- leading to two conformal models in D=2,
- which are T-dual to each other.

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More results ::

- generalization to multiple (non-abelian) gaugings,
- T-duality is a symmetry of the doubled theory,
- the Buscher rules are reproduced,
- results on conformality of reduced theories,
- WZW models with arbitrary group,
- …