Axion Monodromy and the Weak Gravity Conjecture

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Talk based on A. Hebecker, FR, A. Westphal [arXiv:1512.03768]

Large field Inflation and axions

→ Slow-roll parameters: $\epsilon \sim (V^{'}/V)^2 \ll 1, \eta \sim V^{''}/V \ll 1, M_{pl} \equiv 1.$

- CMB Observables:

$$n_s - 1 = 2\eta - 6\epsilon \simeq 0.04$$
$$r = 16\epsilon \leq 0.08$$

→ Natural choice, $\epsilon \lesssim \eta \sim 10^{-2}$, requires transplanckian field displacements $\Delta\phi \gtrsim 4(\epsilon/10^{-2})^{1/2} M_{Pl}$.

- Shift symmetry needed to ensure flatness over large range→ **Axions** with large decay constants!

- Axions with large periodicities are also important in the mechanism of *Cosmological Relaxation* of EW scale (see Witkowski's talk).

- However, stringy implementations of Axion Inflation exhibits a certain degree of tuning/complexity.

→ Why? [Rudelius/Montero,Uranga,Valenzuela/$\cdots$]

The Weak Gravity Conjecture (WGC)

[Arkani-Hamed,Motl,Nicolis,Vafa'06]

→ Extremal ($Q/M = 1$) black holes should be allowed to decay.

The Electric WGC

Consider a $U(1)$ gauge theory with coupling g coupled to gravity in 4d ($M_{Pl} \equiv 1$). Then there exists an electrically charged particle of mass m_{el} and charge q s.t.:

$$\frac{qg}{m_{el}} \gtrsim 1$$

The conjecture can be generalised to any p-form gauge theory with coupling $e_{p,d}$ in d dimensions coupled to $p-1$-dimensional objects with tension T_{p-1}.

$$\frac{e_{p,d}\, q M_d^{d/2-1}}{T_{p-1}} \gtrsim 1$$

→ The case $p = 0$ constrains/rules out models of Axion Inflation where a transplanckian range arises in the field space of one or more axions [see Arthur's talk].

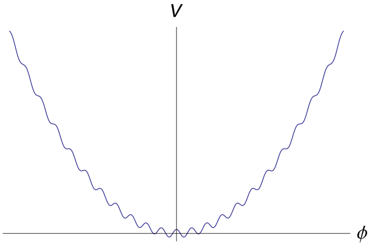
Axion Monodromy [Silverstein,Westphal '08/···]

- However, transplanckian ranges can also be obtained by introducing *monodromies* to extend the compact field space of one axion.
- These models are *not* affected by the WGC for axions.

→ Phenomenological definition:

$$V = \underbrace{\frac{1}{2} m^2 \phi^2}_{\text{monodromy breaking shift symmetry}} + \alpha \cos(\phi/f)$$

→ For $\alpha/(m^2 f^2) > 1$, the potential exhibits local minima.



Domain walls in monodromy potential

→ The presence of local minima separated by "wiggles" can be effectively understood via a dual description:

$$\star \mathbf{F}_4 \equiv \phi.$$

- The corresponding 3-form potential couples to 2-dimensional objects in $4d$ ⇒ Domain walls!

$$\mathcal{L} = \frac{1}{e^2} F_4^2 + \int_{DW} A_3.$$

- F_4 is quantized in units of e: $F_4 = ne, n \in \mathbb{Z}$.
- Across the membrane, the flux changes by one unit: $n \to n+1$.
- The discretuum of points n corresponds to the values of the field ϕ in the cosine wells: $\phi \sim 2\pi n f$.
- Away from membranes, energy density: $\frac{1}{2}F_4^2 \sim \frac{1}{2}n^2 e^2 \to \frac{1}{2}m^2\phi^2$, so $e = 2\pi m f$.

→ see Arthur's talk for Kaloper-Sorbo domain walls arising in axion monodromy

Constraints on Axion Monodromy, pt. 1

- Apply WGC for 3-form potential and domain walls.

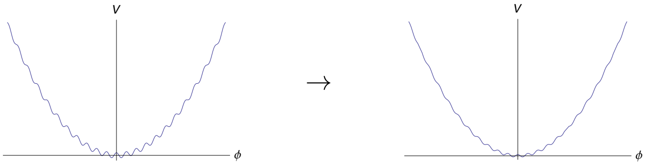
$\rightarrow$ **WGC**: $T_{DW} \lesssim e \Rightarrow \alpha \lesssim m^2$.

- Tension of the heaviest domain walls: $T_{DW} \sim \sqrt{V}\Delta\Phi \sim \sqrt{\alpha} f$.

$\rightarrow$ The WGC requires a small tension, which can be obtained by lowering the height of the wiggles, i.e. α.

- **Crucial**: this is precisely what is needed to have slow roll inflation!

$\Rightarrow$ No tension between WGC and slow-roll$\Rightarrow$no constraint from the electric WGC!



$\rightarrow$ see Ibanez, Montero, Uranga, Valenzuela '15 for constraints on KS domain walls for relaxion

Magnetic WGC

The magnetic WGC

Weakly coupled gauge theories with cutoff Λ and magnetic monopole of charge q_m break down at a scale:

$$\Lambda \lesssim \frac{g}{q_m} \quad \text{parametrically smaller than } M_{Pl} \equiv 1!$$

- The minimally charged magnetic monopole should not be a black hole, i.e. $M_{mon} \sim \Lambda/e^2 \lesssim M_{BH} \sim R \sim \Lambda^{-1}$.
- **Strategy**: generalise this statement to any $(p+1)$-form gauge theory with $d-(p+4)$ magnetic branes.

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1 In analogy to the monopole, the tension of the brane is found by:

$$T \sim (1/e^2_{p,d}) \int F^2_{p+2} \sim \Lambda^{p+1}/e^2_{p,d}$$

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2 Impose:

$$T_{d-(p+4)} \lesssim T_{BH} \sim M_d^{d-2} R^{p+1}.$$

Constraints on Monodromy Inflation, pt. 2

⇒ Specialise to domain walls in $4d$, i.e. $p = 2$:

$$\Lambda \lesssim e^{1/3}$$

→ **Caveat**: there are no (-2)-branes! Assume that the bound can be obtained by "analytical continuation" in p, d.

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Take home

1. **Electric** WGC does not constrain monodromy inflation [see also Ibanez, Montero,Uranga,Valenzuela '15].
2. **Magnetic** WGC limits the field range, but does not forbid phenomenological realisations.

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$\rightarrow$ EFT is valid up to KK-scale: $M_{KK} \sim 1/R$.

- Tension of a q-dimensional object descending from a p-brane in $10D$: $T_q \sim M_s^{p+1} R^{p-q}/g_s$.

$$T_q/M_{KK}^{q+1} \sim M_s^{p+1} R^{p+1}/g_s.$$

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$\rightarrow$ **Take home**: the magnetic WGC as the more fundamental constraint from a ST point of view, therefore we assume that it is valid for any p-form.

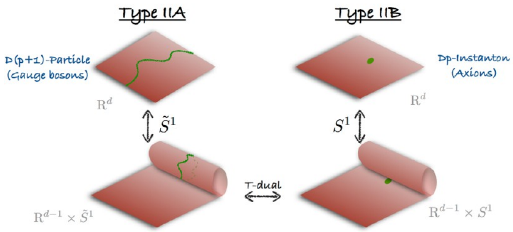
Problems with domain walls

General form of the WGC from extremality argument in d-dimensions:

$$\gamma_{p,d} T_p^2 \leq e_{p,d}^2 q^2 M_d^{d-2}$$

→ $\gamma_{p,d} = 0$ for axions ($p = 0$) and strings ($p = d - 2$). For domain walls ($p = d - 1$) the inequality cannot be solved.

→ Strategy for axions [Picture taken from Gary Shiu's talk at StringPheno 2015].



The WGC as a geometric constraint

- **Idea**: in string compactifications, the WGC translates into a constraint on the geometry of a chosen CY X
- **Our strategy**:
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  3. Apply previously found constraint on the geometry to get the WGC for $q-p$ dim. objects in $4d$.

  → No need of string dualities! (see Brown,Cottrell,Shiu,Soler/Ibanez, Montero, Uranga, Valenzuela '15)

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1. **Phenomenology**:
   - Models of axion monodromy (inflation/relaxation) exhibit low energy "wiggles", which correspond to $4d$ domain walls.
   - The WGC can be applied to these domain walls:
     - **Electric WGC**: no constraints.
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- Therefore, the magnetic WGC should be seen as the more fundamental constraint.
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- **Many thanks!**

Application to Axion Inflation

1 For 0-form gauge fields, i.e. axions:

$$\mathcal{L}_{axions} \sim f^2 \int (\partial\phi)^2 + S_{inst}, \tag{1}$$

so the gauge coupling is $1/f$ and the tension is S_inst.

$\Rightarrow$ WGC for axions:

$$f \lesssim 1/S \lesssim 1, \tag{2}$$

because $S > 1$ in a calculable regime.

- Simplest model: single field natural inflation with $f > 1$ is ruled out!
- Models with two or more axions with subplanckian decay constants are also constrained.
- Monodromy models are not constrained, because they have a single axion with subplanckian decay constant.

WGC as a geometric constraint

w^i basis of $H^p(X, \mathbb{Z})$.

$$q_i^k = \int_{\Sigma_k} w^i = \int_X w^i \wedge w^k, \tag{3}$$

$$K_{ij} \equiv \int_X w_i \wedge \star w_j. \tag{4}$$

$$|q^\sigma|^2 \equiv K^{ij} q_i^\Sigma q_j^\Sigma \tag{5}$$