# Freiburg Lectures 2016

**Graduiertenkolleg** "Mass and Symmetries after the Discovery of the Higgs Particle at the LHC"

# Tracking and Tracking Detectors


Norbert Wermes
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# Outline

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## Lecture 1

### Tracking

- momentum measurement
- vertex measurement
- influence of multiple scattering
- errors and what to do …

## Lectures 2 & 3 & 4

### Tracking Detectors

- the signal (and the noise)
- spatial resolution with structured electrodes
- gaseous detectors
- semiconductor detectors
- pixel detectors ... status and future

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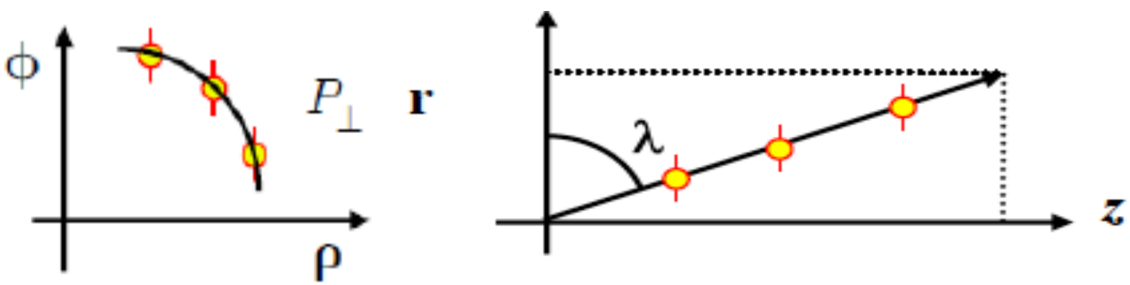
## Lecture 1

## Tracking





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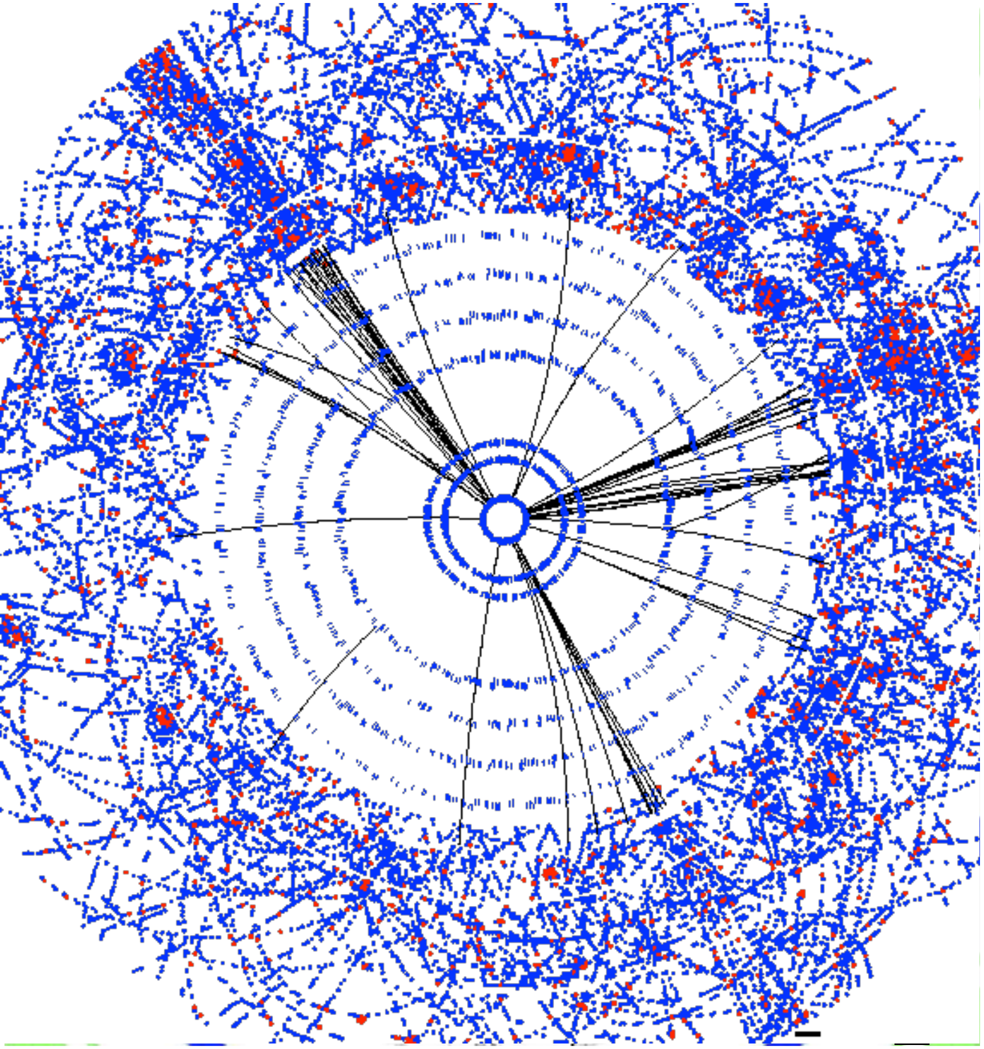
# Content Lecture 1

- Tasks of trackers
- Motion in magnetic field
- (Tracking) Spectrometers
  - **Dipole**
  - **Solenoid**
  - **Toroid**
- some examples
- Trajectories
  - straight lines
  - helix
- Track models
  - straight line fits
  - circular fits
  - matrix formalism
- Use cases
  - Dipole tracking
  - Solenoid tracking
  - **Momentum resolution**
  - Determination of the sign of charge
- **Multiple scattering**
  - Momentum resolutions at low momenta
- Vertex detection
  - **Extrapolation error**
  - Impact parameter resolution at low momenta

# "Tracking"



$H \to b\bar{b}$

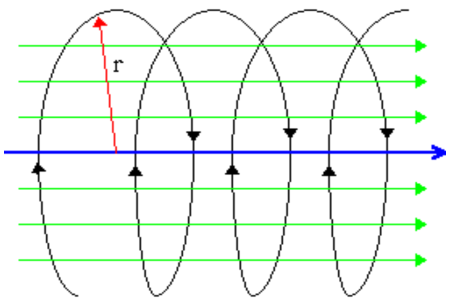
- ❑ find track patterns in an ensemble of measured hits
  - algorithmic pattern recognition (including noise hits -> Kalman filter)
- ❑ find best estimate for the trajectory of a particle's path
- ❑ measure the curvature -> momentum
- ❑ measure the (polar) angle
- ❑ extrapolate to origin -> vertex (1st, 2nd)
- ❑ estimate the precision of the momentum or vertex measurement
- ❑ determine the effect of scattering in matter in the precision
- ❑ include systematic effects (like mis-alignment)

see also **Lecture by Markus Elsing** next week

# Aims .. cont'd

- Recognition and reconstruction of charged particles trajectories ( tracks )
- Measure ( not a full list, but addressed here )

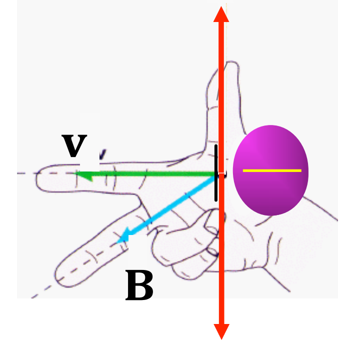
**momentum (in magnetic field)**




p = 0.3·B·r

**the sign of the charge**

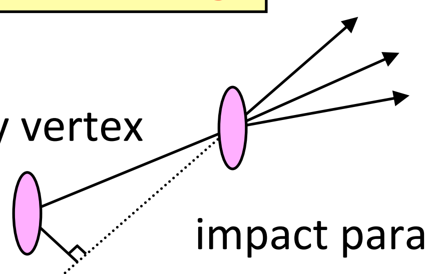
$\vec{F} = q\,(\vec{v} \times \vec{B})$




**lifetime and lifetime "tag"**

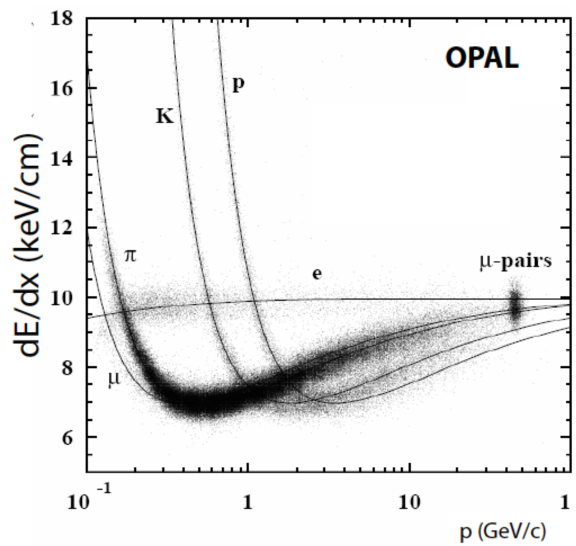
secondary vertex

primary vertex



impact parameter

**particle ID (mass),**
**not necessarily with the same detector**




specific ionisation loss

$p = m_o \gamma \beta$

$dE/dx = fct(\gamma\beta)$

not topic of lecture

# A spectrometer -> momentum measurement in B-field

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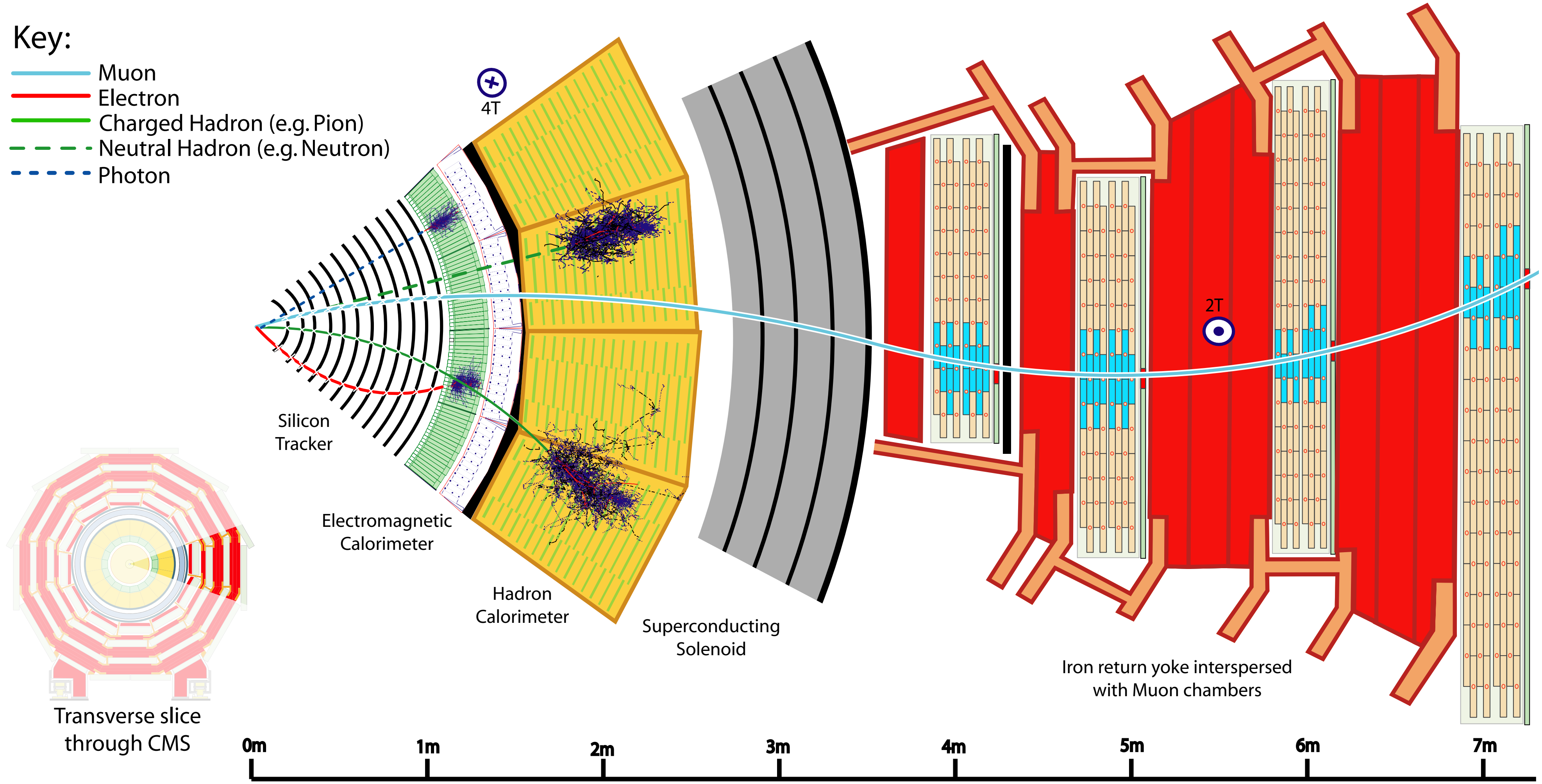

# Example: Muon Tracking in CMS and ATLAS

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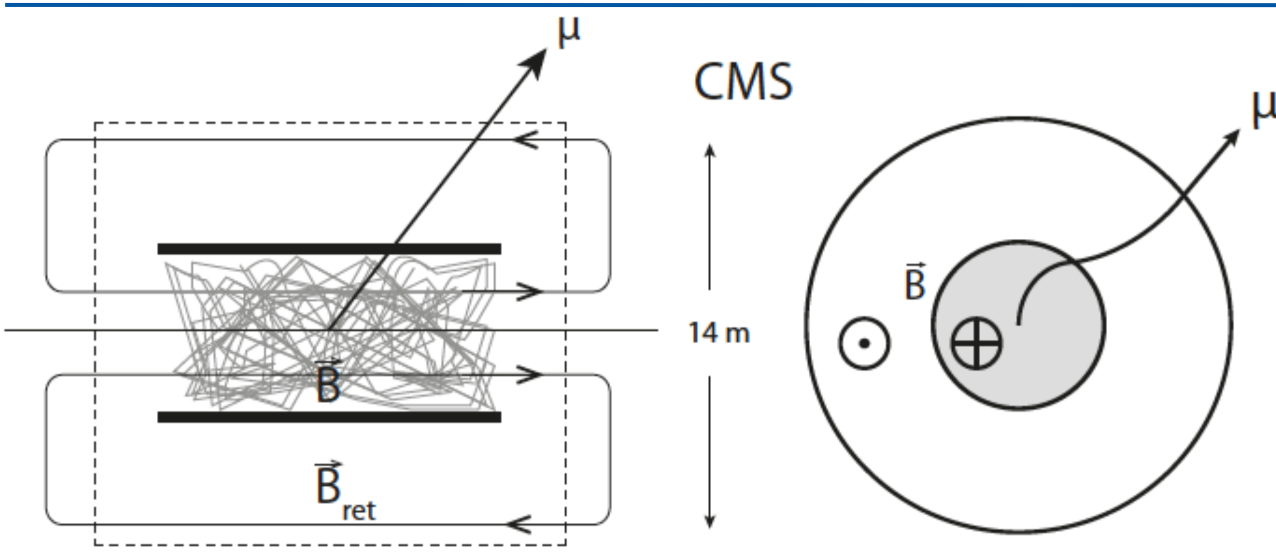
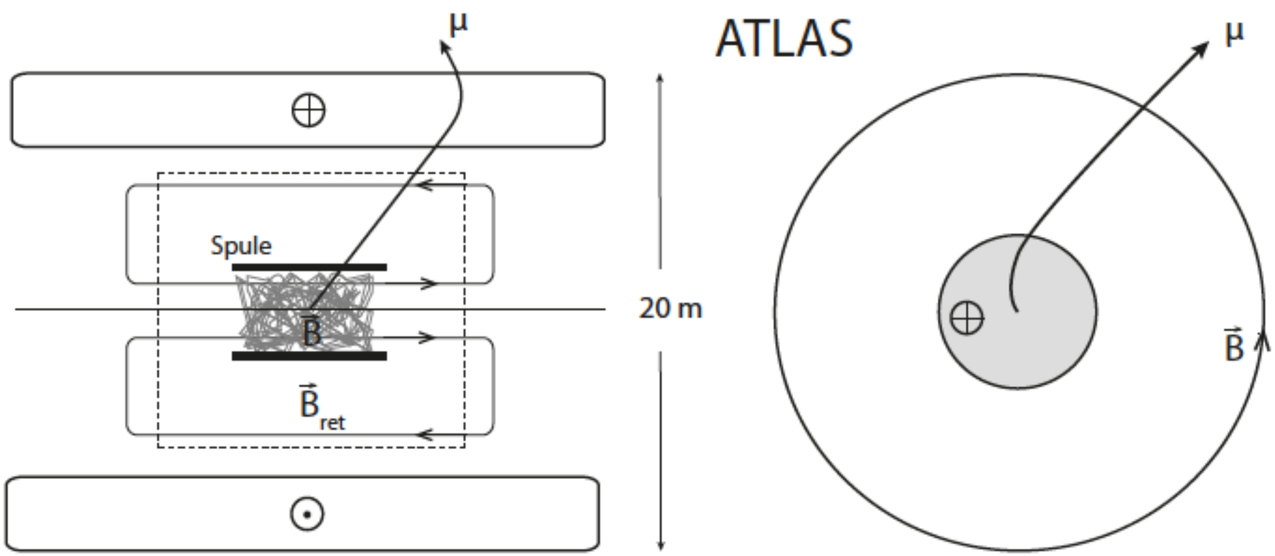



**CMS**

measurement of momentum in tracker <u>and</u> B return flux;

Solenoid with Fe flux return

Property: μ tracks point back to vertex in r-z plane



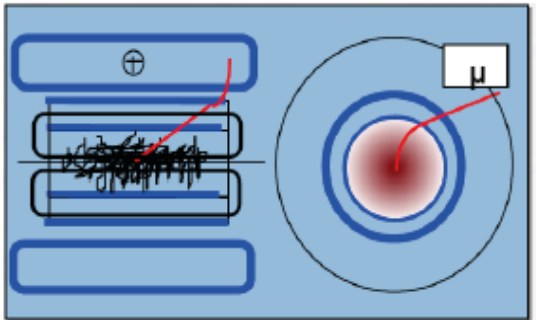

**ATLAS**

standalone μ momentum measurement; Air-core toroid safe for high multiplicities;

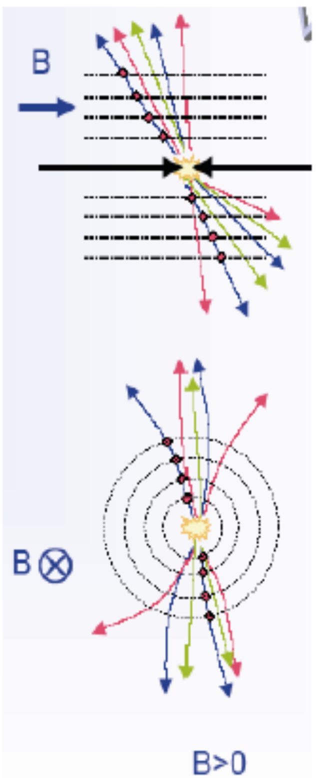
Property: $\sigma_p$ flat with η

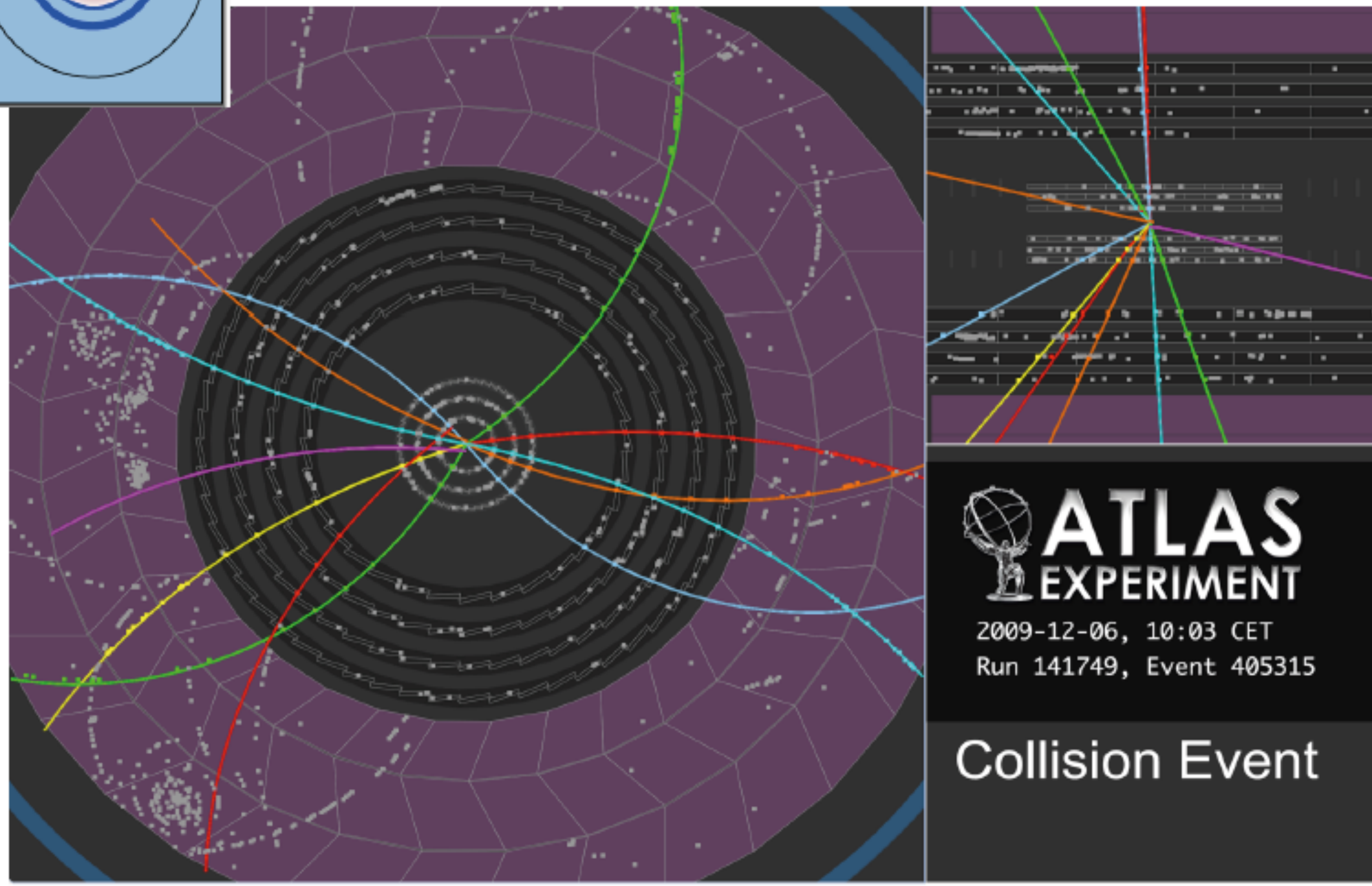
# Tracking in ATLAS






ATLAS: (air-core) toroid magnet
+ inner solenoid




http://atlas.web.cern.ch/Atlas/public/EVTDISPLAY/events.html

# Equation of Motion in a (homogeneous) magnetic field

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$$\vec{F} = \dot{\vec{p}} = q\left(\vec{v} \times \vec{B}\right) \quad \Rightarrow \quad \dot{\vec{v}} = \frac{q}{\gamma m}\left(\vec{v} \times \vec{B}\right)$$

differential equation in $\vec{v}$

- solution is a rotating vector $\vec{v}_T$ in plane perpendicular to $\vec{B}$ $\quad$ $\vec{v}_{||}$ is unchanged

$$B_1 = B_2 = 0,\ B_3 = B > 0$$

$$\omega_B = \frac{|q|\,B}{\gamma m}$$

⇨

$$\begin{aligned} v_1 &= v_T \cos(\eta\,\omega_B\,t + \psi) \\ v_2 &= -v_T \sin(\eta\,\omega_B\,t + \psi) \\ v_3 &= v_3 \end{aligned} \qquad \eta = \frac{q}{|q|}$$

equations also hold relativistically $\quad \omega_B = \omega_B(\gamma); E = \gamma m$

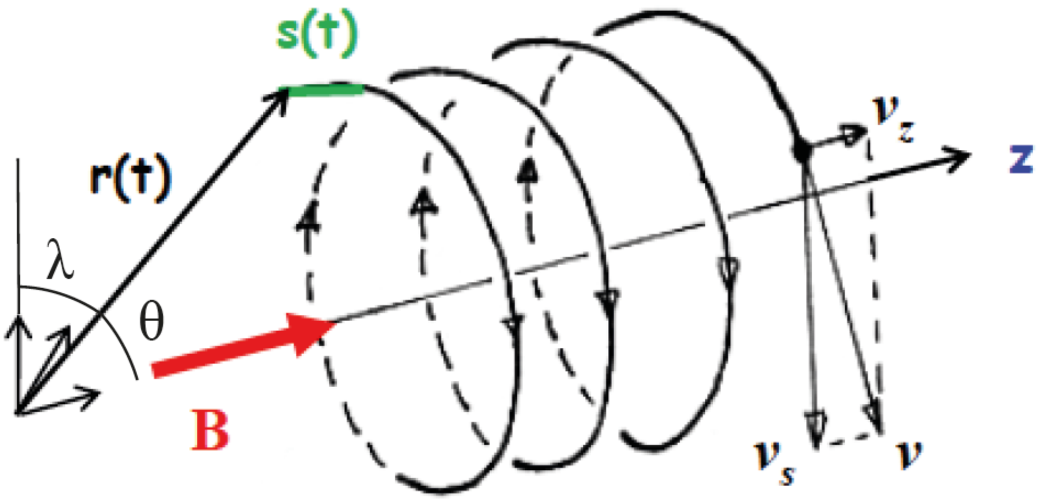
- integration yields spatial trajectory

$$\begin{aligned} x_1 &= \frac{v_T}{\eta\,\omega_B} \sin(\eta\,\omega_B\,t + \psi) + x_{10} \\ x_2 &= \frac{v_T}{\eta\,\omega_B} \cos(\eta\,\omega_B\,t + \psi) + x_{20} \\ x_3 &= v_3\,t + x_{30} \end{aligned}$$

curvature radius

$$R = \sqrt{(x_1 - x_{10})^2 + (x_2 - x_{20})^2} = \frac{v_T}{\omega_B} = \frac{\gamma\,m\,v_T}{|q|\,B} = \frac{p_T}{|q|\,B}$$

$$p_T = 0.3 \times B \times R$$




$$\sin\theta = \frac{p_T}{p}$$

$$\cos\lambda = \frac{p_T}{p} = \pi/2 - \theta$$

(dip angle)

using $q = 1.6 \times 10^{-19} C$ $\quad J = \frac{1}{1.6\ 10^{-19}} eV$ $\quad \frac{s}{m} = \frac{1}{c} \cdot 3 \cdot 10^8$

$$p(GeV/c) = 0.3\ B(T)\ R(m); \qquad p = \frac{p_T}{sin\theta}$$

q -> 0.3 |z|

# Magnetic Spectrometers (= mom. measurement)

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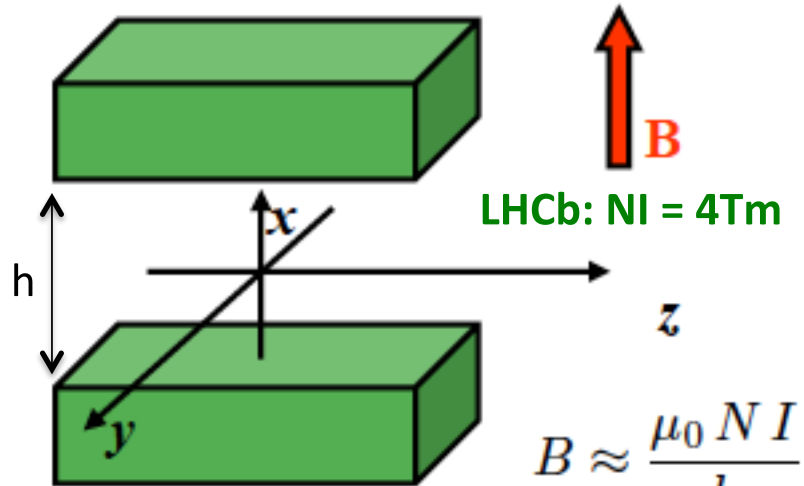
**Almost all HEP experiments at accelerators have a magnetic spectrometer to measure the momentum of charged particles.**

**main configurations:**

- dipole
- solenoid
- toroid

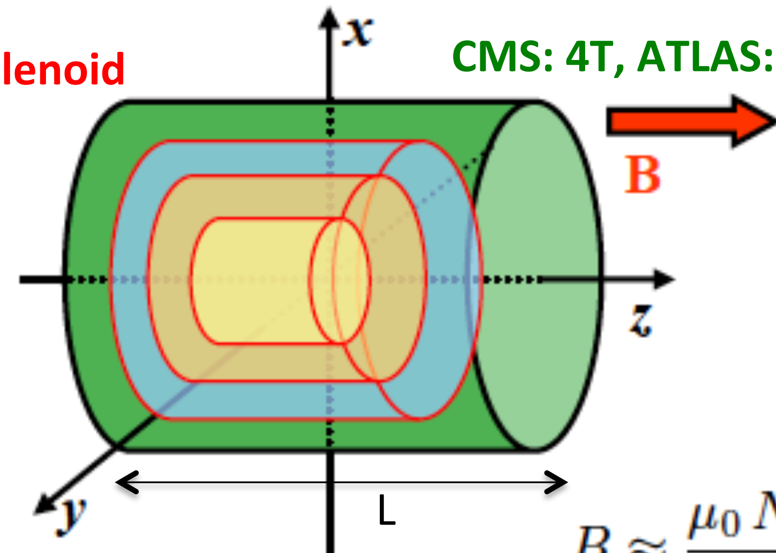
$$N\,I = \frac{1}{\mu_0\,\mu_{\mathrm{Luft}}}\int_{\mathrm{Luft}} \vec{B}\,d\vec{l} + \frac{1}{\mu_0\,\mu_{\mathrm{Fe}}}\int_{\mathrm{Fe}} \vec{B}\,d\vec{l}$$

**Dipole**




$$B \approx \frac{\mu_0\,N\,I}{h}$$

**Solenoid**




$$B \approx \frac{\mu_0\,N\,I}{L}$$

❑ **rectangular symmetry**

- deflection in $y$ - $z$ plane
- tracking detectors are arranged in parallel planes along z
- measurement of curved trajectories in $y$ - $z$ planes at fixed $z$

❑ **cylindrical symmetry**

- deflection in $x$ –$y$ ( r - ϕ ) plane
- tracking detectors are arranged in cylindrical shells along r
- measurement of curved trajectories in r-ϕ planes at fixed r

# Magnetic Spectrometers (= mom. measurement)

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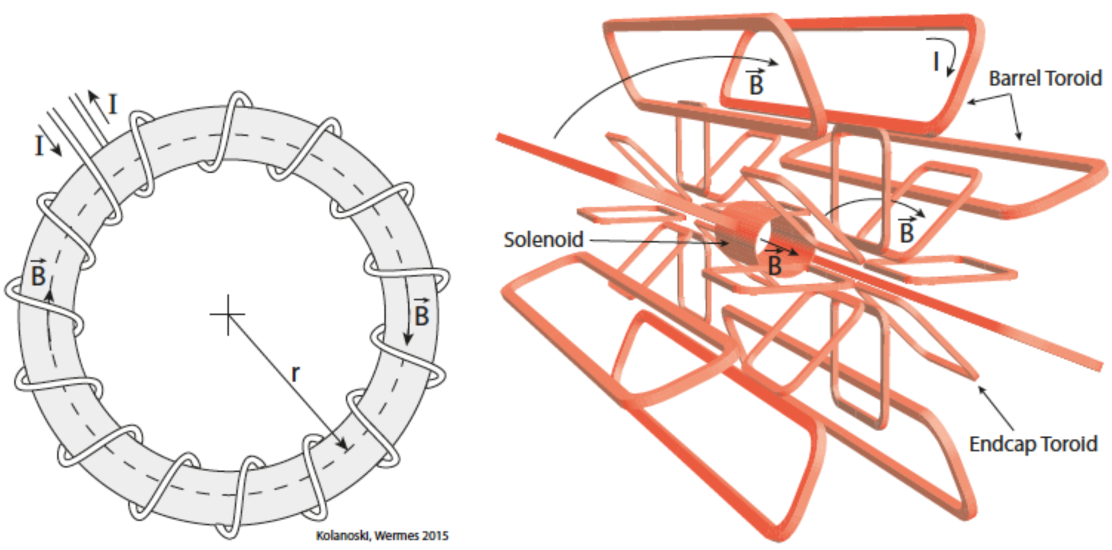
**Almost all HEP experiments at accelerators have a magnetic spectrometer to measure the momentum of charged particles.**

**main configurations:**

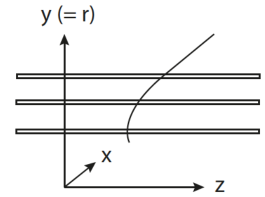
- **dipole**
- **solenoid**
- **toroid**

$$N\,I = \frac{1}{\mu_0\,\mu}\oint \vec{B}\,d\vec{l} = \frac{1}{\mu_0\mu}\,B(r)\,2\pi r$$

## Toroid



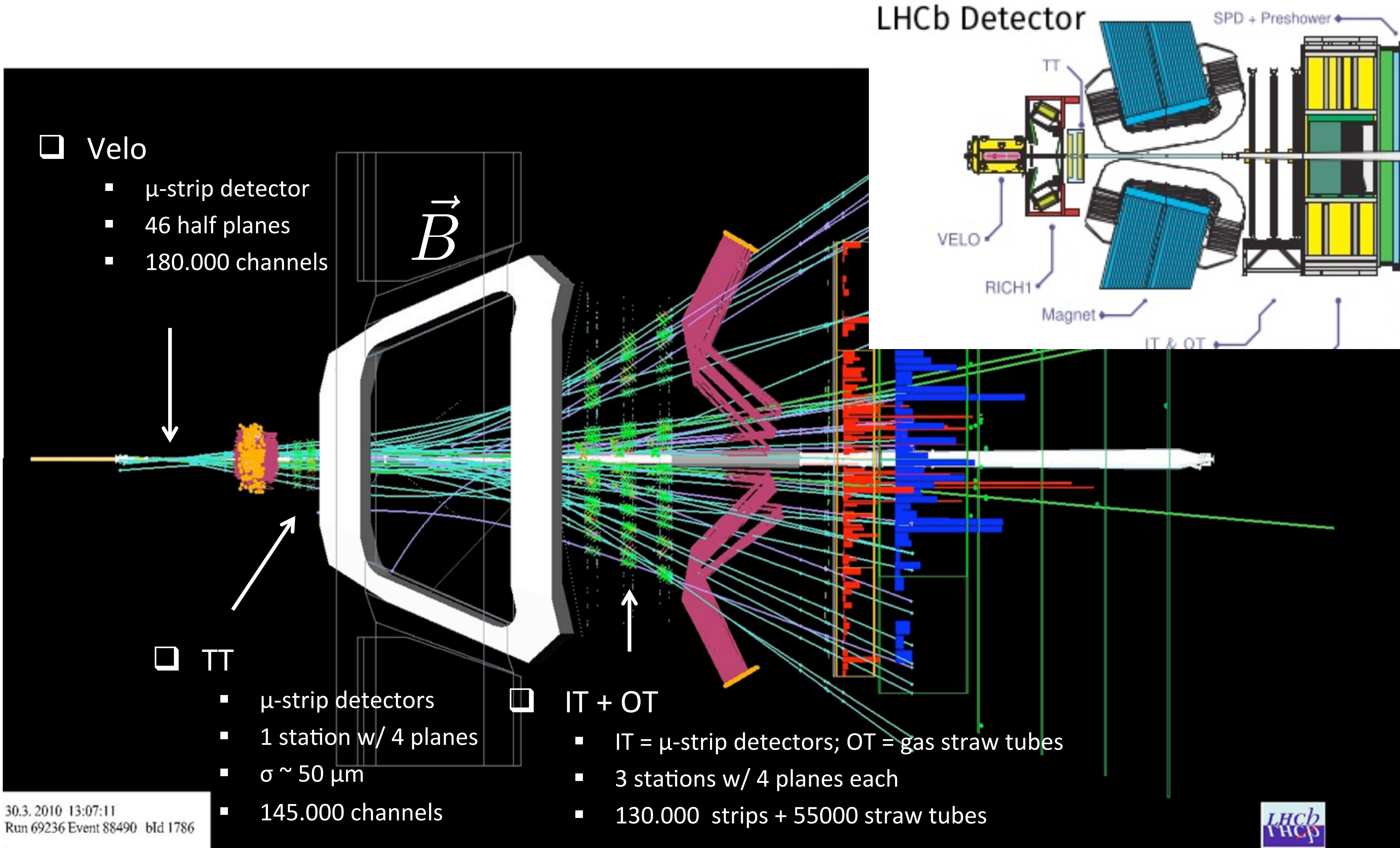

(a) (b)




**ATLAS: 0.5 T**

- **azimuthal symmetry**
  - deflection in (r - z) - plane
  - tracking detectors are (in ATLAS) also arranged in cylindrical shells
  - but measurement of curved trajectories in r-z planes at fixed r

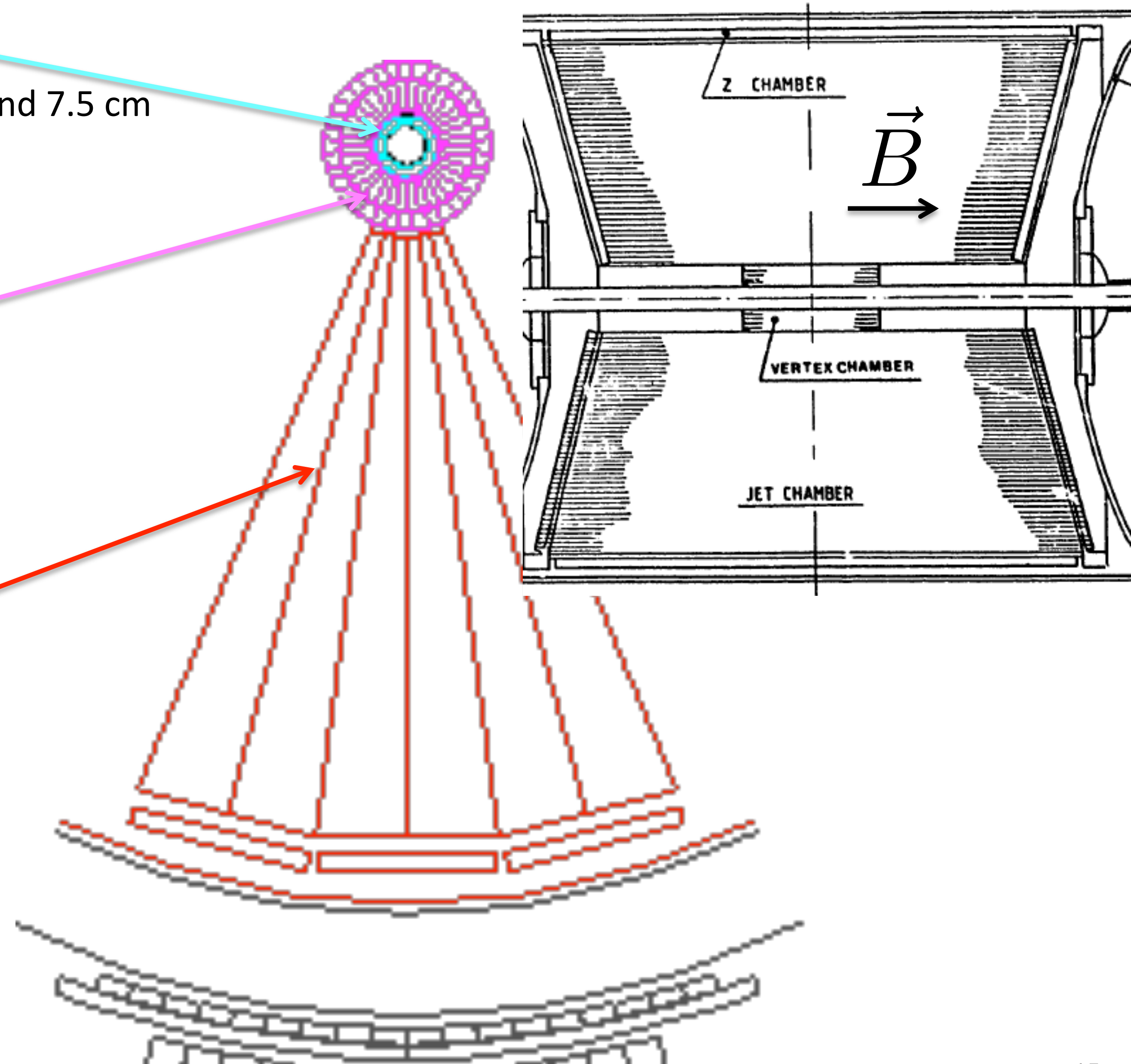
# Example: Tracking in LHCb (Dipole)



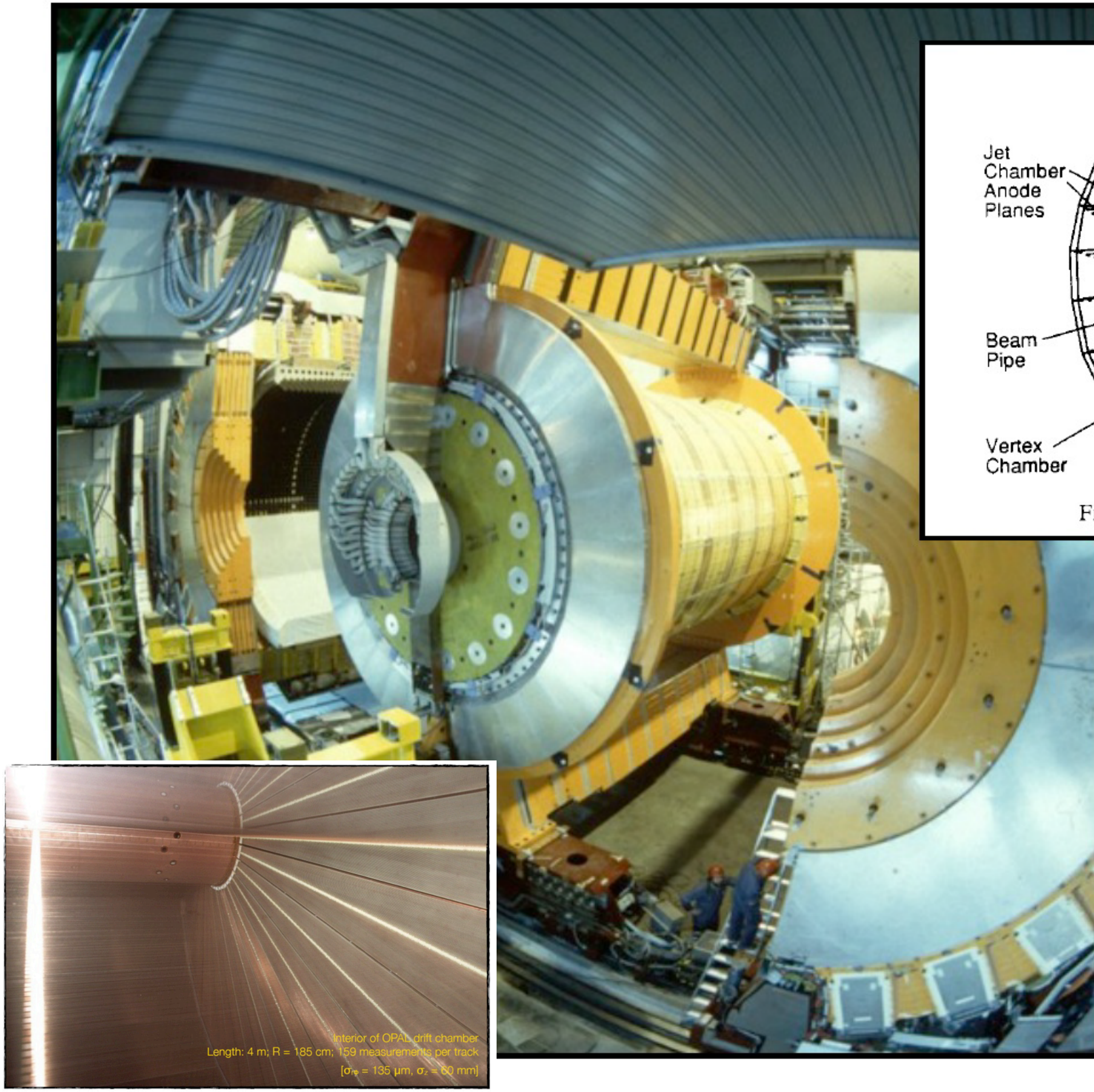

- Velo
  - µ-strip detector
  - 46 half planes
  - 180.000 channels
- TT
  - µ-strip detectors
  - 1 station w/ 4 planes
  - σ ~ 50 µm
  - 145.000 channels
- IT + OT
  - IT = µ-strip detectors; OT = gas straw tubes
  - 3 stations w/ 4 planes each
  - 130.000 strips + 55000 straw tubes

# Example: Tracking in OPAL (LEP) (solenoid)

- **Micro Vertex Detector (Si)**
  - 2 double-barrels @ r = 6.1 and 7.5 cm
  - strip pitch 25 µm
  - $\sigma_{r\phi}$= 5-10 µm $\sigma_z$ = 5-10 µm
- **Vertex Chamber (gas)**
  - 648 wires
  - 36 axial and 36 stereo cells
  - stereo angle 4°
  - $\sigma_{r\phi}$= 50 µm, $\sigma_z$= ~700 µm
- **Jet Chamber (gas)**
  - 7600 wires in 24 sectors
  - 159 points on track
  - $\sigma_{r\phi}$= 120 µm
    $\sigma_z$= 4 cm (per wire)



# OPAL Jet Chamber



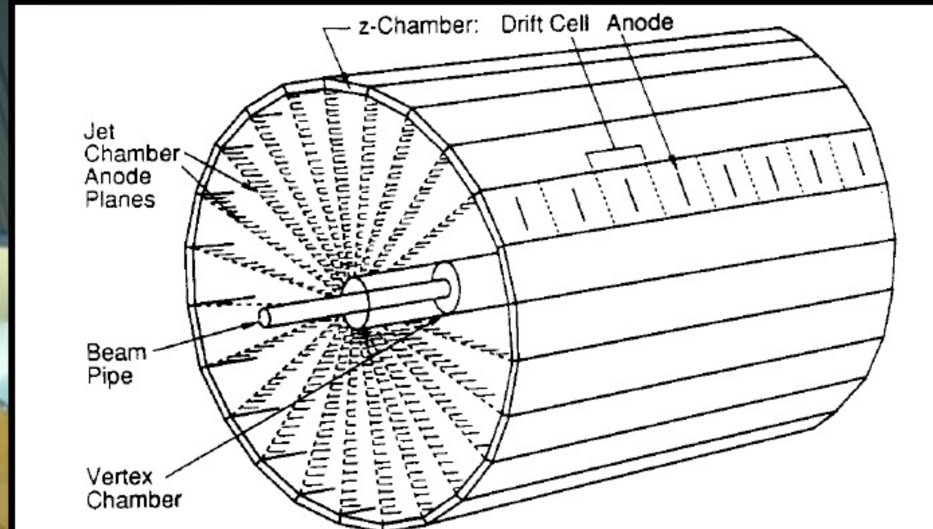
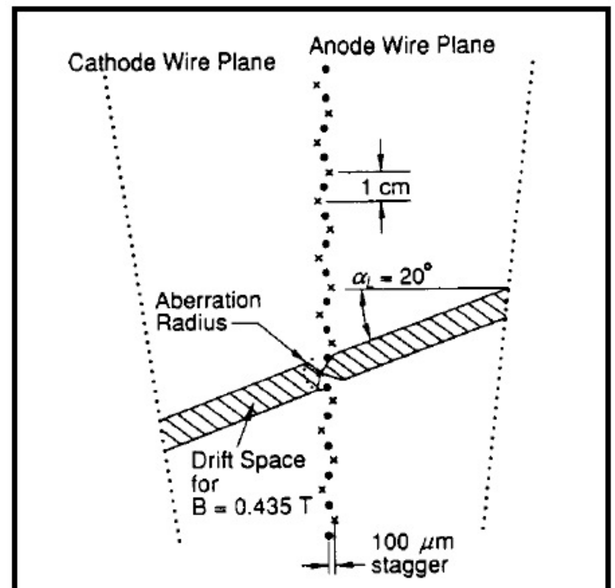
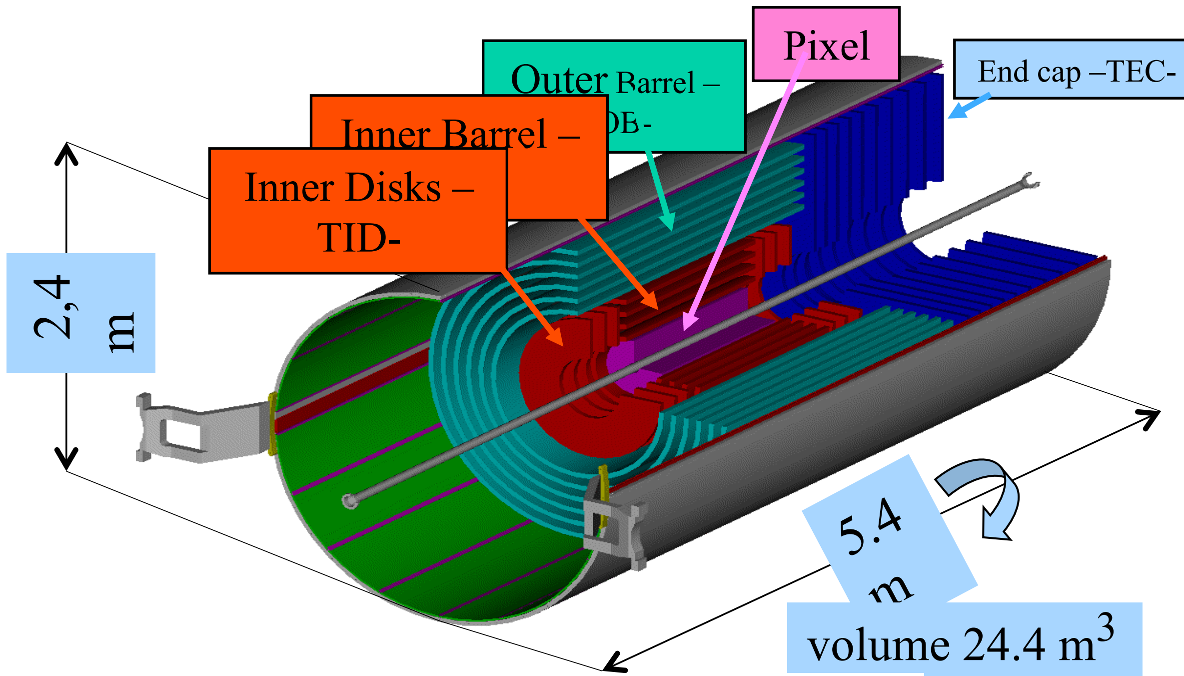



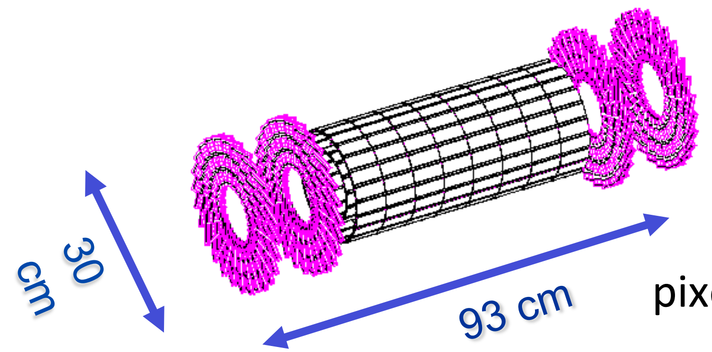

Fig. 1. Schematic view of the OPAL central detector.



# Example: Tracking in CMS (solenoid)

- **Pixel Detector**
  - 2 barrels, 2 disks: $40\times10^6$ pixels
  - barrel radii: 4.1, ~10 cm
  - pixel size 100×150 μm
  - $\sigma_{r\phi}$= 10 μm $\sigma_z$ = 30 μm
- **Inner Silicon Strip Tracker**
  - 4 barrels, many disks: $2\times10^6$ strips
  - strip pitch 80,120 μm
  - $\sigma_{r\phi}$= 25 μm $\sigma_z$ = 230 μm
- **Outer Silicon Strip Tracker**
  - 6 barrels, many disks: $7\times10^6$ strips
  - barrel radii: max 120 cm
  - strip pitch 80, 120 μm
  - $\sigma_{r\phi}$= 35 μm $\sigma_z$ = 530 μm



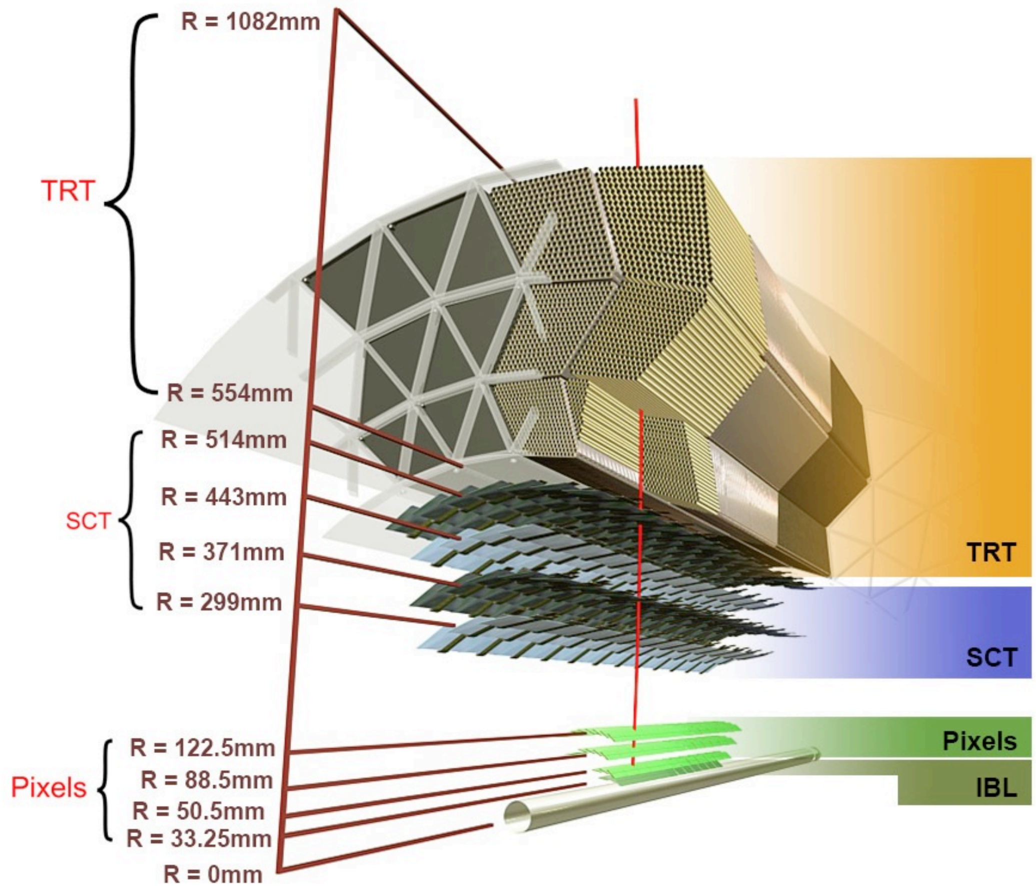



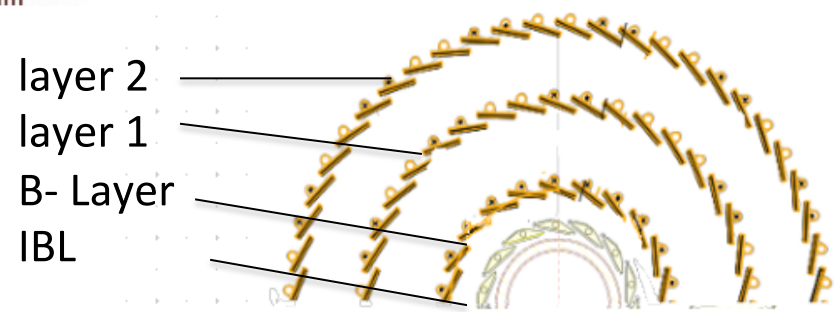

pixel detector

# Example: Tracking in ATLAS (solenoid)



- **Pixel Detector**
  - 4 barrels, 3+3 disks: $80\times10^6$ pixels
  - barrel radii: 3.3, 5.1, 8.9, 12.3 cm
  - pixel size $50\times400$ μm (50x250 μm)
  - $\sigma_{r\phi}$= 8-10 μm   $\sigma_z$ = 100 μm (70 μm)
- **SCT**
  - 4 barrels, disks: $6.3\times10^6$ strips
  - barrel radii:30, 37, 44 ,51 cm
  - strip pitch 80 μm
  - stereo angle ~40 mr
  - $\sigma_{r\phi}$= 16 μm   $\sigma_z$ = 580 μm
- **TRT**
  - barrel: 55 cm < R < 105 cm
  - 36 layers of straw tubes
  - $\sigma_{r\phi}$= 170 μm
  - 400.000 channels



# Example: Tracking in ATLAS μ-Spectrometer (toroid)

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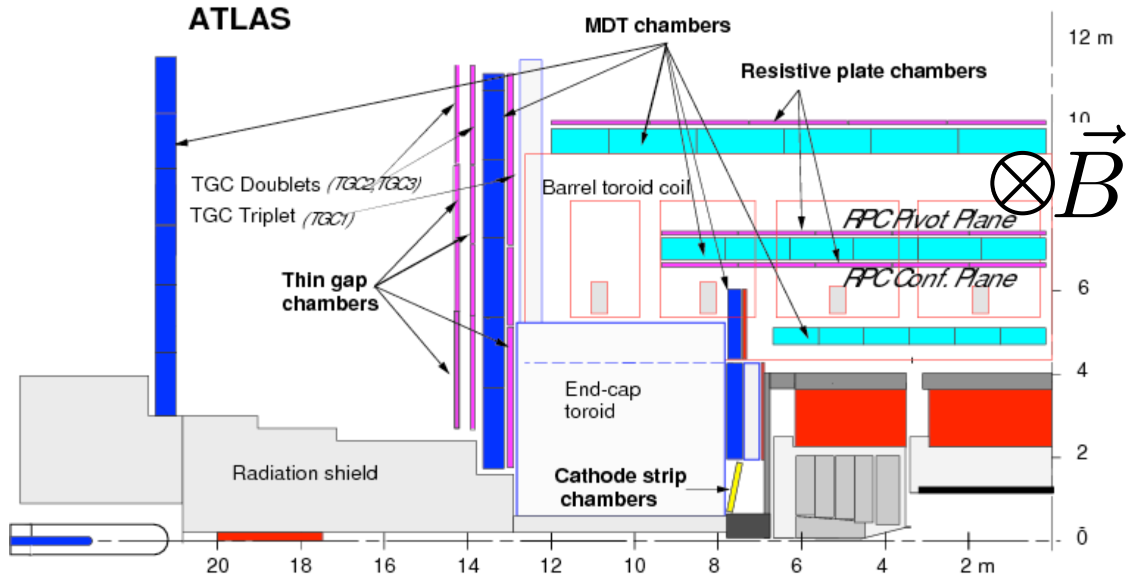
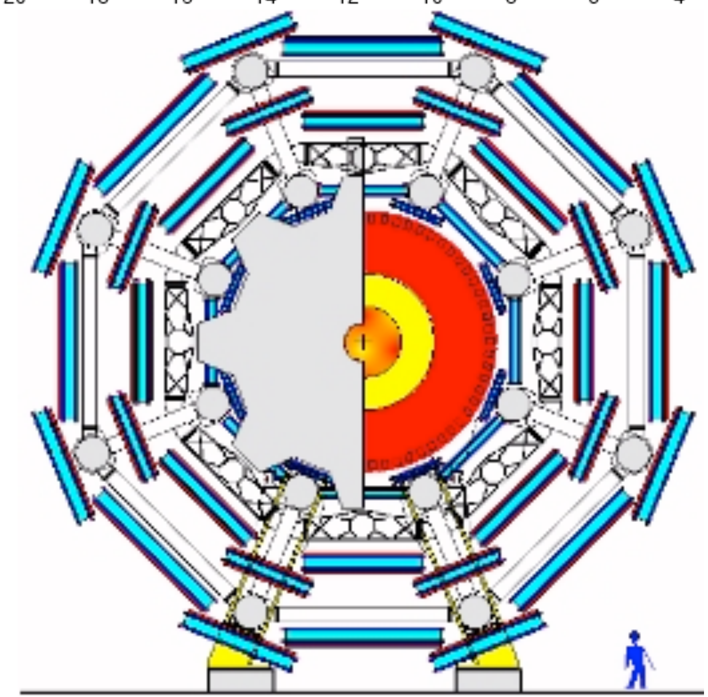
- **MDT = monitored drift tubes**
  - precision tracking
  - $\sigma_z$ = 35 μm (intrinsic)
  - 1088 chambers, 350 000 channels
- **CSC = cathode strip chambers**
  - precision tracking
  - $\sigma_r$ = 40 μm, $\sigma_\varphi$ = 5 mm (intr.)
  - 32 chambers, 31 000 channels

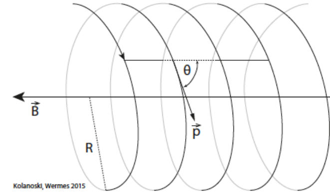


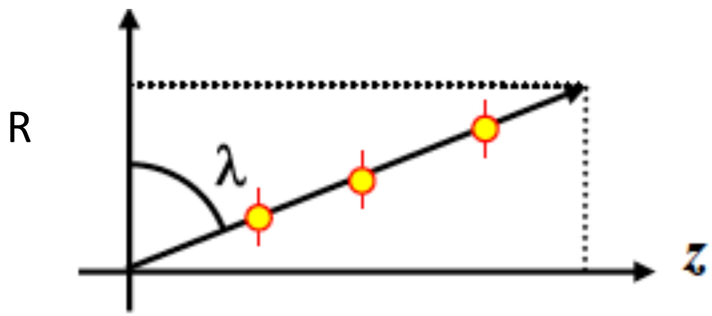
- **RPC = resistive plate chambers**
  - triggering
  - 600 chambers, 370 000 channels
  - $\sigma_z$ = 10 mm, $\sigma_\phi$ = 10 mm
- **TGC = Thin Gap Chambers**
  - triggering
  - 3600 chambers, 320 000 channels
  - $\sigma_r$ = 2-6 mm, $\sigma_z$ = 3-7 mm

(see lecture 2)

# Trajectories




- straight in space

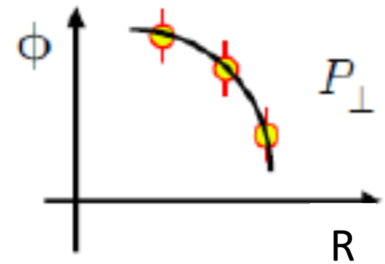


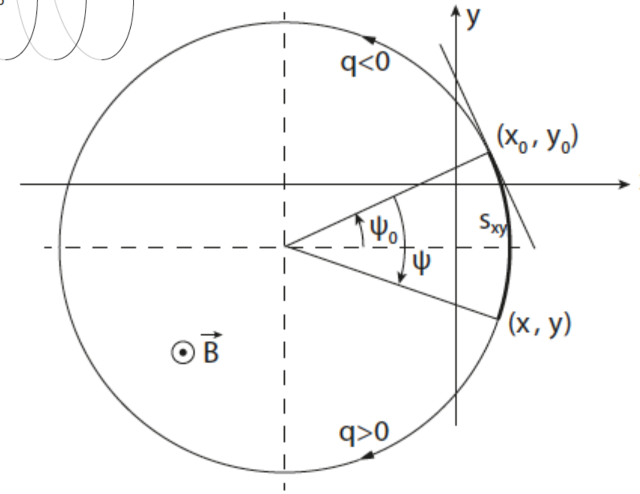

$$y = t_{yx}x + y_0$$

$$z = t_{zx}x + z_0$$

4 parameters

- helix




$$x = x_0 + R\left(\cos(\psi_0 - \eta\psi) - \cos\psi_0\right)$$

$$y = y_0 + R\left(\sin(\psi_0 - \eta\psi) - \sin\psi_0\right)$$

$$z = z_0 + \frac{\psi R}{\tan\theta}.$$

6->5 parameters
w/ $d_0^2 = x_0^2 + y_0^2$

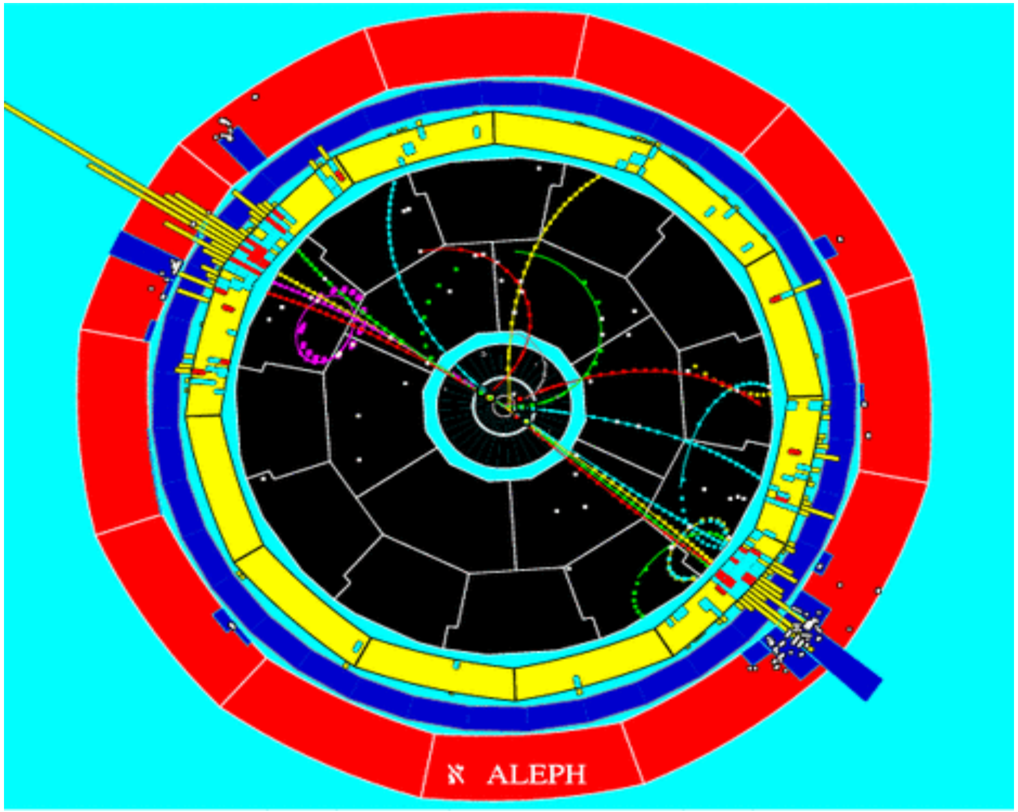
The representation of the circular projection can be series expanded (large R) => parabolic eq.

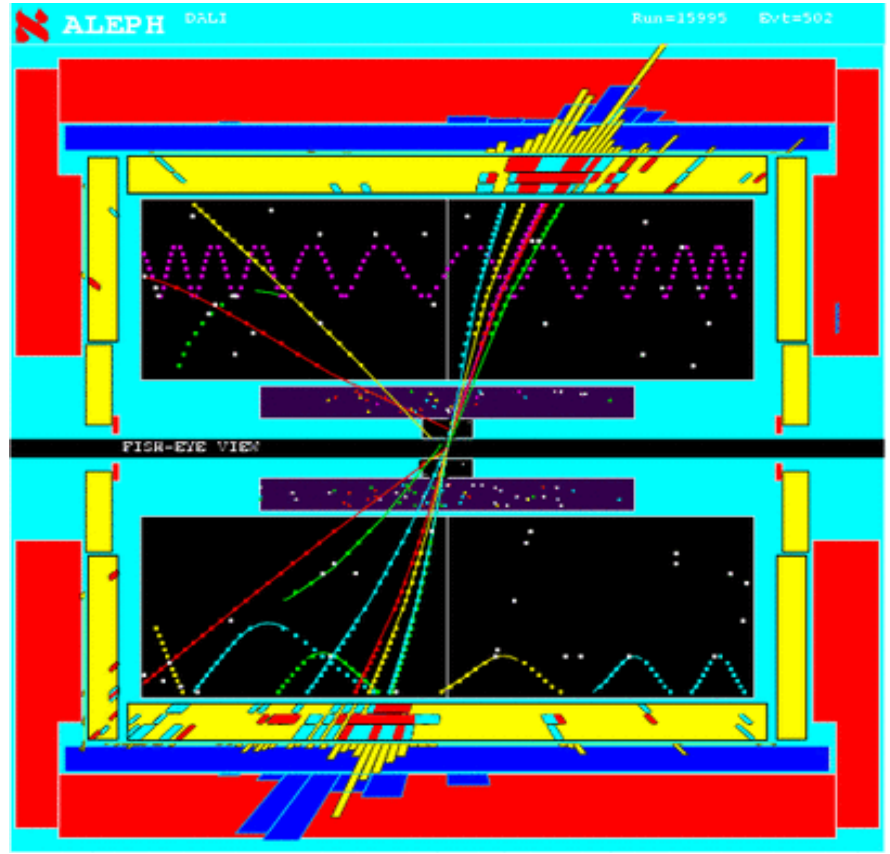
$$y = y_0 + \sqrt{R^2 - (x - x_0)^2}$$

$$y = y_0 + R\left(1 - \frac{(x-x_0)^2}{2R^2} + \dots\right) = \left(y_0 + R - \frac{x_0}{2R^2}\right) + \frac{x_0}{R}x - \frac{1}{2R}x^2 + \dots$$

$$= a + bx + \frac{1}{2}cx^2 \approx a + bx \text{ (for very large R)}$$

# The Helix … seen in an experiment




For small momenta y is a periodic function of z
For large momenta we have a straight line as a function of z

# What we need to do …

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- **Once we have measured the transverse momentum and the dip angle the total momentum is**

$$P = \frac{P_\perp}{\cos\lambda} = \frac{0.3BR}{\cos\lambda}$$

- **The error on the momentum is given by the measurement errors on the curvature radius R and the dip angle λ**

$$\frac{\partial P}{\partial R} = \frac{P_\perp}{R}$$

$$\frac{\partial P}{\partial \lambda} = -P_\perp \tan\lambda$$

$$\left(\frac{\Delta P}{P}\right)^2 = \left(\frac{\Delta R}{R}\right)^2 + (\tan\lambda\Delta\lambda)^2$$

relative error (%)

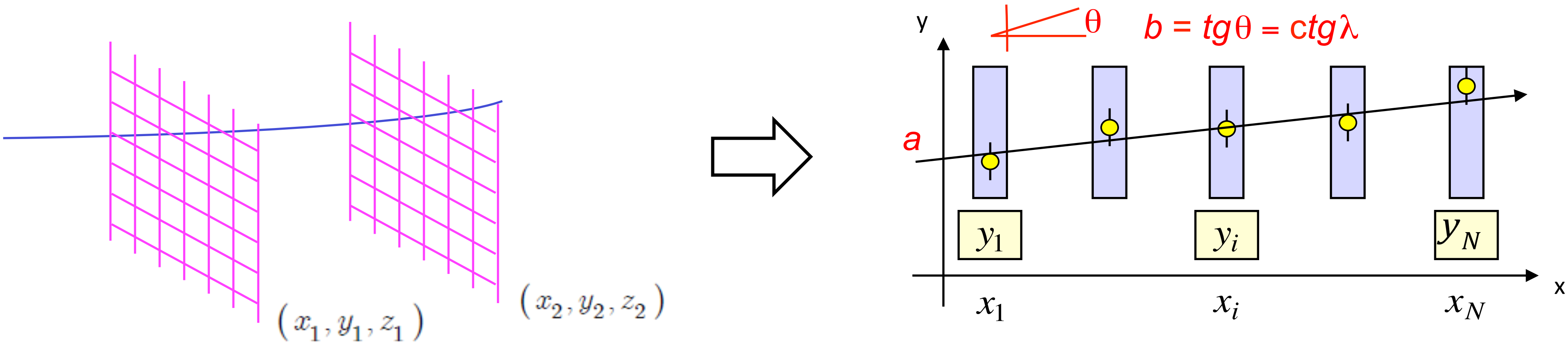
- **We need to study (for solenoid magnets)**
  - the error on the radius measured in the bending plane r - ϕ
  - the error on the dip angle in the r - *z* plane

- **… and also**
  - The contribution of multiple scattering to the momentum resolution

- **Comment:**
  - in a hadron collider like LHC the main emphasis is on transverse momentum measurement
  - elementary processes take place among partons that are not at rest in the laboratory frame
  - → use momentum conservation only in the transverse plane

> 25 mins?

# Using a track model …




N measurements

$$S = \sum_{i=1}^{N} \sum_{j=1}^{N} (\xi_i^{meas} - \xi_i^{fit})\ V_{y,ij}^{-1}\ (\xi_j^{meas} - \xi_j^{fit}) \quad = \sum_{i=1}^{N} \frac{(\xi_i^{\text{mess}} - \xi_i^{fit}(\theta))^2}{\sigma_i^2}$$

covariance matrix

if V is diagonal

# in general: track fit in matrix formalism (x-y space)

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- **f be a linear function of the parameters $\theta_i$:** $f(x|\theta) = \theta_1 f_1(x) + \ldots + \theta_m f_m(x) = \sum_{j=1}^{m} \theta_j f_j(x)$
  (i.e. true for both discussed cases)
- **then the expectation values for the measurement points at positions $x_i$ are:**

$$\eta_i = \theta_1 f_1(x_i) + \ldots + \theta_m f_m(x_i) = \sum_{j=1}^{m} \theta_j f_j(x_i) = \sum_{j=1}^{m} H_{ij} \theta_j$$

$H_{ij}$ → n x m matrix

- **the minimization requirement then reads:**

$$\Rightarrow \quad S = (\vec{y} - H\theta)^T V_y^{-1} (\vec{y} - H\theta) \longrightarrow \text{min}$$

**with solution**

$$\hat{\theta} = \underbrace{\left(H^T V_y^{-1} H\right)^{-1} H^T V_y^{-1}}_{=:\,A} \vec{y} = A\vec{y} \quad \Rightarrow \quad \hat{y} = \sum_{j=1}^{m} \hat{\theta}_j f_j(x)$$

best coord. estimate for given x

**and errors**

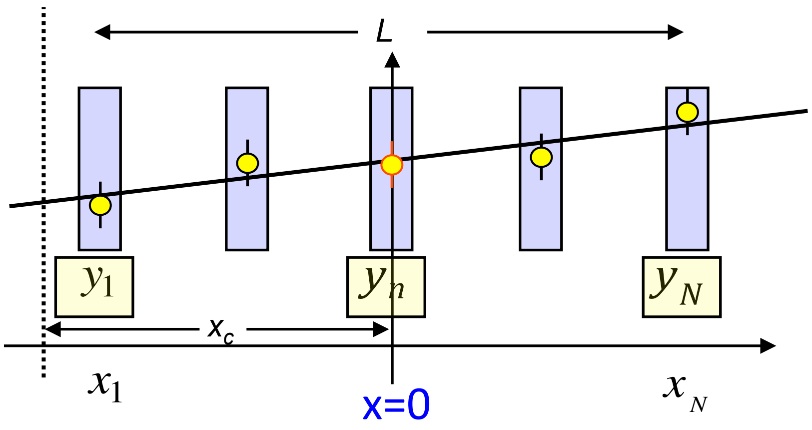
$$V_\theta = A V_y A^T = \left(H^T V_y^{-1} H\right)^{-1}$$

(error propagation = linear trafo of $V_y$)

$$\Rightarrow \quad \sigma_y^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial y}{\partial \theta_i} \frac{\partial y}{\partial \theta_j} V_{\theta,ij} = \sum_{i=1}^{m} \sum_{j=1}^{m} f_i(x) f_j(x) V_{\theta,ij}$$

error of best fit coordinate

# Application to a straight line




N measurements $x_N - x_1 = L\,,$

$$x_i = x_1 + (i-1)\,\frac{L}{N-1} \qquad x_c = \frac{x_1 + x_N}{2} = 0$$

$$f_1(x) = 1,\ f_2(x) = x \qquad \theta_1 = a,\ \theta_2 = b$$

$$\Rightarrow \quad y = f(x|\theta) = a + b\,x$$

fit $\Rightarrow$ $\hat{y} = \sum_{j=1}^{m} \hat{\theta}_j\, f_j(x)$ best estimate of y for a given x

with errors (on parameters)

$$V_\theta = A\,V_y\,A^T = \left(H^T\,V_y^{-1}\,H\right)^{-1} \Rightarrow \begin{cases} \sigma_a^2 = \dfrac{\sigma^2}{N} \\ \sigma_b^2 = \dfrac{\sigma^2}{N}\,\dfrac{12(N-1)}{(N+1)\,L^2} \\ \sigma_{ab} = 0 \end{cases}$$

cf. choice of coord. system

dipole

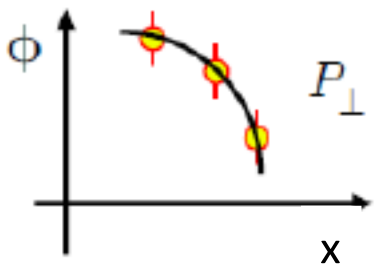
imp.par

... and errors on position estimates

$$\sigma_y^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial y}{\partial \theta_i}\,\frac{\partial y}{\partial \theta_j}\,V_{\theta,ij} \Rightarrow \sigma_y^2 = \sigma_a^2 + x_0^2\,\sigma_b^2 = \frac{\sigma^2}{N} + \frac{\sigma^2}{N}\,\frac{12(N-1)}{(N+1)}\,\frac{x_0^2}{L^2}$$

$x_0$ = a specificly chosen x - value

# Application to a linearized circle




$$y = y_0 + \sqrt{R^2 - (x - x_0)^2}$$

$$\Rightarrow y \approx a \; + \; bx \; + \; \frac{1}{2} c x^2$$

errors

$$V_\theta = A\, V_y\, A^T = \left(H^T\, V_y^{-1}\, H\right)^{-1} \quad \Rightarrow$$

$$\begin{cases} \sigma_a^2 = \sigma^2 \dfrac{3N^2 - 7}{4(N-2)N(N+2)} \\ \sigma_b^2 = \dfrac{\sigma^2}{L^2} \dfrac{12(N-1)}{N(N+1)} \\ \sigma_c^2 = \dfrac{\sigma^2}{L^4} \dfrac{720(N-1)^3}{(N-2)N(N+1)(N+2)} \\ \sigma_{ab} = \sigma_{bc} = 0 \\ \sigma_{ac} = \dfrac{\sigma^2}{L^2} \dfrac{30N}{(N-2)(N+2)} \end{cases}$$

$$\sigma_y^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial y}{\partial \theta_i} \frac{\partial y}{\partial \theta_j} V_{\theta,ij}$$

$$\Rightarrow \quad \sigma_y^2 = \sigma_a^2 + x_0^2\, \sigma_b^2 + \frac{1}{4} x_0^4\, \sigma_c^2 + x_0^2\, \sigma_{ac}$$

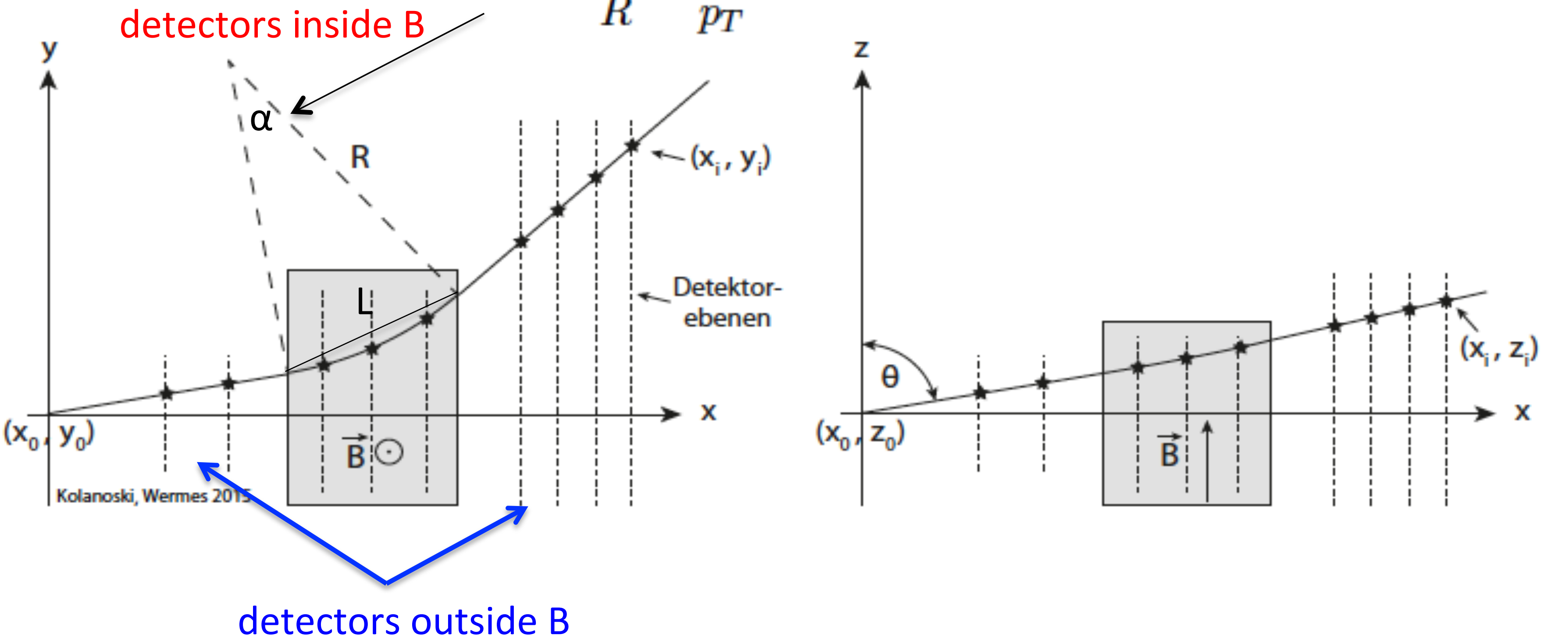
$$= \frac{\sigma^2}{N} \left( 1 + \frac{x_0^2}{L^2} \frac{12(N-1)}{(N+1)} + \frac{x_0^4}{L^4} \frac{180(N-1)^3}{(N-2)(N+1)(N+2)} + \frac{x_0^2}{L^2} \frac{30N^2}{(N-2)(N+2)} \right)$$

$x_0$ = a specificly chosen x - value

# Use cases: tracking with a dipole field

**Dipole spectrometer:** need a minimum of 3 points in bending plane and 2 in plane perpendicular

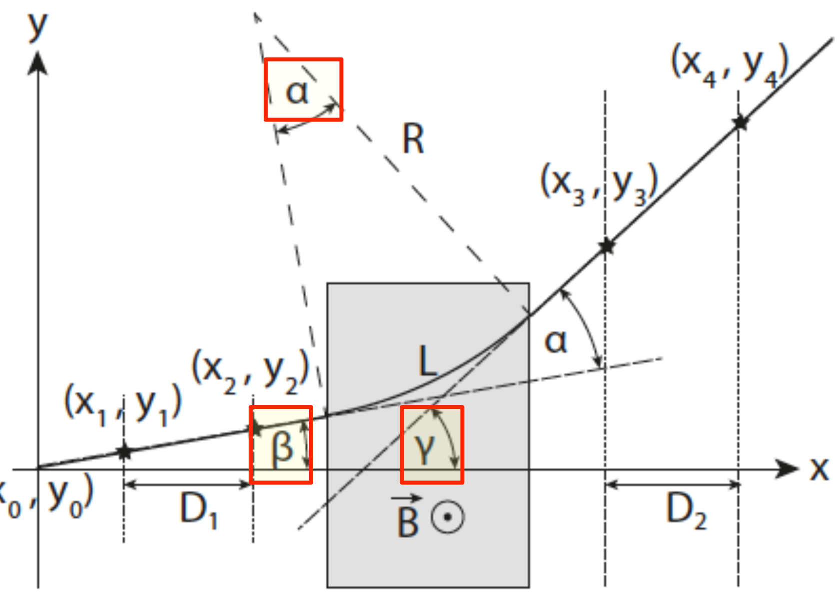
$$\alpha = \frac{L}{R} = \frac{q}{p_T} L B$$




- ❑ two straight line fits
- ❑ get bending α from difference of the slopes before and after magnet
- ❑ track model can use that straight lines should be tangential to the circle section in B
- ❑ measurements inside B are not mandatory

# Use cases: tracking with a dipole field






Kolanoski, Wermes 2015

$$\alpha = \frac{L}{R} = \frac{q}{p_T} L B \quad \propto \int_L B dl$$

bending power

$$\alpha = \gamma - \beta$$

$$\tan\beta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{D_1}$$

$$\tan\gamma = \frac{y_4 - y_3}{x_4 - x_3} = \frac{y_4 - y_3}{D_2}$$

$$D_1 = D_2 = D$$

(lever arm)

resolution for 2+2 measurements

$$\sigma_{\tan\beta} = \sqrt{2}\,\frac{\sigma_{\text{mess}}}{D_1}$$

$$\sigma_{\tan\gamma} = \sqrt{2}\,\frac{\sigma_{\text{mess}}}{D_2}$$

assume equal
$\sigma = \sigma_{\text{mess}}$

$$\sigma_\alpha = \sqrt{2}\,\sigma_{\text{mess}}\sqrt{\frac{1}{D_1^2} + \frac{1}{D_2^2}} = \frac{2\,\sigma_{\text{mess}}}{D}$$

resolution (for 2 x N measurements)

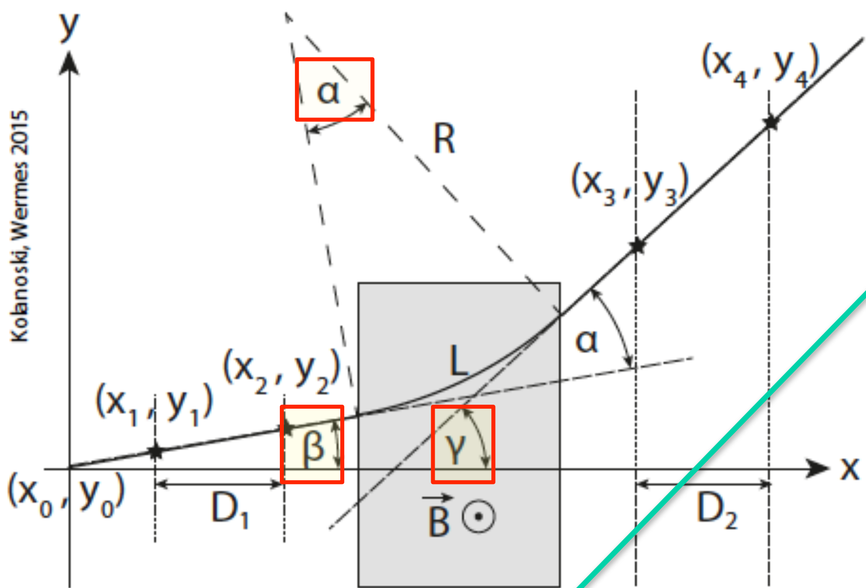
with
(from p. 25)

$$\sigma_{slope} = \frac{\sigma_{\text{mess}}}{D}\sqrt{\frac{12(N-1)}{N(N+1)}}$$

$$\Rightarrow \quad \sigma_\alpha = \sqrt{\frac{24(N-1)}{N(N+1)}}\,\frac{\sigma_{\text{mess}}}{D}$$

$$= 2\,\frac{\sigma_{mess}}{D}\,(for\ N = 2)$$

# Use cases: tracking with a dipole field




$$\alpha = \frac{L}{R} = \frac{q}{p_T} L\, B \quad \propto \int_L B dl$$

bending power

$$\alpha = \gamma - \beta$$

$$\tan\beta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{D_1}$$

$$\tan\gamma = \frac{y_4 - y_3}{x_4 - x_3} = \frac{y_4 - y_3}{D_2}$$

$$D_1 = D_2 = D$$

(lever arm)

$$dp_T = \frac{p_T^2}{qLB} d\alpha \quad \Rightarrow \quad \frac{\sigma_{p_T}}{p_T} = \frac{\sigma_{\mathrm{mess}}\, p_T}{0.3\,|z|\, L\, B\, D} \sqrt{\frac{24(N-1)}{N(N+1)}}$$

momentum resolution with dipole field

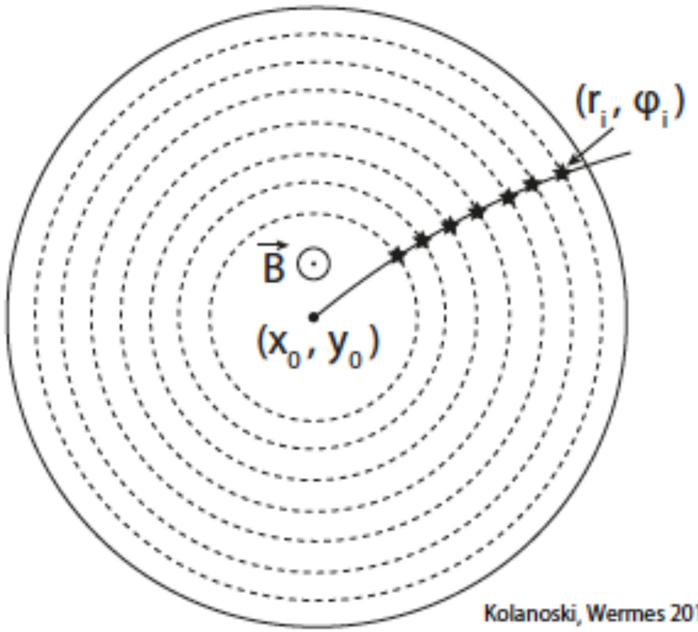
example: N = 5, $\sigma_{mess}$ = 100 µm, D = 1m, B = 1.5 T, L = 2 m

$$\Rightarrow \quad \frac{\sigma_{p_T}}{p_T} = 0.2 \cdot 10^{-3}\, p_T/(\mathrm{GeV/c})$$

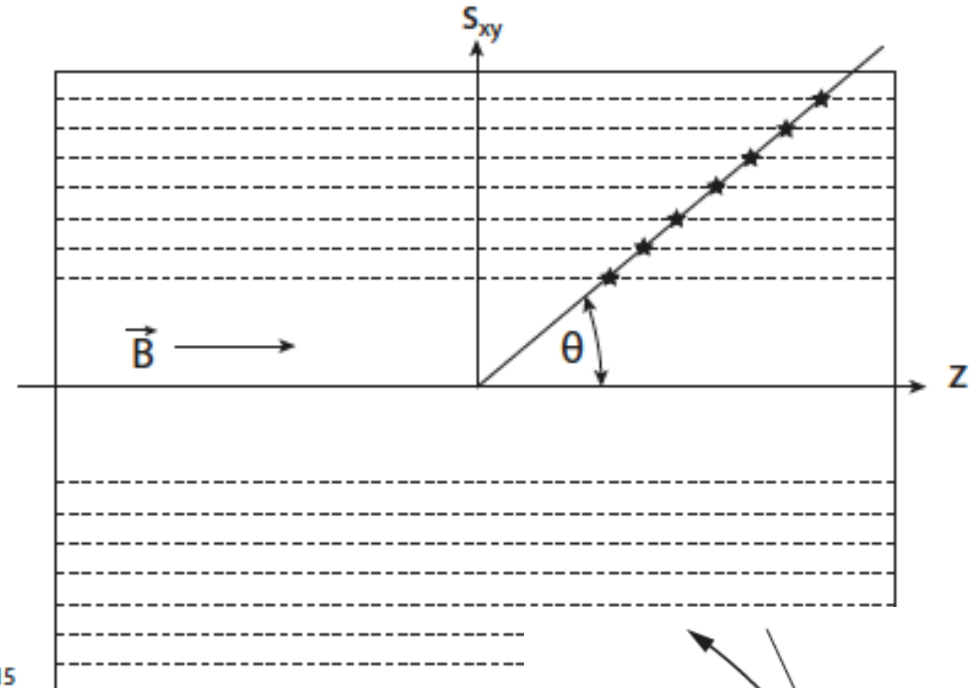
i.e. 0.2% @ 10 GeV/c

# Use cases: solenoid

**Solenoid spectrometer:** can often use beam center as “beam constraint”



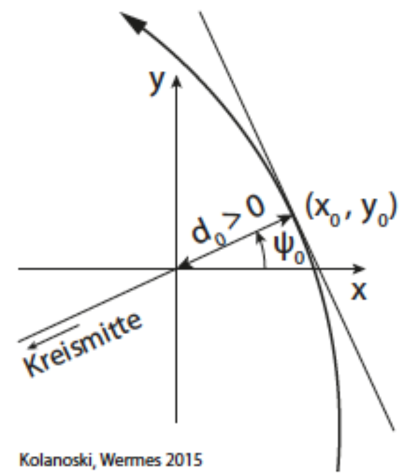

Kolanoski, Wermes 2015




**measure**: $(r_i, \phi_i)$ or $(r_i\ \phi_i, z_i)$
**helix model** with **5 parameters:**

$$\kappa = \frac{\pm 1}{R}, \psi_0, d_0, \theta, z_0$$

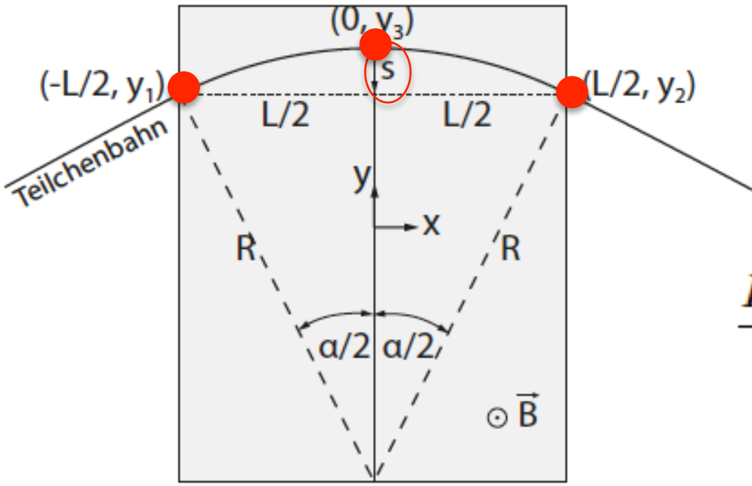
$$x_0 = d_0 \cos \psi_0 \,, \qquad y_0 = d_0 \sin \psi_0$$




Kolanoski, Wermes 2015

# Use cases: solenoid

the sagitta s




$$p_T = |q|\, B\, R = \frac{q\, B}{\kappa}$$

The precision of a measurement of the momentum in a homogeneous B-field is determined by the precision with which the sagitta s can be measured.

$$\left.\begin{aligned} \frac{R-s}{R} &= \cos\frac{\alpha}{2} \approx 1 - \frac{\alpha^2}{8} \\ \frac{L}{2\,R} &= \sin\frac{\alpha}{2} \approx \frac{\alpha}{2} \end{aligned}\right\} s = \frac{R\,\alpha^2}{8} = \frac{1}{8}\frac{L^2}{R} = \frac{1}{8} L^2\, |\kappa|$$

hence $\sigma_\kappa = \frac{8}{L^2}\,\sigma_s$

if we measure y at the three distinct points ● we have (N=3):

$$s = y_3 - \frac{y_1 + y_2}{2} \quad \Rightarrow \quad \sigma_s = \sqrt{\sigma^2_{\rm mess} + \frac{1}{4}\, 2\, \sigma^2_{\rm mess}} = \sqrt{\frac{3}{2}}\,\sigma_{\rm mess} \quad \Rightarrow \quad \sigma_\kappa = \frac{\sqrt{96}}{L^2}\sigma_{mess}$$

for N measurements use linearized circle approximation and matrix formalism (see p. 26)

for N > 10

$$\sigma_\kappa = \frac{\sigma_{\rm mess}}{L^2}\sqrt{\frac{720(N-1)^3}{(N-2)N(N+1)(N+2)}} \approx \frac{\sigma_{\rm mess}}{L^2}\sqrt{\frac{720}{N+4}}$$

# Use cases: solenoid

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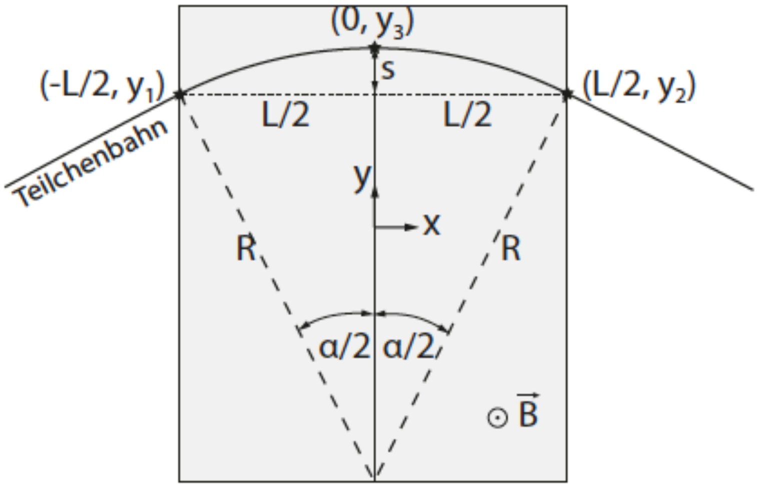
## the sagitta s




$$p_T = |q|\, B\, R = \frac{q\,B}{\kappa}$$

$$\Rightarrow \quad \sigma_{p_T} = \frac{p_T^2}{|q|\,B}\sigma_\kappa \overset{(!)}{=} \frac{p_T^2}{0.3\,|z|\,B}\sigma_\kappa$$

Hence we get for N equidistant points:

$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{\mathrm{mess}} = \frac{p_T}{0.3|z|}\frac{\sigma_{\mathrm{mess}}}{L^2 B}\sqrt{\frac{720}{N+4}}$$

Gluckstern formula

NIM 24 (1963) 381

$$[p_T] = \mathrm{GeV/c}, \; [L] = \mathrm{m}, \; [B] = \mathrm{T}$$

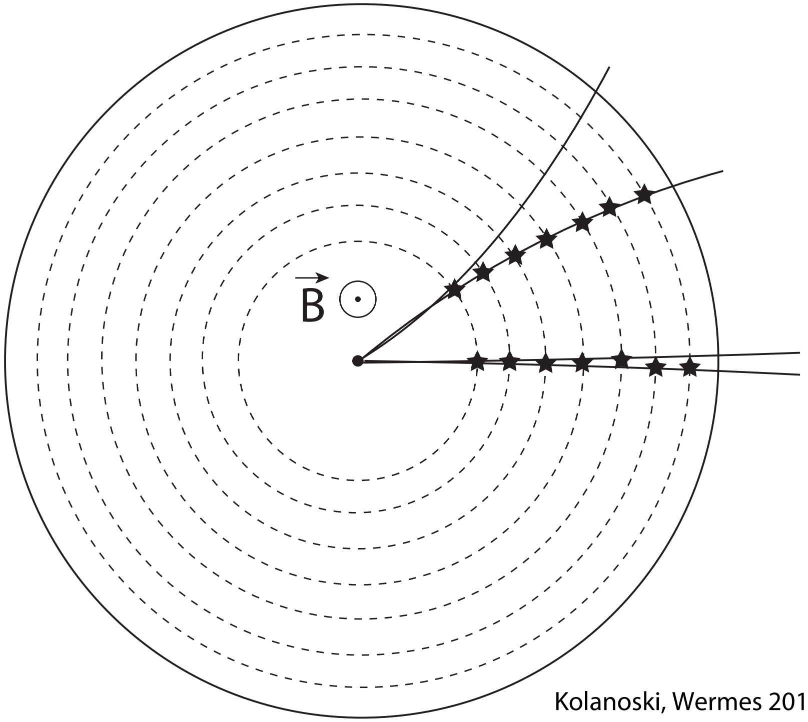
dependencies for optimization

- optimize $\sigma_{mess}$ until other effects dominate (e.g. multiple scattering, next)
- $1/L^2$ : the longer the better
- better with N, but only $1/\sqrt{N}$
- linear better with B-field strength (!) … but more confusion if many tracks
- rel. error ~$p_T$ (=> $p_T$ of very stiff tracks cannot be measured) .

Note: resolution not symmetric and not gaussian in $p_T$, only ~in κ -> generate random no's. in κ

# The sign of charge of a particle






Kolanoski, Wermes 2015

- The largest momenta that can be resolved (with nσ) wrt the sign of charge can be calculated with the condition

$$\frac{1}{R} = |\kappa| = \frac{0.3B}{p_T} > n\sigma_\kappa \Rightarrow |p_t| < \frac{0.3B}{n\sigma_\kappa}$$

- with $\sigma_\kappa \approx \frac{\sigma_{\mathrm{mess}}}{L^2}\sqrt{\frac{720}{N+4}}$

we get

$$p_T < \frac{0.3\,z\,B\,L^2}{n\sigma_{\mathrm{mess}}}\sqrt{\frac{N+4}{720}}$$

example: N = 12, L=1m, B = 1T, $\sigma_{mess}$ = 100 µm, z = 1 => $p_T$ < 220 GeV/c for n = 2

# The influence of Multiple Coulomb Scattering

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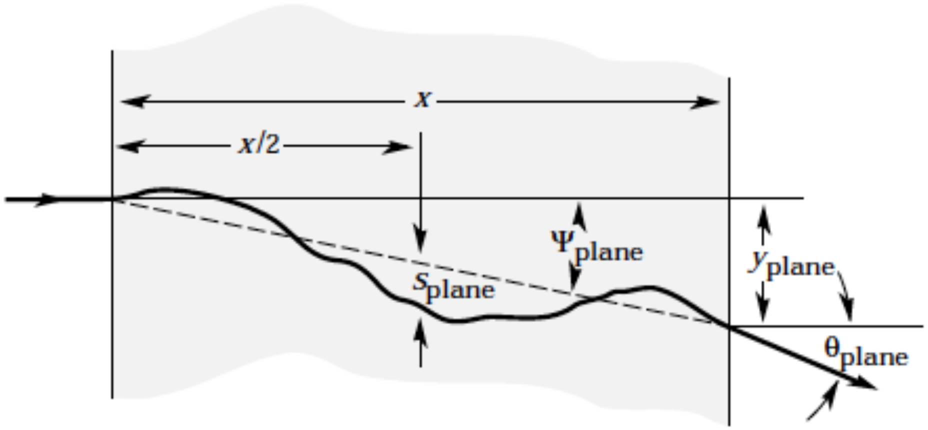
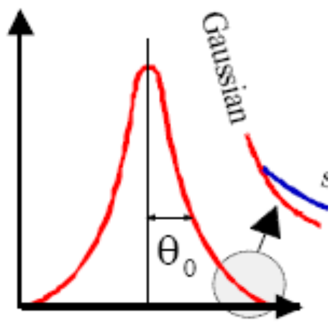



charged particles undergo multiple Coulomb scattering processes when passing through matter

average deviation in a thickness x constitutes additional contribution to sagitta:

$$\langle s_{plan} \rangle = \frac{1}{4\sqrt{3}}\, x\, \theta_0 \quad \approx \sigma_{sagitta}$$

- The scattering angle has a distribution that is almost gaussian
- At large angles deviations from gaussian distributions appear that manifest as a long tail going as $\sin^{-4}\theta/2$ (Moliere theory).
- In "thick" detectors the distribution of the lateral displacement $y_{plane}$ should also be considered.




$$\frac{1}{\sin^4 \frac{\theta_p}{2}}$$

radiation length

$$\theta_0 = \frac{13.6\,\mathrm{MeV/c}}{p\beta} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln \frac{x}{X_0}\right)$$

$$\sigma_\kappa = \frac{8}{L^2}\sigma_s = \frac{8}{L^2}\frac{1}{4\sqrt{3}}\, x\, \theta_0 \overset{x \approx L}{=} \sqrt{\frac{4}{3}}\frac{\theta_0}{L} = \frac{0.0136\,\mathrm{GeV/c}}{p\,\beta\,L} z \sqrt{\frac{L/\sin\theta}{X_0}} \left(\sqrt{1.33} - \sqrt{1.43}\right)$$

N=3    N > 10

# Total momentum resolution

$= \frac{0.0136 \cdot \sqrt{1.43}}{0.3}$

$$\sigma_{p_T} = \frac{p_T^2}{|q|\,B}\sigma_\kappa = \frac{p_T^2}{0.3\,|z|\,B}\sigma_\kappa \quad \Rightarrow$$

in GeV/c, Tesla units

$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{MS} = \frac{0.054}{L\,B\,\beta}\sqrt{\frac{L/\sin\theta}{X_0}}$$

$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{mess} = \frac{p_T}{0.3|z|}\frac{\sigma_{mess}}{L^2 B}\sqrt{\frac{720}{N+4}}$$ for N>10

$$[p_T] = \mathrm{GeV/c},\ [L] = \mathrm{m},\ [B] = \mathrm{T}$$

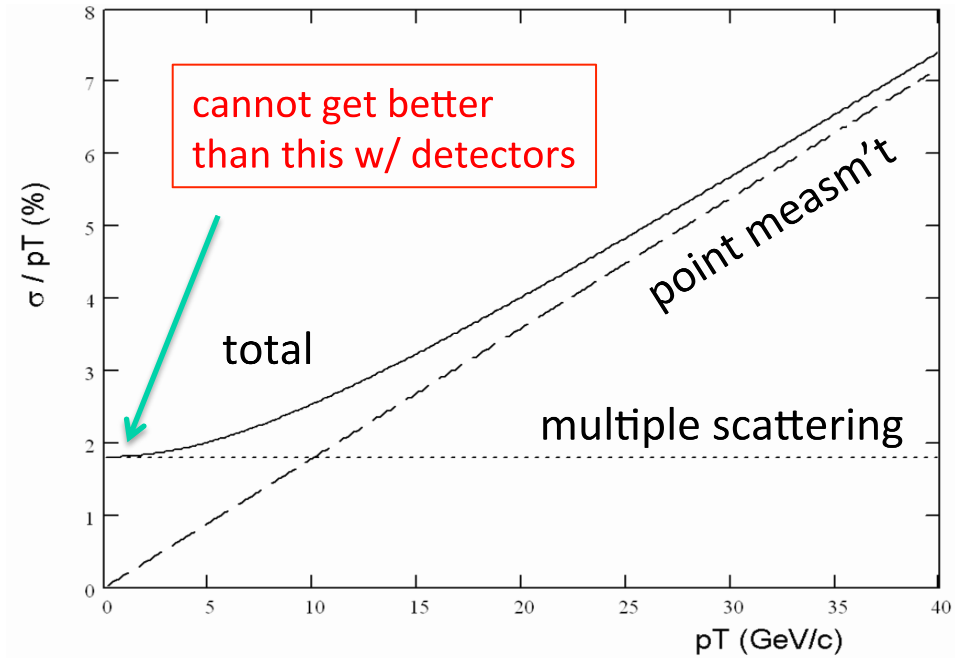
$$\frac{\sigma_{p_T}}{p_T} = \sqrt{\left(\frac{\sigma_{p_T}}{p_T}\right)^2_{mess} + \left(\frac{\sigma_{p_T}}{p_T}\right)^2_{MS}}$$

example: OPAL

L = 1.6 m, B = 0.435 T, N = 159, $\sigma_{mess}$ = 135 µm

$$\frac{\sigma_{p_T}}{p_T} = \sqrt{(0.0015\,p_T)^2 + (0.02)^2}$$

$p_T$ in GeV

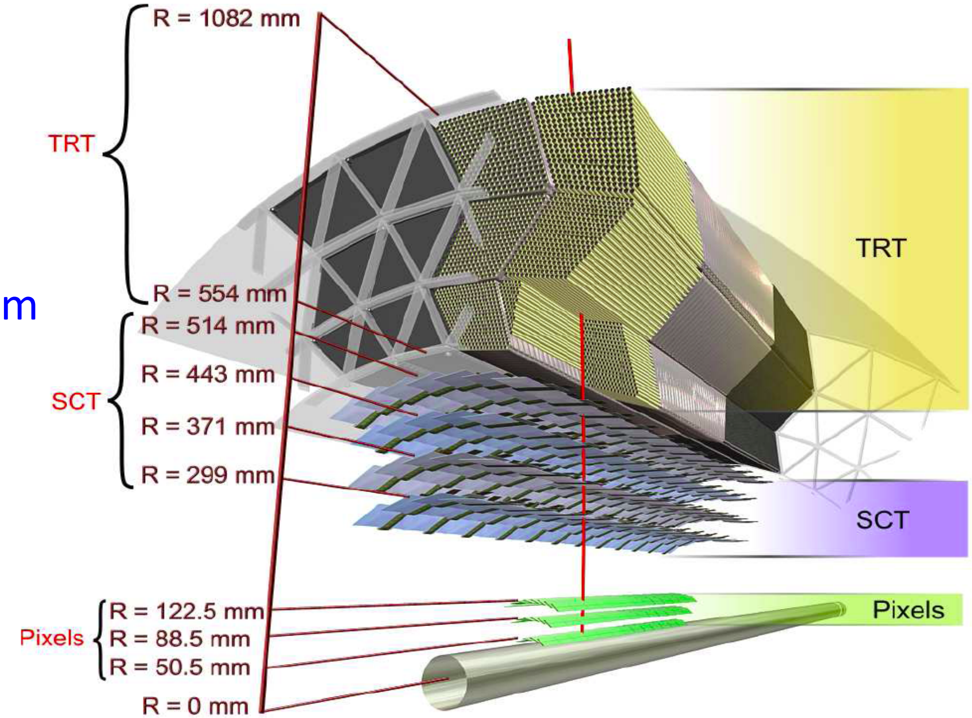


# Example: Momentum Resolution in ATLAS

- We can now give a rough estimate of the momentum resolution of the ATLAS tracking systems (pixels + SCT + TRT)

**Simplifications:**

- assume high momenta (no MS)
- $R_{min}$ = 5.05 cm, $R_{max}$ = 1082cm
- pixels (5cm to 12cm) N=3 (up to 2012), σ = 12µm
- SCT (30cm to 55cm) N=4 layers, σ = 16 µm
- TRT (55cm to 105 cm) N = 36, σ = 170 µm
  → use as a single point with σ = 28 µm
  at R = 80 cm (= $R_{max} \Rightarrow$ L = 75 cm)
- N = 3 + 4 + 1 = 8
- σ = 12, 16, 28 µm → < σ > ~16 µm




$$\sigma_\kappa \approx \frac{\sigma_{\text{mess}}}{L^2} \times 7.56$$

$$\sqrt{\frac{720(N-1)^3}{(N-2)N(N+1)(N+2)}}$$

# ATLAS Momentum Resolution (cont'd)

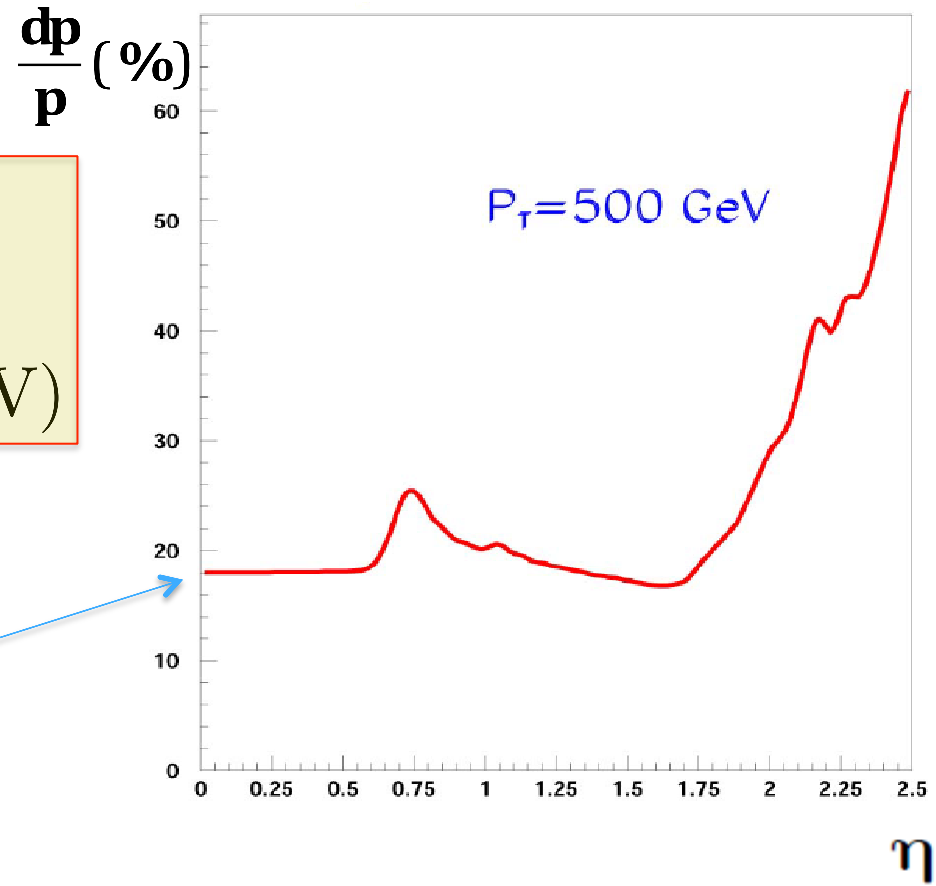
B = 2 T

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_{mess}}{0.3BL^2}\, p_T \times 7.56$$
$$= 3.6 \times 10^{-4}\ p_T/(\text{in GeV})$$

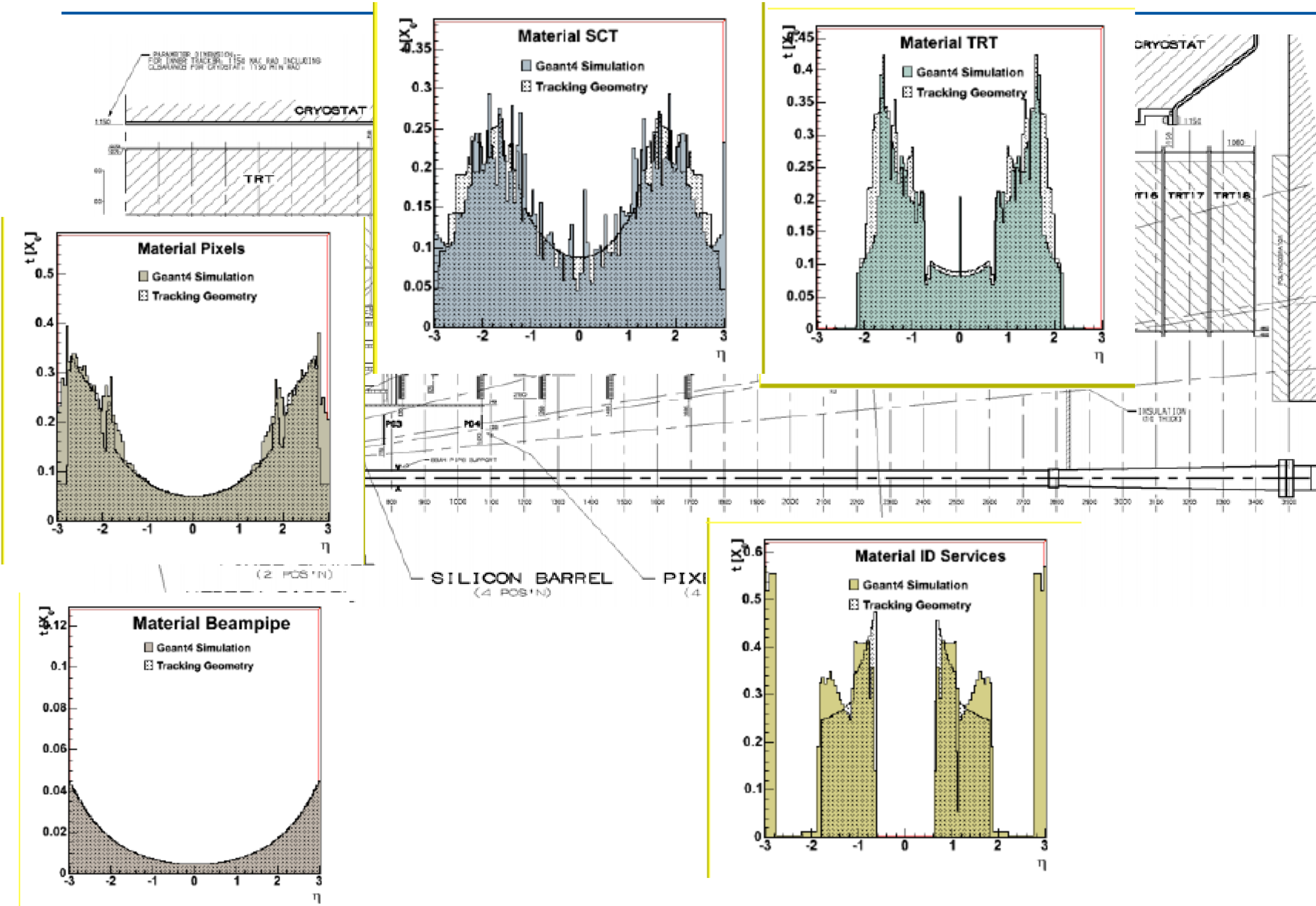
at $p_T$ = 500 GeV

$$\frac{dp}{p} = 18\%$$

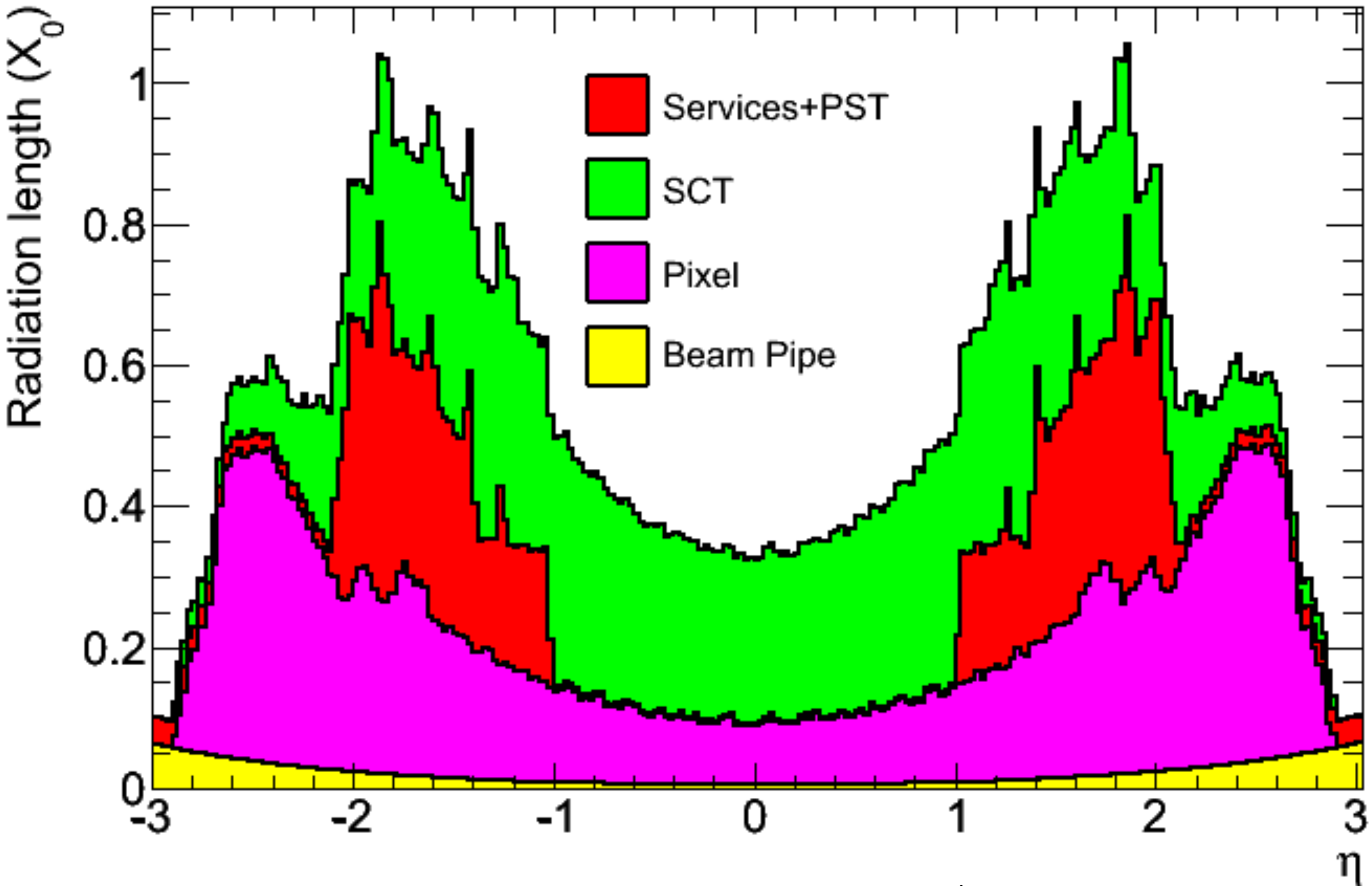
ATLAS Simulation



# … why is the resolution not constant ?



# Material in ATLAS

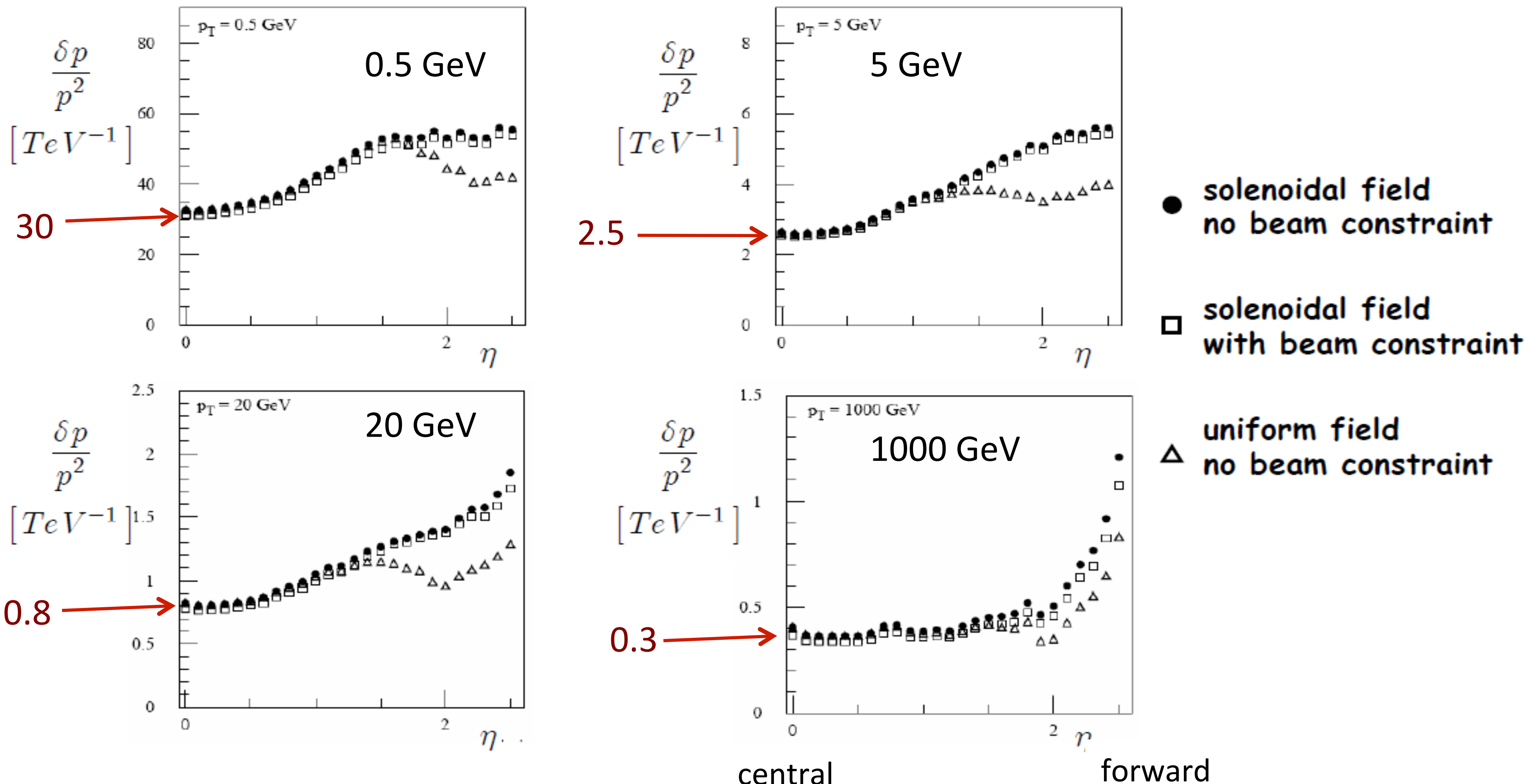



$$\eta = -\ln \tan\theta/2$$

# Momentum Resolution with M.S. in ATLAS

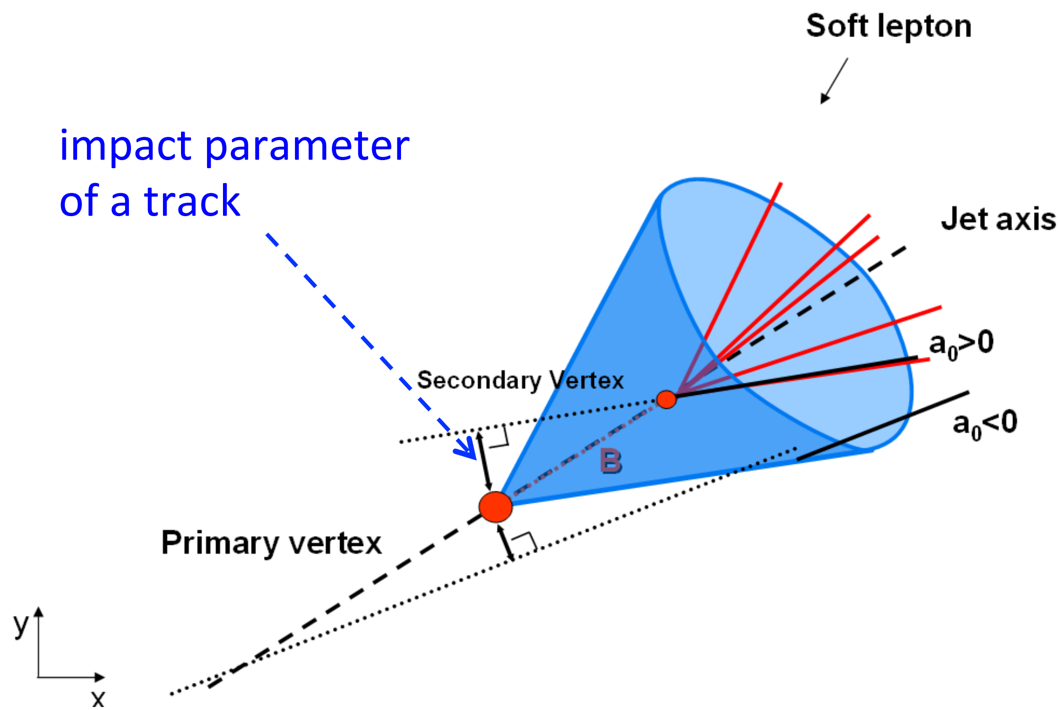
$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{\text{mess}} = \frac{p_T}{0.3|z|}\frac{\sigma_{\text{mess}}}{L^2 B}\sqrt{\frac{720}{N+4}} \qquad \frac{\sigma_{p_T}}{p_T} = \sqrt{\left(\frac{\sigma_{p_T}}{p_T}\right)^2_{\text{mess}} + \left(\frac{\sigma_{p_T}}{p_T}\right)^2_{\text{streu}}}$$

as a function of η



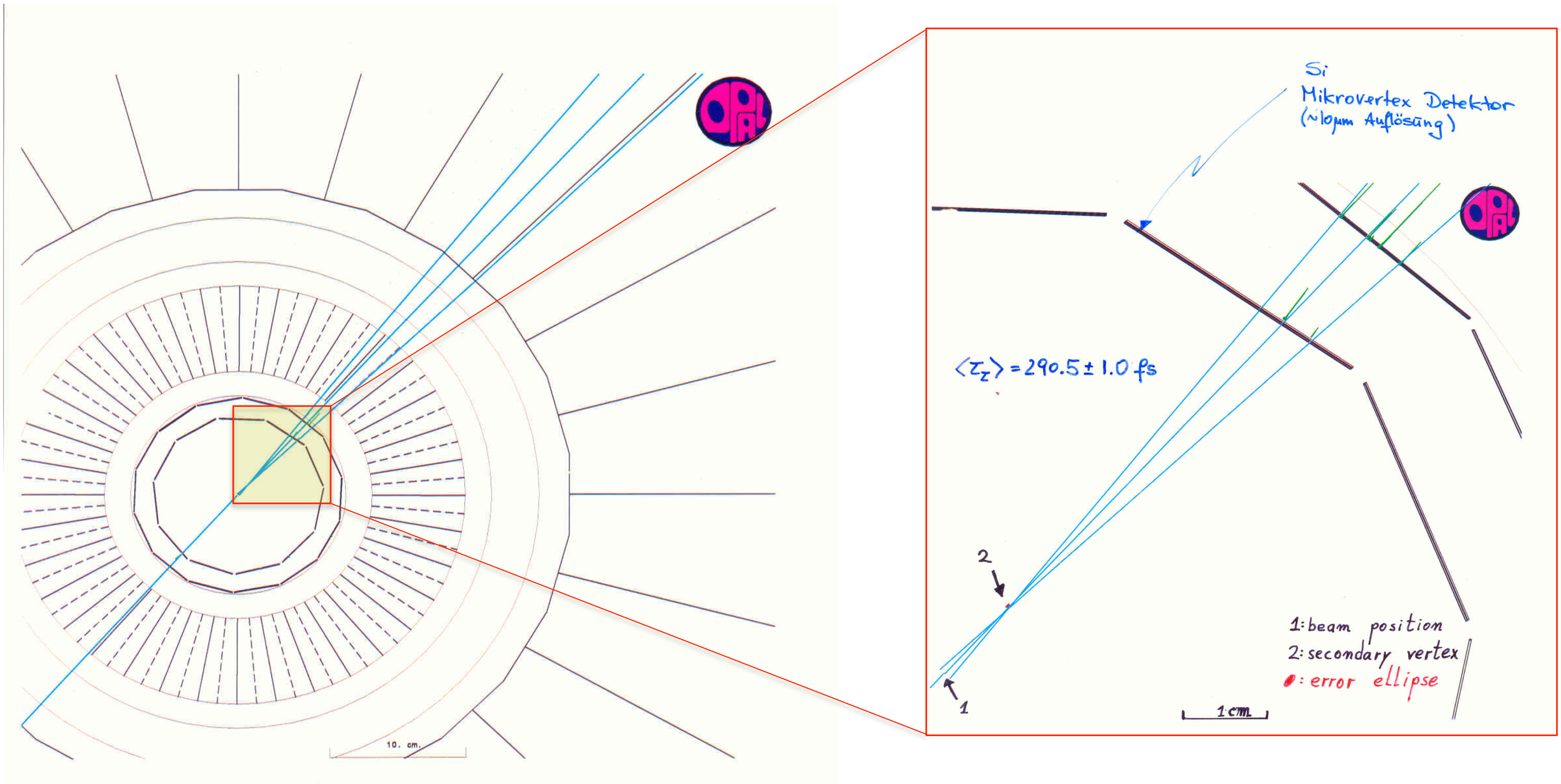
# Vertex detection and measurement

vertex detectors measure the "impact parameter"

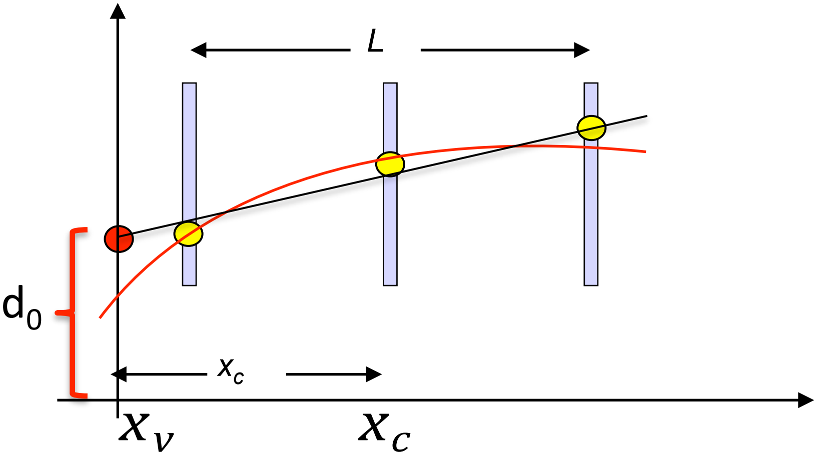



skip to end?

# Extrapolation to the (primary) vertex



# Impact parameter resolution




must extrapolate to vertex either using

$$y = a + bx$$

or $$y = a + bx + \frac{1}{2}cx^2$$ i.e. with curvature

$$d_0 \approx y - y_0$$

extrapolation dist. / meas. distance

$$r := \frac{x_0}{L}$$

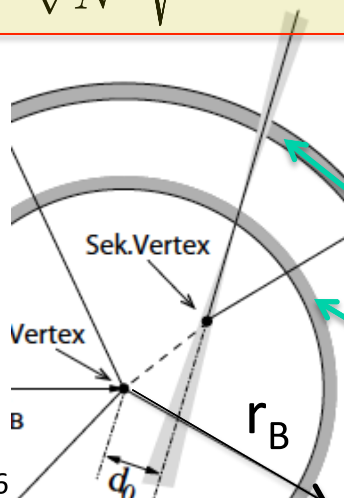
linear

$$\sigma_{d_0} = \sigma_y = \sqrt{\sigma_a^2 + x_0^2\,\sigma_b^2} = \frac{\sigma_{\text{mess}}}{\sqrt{N}}\sqrt{1 + \frac{12(N-1)}{(N+1)}r^2}$$

parabolic

$$\sigma_{d_0} = \frac{\sigma_{\text{mess}}}{\sqrt{N}}\sqrt{1 + r^2\frac{12(N-1)}{(N+1)} + r^4\frac{180(N-1)^3}{(N-2)(N+1)(N+2)} + r^2\frac{30N^2}{(N-2)(N+2)}}$$

multiple scattering:

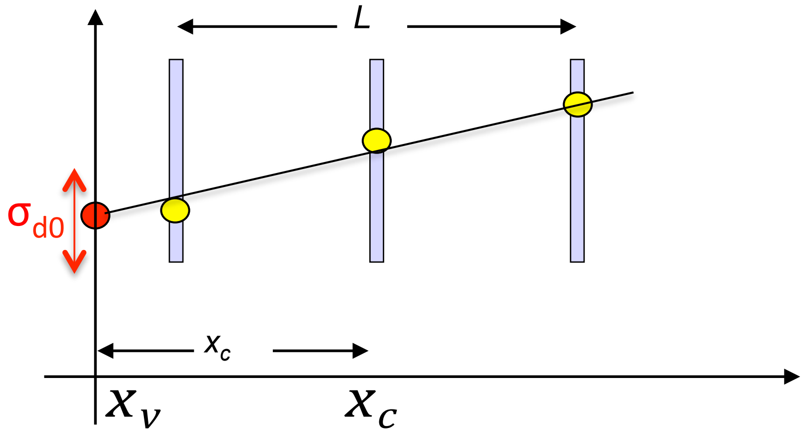



1st detector layer

beam pipe

$$\sigma_{d_0}^{MS} = \theta_0\, r_B$$

# What we conclude …




here only linear case

$$\sigma_{d_0} = \frac{\sigma_{\text{mess}}}{\sqrt{N}} \sqrt{1 + \frac{12(N-1)}{(N+1)} \frac{x^2}{L^2}} \oplus \theta_0 r_B$$

We should have
- small measurement errors $\sigma_{mess}$
- large lever arm *L*
- place first plane as near as possible to the production point: small *x*

Increasing the number of points also improves the resolution on $d_0$ but only as $1/\sqrt{N}$

The technology most often used is
Si - detectors

**PRO** – high resolution $\sigma_{mess} \sim 10\ \mu m$

**CON** - expensive
- small *N*
- small *L*
- small $X_0$ => large $\theta_0$

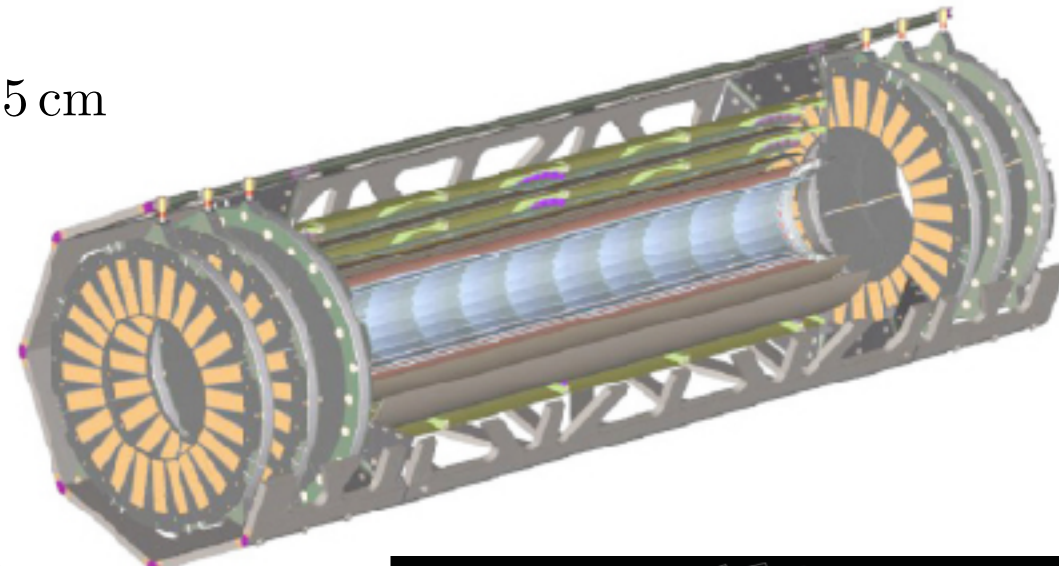
# Example: ATLAS Pixel Detector

$$N = 3,\ \sigma = 10\mu m$$

$$x_1 = 5.05\text{cm},\ \mathrm{x}_2 = 8.85\,\text{cm},\ \mathrm{x}_3 = 12.25\,\text{cm}$$

$$L = 7.3\text{cm},\ r = x_2/L = 1.22$$

$$\sqrt{1 + \frac{12(N-1)}{(N+1)} r^2} = 3.15$$



Impact parameter resolution

$$\sigma_{d_0} = 18.2\,\mu\text{m}$$ (linear, i.e. no field)

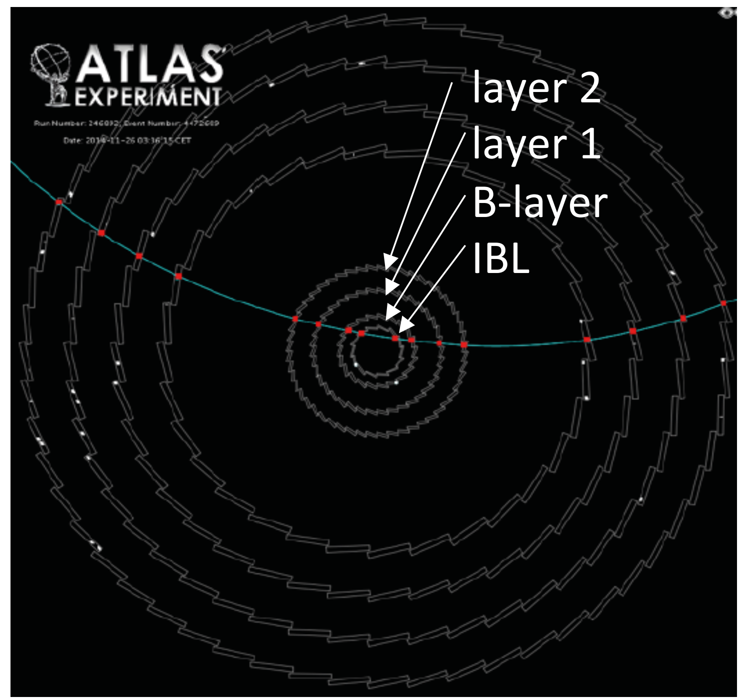
$$\sigma_{d_0} = 45.5\,\mu\text{m}$$ (extrapolation with B-field)

Note

- if curvature is used for extrapolation with N=3 the error on $d_0$ gets larger by a factor ~2.5.
- however, curvature is measured by the entire inner detector => error on $d_0$ similar to linear case

With new IBL (N=4, $x_0$ = 3.55 cm)

$$\sigma_{d_0} = 12.5\,\mu\text{m}$$



# Tracker resolutions with Multiple Scattering

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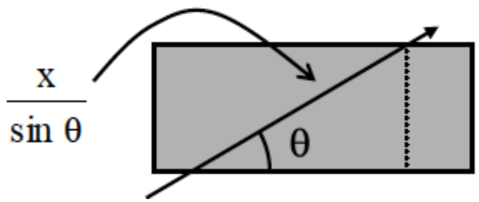
- For low momentum tracks the momentum resolution and the impact parameter resolution are dominated by multiple scattering
- The amount of material actually traversed by the particles depends on the polar angle




- the momentum resolution tends to

$$\frac{\sigma_p}{p^2} \rightarrow k_p \frac{\sqrt{x/X_0}}{p\sqrt{\sin\theta}}$$

- the impact parameter resolution tends to

$$\sigma_{d_0} \rightarrow k_{d_0} \frac{\sqrt{x/X_0}}{p\sqrt{\sin\theta}}$$

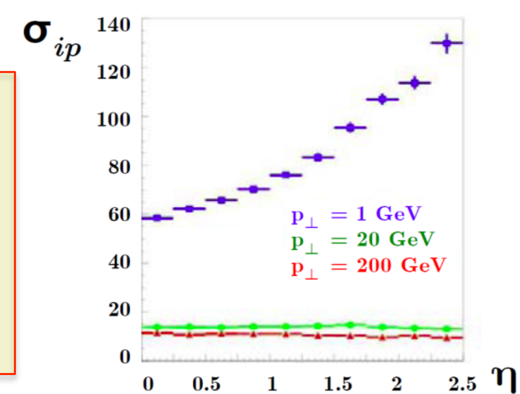
- Since the MS error and the point measurement error are independent of each other, the total error is the sum in quadrature of the 2 terms with and w/o MS
- For the ATLAS detector Monte Carlo studies have shown that the resolutions on momentum and impact parameter can be parametrized as

$$\frac{\sigma_{p_T}}{p_T^2} = 0.00036 \oplus \frac{0.013}{p_T\sqrt{\sin\theta}} (\text{GeV})^{-1}$$

*or*

$$\frac{\sigma_{p_T}}{p_T} = 0.04\%\, p_T \oplus \frac{1.3\%}{\sqrt{\sin\theta}} (\text{GeV})^{-1}$$

$$\sigma_{d_0} = 11\mu m \oplus \frac{73\mu m}{p_T\sqrt{\sin\theta}}$$



# Literature on Tracking

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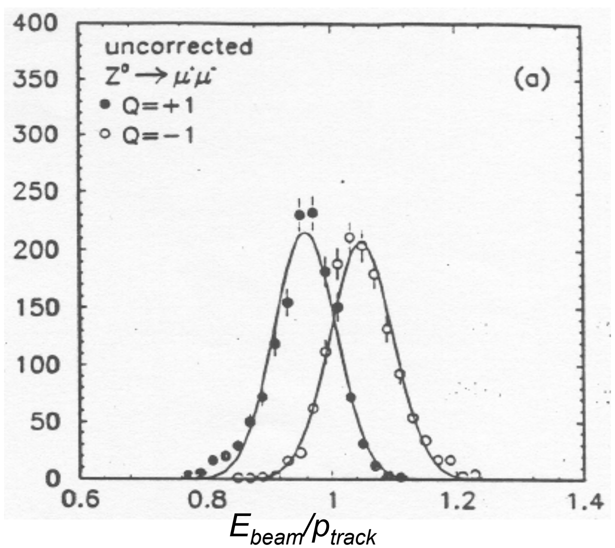
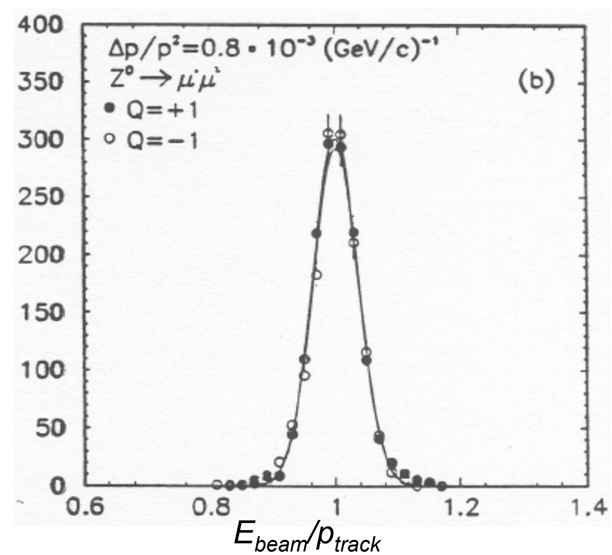
# Backup

# Systematic Effects: Misalignment

- As an example consider a systematic effect as seen in the ALEPH TPC
- The resolution can be studied using muon pairs produced in $e^+ e^-$ annihilation at the $Z^0$ peak
- The muons are produced back to back and have exactly half the c.m. energy
- The plot shows the momentum reconstructed separately for positive and negative muons
- As the plot clearly shows, the error is quite large
- To account for this error $\delta \sim 1\ mm$ needed !
- => Magnetic field distortion !
- A correction procedure is essential !
- The following plot shows the same distribution after proper magnetic distortion corrections are applied



# Kalman Filter

- Applications of Kalman Filter:
  - navigation
  - radar tracking
  - sonar ranging
  - satellite orbit computation
  - stock prize prediction
- It is used in all sort of fields
  - Eagle landed on the moon using KF
  - Gyroscopes in airplanes use KF
- Usually the problem is to estimate a "state" of some sort and its uncertainty
  - location and velocity of airplane
  - track parameters of charged particles in HEP experiments
- However we do not observe the "state" directly.
- We only observe some measurements from sensors which are noisy:
  - radar tracking
  - charged particle tracking detectors
- As an additional complication the state evolves (in time or in propagation) with its own uncertainties: process stochastic noise
  - deviation from trajectory due to random wind (radar)
  - multiple scattering (HEP)
- In case of tracking in HEP instead of time we consider the evolution of the track parameter at the discrete subsequent layers, where the detectors perform the measurement

# Kalman Filter

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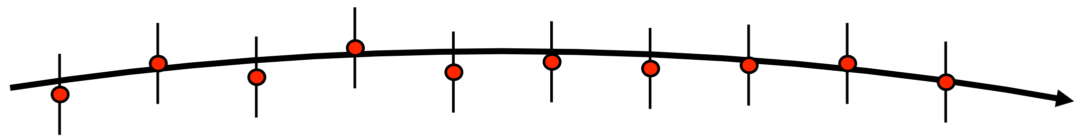
- It is best to start the filter from the end of the track

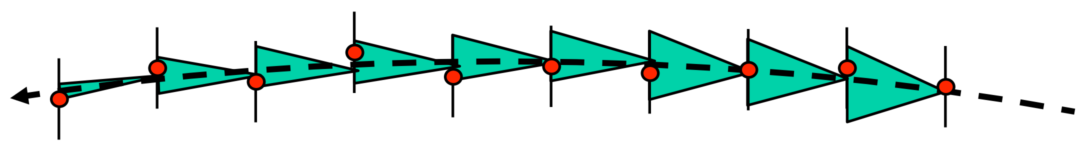
- After all the measurements have been used (filtered) it is possible to build a procedure that
  - uses the (stored) intermediate results of the filter
  - gives the best parameter estimation at any point
- This is the ”smoother” algorithm

production vertex



direction of flight →

production vertex



← direction of filter