Relaxion with multiple axions

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KC, S.H. Im, arXiv:1511.00132 (JHEP 01 (2016) 149)

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Outline

- Relaxion solution to the weak scale hierarchy problem:

  Hierarchical relaxion scales

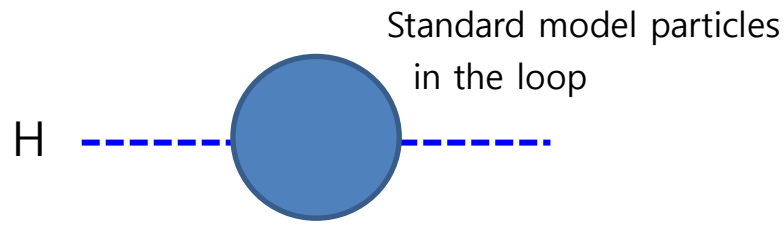
- Generating hierarchical relaxion scales with multiple axions

- UV completed relaxion model with high scale SUSY

- Conclusion

Hierarchy problem:

Why is the Higgs boson so light compared to the Planck or GUT scale?



➔ $$\delta m_H^2 = \left(-3y_t^2 + 3\lambda + \frac{9g_2^2 + 3g_1^2}{8} + \ldots\right)\frac{\Lambda^2}{16\pi^2}$$

Proposed solutions:

1) New physics near the weak scale?

   SUSY, Composite Higgs, Extra Dim, ...

2) Anthropic selection with multiverse?

3) Relaxion?

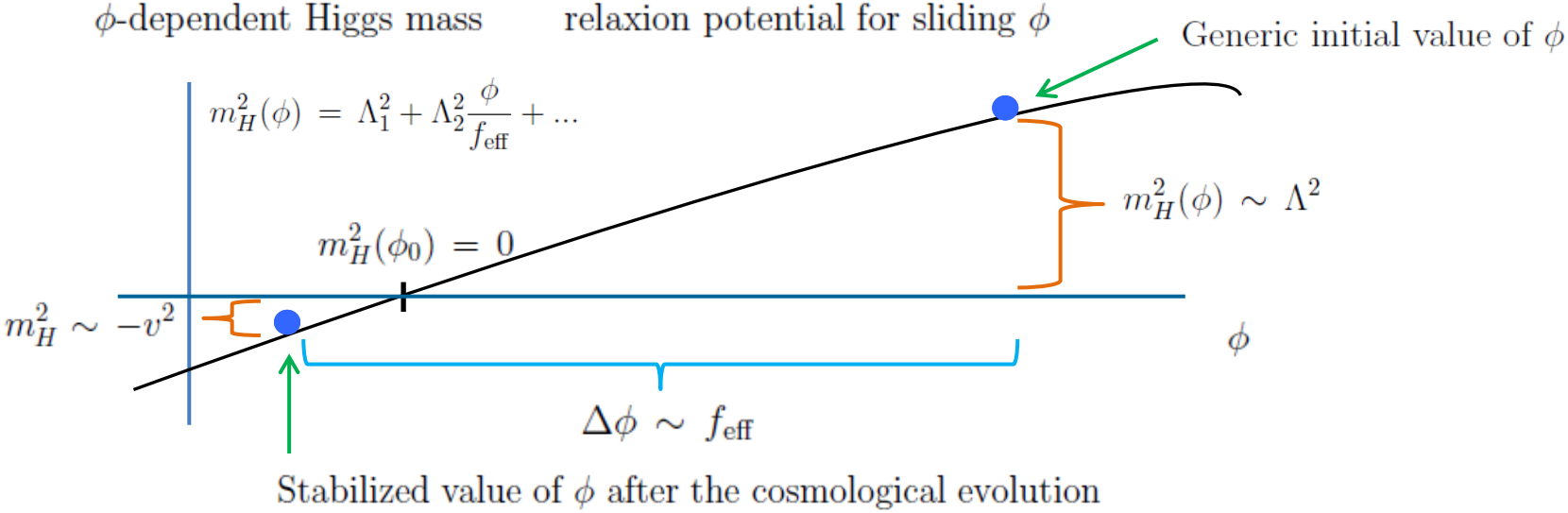
4) ...

Relaxion mechanism for the hierarchy problem

Graham, Kaplan, Rajendran '15

A pseudo-Nambu-Goldstone boson (=**relaxion**) ϕ which scans cosmologically the Higgs boson mass over the cutoff scale $\Lambda \gg v = 174\ \mathrm{GeV}$:

$$V_0 = \underbrace{\left(\Lambda_1^2 + \Lambda_2^2 \frac{\phi}{f_{\rm eff}} + \ldots\right)}_{\phi\text{-dependent Higgs mass}} |H|^2 + \underbrace{\frac{\phi}{f_{\rm eff}} \Lambda_3^4 + \ldots}_{\text{relaxion potential for sliding } \phi} \quad \text{with the UV cutoff } \Lambda \sim \Lambda_1 \sim \Lambda_2 \sim \Lambda_3$$

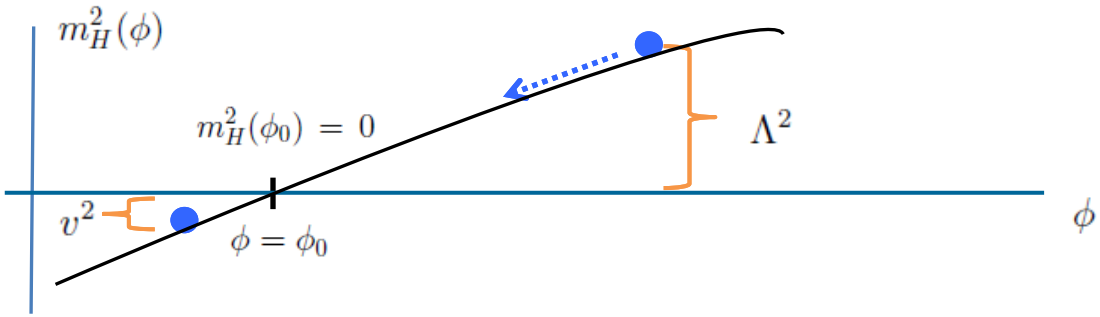


* Mass scales: Higgs mass cutoff Λ and the relaxion scale $\Delta\phi \sim f_{\rm eff} \gtrsim \Lambda$

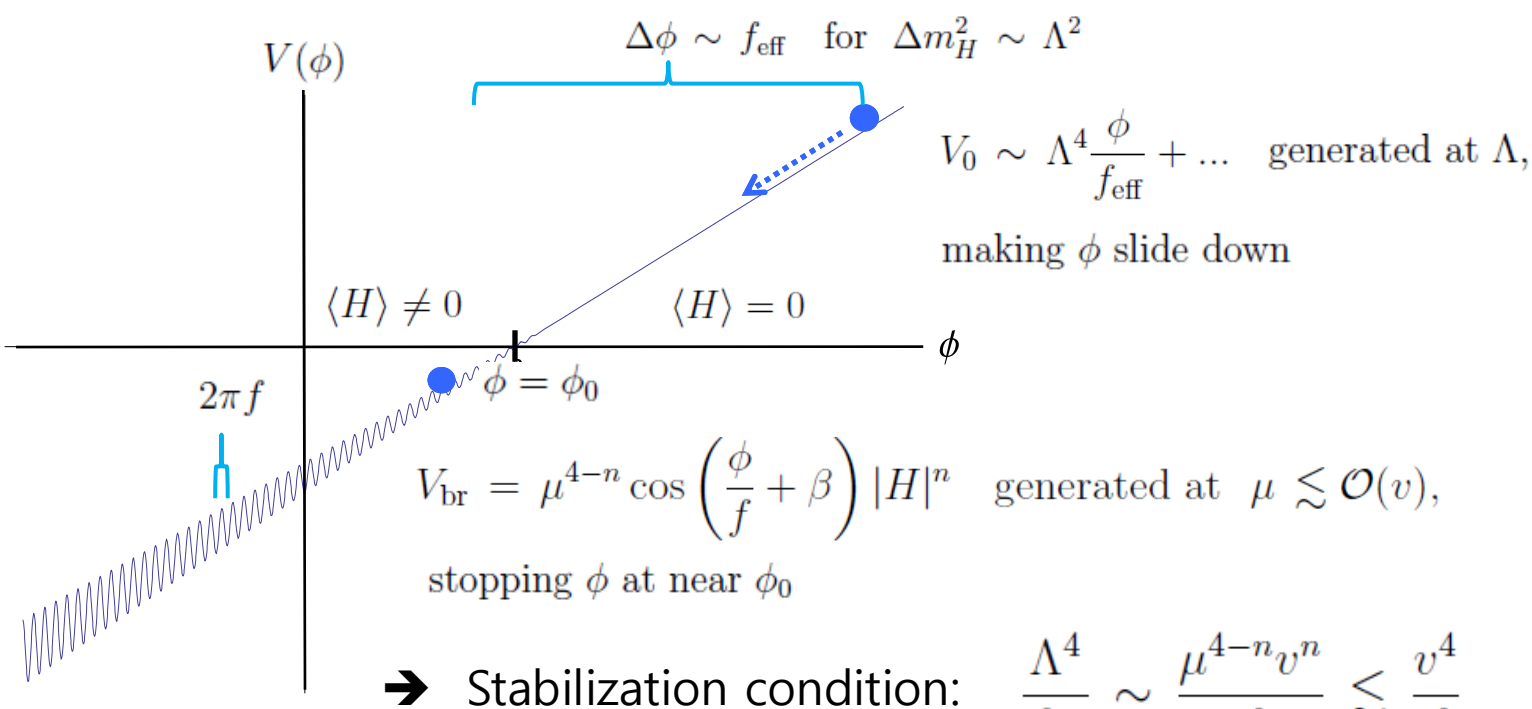
* The full potential can be periodic in ϕ, so that the relaxion is an axion-like field:

$$V_0 = \left(\Lambda_1^2 + \Lambda_2^2 \cos\left(\frac{\phi}{f_{\rm eff}}\right)\right) |H|^2 + \Lambda_3^4 \cos\left(\frac{\phi}{f_{\rm eff}} + \delta\right) + \ldots$$

To complete the scheme, we need to stabilize the relaxion at near ϕ_0 .



This is achieved by introducing an additional Higgs-dependent relaxion potential, which involves another relaxion scale f : Back-reaction potential V_{br}



➔ Stabilization condition: $\frac{\Lambda^4}{f_{\rm eff}} \sim \frac{\mu^{4-n} v^n}{f} \lesssim \frac{v^4}{f}$

Relaxion stabilization condition: $\frac{\Lambda^4}{f_{\rm eff}} \sim \frac{\mu^{4-n} v^n}{f} \lesssim \frac{v^4}{f}$

$$(\Lambda = \text{Higgs mass cutoff}, \quad v = \text{weak scale})$$

➔ Relaxion scale hierarchy: $\frac{f_{\rm eff}}{f} \gtrsim \left(\frac{\Lambda}{v}\right)^4 \quad (\Lambda \gg v = 174\,\text{GeV})$

($\frac{f_{eff}}{f} \sim 10^{12} \left(\frac{\Lambda}{v}\right)^4$ for V_{br} generated by QCD dynamics)

The relaxion mechanism replaces the weak scale hierarchy with the other hierarchy in relaxion scales.

The key point is that $f \ll f_{\rm eff}$ is stable against radiative corrections, and therefore technically natural, while $v \ll \Lambda$ is not!

Yet the required hierarchy for relaxion scales appears to be too large, e.g.

$$\Lambda \sim 10^2 v \sim 10 \text{ TeV} \quad \Rightarrow \quad f_{\rm eff} \gtrsim 10^8 f$$

$(f_{eff} > 10^{20} f)$

which calls for an explanation.

Hierarchical axion couplings from multiple axions

<u>A simple model to generate hierarchical axion scales</u>

SU(N) gauge theory with softly broken SUSY and an elementary axion ϕ_1

quantum gravity,
stringy instantons, ..?

At UV scales well above the SU(N) confinement scale M,

$$\mathcal{L} = -\frac{1}{4g^2}F^{a\mu\nu}F^a_{\mu\nu} - i\bar{\lambda}D\lambda + \frac{1}{2}\partial^\mu\phi_1\partial_\mu\phi_1 - \frac{1}{2}\left(m_\lambda\lambda\lambda + \text{h.c}\right) + \frac{1}{32\pi^2}\frac{\phi_1}{f_1}F^{a\mu\nu}\tilde{F}^a_{\mu\nu} + \mu^4_{\text{UV}}\cos\left(\frac{\phi_1}{f_1}\right)$$

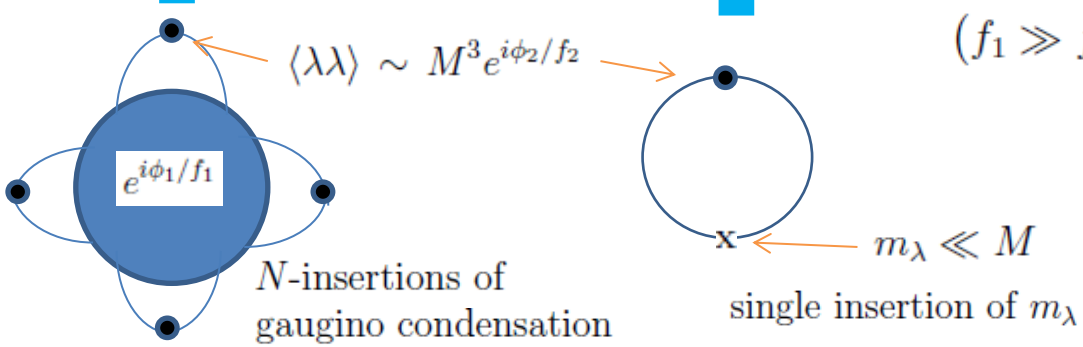
At scales $\sim M$, gaugino condensation is formed, producing a composite axion ϕ_2:

$$\langle\lambda\lambda\rangle \sim M^3 e^{i\phi_2/f_2} \quad \text{with} \quad f_2 \sim M \ll f_1 \qquad \left(\frac{\phi_1}{f_1} \equiv \frac{\phi_1}{f_1} + 2\pi,\ \frac{\phi_2}{f_2} \equiv \frac{\phi_2}{f_2} + 2\pi\right)$$

Axion potential:

$$V = -M^4\cos\left(\frac{\phi_1}{f_1} + N\frac{\phi_2}{f_2}\right) - m_\lambda M^3\cos\left(\frac{\phi_2}{f_2}\right) - \mu^4_{\text{UV}}\cos\left(\frac{\phi_1}{f_1}\right) + \ldots$$

$$\left(f_1 \gg f_2 \sim M \gg m_\lambda, \quad \mu^4_{\text{UV}} \sim m_\lambda M^3\right)$$



At UV scales: $V_{\rm UV} = -\mu_{\rm UV}^4 \cos\left(\frac{\phi_1}{f_1}\right)$

At scales just below the SU(N) confinement scale M:

$$V = -M^4 \cos\left(\frac{\phi_1}{f_1} + N\frac{\phi_2}{f_2}\right) - m_\lambda M^3 \cos\left(\frac{\phi_2}{f_2}\right) - \mu_{\rm UV}^4 \cos\left(\frac{\phi_1}{f_1}\right) + \ldots$$

$$(f_1 \gg f_2 \sim M \gg m_\lambda, \quad M^4 \gg m_\lambda M^3 \sim \mu_{\rm UV}^4)$$

(ϕ_1 = light elementary axion, ϕ_2 = heavy composite axion)

Integrate out ϕ_2 to derive the low energy effective potential of ϕ_1:

$$\frac{\phi_2}{f_2} \approx -\frac{1}{N}\frac{\phi_1}{f_1} \quad \Rightarrow \quad V_{\rm eff} \approx -m_\lambda M^3 \cos\left(\frac{\phi_1}{\underbrace{N f_1}_{f_{\rm eff}}}\right) - \mu_{\rm UV}^4 \cos\left(\frac{\phi_1}{f_1}\right)$$

Although we started at UV scales with a single axion scale $\boldsymbol{f_1}$, in the low energy limit there appear two axion scales with a bigger scale $\boldsymbol{f_{\rm eff} = N f_1}$.

Interpretation 1: **Alignment** Kim, Nilles, Peloso, 05

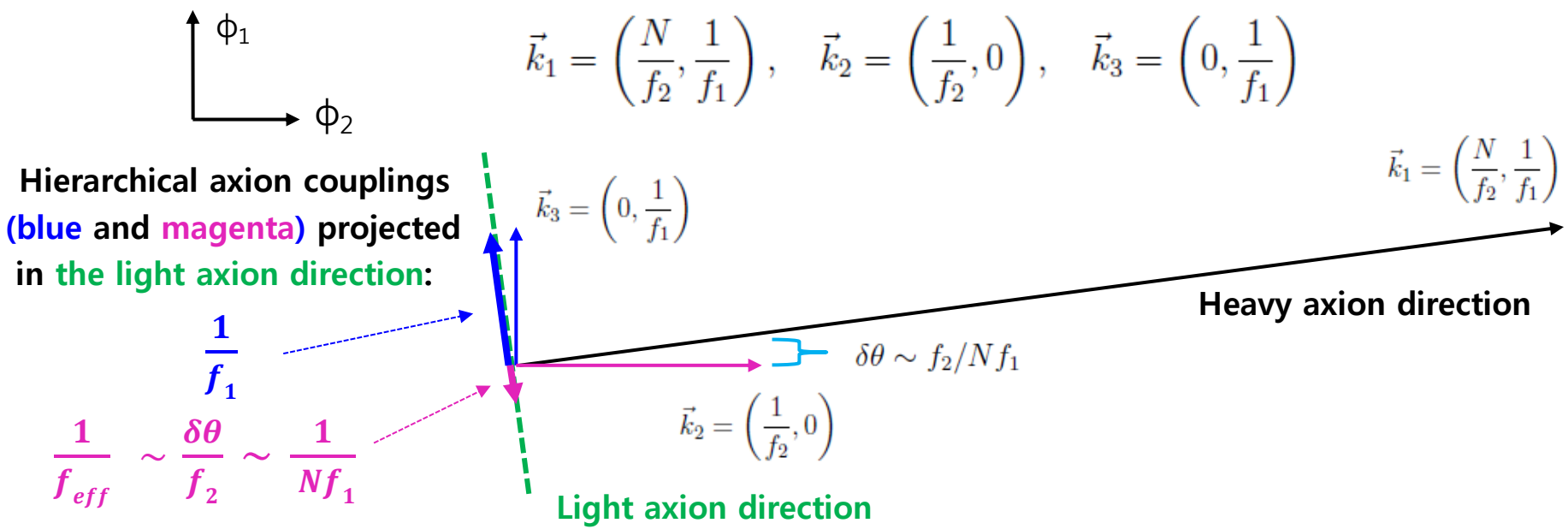
$$V = -M^4 \cos\left(\frac{\phi_1}{f_1} + N\frac{\phi_2}{f_2}\right) - m_\lambda M^3 \cos\left(\frac{\phi_2}{f_2}\right) - \mu_{\rm UV}^4 \cos\left(\frac{\phi_1}{f_1}\right) + \ldots$$

$$\Rightarrow \quad V_{\rm eff} = -m_\lambda M^3 \cos\left(\frac{\phi_1}{f_{\rm eff}}\right) - \mu_{\rm UV}^4 \cos\left(\frac{\phi_1}{f_1}\right) \qquad (f_{\rm eff} = N f_1)$$

In the limit N ≫ 1, the resulting axion scale hierarchy $f_{\rm eff}/f_1 \approx N \gg 1$ can be understood as a consequence of the aligned couplings of ϕ_1 and ϕ_2 :

Three axion couplings (1/scales) with $\mathbf{k_1}$ and $\mathbf{k_2}$ aligned to be nearly parallel

$$\vec{k}_1 = \left(\frac{N}{f_2}, \frac{1}{f_1}\right), \quad \vec{k}_2 = \left(\frac{1}{f_2}, 0\right), \quad \vec{k}_3 = \left(0, \frac{1}{f_1}\right)$$

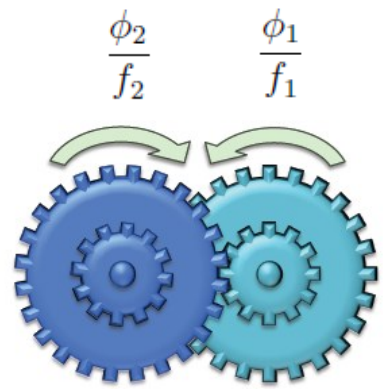


Interpretation 2: **Clockwork**

KC, Kim, Yun '14; KC, Im, arXiv:1511.00132; Kaplan, Rattazzi, arXiv:1511.01827

$$V = -M^4 \cos\left(\frac{\phi_1}{f_1} + N\frac{\phi_2}{f_2}\right) - m_\lambda M^3 \cos\left(\frac{\phi_2}{f_2}\right) - \mu_{\rm UV}^4 \cos\left(\frac{\phi_1}{f_1}\right) + \ldots$$

$$\frac{\phi_2}{f_2} \approx -\frac{1}{N}\frac{\phi_1}{f_1} \quad :$$



Field range enhanced by clockwork: $\Delta\phi_2 = 2\pi f_2$ ➔ $\Delta\phi_1 = 2\pi N f_1$

Effective axion potential over the enhanced field range $\Delta\phi_1 = 2\pi N f_1 = 2\pi f_{\rm eff}$:

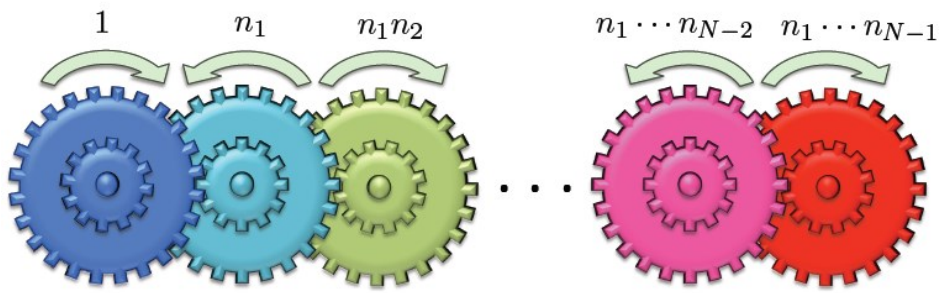
$$V_{\rm eff} = -m_\lambda M^3 \cos\left(\frac{\phi_1}{f_{\rm eff}}\right) - \mu_{\rm UV}^4 \cos\left(\frac{\phi_1}{f_1}\right) \quad (f_{\rm eff} = N f_1)$$

Clockwork scheme allows a straightforward generalization which can yield an **exponentially big hierarchy** of axion couplings, which might be required for the relaxion mechanism:

$$\Omega_1\left(\frac{\phi_1}{f_1}+n_1\frac{\phi_2}{f_2}\right)+\Omega_2\left(\frac{\phi_2}{f_2}+n_2\frac{\phi_3}{f_3}\right)+\ldots+\Omega_{N-1}\left(\frac{\phi_{N-1}}{f_{N_1}}+n_{N-1}\frac{\phi_N}{f_N}\right)+V_1\left(\frac{\phi_1}{f_1}\right)+V_N\left(\frac{\phi_N}{f_N}\right)$$

$$(\Omega_i \gg V_1, V_N)$$

➔ $$\frac{\phi_1}{f_1}=-n_1\frac{\phi_2}{f_2},\quad \frac{\phi_2}{f_2}=-n_2\frac{\phi_3}{f_3},\quad \ldots\quad \frac{\phi_{N-1}}{f_{N-1}}=-n_{N-1}\frac{\phi_N}{f_N}$$



➔ Light axion ϕ with an exponentially enhanced field range and hierarchical couplings in the low energy limit:

$$\Delta\phi = 2\pi n_1 n_2 \ldots n_{N-1} f_1 \sim 2\pi e^N f_1 \quad (n_i = \mathcal{O}(1))$$

$$V_{\text{eff}} = V_1\left(\frac{\phi}{f_1}\right)+V_N\left(\frac{\phi}{f_{\text{eff}}}\right) \quad (f_{\text{eff}} = n_1 n_2 \ldots n_{N-1} f_1 \sim e^N f_1)$$

UV completed relaxion for high scale SUSY

KC, Im, arXiv:1511.00132

* Multiple axions:

N global $U(1)$ symmetries spontaneously broken at $f \sim m_{\rm SUSY}$ or $\sqrt{m_{\rm SUSY} M_P}$ by soft SUSY-breaking mass:

$$U(1)_i: \quad X_i \to e^{-2i\alpha_i} X_i \left(\text{or } e^{-3i\alpha_i} X_i\right), \quad Y_i \to e^{i\alpha_i} Y_i \qquad (i = 1, 2, ..., N)$$

$$\Rightarrow \qquad W_1 = \lambda_i Y_i X_i^2 \text{ or } \frac{Y_i X_i^3}{M_P}$$

$$V = -m_{\rm SUSY}^2 |X_i|^2 + \lambda_i^2 |X_i|^4 \left(\text{or } \frac{|X_i|^6}{M_P^2}\right) + ...$$

$$\langle X_i \rangle \sim \langle Y_i \rangle \sim m_{\rm SUSY} \text{ or } \sqrt{m_{\rm SUSY} M_P}$$

(All axions begin to appear at low scales well below M_P, after SUSY breaking is taken into account, so there is no tension with WGC.)

* Exponentiated clockwork

Hidden YM sector with gauge group: $G = \prod_{i=1}^{N-1} SU(k_i)$

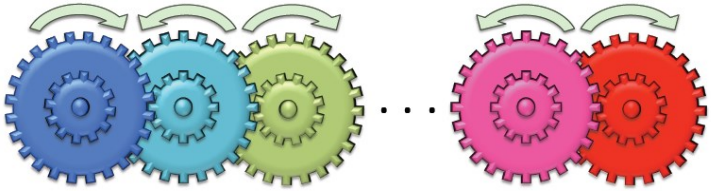
charged matter superfields: $\Psi_i + \Psi_i^c, \quad \Phi_{ia} + \Phi_{ia}^c \quad (i = 1, 2, ..., N-1;\ a = 1, 2, ..., n_i)$

$$\text{with} \quad W_2 = \sum_{i=1}^{N-1} \left(X_i \Psi_i \Psi_i^c + X_{i+1} \Phi_{ia} \Phi_{ia}^c \right)$$

For $f \gtrsim \Lambda_{\rm dyn} \gtrsim m_{\rm SUSY} \quad (\Lambda_{\rm dyn} = \text{confinement scale of } G)$,

$$W_3 = \Lambda_{\rm dyn}^3 \exp\left(-\frac{T_i + n_i T_{i+1}}{k_i} \right) \quad \left({\rm Im}(T_i) = \frac{\phi_i}{f_i} \right)$$

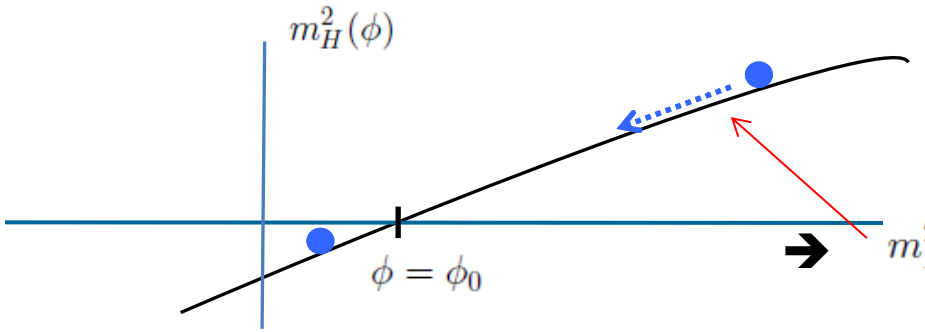
$$\Rightarrow \quad V = \frac{8\pi^2}{k_i} m_{\lambda_i} \Lambda_{\rm dyn}^3 \cos\left(\frac{1}{k_i} \left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) \right)$$



$$\Rightarrow \quad \frac{\phi_i}{f_i} = (-1)^{i-1} \left(\prod_{j=i}^{N-1} n_j \right) \frac{\phi}{f_{\rm eff}} \quad (\phi = \text{lightest axion} = \text{relaxion})$$

$$f_{\rm eff} = \sqrt{\sum_{i=1}^{N} \left(\prod_{j=i}^{N-1} n_j^2 \right) f_i^2} \sim \left(\prod_{j=1}^{N-1} n_j \right) f_1 \sim e^{\xi N} f_1 \quad (\xi = \mathcal{O}(1))$$

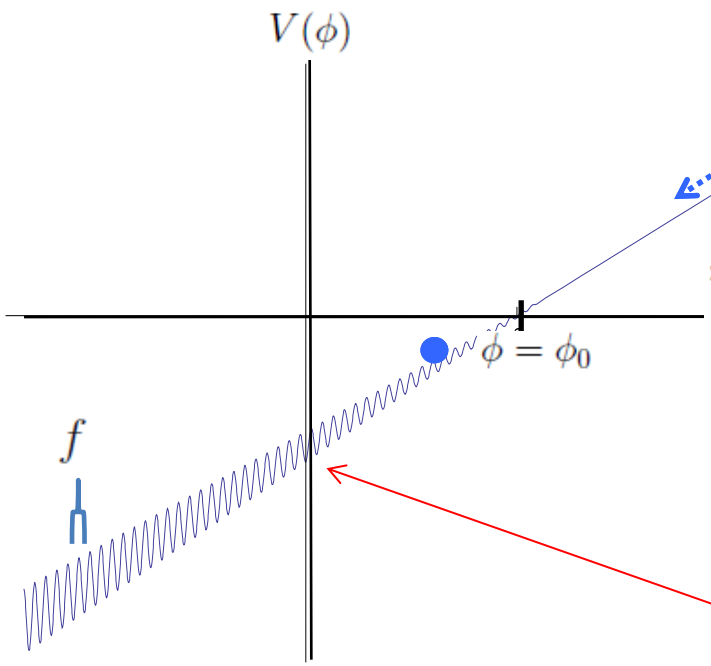
* Relaxion-dependent Higgs mass and the relaxion potential from the μ-term



$$\Delta W = (X_N + X_{N-1})\, H_u H_d \quad \text{or} \quad \frac{(X_N^2 + X_{N-1}^2) H_u H_d}{M_P}$$

$$\Delta K = X_N X_{N-1}^* \quad \text{or} \quad \frac{X_N^2 X_{N-1}^{*2}}{M_P^2}$$

$$\rightarrow \quad m_H^2 = c_1 m_{\rm SUSY}^2 + c_2 m_{\rm SUSY}^2 \cos\left(2(n_{N-1}+1)\frac{\phi}{f_{\rm eff}} + \delta\right)$$



$$V_0 = c_0 m_{\rm SUSY}^4 \cos\left(2(n_{N-1}+1)\frac{\phi}{f_{\rm eff}} + \tilde{\delta}\right) + \ldots$$

$$\left(\frac{1}{f_{\rm eff}} \sim e^{-\xi N}\frac{1}{f} \quad \text{with} \quad \xi = \mathcal{O}(1), \quad c_i = \mathcal{O}(1)\right)$$

* Backreaction potential to stop the relaxion

Another hidden color which confines at $\Lambda_{\rm HC} \sim$ weak scale with hidden colored matter $L + L^c, N + N^c$ having

$$W_{\rm br} = H_u L N^c + H_d L^c N + X_1 L L^c \quad \left(\text{or } \frac{X_1^2}{M_P} L L^c\right)$$

$$\rightarrow \quad V_{\rm br} \sim \frac{\Lambda_{\rm HC}^3}{m_{\rm SUSY}} |H|^2 \cos\left(\frac{\phi}{f} + \delta_1\right)$$

Conclusion

* Relaxion mechanism which was recently proposed as a new solution to the weak scale hierarchy problem requires a big (but technically natural) hierarchy among the involved relaxion scales.

* Clockwork scheme with multiple axions can yield an exponentially big hierarchy among the effective couplings (scales) of light axion.

* Clockwork scheme to generate hierarchical relaxion scales can find a natural UV completion in SUSY models with $m_{SUSY} >>$ weak scale.