Testing Markovian MC with $\alpha(k^T)$

M. Skrzypek

on behalf of IFJ-PAN group

IFJ-PAN, Kraków, Poland

DESY–Hanburg 25 October 2006

Partly supported by EU grant MTKD-CT-2004-510126

Intro: R&D on MC solutions of QCD Evolution in Cracow

Monte Carlo solutions of the QCD $\overline{MS}$ DGLAP evolution:

- LL massless Markovian MC, precision $\sim 10^{-3}$, APP B35 745 (2004).
- NLO massless Markovian MC, APP B37 1785 (2006), NEW $\rightarrow$ WP

Constrained Monte Carlo (CMC) algorithms for DGLAP evolution:

- CMC (non-Markovian) class II, NP B135 338 (2004); APP B36 2979 (2005).
- Constrained MC (non-Markovian) class I, CPC 175 511 (2006).

Evolution in the rapidity space:

- Markovian evolution and constrained evolution, NEW $\rightarrow$ MS, StJ
- $\alpha_S(q(1-z))$ and $\alpha_S(qx(1-z)/z)$, NEW $\rightarrow$ MS
- CCFM evolution, NEW $\rightarrow$ StJ
- Joining two hemispheres, NEW $\rightarrow$ StJ

Framework for fitting PDFs: NEW $\rightarrow$ PS

Introduction

We all know evolution equation (DGLAP-type)

$$\partial_t D(x,t) = \alpha_S P(x) \otimes^x D(x,t)$$

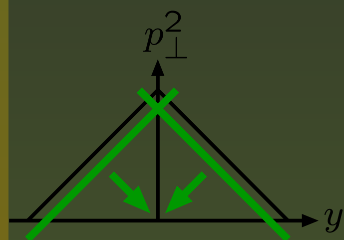
but

- What exactly is t ??
  i.e. what kind of ordering do we have ??
  and how the kinematics is reconstructed ??
- What exactly is the argument of α_S ??
  i.e. which non-leading corrections do we include ??
  (depends on the meaning of t as well)

Evolution time (transparency from T. Sjöstrand)

Ordering variables in final-state radiation

PYTHIA: $Q^2 = m^2$



large mass first

$\Rightarrow$ "hardness" ordered

coherence brute force

covers phase space

ME merging simple

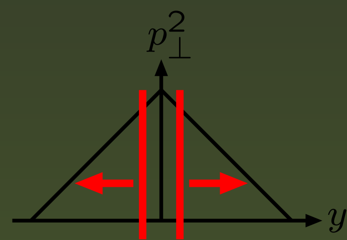
g $\to$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $m^2 \to -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



large angle first

$\Rightarrow$ **hardness not ordered**

coherence inherent

gaps in coverage

ME merging messy

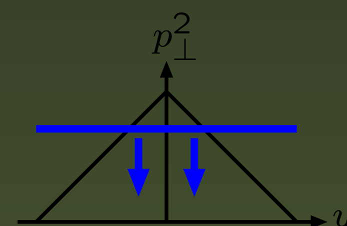
g $\to$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $\theta \to \theta$

ARIADNE: $Q^2 = p_\perp^2$



large $p_\perp$ first

$\Rightarrow$ "hardness" ordered

coherence inherent

covers phase space

ME merging simple

g $\to$ q$\overline{\text{q}}$ **messy**

Lorentz invariant

can stop/restart

ISR: more messy

Coupling constant

Some choices of argument of α_S

- DGLAP – no sub-leading corrections

$$\alpha_S(e^t)$$

- Amati, Bassetto, Ciafaloni, Marchesini, Veneziano; Collins, Soper, Sterman . . .
  – all soft non-leading corrections

$$\alpha_S(e^t(1-z))$$

- CCFM-like – true k_T

$$\alpha_S(e^t x(1-z)/z)$$

Kinematics

Let us define kinematics:

k_i – emitted gluons q_i – virtual parton $q_h^+ = 2E_h$ – initial hadron

for each emitted parton:

$k_i^+ = q_{i-1}^+ - q_i^+ = 2E_h(x_{i-1} - x_i) = 2E_h x_{i-1}(1 - z_i)$

$\eta_i = (1/2)\ln(k_i^+/k_i^-)$ (rapidity)

transverse momentum of parton ($k_i^2 = 0$):

$k_i^T = \sqrt{k_i^+ k_i^-} = k_i^+ e^{-\eta_i} = x_{i-1}(1 - z_i)2E_h e^{-\eta_i}$

Introduce evolution time :

$$t_i = -\eta_i + \ln(2E_h)$$

• This is our choice of evolution time •

k_i^T becomes: $k_i^T = e^{t_i} x_i (1 - z_i)/z_i \quad = e^{t_i}(x_{i-1} - x_i)$

Evolution in rapidity – NEW

$$\partial_t D(x,t) = \int dudz\, \alpha_S\big(e^t x(1-z)/z\big) P(z) D(u,t)\delta(x-zu)$$
$$= \int du/u\, \alpha_S(e^t(u-x)) P(x/u) D(u,t)$$

BEWARE: Landau pole requires cut-off λ on $k^T > \lambda$

$$\lambda < e^t x(1-z)/z \;\rightarrow\; z < e^t x/(e^t x + \lambda) \;<< 1; \quad u > \lambda e^{-t} + x$$

This cut-off influences sum rules!!

To keep normalisation virtual part of the kernel must be adjusted:

$$P^\delta(u,t) = -\int_0^{1-\lambda e^{-t}/u} dz \alpha_S(e^t u(1-z)) z P(z)$$

Evolution in rapidity – algorithm

The Sudakov formfactor is as usual

$$\Phi(t_i, t_{i-1}; u) = \int_{t_{i-1}}^{t_i} dt' \int_0^{1-\lambda e^{-t'}/u} dz \alpha_S(e^{t'} u(1-z)) z P(z)$$

and the probability of a step forward in z is $(u = x_{i-1})$

$$\frac{d\omega(z_i, t_i; i-1)}{dt_i dz_i} = \alpha_S(k_i^T) z_i P^\theta(z_i) \theta_{1-\lambda e^{-t_i}/x_{i-1} \geq z_i} e^{-\Phi(t_i, t_{i-1}; x_{i-1})} \theta_{t_i \geq t_{i-1}}$$

probability of step in t is given by integral over dz_i of the above $d\omega$

$$\frac{d\omega(t_i; i-1)}{dt_i} = \theta_{t_i \geq t_{i-1}} \partial_{t_i} \Phi(t_i, t_{i-1}; x_{i-1}) e^{-\Phi(t_i, t_{i-1}; x_{i-1})}$$

Evolution in rapidity – algorithm – more details

$\Phi(t_i)$ calculable analytically only for $\alpha(k^T)/(1-z)$ part of kernel

$$\Phi_{1/(1-z)}(t_i, t_{i-1}; u) = (2/\beta_0)\left[\rho(t_i + \ln u) - \rho(t_{i-1} + t_u)\theta_{t_{i-1}+t_u>t_\lambda}\right]$$

$$\rho(t) = \hat{t}(\ln \hat{t} - \ln \hat{t}_\lambda - 1) + \hat{t}_\lambda; \quad \hat{t} = t - \ln \Lambda_0, \quad t_\lambda = \ln \lambda, \quad t_u = \ln u$$

But even $\Phi_{1/(1-z)}(t_i)$ cannot be inverted analytically (to generate t_i).

Fast numerical routine writen and used. All in LO approx.

Non-singular part of $\Phi(t_i)$ not integrable analytically. 1-d integral done numerically. No numerical inverting – implemented as weight

$$\Phi_F(t_i, t_0; u) = \int_{t_\lambda - t_1}^{\min(t_u, t_\lambda - t_0)} dv F(v) \ln \frac{t_1 + v}{t_\lambda} + \theta_{t_0+t_u>t_\lambda} \int_{t_\lambda - t_0}^{t_u} dv F(v) \ln \frac{t_1 + v}{t_0 + v}$$

Evolution in rapidity – Master equation

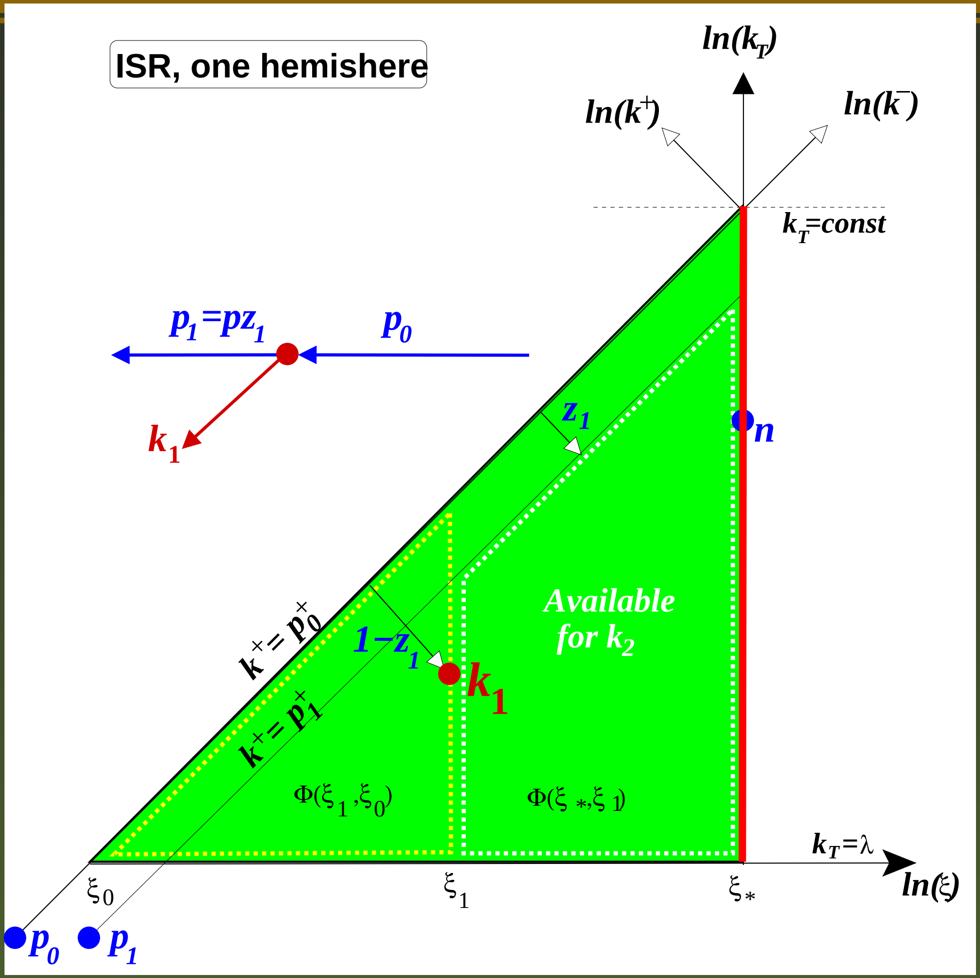
Denote: $\xi = e^{-\eta}, \quad k_i^+ = 2E_h x_{i-1}(1-z_i), \quad \xi_0 = \lambda,$
$\tilde{\mathbf{P}}(k,z,x) = z(1-z)\mathbf{P}(k,z,x)$

Master formula for ISR gluonstrahlung with rapidity ordering

$$
\begin{aligned}
\tilde{D}(\xi,x) &= e^{-\Phi(\xi,\xi_0)}\delta(1-x) + \\
&+ \sum_{n=0}^{\infty} e^{-\Phi(\xi|\xi_n,x)} \left(\prod_{i=1}^{n} \int_{\xi_{i-1}}^{\xi} \frac{d\xi_i}{\xi_i} \int_{\lambda/\sqrt{\xi_i}}^{2E_h x_{i-1}} \frac{dk_i^+}{k_i^+} \int \frac{d\varphi_i}{2\pi} \right) \\
&\times \left(\prod_{i=1}^{n} \tilde{\mathbf{P}}(k_i, z_i, x_{i-1}) e^{-\Phi(\xi_i|\xi_{i-1},x_{i-1})} \right) \delta_{x=\prod_{i=1}^{n} z_i}
\end{aligned}
$$

For the α_S in quark-gluon transitions we keep the usual DGLAP-type argument $\alpha_S(e^t)$ and do not apply any finite IR cut-offs. Are there any good reasons to modify this attitude?

Single emission in a detail



Integration domains of $\Phi(\xi_1|\xi_0, x_0)$ and $\Phi(\xi_*|\xi_1, x)$ are triangle and trapezoid

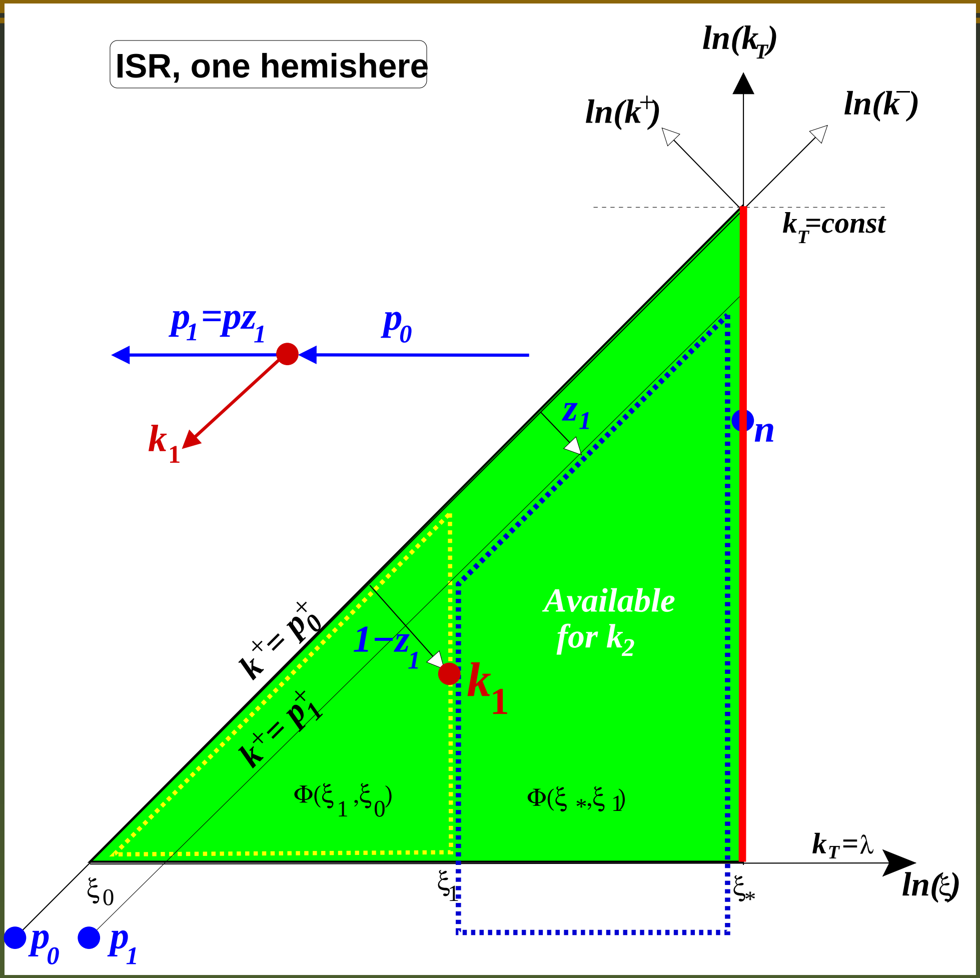
$$\tilde{D}(\xi, x)_{n=1} = \int_{\xi_0}^{\xi} \frac{d\xi_1}{\xi_1} \int_{\lambda/\sqrt{\xi_1}}^{2E_h} \frac{dk_1^+}{k_1^+} \int \frac{d\varphi_1}{2\pi} e^{-\Phi(\xi|\xi_1,x)} \tilde{\mathbf{P}}(k_1, z_1) e^{-\Phi(\xi_1|\xi_0,x_0)} \delta_{x=z_1}$$

Intermediate step: $\alpha_S\big(e^t(1-z)\big)$

At first we implemented evolution with $\alpha_S\big(e^t(1-z)\big)$ – it is easier:

- α_S does NOT depend on x (depends on z only)
- Cut-off related to Landau pole is $\lambda \le e^{t_i}(1-z_i) = k_i^T/x_{i-1}$. Therefore k^T can drop below λ, down to $k_i^T \ge \lambda x_{i-1}$

Intermediate step: $\alpha\left(e^t(1-z)\right)$

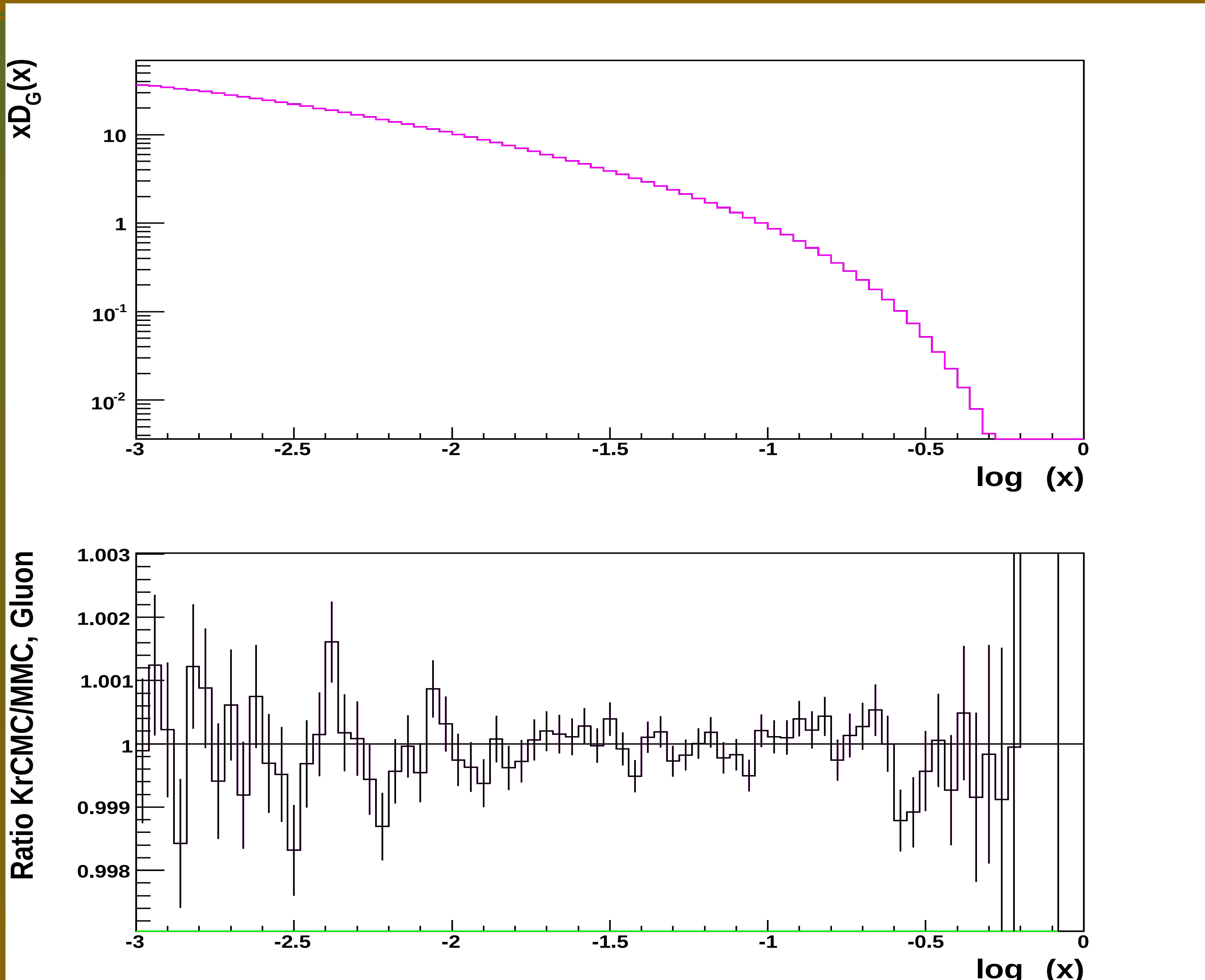


In this scenario IR boundary is not $k_i^T > \lambda$ but $k_i^T > x_{i-1}\lambda$, i.e. $1 - z_i > \frac{\lambda}{p_0^+\sqrt{\xi_i}}$. It is depicted above, see blue line defining phase space of the next (2nd) emission.

Numerical results

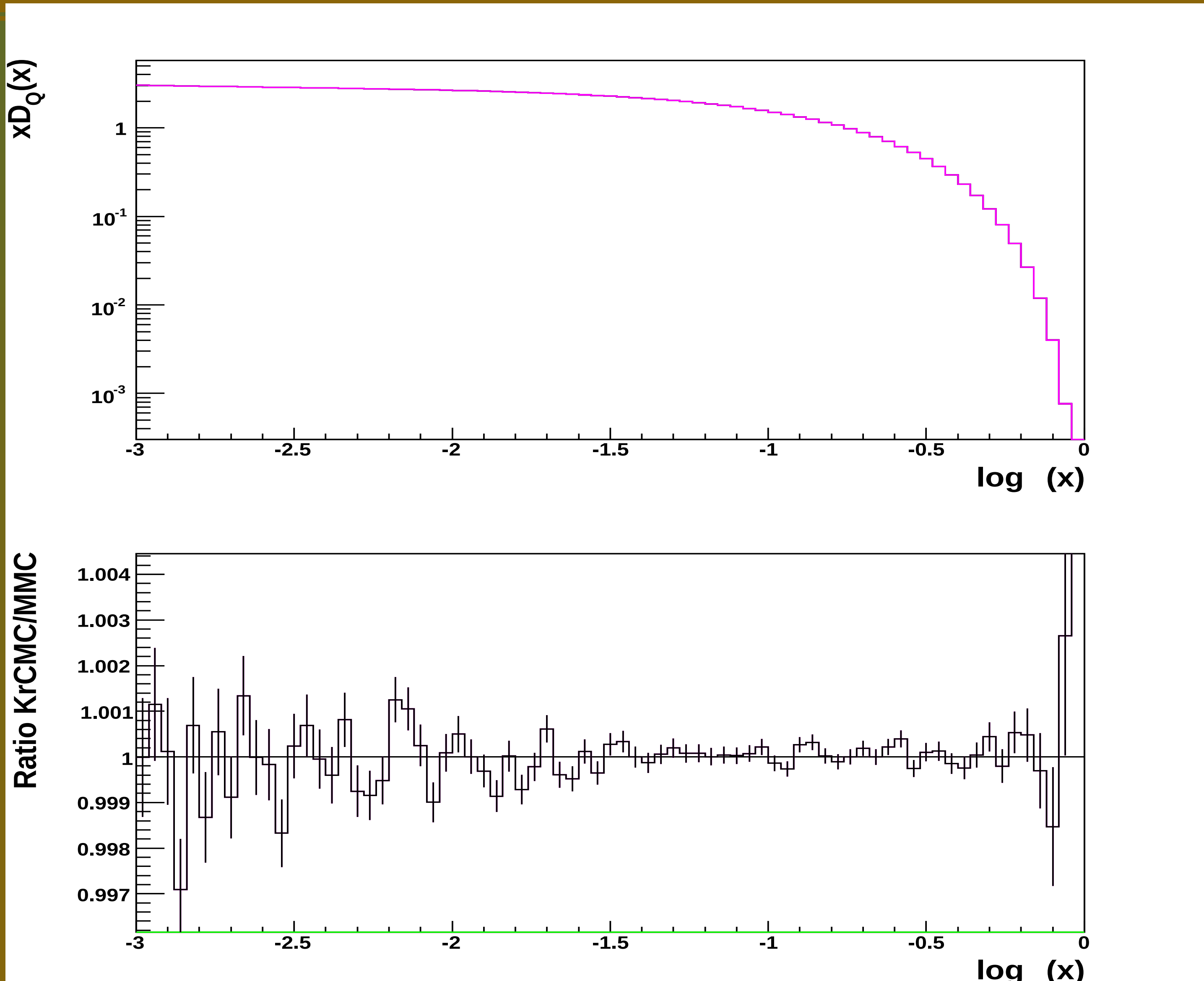
- The full $\alpha(k^T)$ evolution has been implemented on the top of the $\alpha_S\big(e^t(1-z)\big)$ algorithm in two independent ways in the Markovian MC called MMC.
- In parallel the same full $\alpha(k^T)$ evolution is implemented in the Constrained MC called KrCMC (see talk by S. Jadach).
- Results of each program have been used for cross-checks of the other. It is powerfull, albeit not quite straightforward method of testing!

Comparison MMC/KrCMC



Gluon PDF at Q=1000GeV, agreement better than 1×10^{-3}

Comparison MMC/KrCMC



Quark PDF at Q=1000GeV, agreement better than 1×10^{-3}

Summary and outlook

- Evolution ordered in rapidity space has been succesfully implemented in Markovian MC
  - for $\alpha(e^t(1-z))$
  - for $\alpha\big(e^t x(1-z)/z\big)$ i.e. *almost* all-loop CCFM – two algorithms (*“almost CCFM”* – non-Sudakov formfactor still missing)
- Implementation has been tested to the precision of 1 per mille by comparing against similar implementation in Constrained MC (KrCMC)
- Immediate plans:
  - More numerical tests
  - Beter treatment of the Quark-Gluon transitions (beyond DGLAP) ??
- Other plans:
  - Inclusion of NLO evolution