# Propagation of uncertainty in the Parton Shower framework

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# Objective

- To quantify the effects of varying different features of the parton shower
- Must leave delicate technical features of original parton shower implementation untouched
- Applicable to a general class of parton shower implementations
- We wish to keep control of the variations of perturbative physics through the whole event simulation

# Method

We start with a general class of probability distributions, written as a functional

$$\mathcal{P}[\boldsymbol{\varphi}(\vec{y})] = F_R[\boldsymbol{\varphi}(\vec{y}), B(\vec{y})] \times \exp\left(-\int d^n\,\vec{y}'\,F_V[\boldsymbol{\varphi}(\vec{y}'), B(\vec{y})]\right).$$

Can vary according to

- Functionals, $\boldsymbol{\varphi}(\vec{y})$
- Boundaries on integrals, $B(\vec{y})$

# Method

If we vary this distribution according to some vector $\Delta\boldsymbol{\varphi}$ we find $1 + \frac{\Delta\mathcal{P}}{\mathcal{P}}$ is

$$\left(1 + \frac{\Delta F_R[\boldsymbol{\varphi}(\vec{y})]}{F_R[\boldsymbol{\varphi}(\vec{y})]}\right) \exp\left(-\int d^n\, \vec{y}'\, \Delta F_V[\boldsymbol{\varphi}(\vec{y}')]\right),$$

where

$$\Delta F_{R/V}[\boldsymbol{\varphi}(\vec{y})] = F_{R/V}[\boldsymbol{\varphi}(\vec{y}) + \Delta\boldsymbol{\varphi}(\vec{y})] - F_{R/V}[\boldsymbol{\varphi}(\vec{y})].$$

# Method

If one chooses

$$F[\boldsymbol{\varphi}(\vec{y}) + \Delta\boldsymbol{\varphi}(\vec{y})] = F'[\boldsymbol{\varphi}(\vec{y})],$$

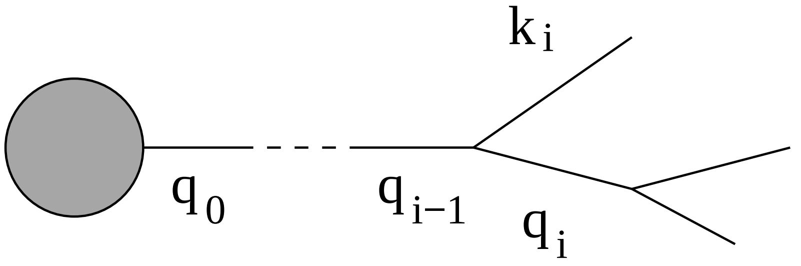
then this method is equivalent to standard Monte Carlo reweighting.

We now look at some examples

- Measurement uncertainty in $\alpha_S$
- Relaxing collinear approximation
- Inclusion of NLO kernel
- Changing kinematics

# Kinematics and evolution

Use variables of Herwig++, $z, \tilde{q}$




$$
\begin{aligned}
q_i &= \alpha_i p + \beta_i n + q_{\perp i}, \\
z_i &= \frac{\alpha_i}{\alpha_{i-1}}, \boldsymbol{p}_{\perp i} = \boldsymbol{q}_{\perp i} - z_i \boldsymbol{q}_{\perp i-1}. \\
\tilde{q}_i^2 &= \frac{\boldsymbol{p}_{\perp i}^2}{z_i^2(1-z_i)^2} + \frac{\mu^2}{z_i^2} + \frac{Q_g^2}{z_i(1-z_i)^2}
\end{aligned}
$$

# Kinematics and evolution

Using these variables we find the probability of branching given by

$$dB(q \to qg) = \frac{C_F}{2\pi}\alpha_S\left(z^2(1-z)^2\tilde{q}^2\right)\frac{d\tilde{q}^2}{\tilde{q}^2}dzP_{qq}(z,\tilde{q}^2).$$

We now stick to the final state shower (for simplicity) and find

$$\begin{aligned} F_R[\boldsymbol{\varphi}(\vec{y}),\vec{y}] &= F_V[\boldsymbol{\varphi}(\vec{y}),\vec{y}] = \frac{1}{2\pi\tilde{q}^2}\alpha_S(z,\tilde{q}^2)P_{qq}(z,\tilde{q}^2) \\ &\quad \times\theta(z^+ - z)\theta(z - z^-)\theta(\tilde{q}_{i-1}^2 - \tilde{q}^2). \end{aligned}$$

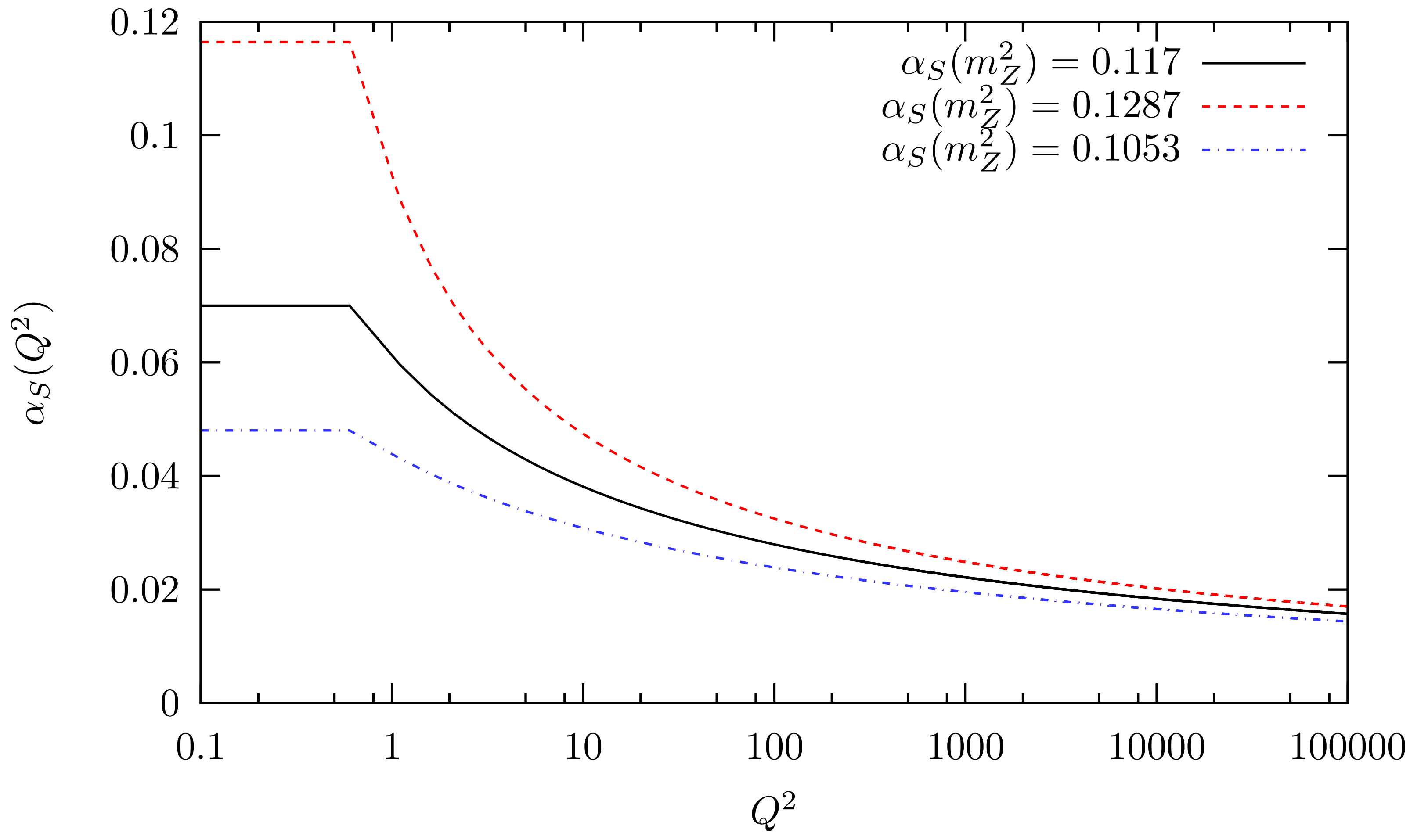
# Uncertainty in Strong Coupling

For a first example we studied the effect of the uncertainty of $\alpha_S(M_Z^2)$ throughout the parton shower.

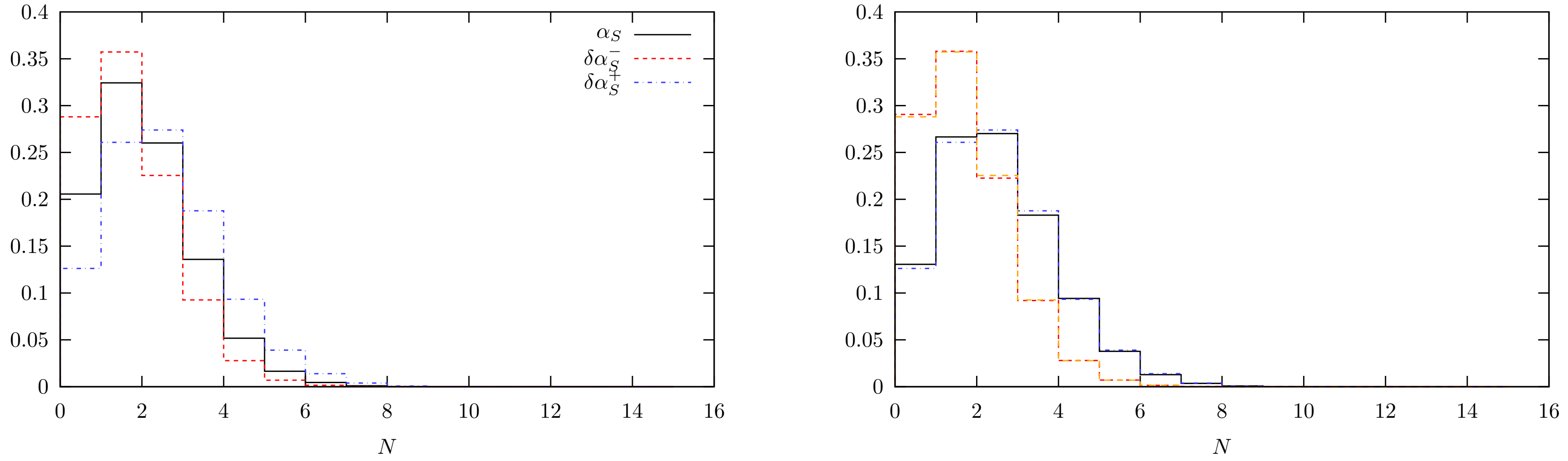
- Used $\alpha_S(M_Z^2) \pm \delta\alpha_S(M_Z^2)$ to compute $\Lambda_{QCD} \pm \delta\Lambda_{QCD}^{\pm}$
- Used 2-loop running coupling
- Froze value of running coupling at $Q^2 = 0.630$ GeV$^2$
- Assumed 10% uncertainty in measured value, $\alpha_S(M_Z^2) = 0.117$

# Running Coupling

The value of the running coupling due to uncertainty in the measured value at $M_Z^2$.

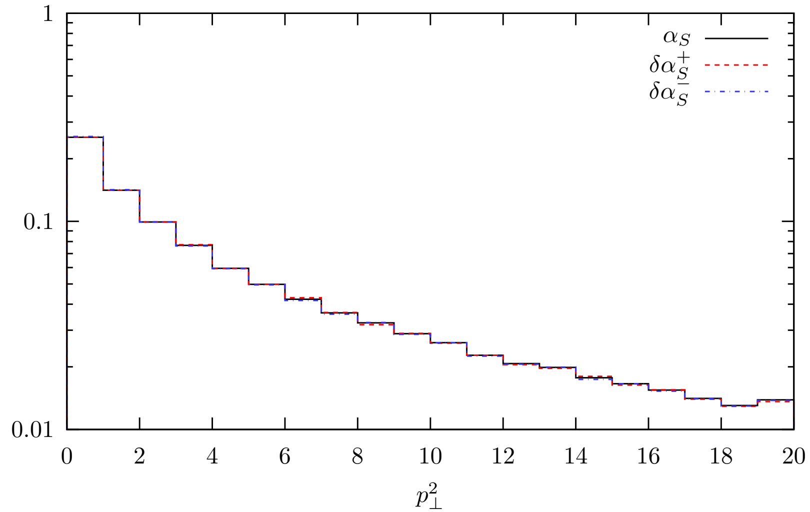


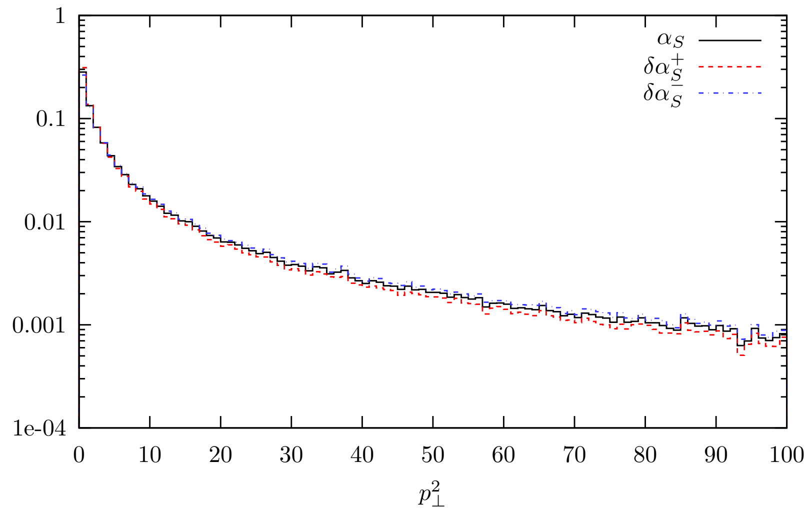
# Running Coupling




Distribution of $N$, the number of emissions.

# Running Coupling




Distribution of $p_\perp$ of each emission.

# Quasi-Collinear Approximation

The kernel in the quasi-collinear approximation is

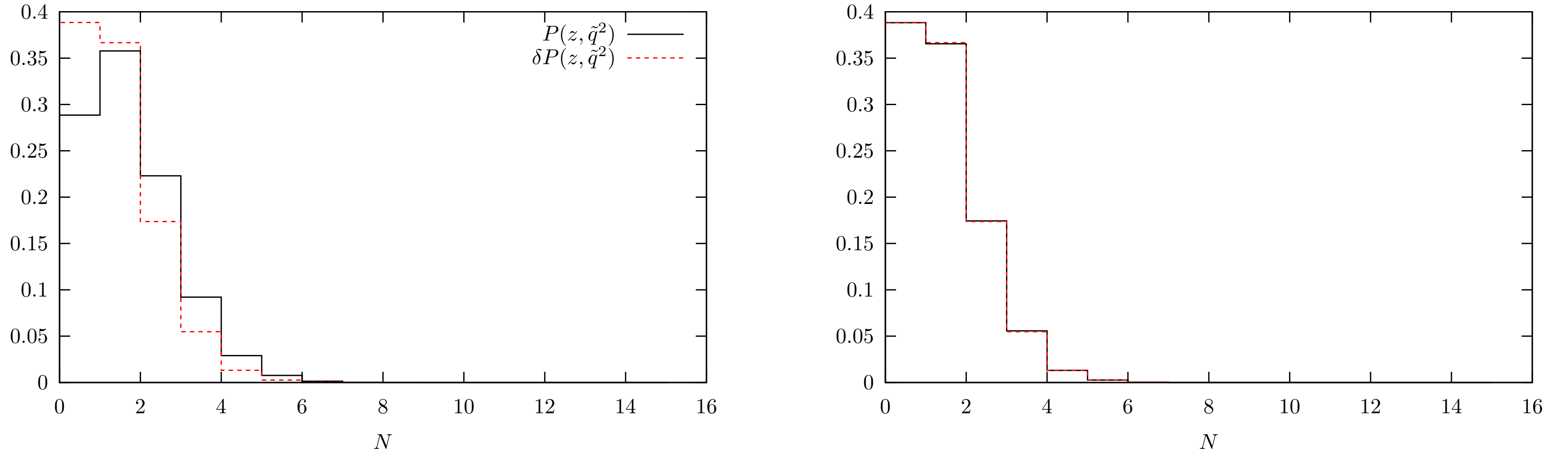
$$P_{qq}(z,\tilde{q}^2) = C_F\left(\frac{1+z^2}{1-z} - \frac{2m^2}{z(1-z)\tilde{q}^2}\right).$$

If we define $\Delta\varphi$ to be the additional term, we can compute the weight by

$$\Delta F[(\alpha_S, P_{qq})(z,\tilde{q}^2)] = \frac{1}{2\pi\tilde{q}^2}\alpha_S(z,\tilde{q}^2)\Delta P_{qq}(z,\tilde{q}^2),$$

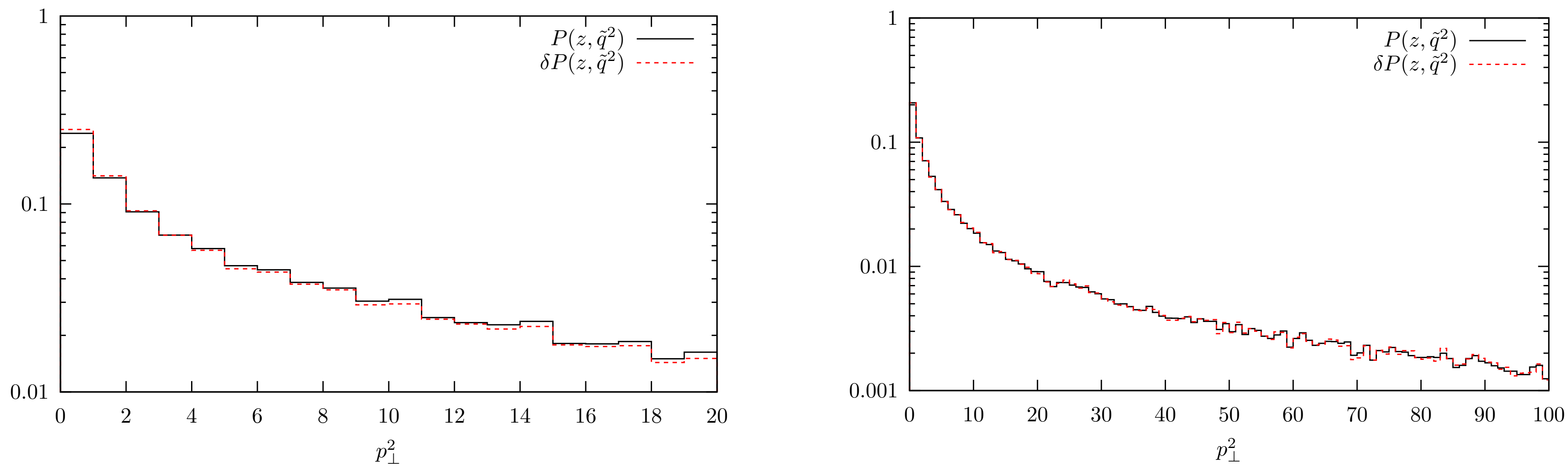
and $\Delta P_{qq}$ given by the additional term in the kernel.

# Quasi-Collinear Kernel




Distribution of $N$, the number of emissions.

# Quasi-Collinear Kernel




Distribution of $p_\perp$ of each emission.

# NLO Kernel

We can also use this to compute the effect of the NLO kernel. In this case we find

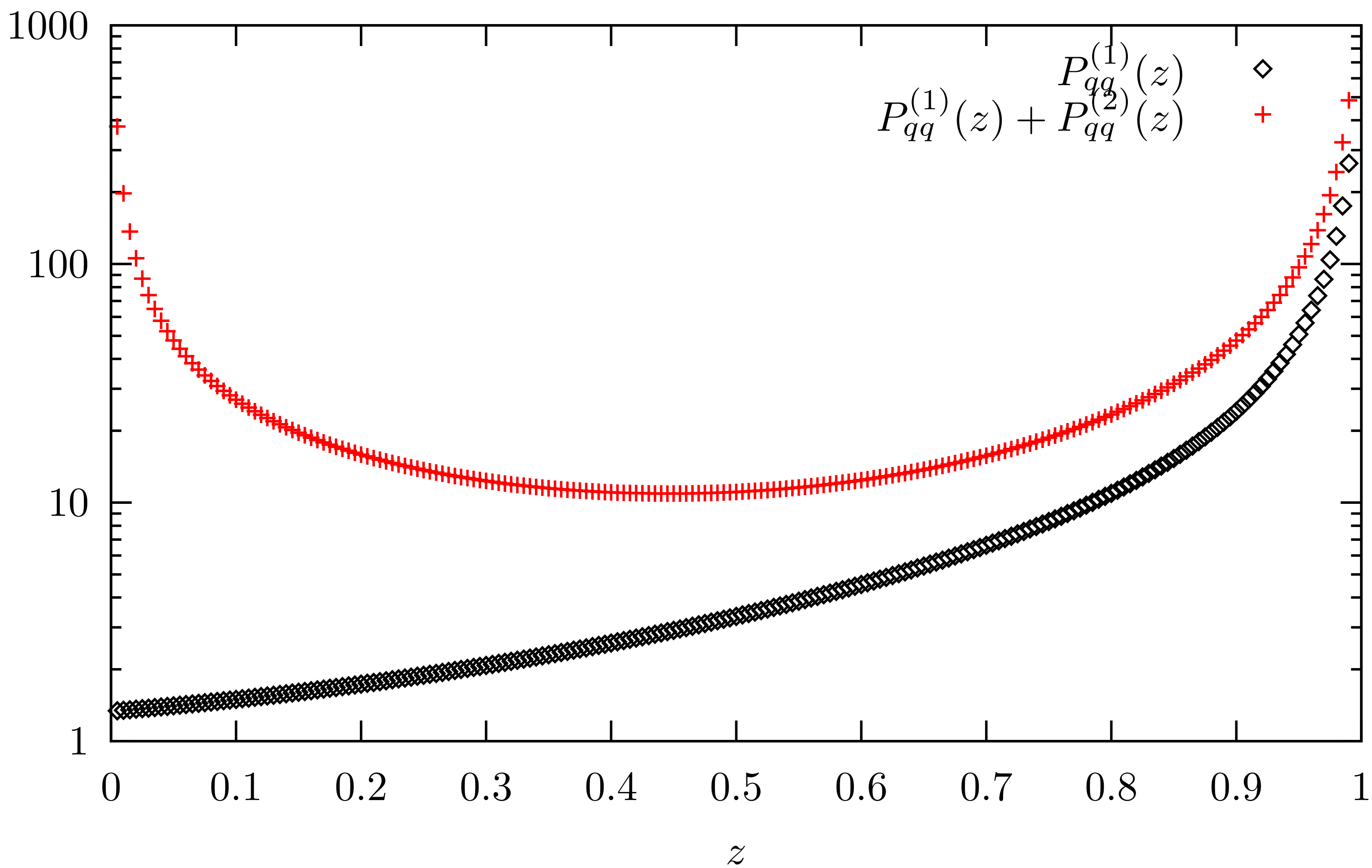
$$\Delta F = \alpha_S^2(z, \tilde{q}^2)\Delta P_{qq}^{(2)}(z, \tilde{q}^2).$$

We fix $P_{qq}^{(2)} = 0$ and

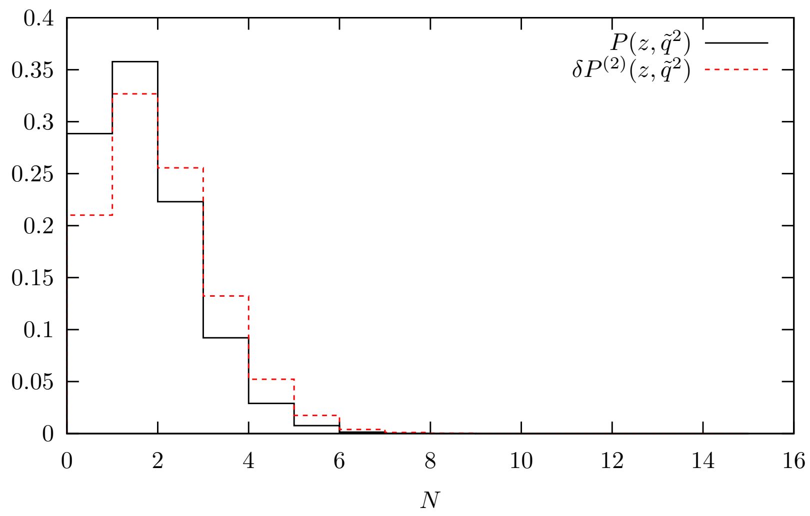
$$\Delta P_{qq}^{(2)}(z, \tilde{q}^2) = P_{qq}^{S(2)}(z) + P_{qq}^{V(2)}(z).$$

# NLO Kernel

Comparison of LO and NLO kernel for $q \to qg$.

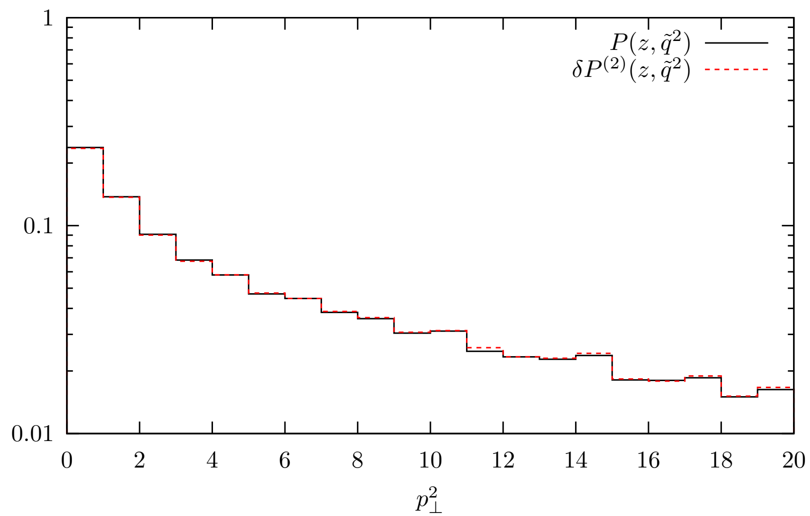


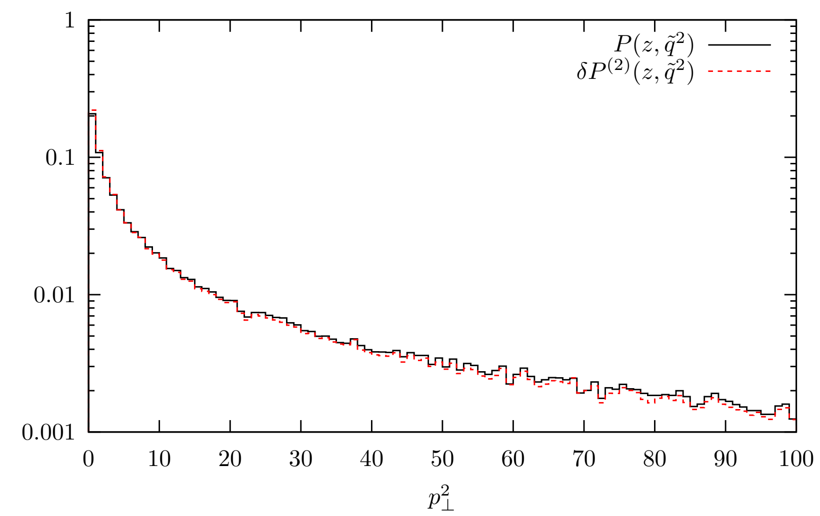
# NLO Kernel




Distribution of $N$, the number of emissions.

# NLO Kernel




Distribution of $p_\perp$ of each emission.

# Change of Kinematics

We also want to change kinematics and evolution ordering through the alternative weight

- Phase spaces of emissions are not identical
- Methods of reconstruction are not the same
- Orderings differ
- Infra-red cutoffs differ

# Pythia-like Kinematics

We now turn to a Pythia-like shower. Evolution variables are

$$Q_i^2 = q_{i-1}^2,$$
$$z_i = \frac{E_i}{E_{i-1}},$$

It is necessary to reconstruct 4-momenta throughout evolution to find bounds on $z$, thus we must reweight after the shower is complete.

# Pythia-like Kinematics

We take advantage of the analytic behaviour of the Sudakov form factor

$$\Delta(t, t_0) = \Delta(t, t_1)\Delta(t_1, t_0),$$

and compute a Sudakov weight

$$w_\Delta = \frac{\Delta_P(Q^2_{\mathrm{max}}, Q^2_0)}{\Delta_H(\tilde{q}^2_{\mathrm{max}}, \tilde{q}^2_0)}.$$

We must compute the cutoff $Q_0^2$ as a function of $\tilde{q}_0^2$.

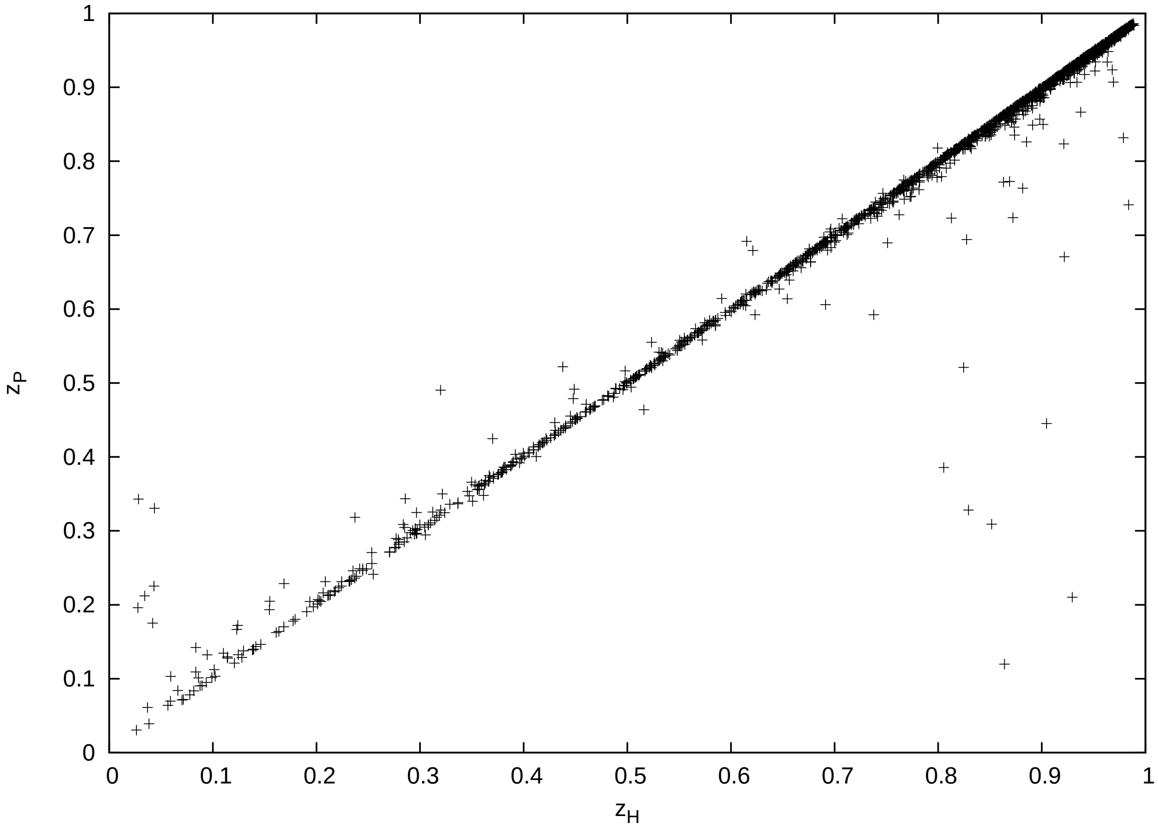
# Pythia-like Kinematics

We also have the weight for each real emission as

$$w_i = \frac{P_{qq}(z_{Pi})}{P_{qq}(z_{Hi})} \frac{\tilde{q}_i^2}{Q_i^2} \mathcal{J}(Q_i^2, z_{Pi}) \times \theta(Q_{i-1}^2 - Q_i^2)\theta(z_P^+ - z_{Pi})\theta(z_{Pi} - z_P^-),$$

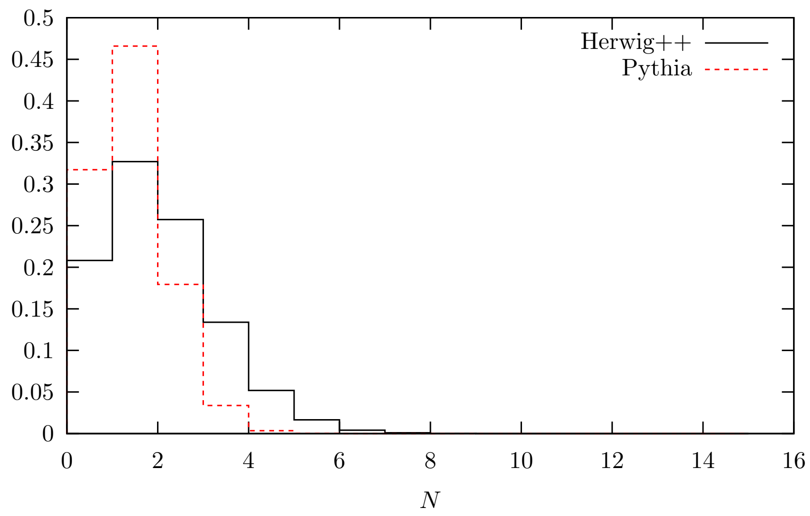
where $\mathcal{J}$ is the Jacobian factor for translating between the two sets of variables.

# Kinematics Transformation



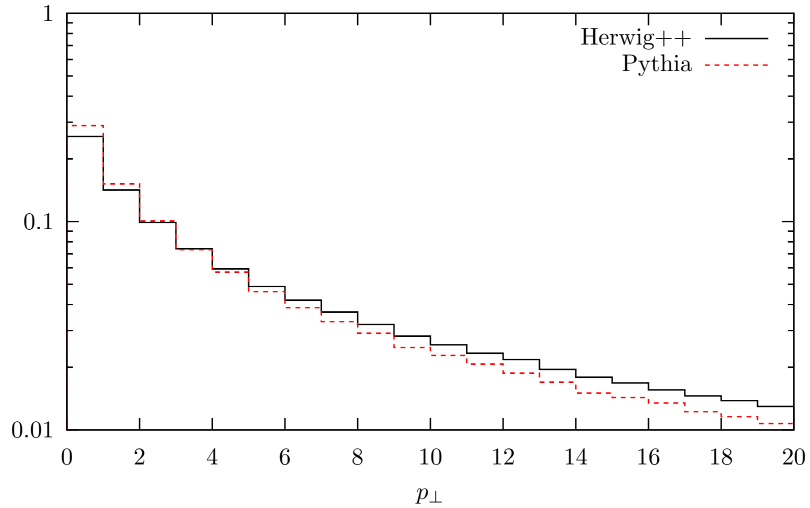

Comparison of $z_P$ and $z_H$.

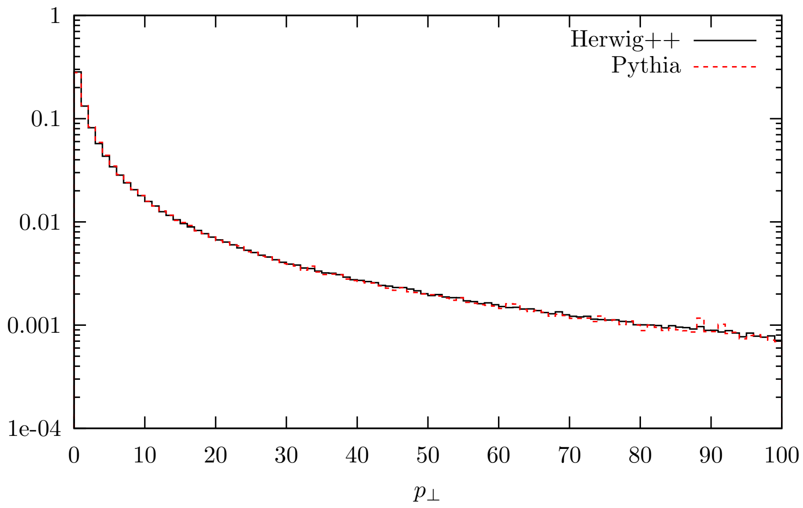
# Kinematics Transformation




Distribution of $N$, the number of emissions.

# Kinematics transformation




Distribution of $p_\perp$ of each emission.

# Conclusion

- Can control effect of perturbative QCD in Monte Carlo
- Direct study of reweighted results vs. full implementation can highlight physical differences between methods
  - Kinematics bounds
  - Evolution ordering
- Could be used to direct research in regions where Monte Carlo's fail to match data