QED PHOTOS

an instructive example for QCD?

Z. Wąs

Institute of Nuclear Physics, PAN, Kraków, Poland

- PHOTOS as a Monte Carlo for QED radiative corrections in decays of particles and resonances seem to be limited to very special applications:
- B mesons decays (CKM pheno. Belle BaBar); decays of t Z W H at LHC or LC.
- But it seem to provide unique example of combining parton shower like algorithm with matrix element calculations and input of experimental data.
- Its mathematical foundations are simpler than for QCD but complicated enough!
- *Thus talk may be worth of attention here ...*
- Personally, I find it nice learning vehicle.
- In PHOTOS it is easier to define mathematical buidling blocks, which may help systematize language and optimize solution elsewhere.

My web page is at http://home.cern.ch/wasm

Plan of my talk (parts will be skipped)

- **I will start with general description:** *What PHOTOS is and why?*
- **Technical difficulties with event record**
- **Matrix element: Z decays Matching ME with PS kernel • numerical tests.**
- **Matrix element B decays Matching ME with PS kernel • Numerical tests.**
- **Phase space, element of general construction**
- **General properties of the algorithm development**
- **Language for expansion. Seem to be well known to mathematicians and used in other applications. Is it worth our effort?**
- **I personally enjoy it a lot.**
- **I have learned new words: Tangent spaces, polynomials on the ring (or field). CW-complexes and I have found these elements in PHOTOS.**

Photos is for QED brem in decays!

- PHOTOS (by E.Barberio, B. van Eijk, Z. W., P.Golonka) is used to simulate the effect of radiatiative corrections in decays, since 1989.
- Full events combining complicated tree structure of production and subsequent decays have to be fed in, e.g. help of F77 HEPEVT event record.
- This is often source of technical difficulties as standard is often overruled.
- At every event decay branching, PHOTOS intervene. With certain probability extra photon may be added and kinematics of other particles adjusted.
- PHOTOS works on four-momenta; watch numerical stability.
- I will not talk about those time consuming aspects but about relation of PHOTOS with explicit field theory calculations, n-body phase space, and expansions with respect to leadin-log truncation and eikonal-truncation.
- Program can provide precise results if one invest in **process dependent ME**.

PHOTOS and its history

E. Barberio, B. van Eijk, Z. Was, Comput. Phys. Commun.(1991) ibid. (1994)

See also: P. Golonka et al. hep-ph/0312240 ,

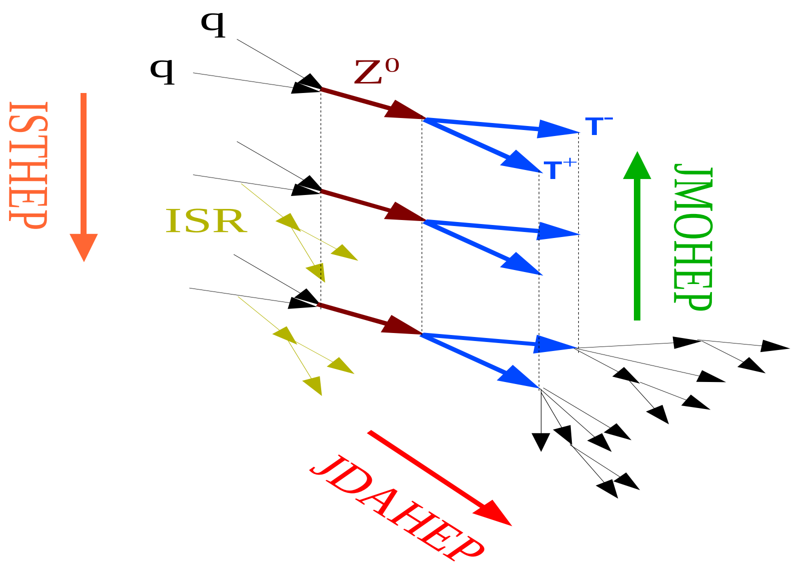
P. Golonka and Z. Was hep-ph/0604232, G. Nanava and Z. Was hep-ph/0607019

- It was developed as single photon emission. starting from MUSTRAAL (F. Berends, R. Kleiss, S. Jadach, Comput. Phys. Commun. (1982)) option for fi nal state bremsstrahlung in Z decay only.
- Factorization of phase space for photonic variables and two-body decay phase space was studied. Similarily for matrix element: process independent kernel was found. Phase space is exact.
- Interference between emission from μ^+ and μ^- is dropped and re-introduced later.
- The algorithm of the antenna type was created. But its precision was not an issue.
- Because of breath-taking precision, I am investing in more mathematical language now.
- For the multiphoton (or not) emission tangent space is constructed.
- In this tangent space, photons emissions are independent one from another and 'parton shower iteration' is algebraically of the fi eld type polynomial.
- With phase-space constraints/Jacobians and ME of some order added, we get polynomial on ring.
- Rules how to expand ring polynomial over algebraically nicer fi eld polynomial exist.
- We have learned already some nice words at least

PHOTOS

- Generally kernels in PHOTOS, are not better than LL. To improve, process dependent weights are needed. Complications for users and me, to be done once necessary only!
- Special weights with complete matrix elements are available now for: $Z \to \mu^+\mu^-$ (2005), and for $B^+ \to K^+\pi^0$, $B^0 \to K^+\pi^-$, etc. (2006 B^0 and $B^\pm$ decays – scalar QED)
- We will see that effects are small, it is suffi cient to keep them for tests only.
- Lots of numerical tests.
- For other decay modes such exercises easy to repeat, if matrix elements are available.
- As consequence: PHOTOS is ready for improvements with measured data as well!
- PHOTOS uses mother-daughter relations in HEPEVT.
- C++ version is prepared but not distributed, nothing like real standard event record exist.
- Program analyze whole event record and may add bremsstrahlung at any branching.
- Appropriately modifi es particles momenta of the whole cascade!
- Algorithm is vulnerable on the way *how* HEPEVT is fi lled in. Any new inconistency and ...
- We will have precision because we have invested in input event record content.
- Next frontier for matrix elements will have to be with 3 body fi nal states with at least 2 of them charged.

Problems With Event Record or finite computer precision



1. Hard process
2. with shower
3. after hadronization
4. Event record overloaded with physics beyond design $\longrightarrow$ gramar problems.
5. Here we have basically LL phenomenology only.

This Is Physics Not F77!

Similar problems are in any use of full scale Monte Carlos, lots of complaints at MC4LHC workshop, `HEPEVTrepair` utility (C. Biscarat and ZW) being probed in D0.

Design of event structure WITH some grammar requirements AND WITHOUT neglecting possible physics is needed NOW to avoid large problems later.

sociology of few k-individualists! Enormous challenge of distributed expertise.

For a Short While Just Believe

That Phase Space is OK

I will come to it much later ...

Let me start with analysis of matrix element to get Shwinger-Dyson like relations.

These types of relations are the foundation of techniques like : exponentiation, structure functions (integrated or not), factorization.

Results of P. Golonka and Z. Was hep-ph/0604232

- From the point of view of matrix element and choice of internal angular variables in case of Z decays, PHOTOS is very close to MUSTRAAL MC by F. Berends S. Jadach and R. Kleiss, CPC 29 (1983) 185.
- The first step to re-introduce NLO terms for Z decay into PHOTOS was to check relations between 4-vectors and angles, see some last transparencies.
- For PHOTOS those angles are defined in complicated way. Incoming beams (defining Z polarisations) have to be along z axis, to install old (1983) NLO formula.
- To match this requirement PHOTOS has to read in events from appropriately orienting Born level events generator.
- The fully differential distribution from MUSTRAAL (used also in KORALZ for single photon mode) reads:

$$X_f = \frac{Q'^2 \alpha (1-\Delta)}{4\pi^2 s} s^2 \quad \left\{ \frac{1}{(k'_+ k'_-)} \left[\frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u) \right] \right\}$$

- Here:

$$
\begin{aligned}
s &= 2p_+ \cdot p_-, \quad & s' &= 2q_+ \cdot q_-, \\
t &= 2p_+ \cdot q_+, \quad & t' &= 2p_+ \cdot q_-, \\
u &= 2p_+ \cdot q_-, \quad & u' &= 2_- \cdot q_+, \\
k'_\pm &= q_\pm \cdot k, \quad & x_k &= 2E_\gamma/\sqrt{s}
\end{aligned}
$$

- The Δ term is responsable for final state mass dependent terms, p_+, p_-, q_+, q_-, k denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.

- after trivial manipulation it can be written as:

$$
X_f = \frac{Q'^2\alpha(1-\Delta)}{4\pi^2 s} s^2 \quad \left\{ \tfrac{1}{(k'_+ + k'_-)} \tfrac{1}{k'_-} \left[\tfrac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \tfrac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u) \right] \right.
$$

$$
\left. + \tfrac{1}{(k'_+ + k'_-)} \tfrac{1}{k'_+} \left[\tfrac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \tfrac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u) \right] \right\}
$$

- In PHOTOS the following expression is used:

$$X_f^{PHOTOS} = \frac{Q'^2\alpha(1-\Delta)}{4\pi^2 s} s^2 \Bigg\{$$

$$\frac{1}{k'_+ + k'_-}\frac{1}{k'_-} \left[(1+(1-x_k)^2)\frac{\mathrm{d}\sigma_B}{d\Omega}\left(s, \frac{s(1-\cos\Theta_+)}{2}, \frac{s(1+\cos\Theta_+)}{2}\right)\right]\frac{(1+\beta\cos\Theta_\gamma)}{2}$$

$$+\frac{1}{k'_+ + k'_-}\frac{1}{k'_+} \left[(1+(1-x_k)^2)\frac{\mathrm{d}\sigma_B}{d\Omega}\left(s, \frac{s(1-\cos\Theta_-)}{2}, \frac{s(1+\cos\Theta_-)}{2}\right)\right]\frac{(1-\beta\cos\Theta_\gamma)}{2}\Bigg\}$$

$$\mathrm{where}: \Theta_+ = \angle(p_+, q_+),\ \Theta_- = \angle(p_-, q_-)$$

$\Theta_\gamma = \angle(\gamma, \mu^-)$ are defined in (μ^+, μ^-)-pair rest frame

- The two expressions define weight to make with PHOTOS complete first order.

- Here we give up the results of 1991 work !

The differences are important

- The two expressions define weight to make out of PHOTOS complete first order.
- The PHOTOS expression separates (i) Final state bremsstrahlung (ii) electroweak parameters of the Born Cross section (iii) Initial state bremsstrahlung that is orientation of the spin quantization axix for Z.
- The constraints can be overcomed but the price is discipline of event record.
- Of course all this has to be understood in context of Leading Pole approximaition. For example initial-final state interference breaks the simplification. Limitations need to be controlled: Phys.Lett.B219:103,1989
- These trivial formulas have a lot do do with this why we have nice picture when up to two unintegrated PDF are used.
- Phase space and NLO ME match that nicely.

Now results of tests:

$Z \rightarrow \mu^+\mu^-$ standard PHOTOS vs. Matrix Element.

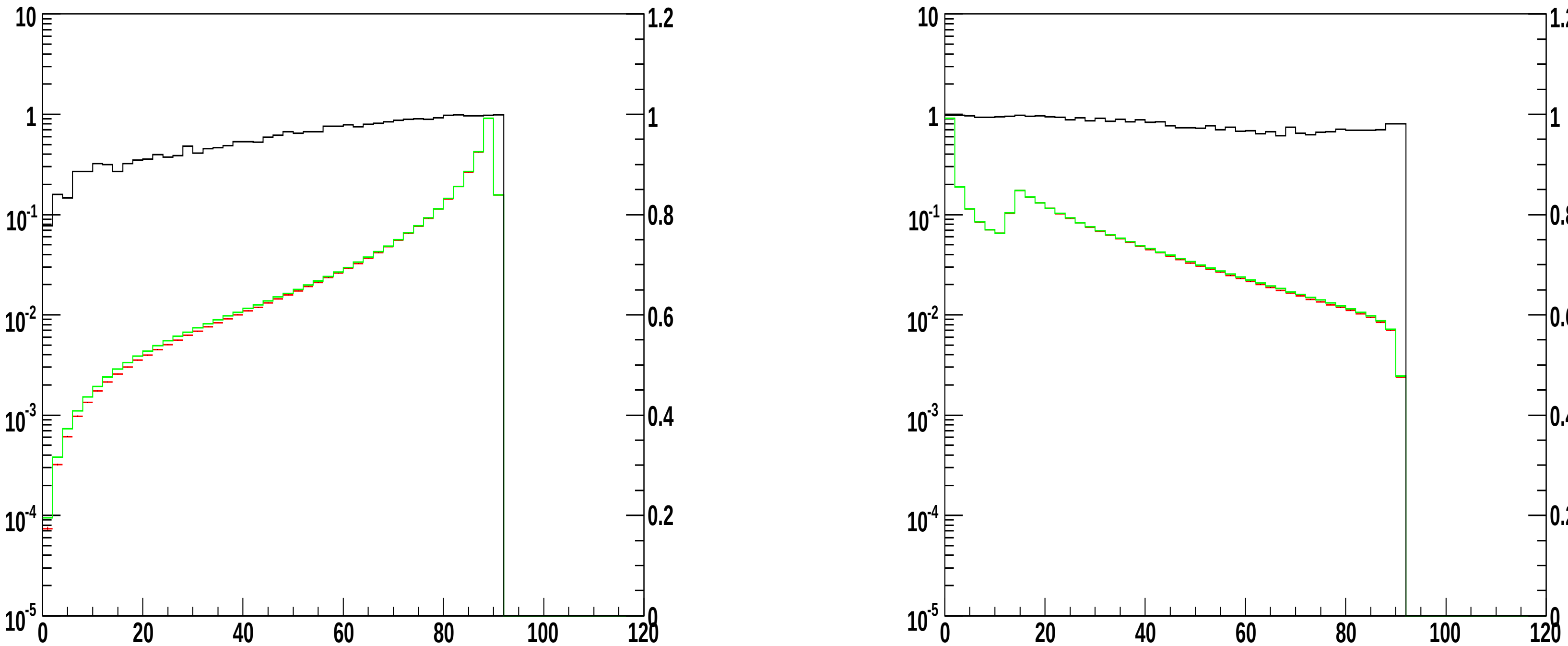


Figure1: Comparison of standard PHOTOS and KORALZ: single photon emission level. On the left hand side the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.00534. On right hand side the invariant mass of $\mu^-\gamma$; SDP=0.00296. The distributions for $\mu^+\gamma$ are identical to $\mu^-\gamma$. The histograms produced by the two programs (logarithmic scale) and their ratio (linear scale, black line) are plotted on both figures. Test1, as defined in Section 3, is used, overall SDP=0.00534, fraction of events with hard photon was 17.4863 $\pm$ 0.0042% for KORALZ and 17.6378 $\pm$ 0.0042% for PHOTOS.

$Z \to \mu^+\mu^-$ PHOTOS vs. Matrix Element.

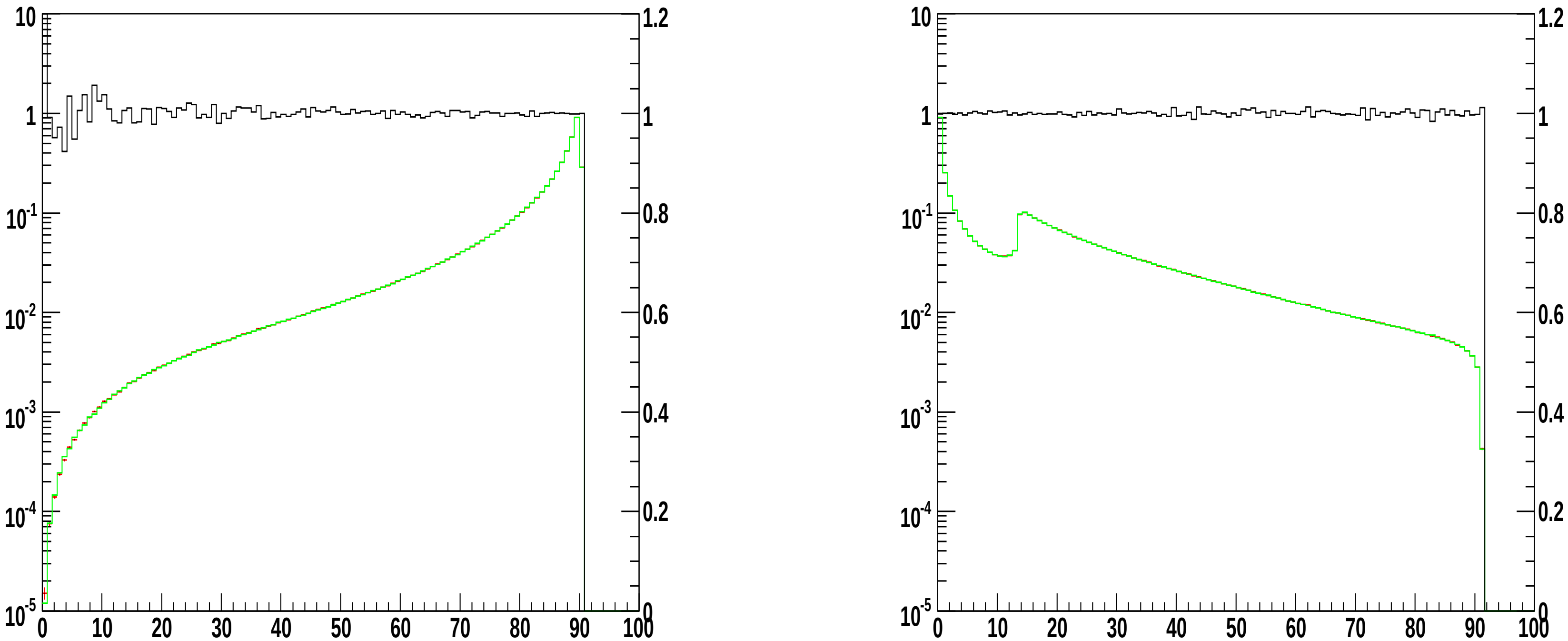


Figure2 Comparisons of improved PHOTOS and KORALZ: single photon emission level. On the left hand side the invariant mass of the $\mu^+\mu^-$ pair. On right hand side the invariant mass of $\mu^-\gamma$ is shown. In both cases differences between PHOTOS and KORALZ are below statistical error. As in Fig 1 distributions for $\mu^+\gamma$ are skipped. Test1, as defined in Section 3, is used, overall SDP=0.0, fraction of events with hard photon was $17.4890 \pm 0.0042\%$ for KORALZ and $17.4926 \pm 0.0042\%$ for PHOTOS.

Bremssthahlung in B decays into pair of scalars

- **The work presented above was an excellent test of technical side of PHOTOS, but not of its universality.**
- **Let us now turn to decays like $B^- \to K^-\pi^0$ or $B^0 \to K^+\pi^-$**
- **In thiese cases threshold mass terms are essential.**
- **Scalar QED can be used for reference calculation, but of course final answer require measured form factors.**
- **I will present work of G. Nanava and Z. Was hep-ph/0607019, based on SANC system by D. Bardin et al.**
- **The matrix element for $B^- \to K^-\pi^0$ read:**
  $d\Gamma^{\,\mathrm{Hard}} = \frac{1}{2\,M}|A^{\mathrm{Born}}|^2 4\pi\alpha \left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q \frac{P.\epsilon}{P.k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$,
- **The matrix element for $B^0 \to K^+\pi^-$ read:**
  $d\Gamma^{\,\mathrm{Hard}} = \frac{1}{2\,M}|A^{\mathrm{Born}}|^2 4\pi\alpha \left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q_2 \frac{k_2.\epsilon}{k_2.k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma),$
- **Renormalization scale is set such that total scalar QED corrections to the width is zero.**

Figure 1: Results from PHOTOS, standard version, and SANC for $B^- \to \pi^0 K^-(\gamma)$ decay are superimposed on the consecutive plots. Standard distributions, as defi ned in the text, are used. Logarithmic scales are used. The distributions from the two plots overlap almost completely. Samples of 10^9 events were used.

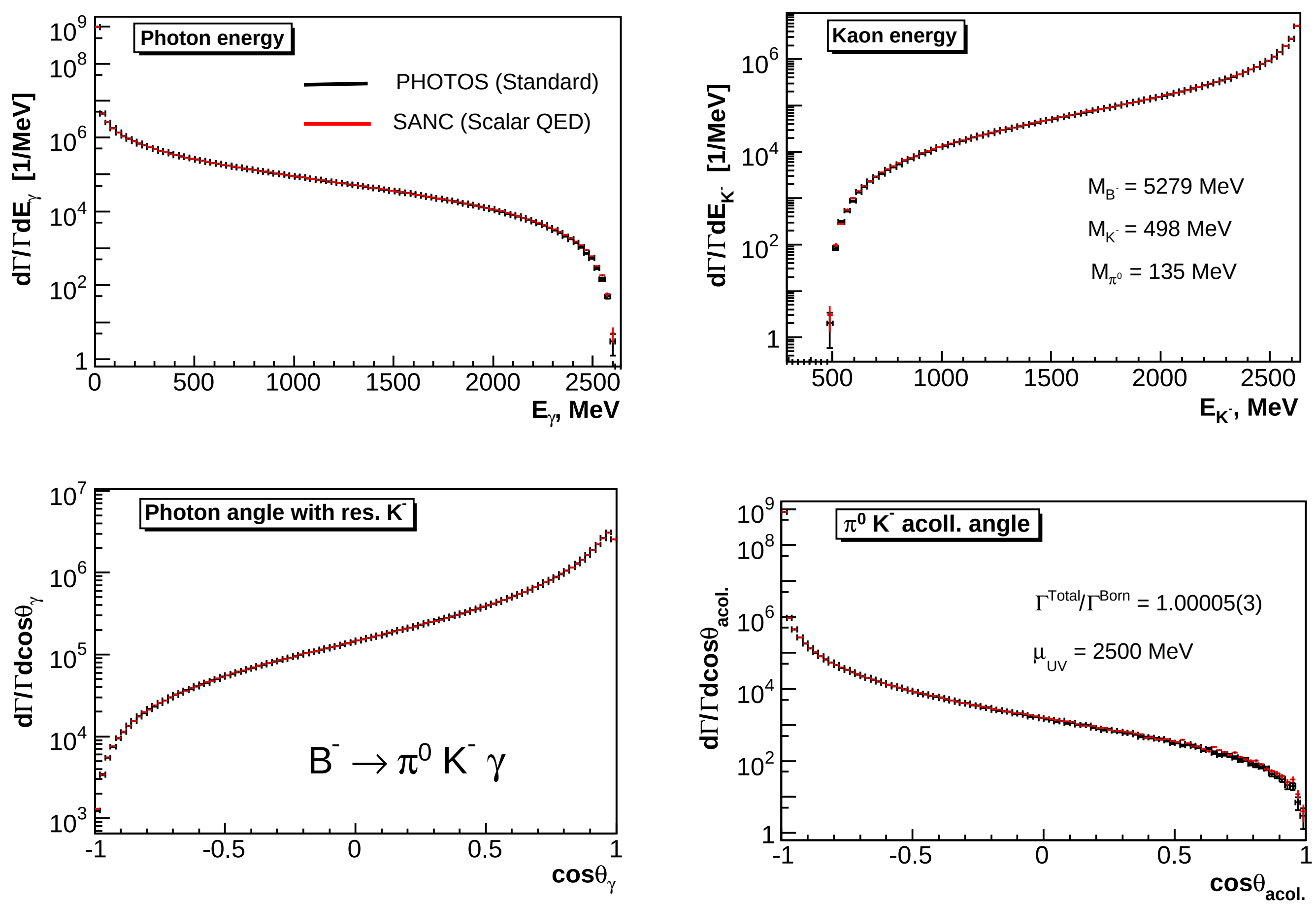


Figure 2: Results from PHOTOS, standard version, and SANC for ratios of the $B^- \to \pi^0 K^-(\gamma)$ distribution in fig.1 are presented. Differences between PHOTOS and SANC are small, but are clearly visible now.

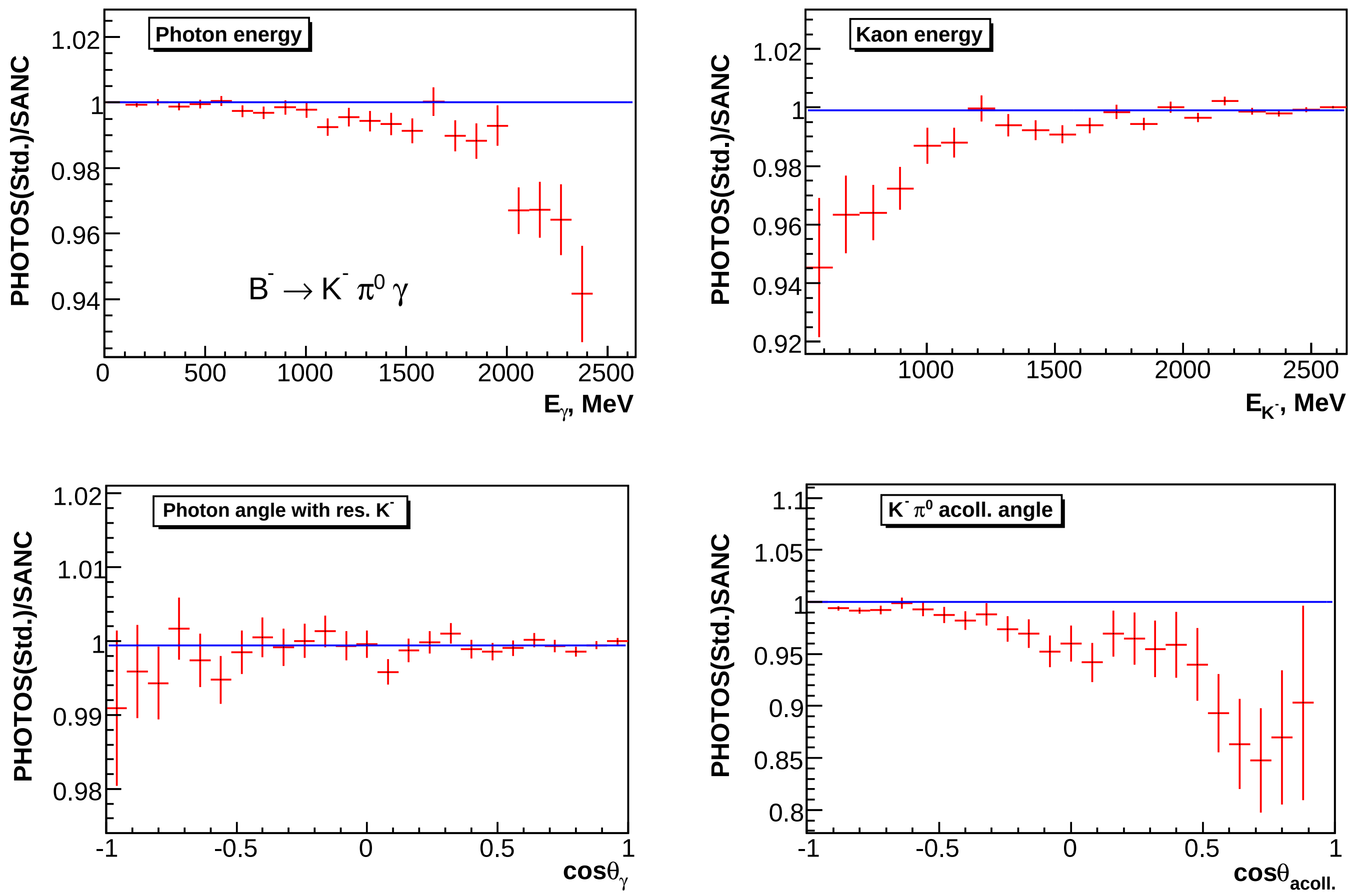


Figure 3: Results from PHOTOS with the exact matrix element, and SANC for ratios of the $B^- \to \pi^0 K^-(\gamma)$ distributions. Differences between PHOTOS and SANC are below statistical error for samples of 10^9 events.

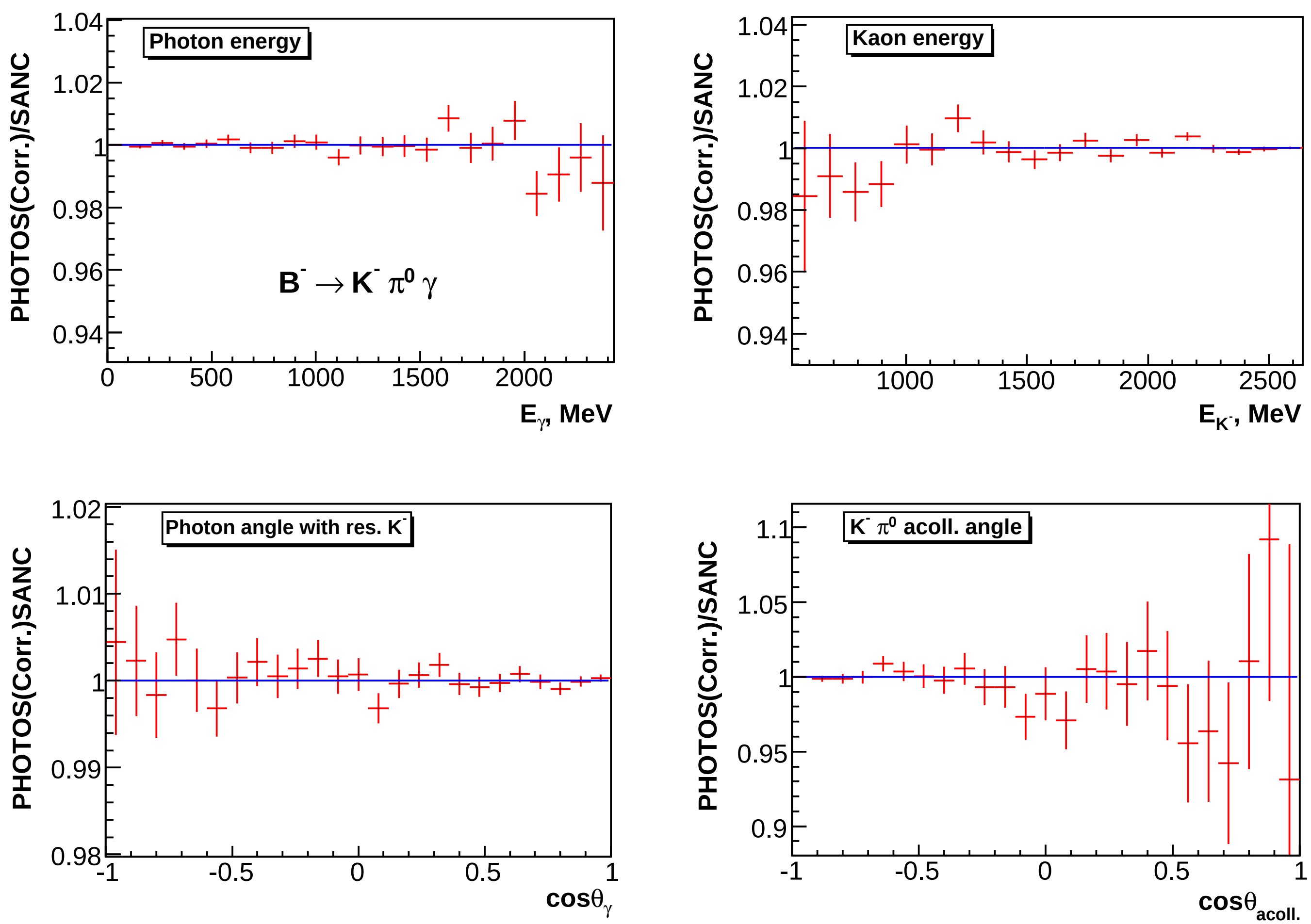


Figure 4: Results from PHOTOS, standard version, and SANC for $B^0 \to \pi^- K^+(\gamma)$ decay are superimposed on the consecutive plots. Standard distributions, as defi ned in the text, are used. Logarithmic scales are used. The distributions from the two plots overlap almost completely. Samples of 10^9 events were used.

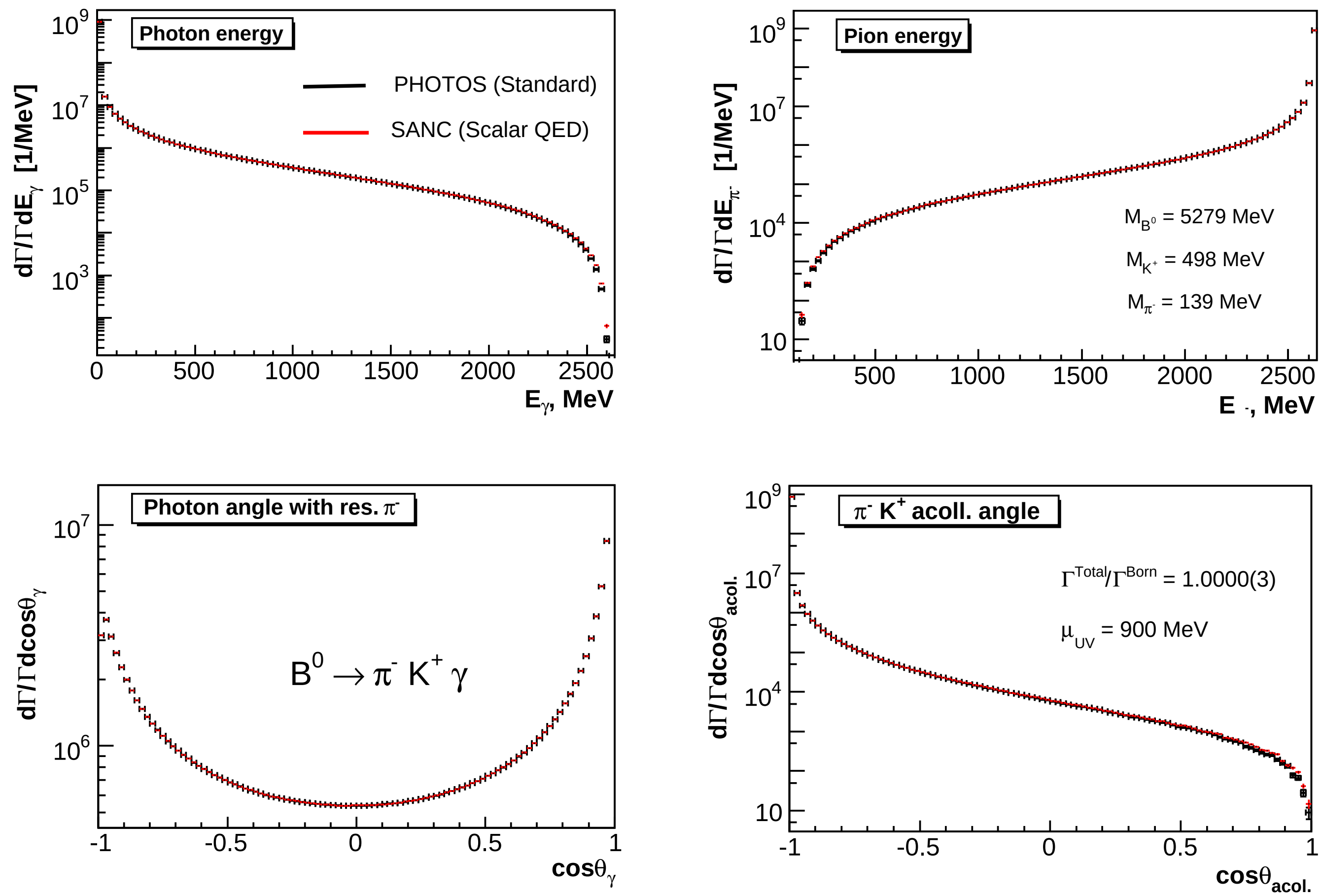


Figure 5: Results from PHOTOS, standard version, and SANC for ratios of the $B^0 \to \pi^- K^+(\gamma)$ distributions in fi g.4 are presented. Differences between PHOTOS and SANC are small, but are clearly visible now.

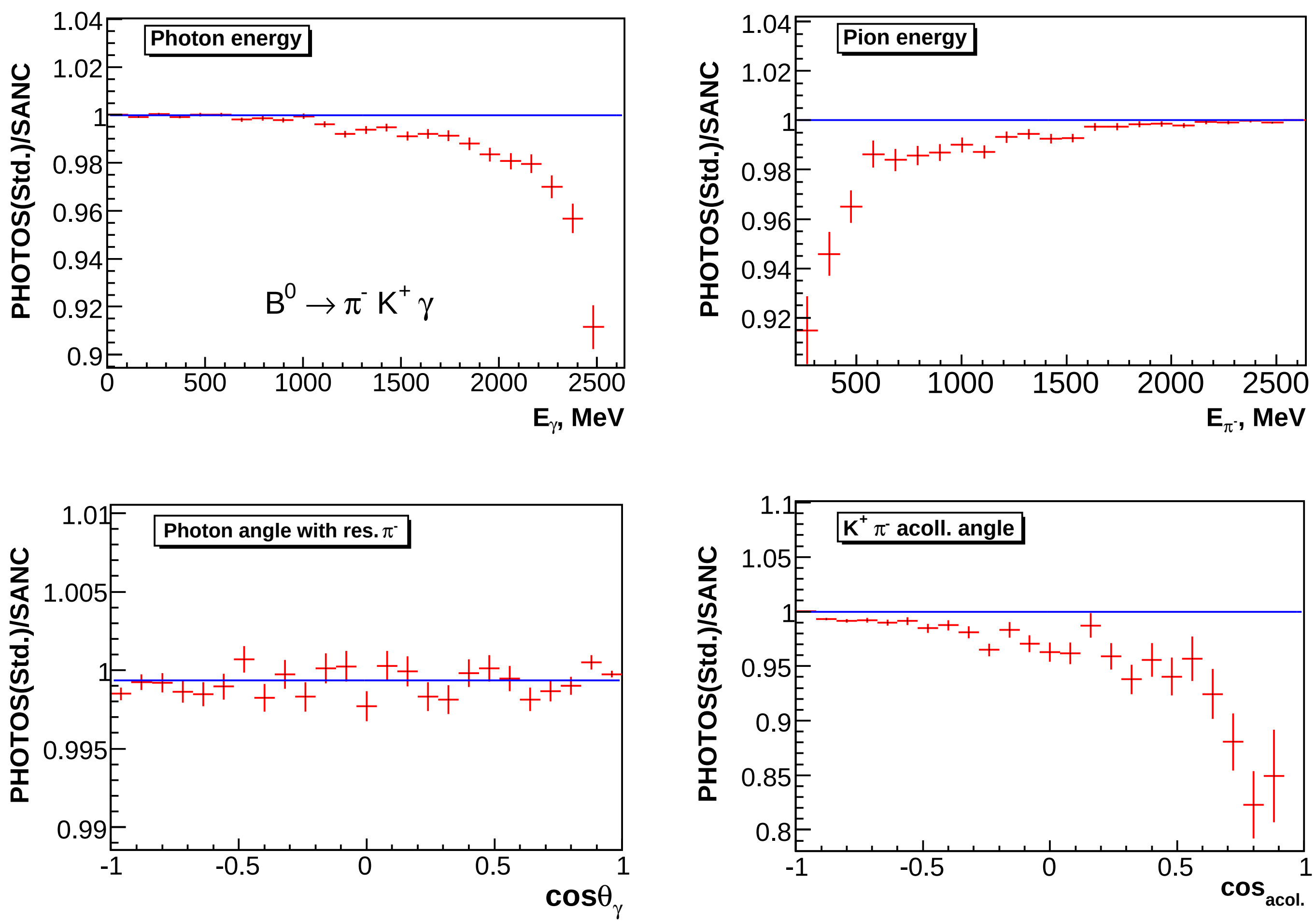
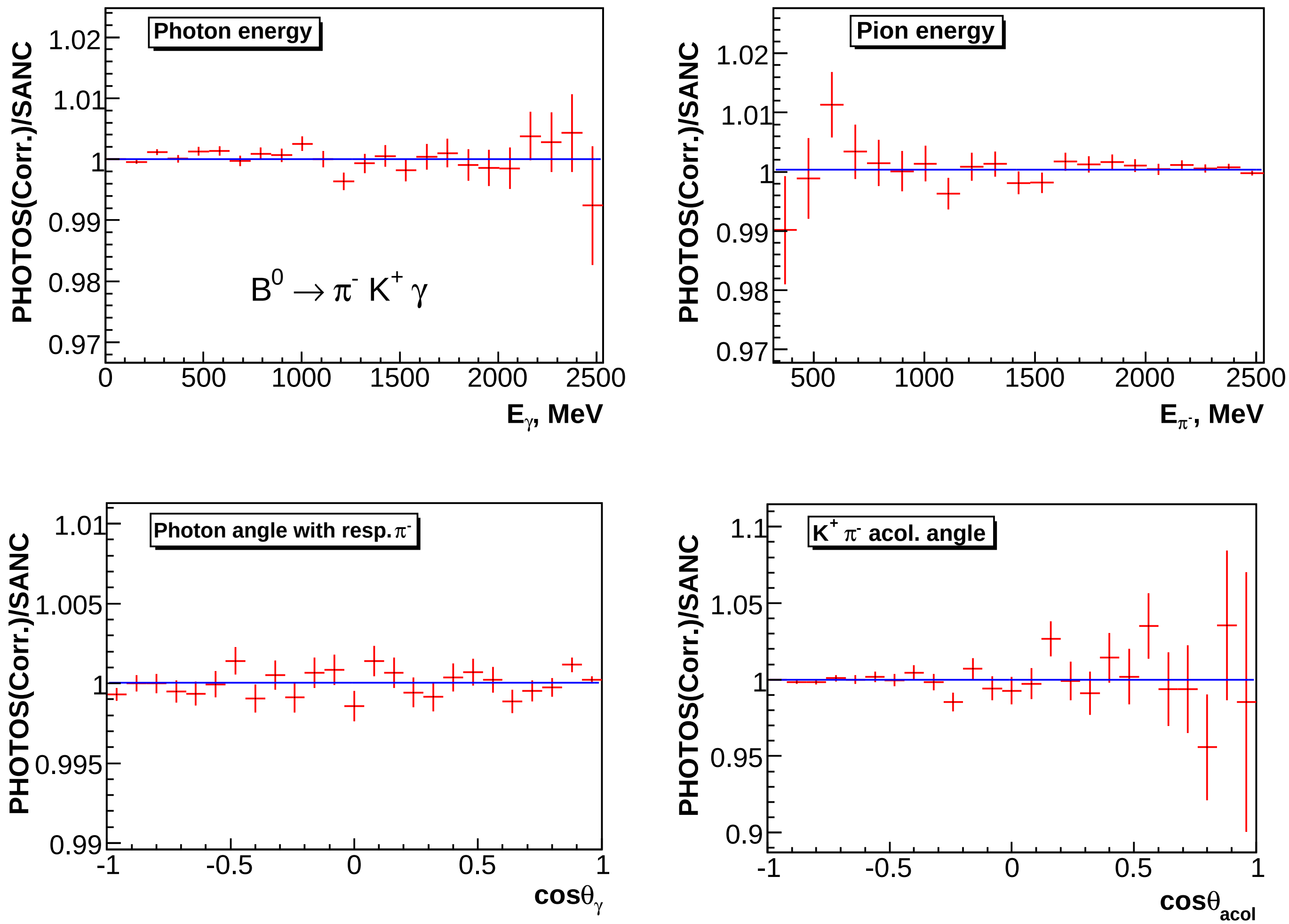


Figure 6: Results from PHOTOS with the exact matrix element, and SANC for ratios of the $B^0 \to \pi^- K^+(\gamma)$ distributions. Differences between PHOTOS and SANC are below statistical error for samples of 10^9 events.



Test of higher orders

$Z \to \mu^+\mu^-$ PHOTOS vs. Matrix Element. γ, γ conf.

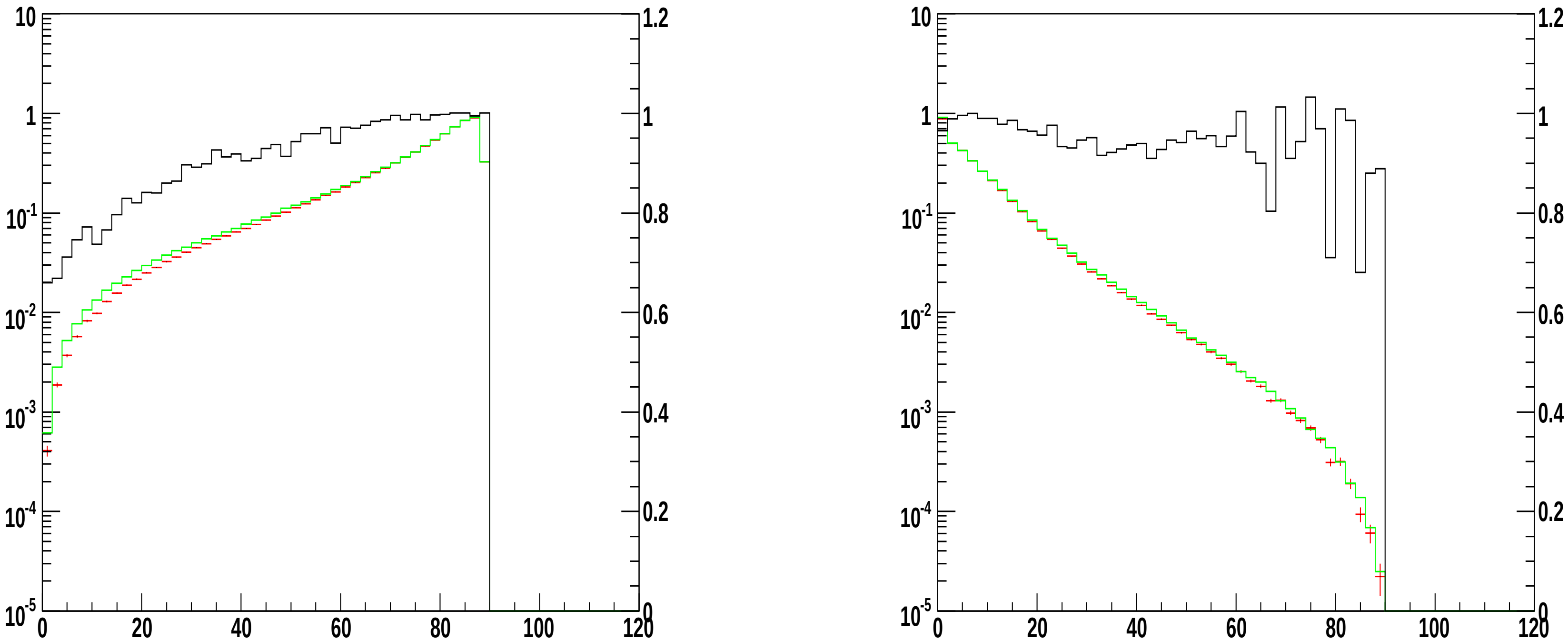


Figure5: Comparisons of standard PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. Two comparison figures of worst agrement were selected from 2 hard photon configurations. On the left hand side the invariant mass of the $\mu^+\mu^-$ pair is shown; SDP= 0.00918. On the right hand side the invariant mass of the $\gamma\gamma$ pair; SDP=0.00268. Test2, as defined in Section 3, is used, overall SDP=0.00918, fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KORALZ and 1.2952 $\pm$ 0.0011% for PHOTOS.

$Z \to \mu^+\mu^-$ NLO PHOTOS vs. Matrix Element γ, γ conf.

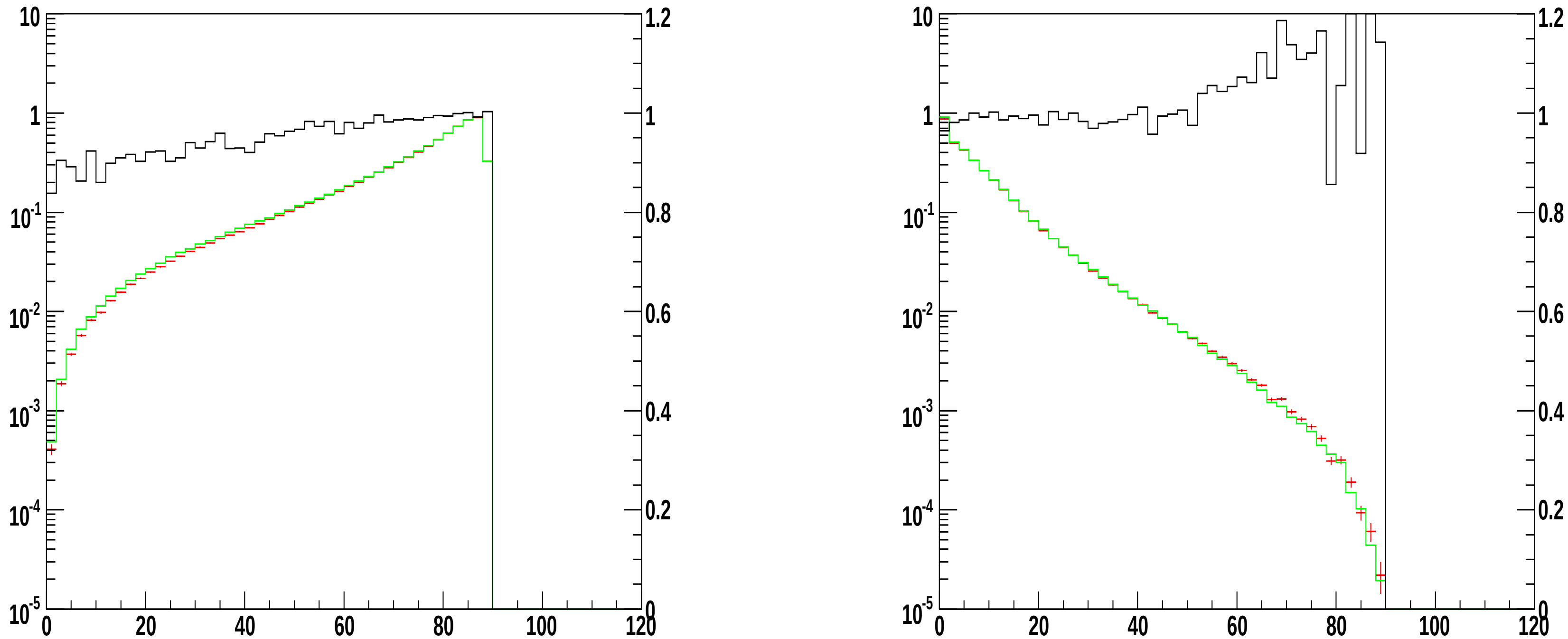
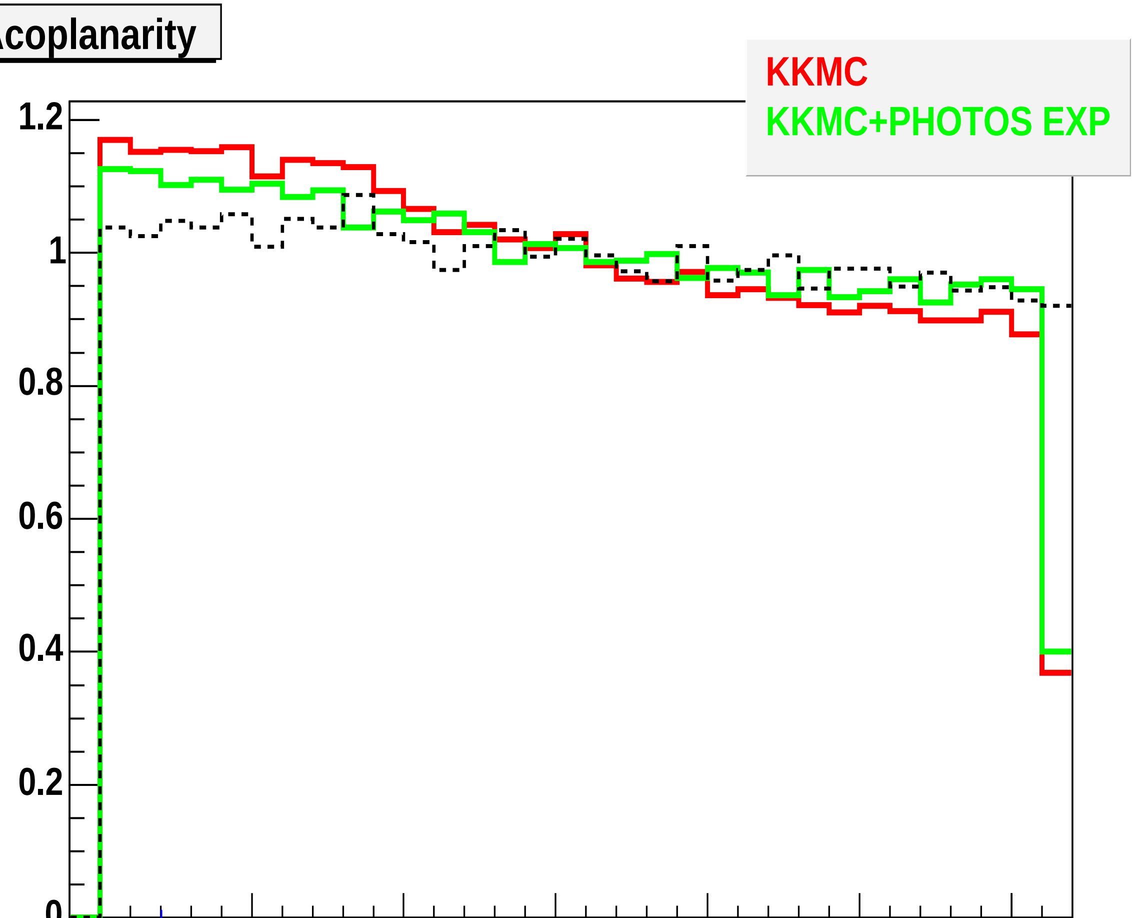


Figure6: Comparisons of improved PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. Two comparison figures of worst agreement were selected from two-hard-photon configurations. On the left hand side the invariant mass of the $\mu^+\mu^-$ pair is shown; SDP= 0.00142. On the right hand side the invariant mass of the $\gamma\gamma$; SDP=0.00293. Test2, as defined in Section 3, was used, overall SDP= 0.00293, fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KORALZ and 1.2868 $\pm$ 0.0011% for PHOTOS.

Acoplanarity distribution – Looks good



Two plane spanned on μ^+ and respectively two hardest photons localized in the same hemisphere μ^+. as Why PHOTOS works so good? KKMC with ME truncated to first order give flat distribution! Answer would require laborious work on ME it was done half-way in ref. Eur. Phys. J. C **44** (2005) 489 for $e^+e^- \to \nu_e \bar{\nu}_e \gamma\gamma$. How to create gauge dependent terms from gauge invariant objects?

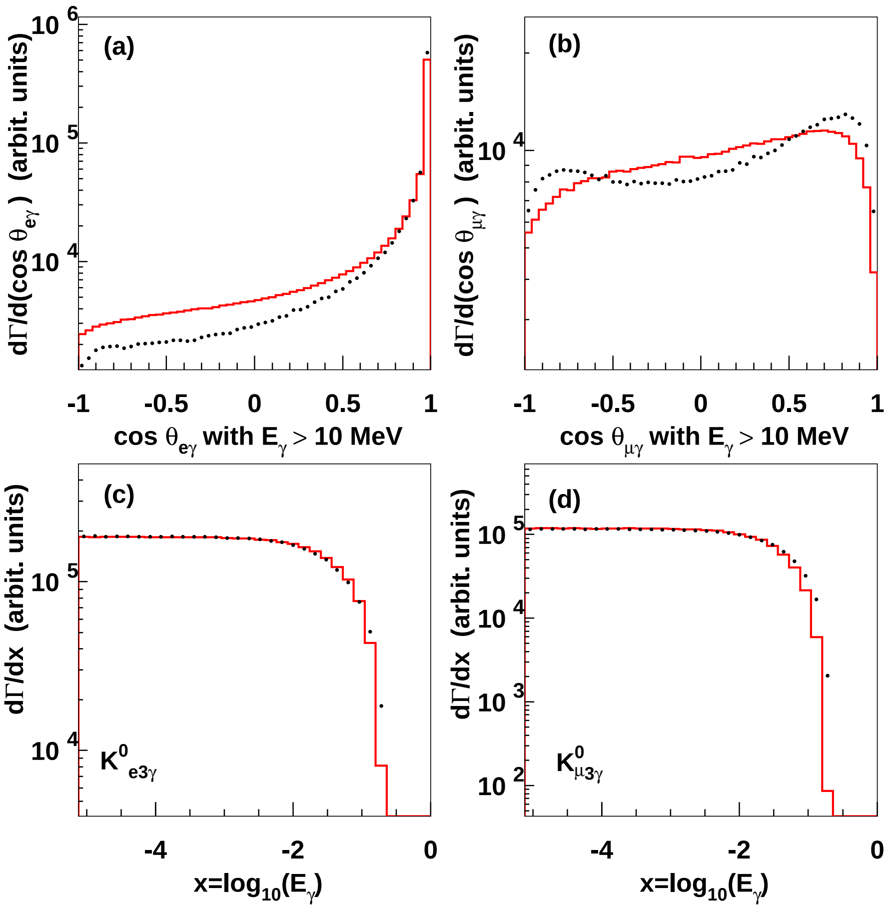
Low quality tests when there are more than two sources

PHOTOS was used by BaBar for up to 10 charged particles in final state

language for QCD?

$K \to \pi l \nu$ in KLOR and PHOTOS: hep-ph:0406006

only on 28 December 2004 we realized that PHOTOS is used for K decays and precision is not suffi cient. Even though, program works not worse than expected.



(a) $\cos(\Theta_{\gamma,l})\ K_{\mu 3}$

(b) $\cos(\Theta_{\gamma,l})\ K_{e3}$

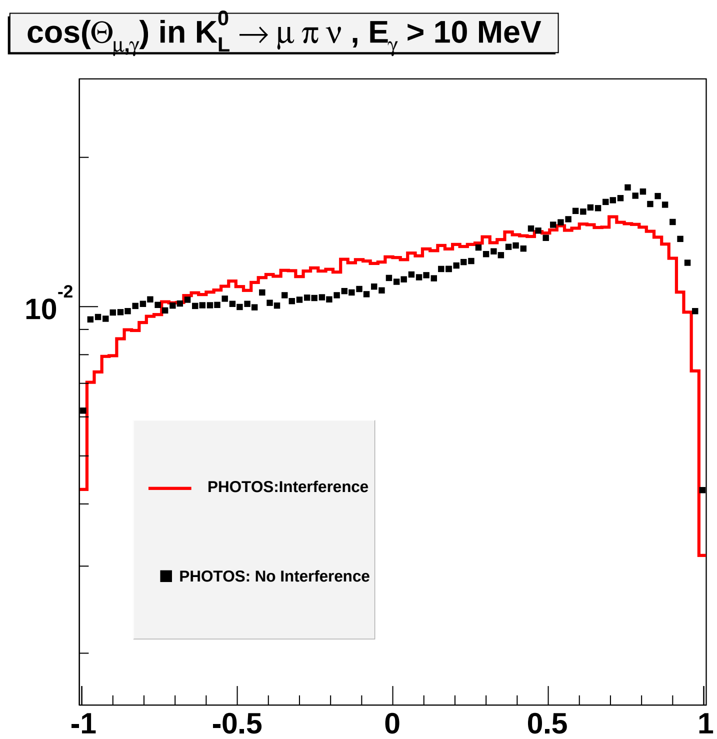
(c) $\log_{10}(E_\gamma)\ K_{\mu 3}$

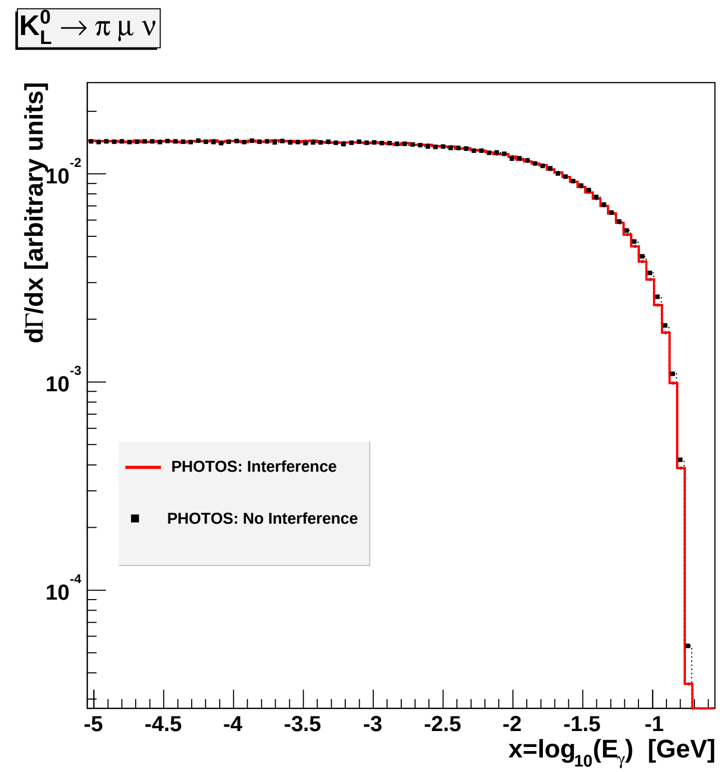
(d) $\log_{10}(E_\gamma)\ K_{e3}$

in KLOR

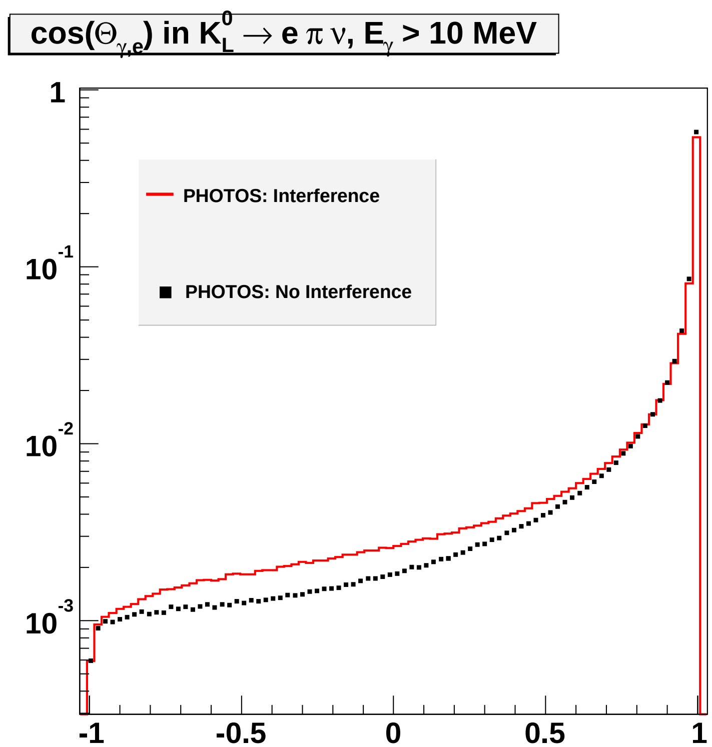
and PHOTOS

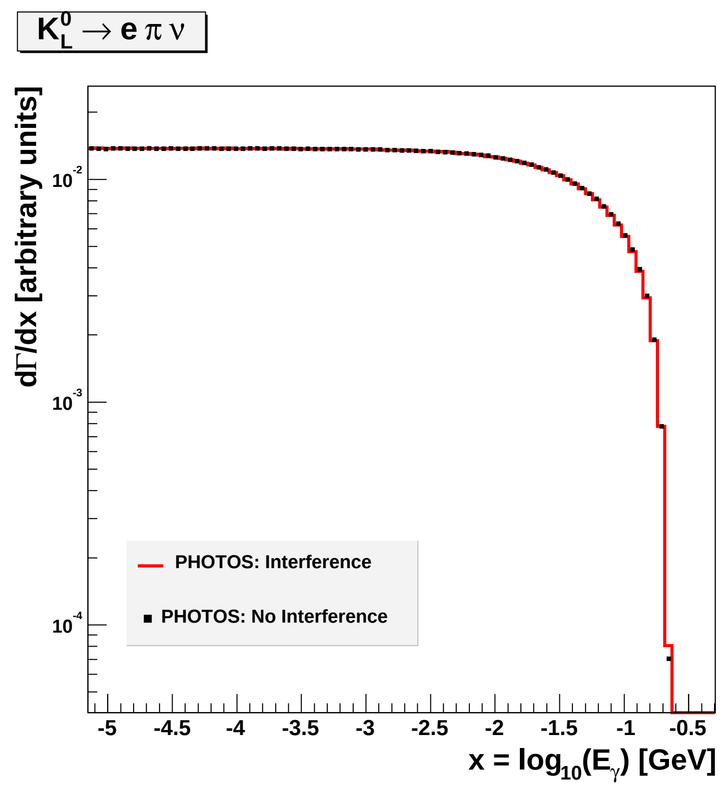
$K \to \pi\mu\nu$ + PHOTOS bremsstrahlung, interference on/off





$K \longrightarrow \pi e \nu$ + PHOTOS bremsstrahlung, interference on/off

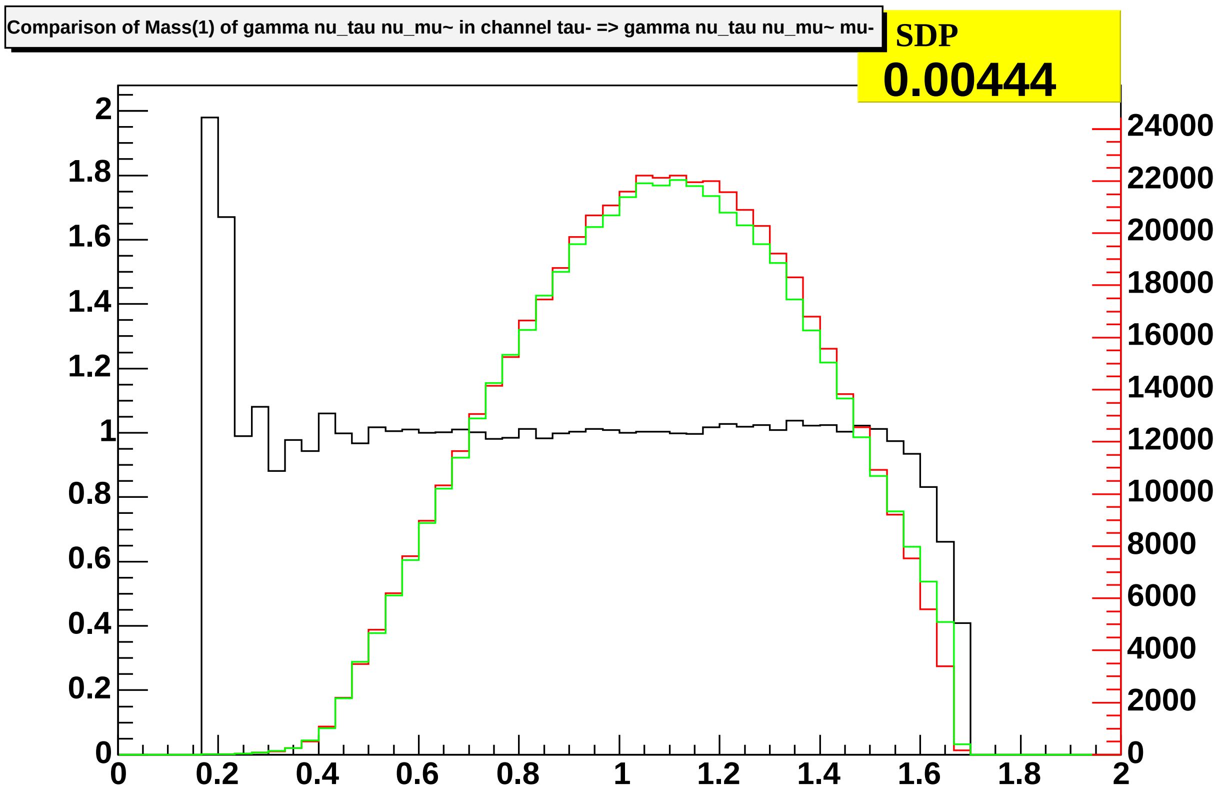




Seems that the interference weight removed the difference to a large degree, but still some inconsistencies at $\cos\Theta \simeq -1$

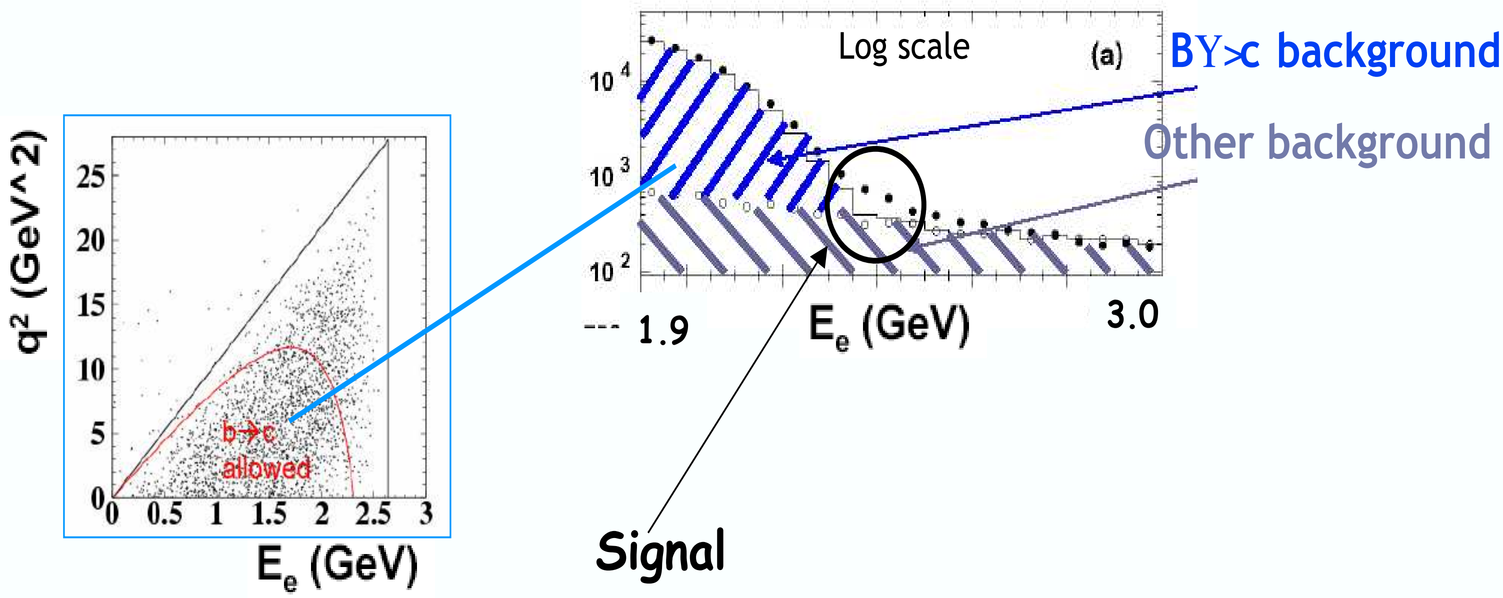
$\tau \to l\nu\bar{\nu}$ PHOTOS vs TAUOLA

Plot of largest difference (basically muon energy spectrum):



The difference in branching ratios are at fraction of permile level. No work on kernel adjustment was performed, agreement was more than sufiicient for phenomenology.

electron endpoint



Systematic uncertainty:
Model dependent signal efficiencies
B_XcℓY background estimation

Phys. Lett. B 621, 28 (2005)

$$\Delta Br(X_u l \nu) = \frac{N(X_u l \nu)}{2 N_{BB}\, \varepsilon_{MC}}$$

$$|V_{ub}| = \sqrt{\frac{(1+\delta_{rad}) \times \Delta Br(X_u l \nu)}{\tau_B} \frac{1}{R}}$$

QED radiative corrections: $b \to u$ endpoint

Lepton momentum region (GeV) In the $\Upsilon(4s)$ frame	PHOTOS correction to partial branching fraction measurement of $b \to u \ell \nu$
1.9-2.6	1.060±0.007
2.2-2.6	1.086±0.010
2.3-2.6	1.096±0.011
2.4-2.6	1.107±0.014

Conclusion on technical tests of PHOTOS

- PHOTOS work exellently for statistical samples of up to 10^9 events
- Matrix element used in generation is explicitly localized
- Even though in case of scalars as decay products, results means not much direct progress in itself, possibility to play with measurements of form-factors opens.
- Standard PHOTOS is most probably sufficient for many years to go.
- In case of doubts or 'big' measuremens technique of validation is prepared.
- Let us go now to technicalities of phase space
- It is an essential step before even talking about extensions to QCD.
- But similar tests for at least 3 body decays with QED interferences would be better!! The ones presented here are physically and technically not sufficient.
- Extra (hopefully non-existing) complications could be ruled out if studies with matrix elements and full phase space control would be completed.

Phase Space: (trivialities but written in quite general form)

Let us recall the element of Lorentz-invariant phase space (*Lips*):

$$
\begin{aligned}
& dLips_{n+1}(P) = \\
& \frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3} \frac{d^3q}{2q^0(2\pi)^3}(2\pi)^4\delta^4\Big(P - \sum_1^n k_i - q\Big) \\
= \quad & d^4p\delta^4(P-p-q)\frac{d^3q}{2q^0(2\pi)^3}\frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3}(2\pi)^4\delta^4\Big(p - \sum_1^n k_i\Big) \\
= \quad & d^4p\delta^4(P-p-q)\frac{d^3q}{2q^0(2\pi)^3} dLips_n(p \to k_1 ... k_n).
\end{aligned}
$$

Integration variables, the four-vector p, compensated with $\delta^4\left(p - \sum_1^n k_i\right)$, and another integration variable M_1 compensated with $\delta\left(p^2 - M_1^2\right)$ are introduced.

Phase Space: (cont.)

$$dLips_{n+1}(P) =$$
$$= \quad \left[k_\gamma dk_\gamma d\cos\theta d\phi \frac{1}{2(2\pi)^3} \right] \times dLips_n(p \to k_1...k_n).$$

If we had l photons accompanying n other particles, the factor in square brackets would be iterated. A statistical factor $\frac{1}{l!}$ would complete the formula for the phase-space parametrization, which is quite similar to the formal expansion of the exponent.

Replace $dLips_n(p \to k_1...k_n)$ in above formula by $dLips_n(P \to k_1...k_n)$ and we obtain a **tangent space**. Photons do not affect other particles' momenta at all. Have no boundaries on energy and are independent one from another.

This expression would be only slightly more complicated if instead of photon massive particle was to be added.

Phase Space: (cont.)

$$dLips_{n+1}(P) =$$
$$\left[4dk_\gamma k_\gamma d\cos\theta d\phi \frac{1}{8(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/p^2, M_{2\ldots n}^2/p^2)}{\lambda^{1/2}(1, m_1^2/P^2, M_{2\ldots n}^2/P^2)}\right]$$
$$\times dLips_n(P \to \bar{k}_1 \ldots \bar{k}_n)$$

The formula should read as follow:

1. Take the distribution of n-body phase space
2. Turn it back into some coordinate variables; choose two sub-groups: here 1 and 2...n.
3. add newly generated variables for photon accordingly to expr. in sqare bracket.
4. construct new kinematical confi guration from all variables. If we had l photons accompanying n other particles, the factor in square brackets would be iterated. A statistical factor $\frac{1}{l!}$ would complete the formula for the phase-space parametrization, which is quite similar to the formal expansion of the exponent.

more detailed description, axes still undef. !

- The differential partial width for the decay into 4+1 particle read:

$$d\Gamma_X = \frac{1}{4M} d\mathrm{Lips}(P; q_i, N)\omega$$

- The phase space distribution is given by the following expression where a compact notation with $q_5 = N$ and $q_i^2 = m_i^2$ is used

$$d\mathrm{Lips}(P; q_1, q_2, q_3, q_4, q_5) = \frac{1}{2^{19}\pi^8 2^4\pi^3} \int_{Q^2_{min}}^{Q^2_{max}} dQ^2 \int_{Q^2_{3,min}}^{Q^2_{3,max}} dQ_3^2$$

$$\int_{Q^2_{2,min}}^{Q^2_{2,max}} dQ_2^2 \quad \times \quad \int d\Omega_5 \frac{\sqrt{\lambda(M^2,Q^2,0)}}{M^2} \int d\Omega_4 \frac{\sqrt{\lambda(Q^2 M^2, Q_3^2, m_4^2)}}{Q^2 M^2}$$

$$\times \quad \int d\Omega_3 \frac{\sqrt{\lambda(Q_3^2, Q_2^2, m_3^2)}}{Q_3^2} \int d\Omega_2 \frac{\sqrt{\lambda(Q_2^2, m_2^2, m_1^2)}}{Q_2^2}$$

$$Q^2 = (q_1 + q_2 + q_3 + q_4)^2, \quad Q_3^2 = (q_1 + q_2 + q_3)^2, \quad Q_2^2 = (q_1 + q_2)^2$$

$$Q_{min} = m_1 + m_2 + m_3 + m_4, \quad Q_{max} = M,$$

$$Q_{3,min} = m_1 + m_2 + m_3, \qquad Q_{3,max} = Q - m_4 \ \ (Q_{3,max} = M),$$

$$Q_{2,min} = m_1 + m_2, \qquad Q_{2,max} = Q_3 - m_3$$

- Terms in blue have to be dropped for $d\mathrm{Lips}(P; q_1, q_2, q_3, q_4)$, and terms in green used instead, the green case is for the original phase space when photon was absent.

Phase Space: (cont.)

$$dLips_{n+l}(P) =$$

$$= \frac{1}{l!}\prod_{i=1}^{l}\left[k_{\gamma_i} dk_\gamma d\cos\theta_i d\phi_i \frac{1}{2(2\pi)^3}\right] \times dLips_n(P \to k_1 ... k_n).$$

- We defi ned **tangent space**. Photons do not affect other particles' momenta. Also, have no boundaries on energy and are independent one from another.
- It is important to realize that one has to control matrix element on the tangent space to defi ne transformation to the real space. Rejection diminish photon mutiplicity.
- Rejection implements changes in phase space density *and* properties of matrix element.
- Rejection is performed photon after photon; phase space is in (principle) trivial.
- It remain to work out how the matrix element for original n particles plus photon(s) should look.

Phase Space: (cont.)

- To move from **tangent space** to exact phase space we perform mandatory rejections (if events are outside phase space boundaries).
- In other cases we perform rejection with the weigt. There are 3 parts of the weight:
- Phase space:

$$WT_1(P, k_1, k_2, k_\gamma) = \frac{\lambda^{1/2}\left(1, \frac{m_1^2}{M^2}, \frac{m_2^2}{M^2}\right)}{\lambda^{1/2}\left(1, \frac{m_1^2}{\tau}, \frac{m_2^2}{\tau}\right)} \frac{2E_\gamma}{M}$$

- Pre-sampler:

$$WT_2(P, k_1, k_2, k_\gamma) = \frac{2(1 - \cos\theta\sqrt{1 - m_R^2})}{1 + (1 - x)^2} \frac{M}{2E_\gamma}$$

- Matrix element (process dependent). As rejection decreases preliminary multiplicity it must include virtual corrections ($|\mathcal{M}|^2_{PHOTOS}$ is somewhat unnecesarily horrible ridiculous and irrelevant piece):

$$wt = \frac{|\mathcal{M}|^2_{exact}}{|\mathcal{M}|^2_{PHOTOS}} \frac{\Gamma^{\rm Born}}{\Gamma^{\rm Total}}.$$

- Some complication arise due to multi-channel generation, especially for multi emissions.

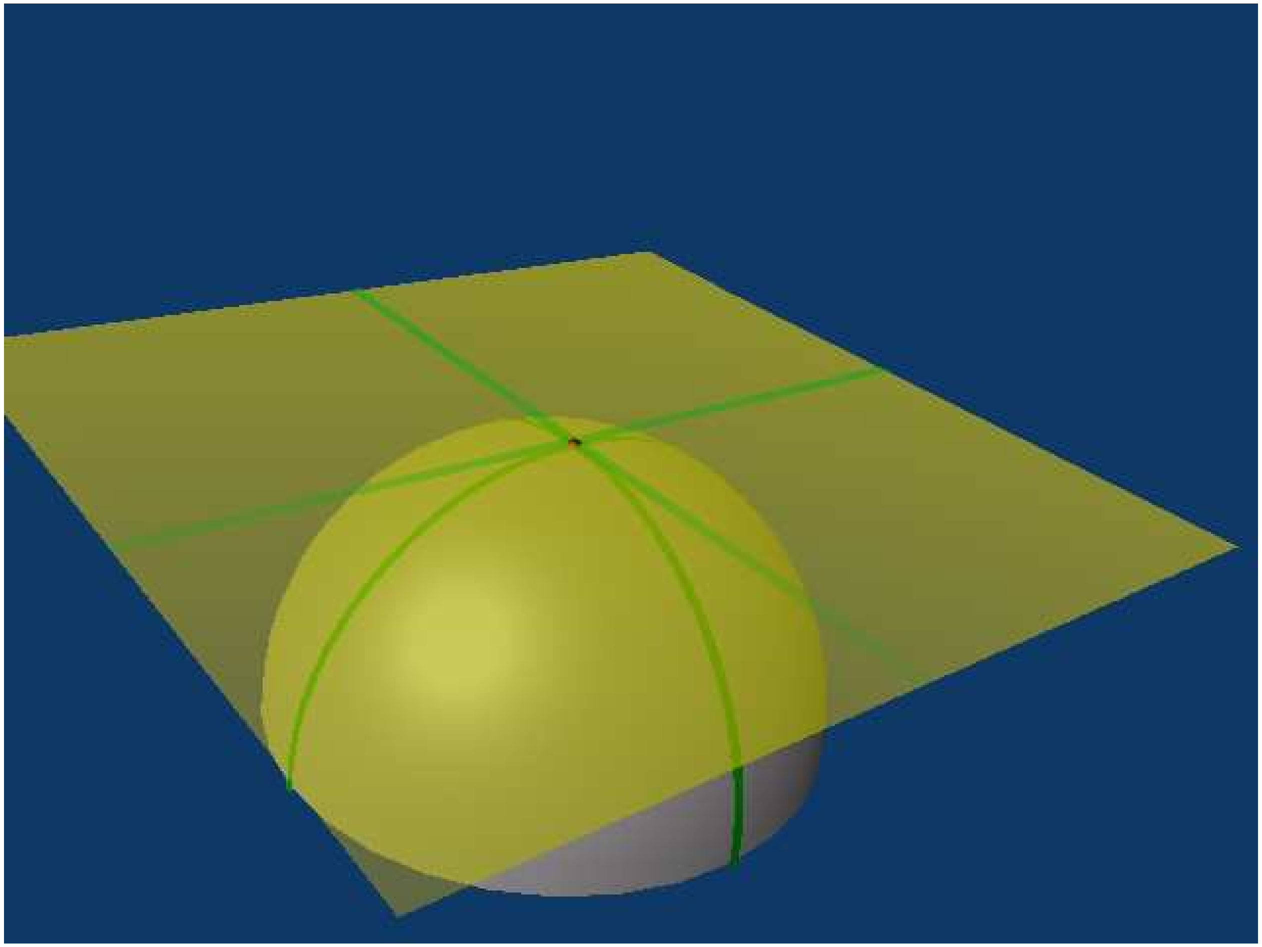
Phase Space: (what is (is not) important for multi-emissions)

- Relation between exact single emission with ME and approximate space established with explicit formulas.
- Solution can be iterated and phase space remain exact
- That would be true even if photon was replaced by massive object.
- Tangent distribution can be easily integrated analytically photon after photon. It leads to Poissonian distribution way away from final one. Huge initial multiplicities
- How arbitrary is the crude solution?
- I do not know ... There is lot of freedom.
- We start from eikonal limit with somewhat excessively large uppeer limit on phase space.
- In that limit each photon is independent from others, also virtual corrections are defined. Together with properties of phase space trivial iteration is the solution.
- Nothing prevents use of eikonal approximation beyond applicability region if it is corrected later ...
- Correcting operation is quite regular (the same structure of IR singularities), so one can use eikonal space as tangent space for building exact distributions.

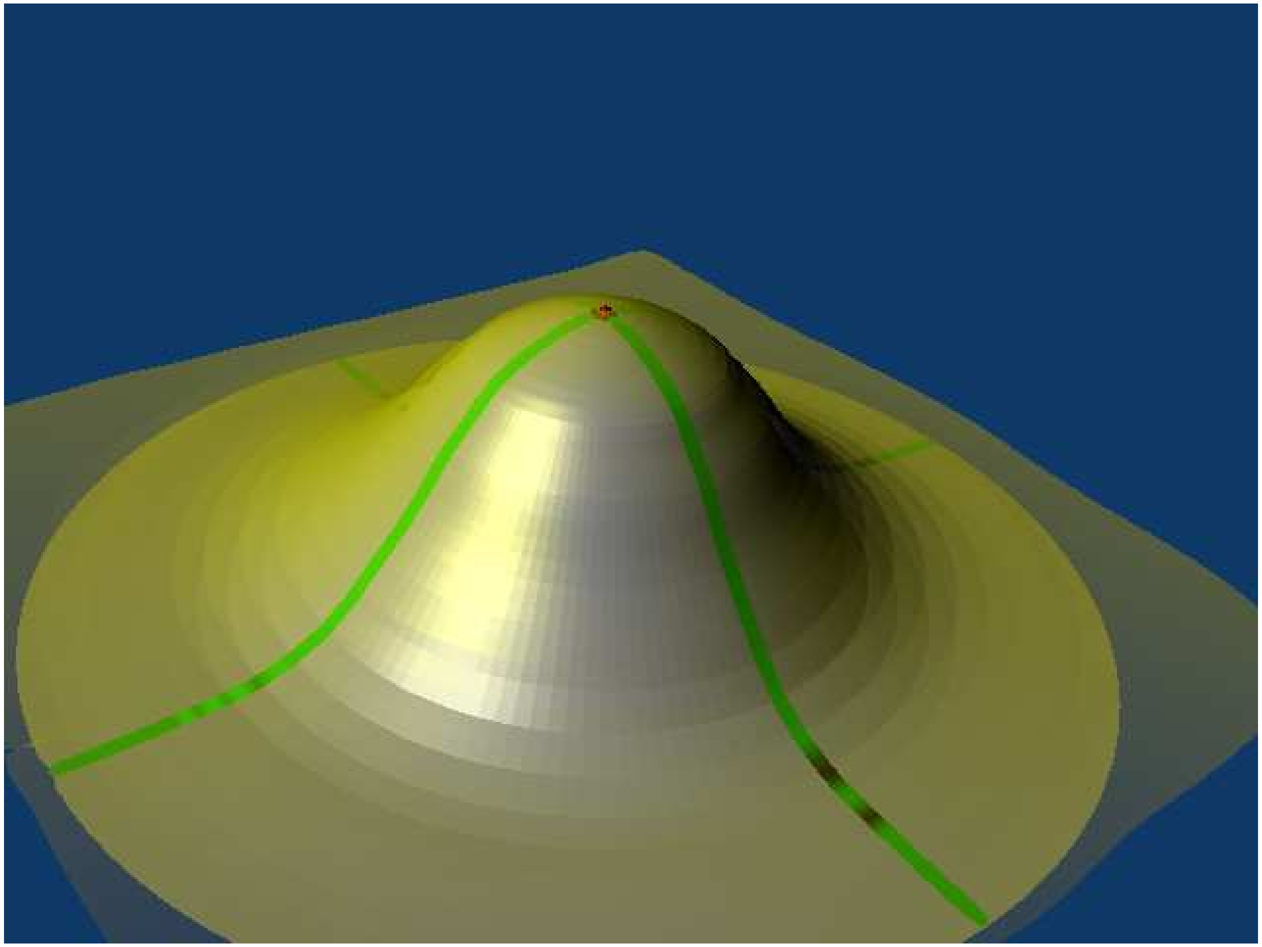
- I may have convinced you that it is possibleto calculate difference between eikonal and exact phase space and ME. For myself I did tests with 10^9 samples.
- Some freedom exist, we have used it to match constraints originating from higher order matrix elements.
- I have played a bit with two hard photons matrix elements from KKMC and earlier from Elzbieta package TOPKI and it turned out that iteration can not only be OK but may mimic some genuine higher order effects, like acoplanarity.
- Exciting toy evironment to understand origins of orderings and what are the conditions to escape such approximations.
- For vitual corrections beyond fi rst order we use sum rules from KLN
- Variant of eikonal approximation exist for QCD as well
- Many other simplifi ed ME as well
- Finally sum rules exist in plentitude
- Structure of singularities is richer than in QED defi nition of soft/collinear equivalencies and request that it should be “the same” for tangent and real space might kill the path.

1. Tangent space for multiphotons – practical approach
2. Accordingly to Poissonian distribution $P(n) = e^{-\lambda} \frac{\lambda^n}{n!}$ with its λ sufficiently large and otherwise arbitrary multiplicity for photons in tangent space is generated.
3. If fixed maximum multiplicity is requested, then crude distribution is a combination of binomial distributions, and $\lambda < 1$ is required.
4. For each photon energy and angular orientation is generated and Jacobians are calculated. Previously generated photons (if any) are treated on equal footing with all other decay products.
5. New configuration in n'+1 body phase space can be now constructed.
6. Rejection is performed individually on every photon from tangent space.
7. We skip refinements necessary due to multibranching.
8. Note also that in this way correlations between consecutive photons are introduced.
9. We skip important point: choice of frames for angular orientation.

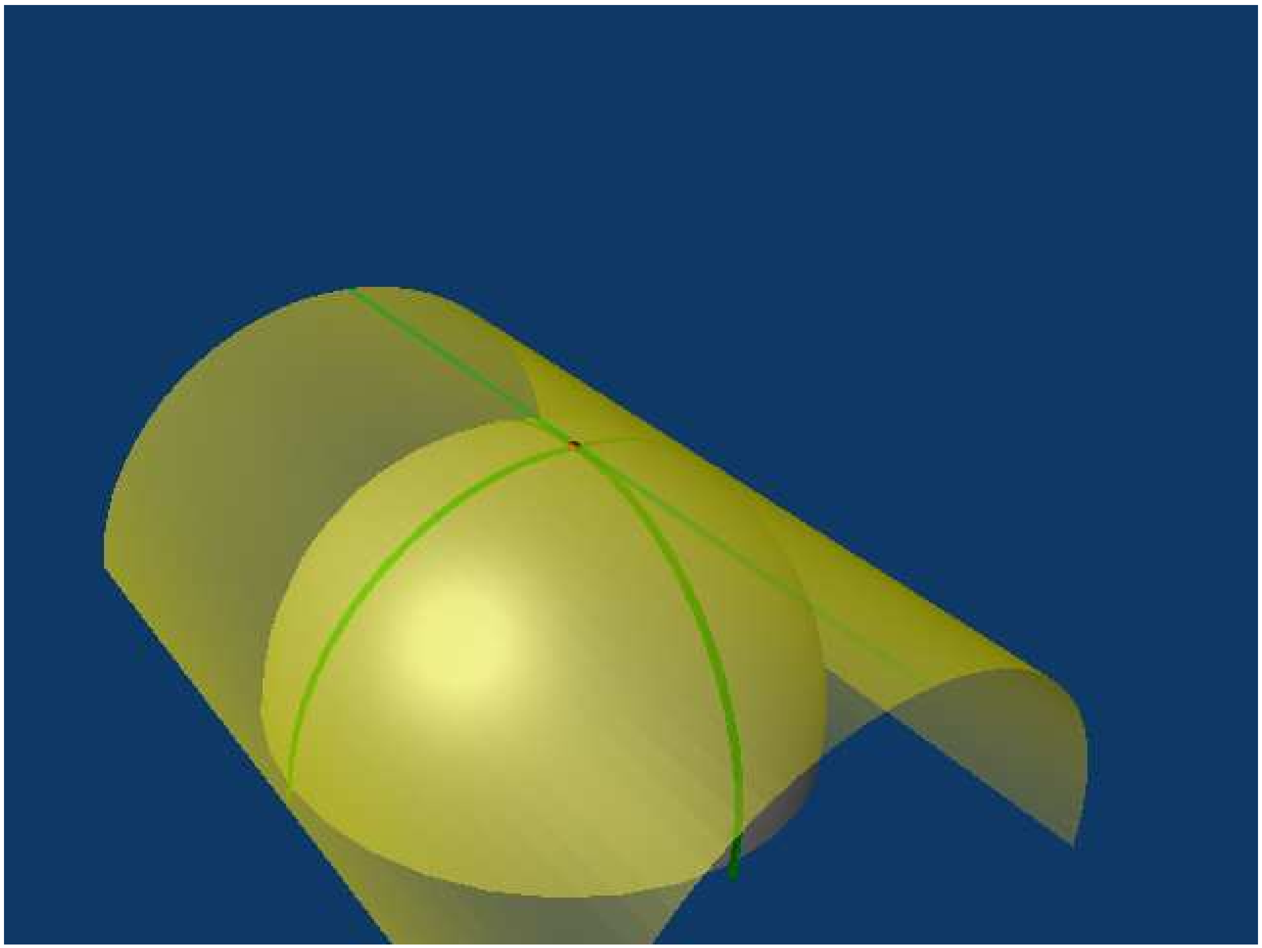
Heuristic plots or CW complexes



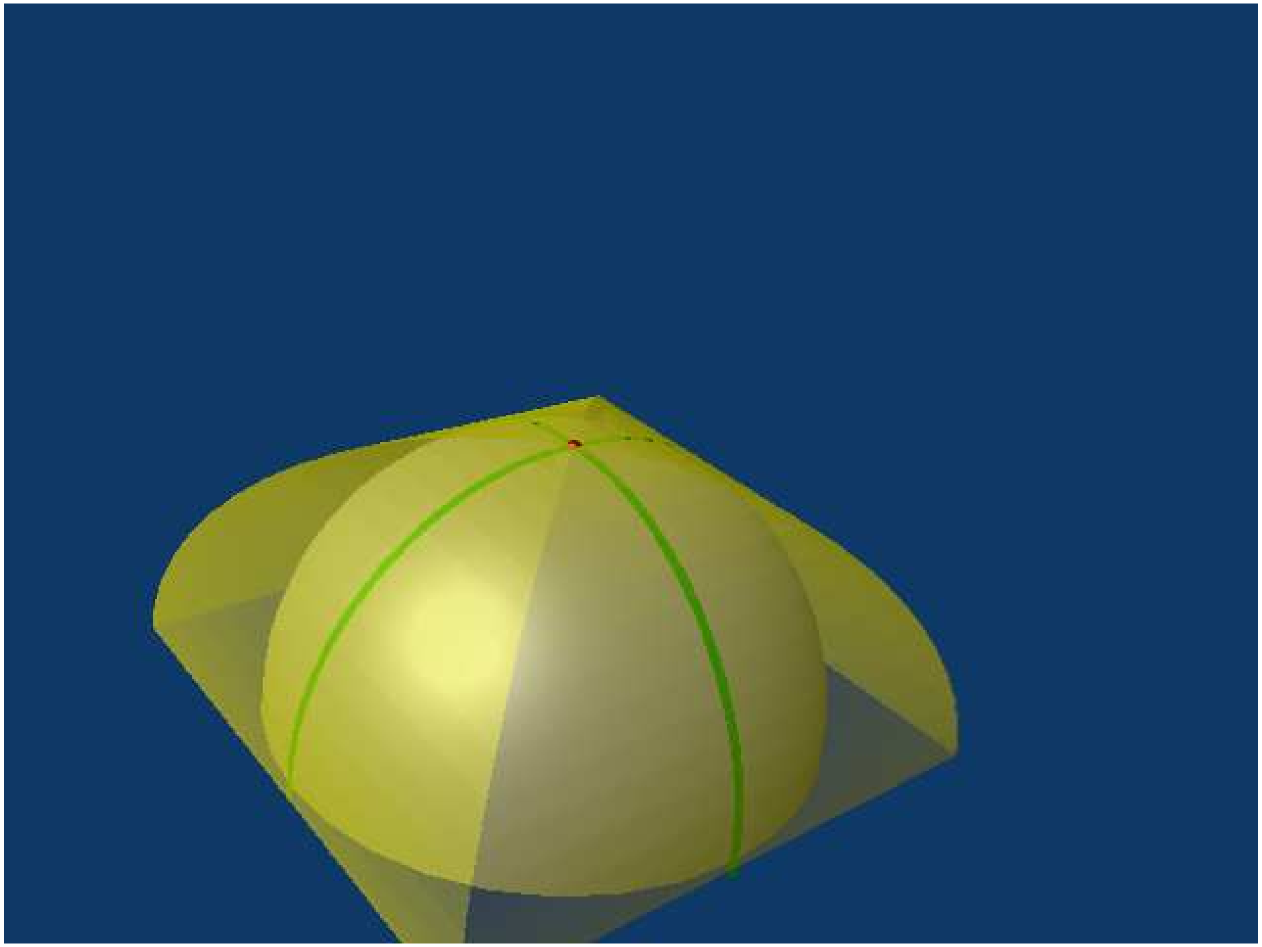
Heuristic plots or CW complexes



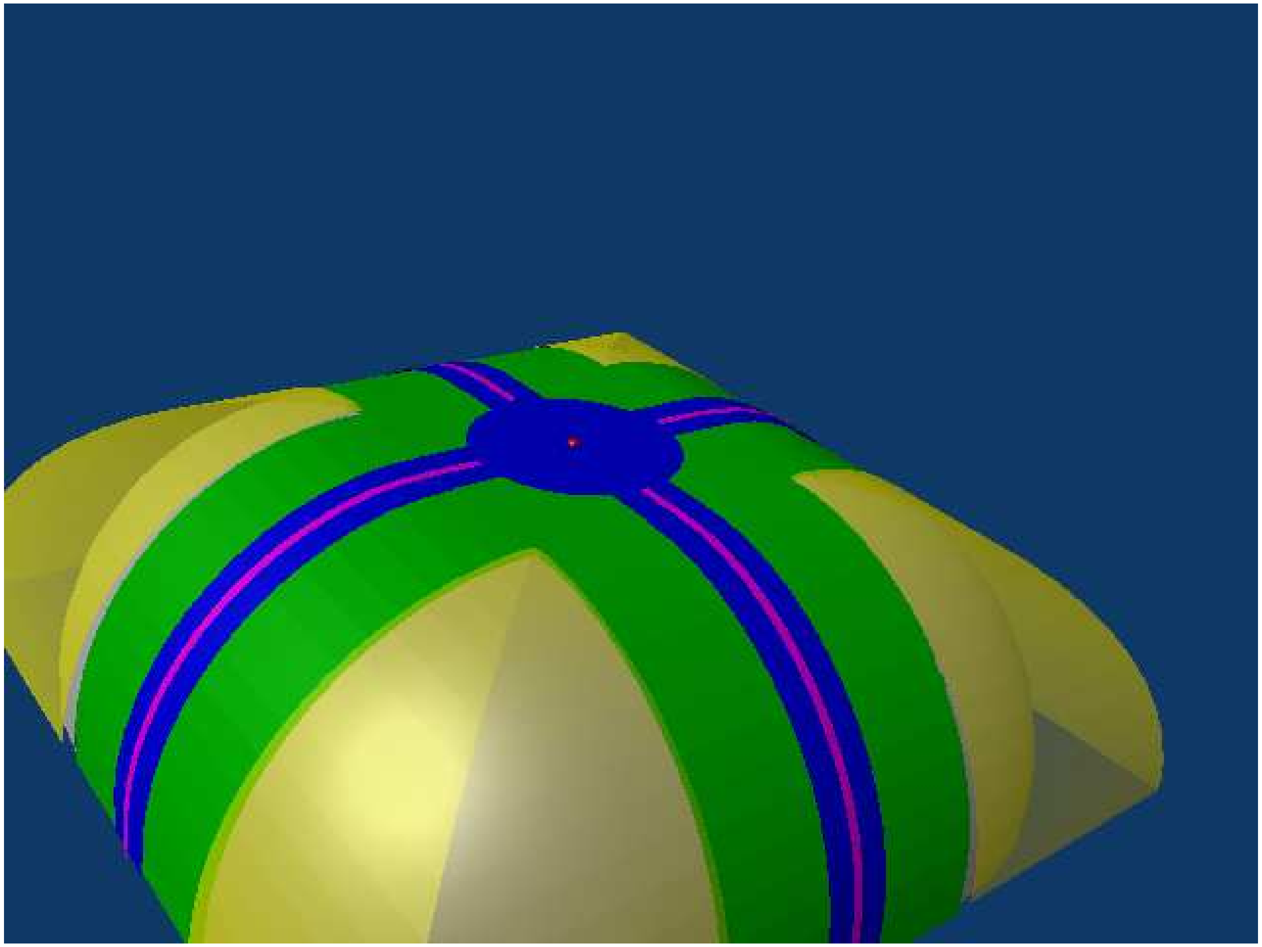
Heuristic plots or CW complexes



Heuristic plots or CW complexes



Heuristic plots or CW complexes



For QED CW-structure is trivial.

Do we have to worry/approximate for QCD?

Summary

- *We have presented PHOTOS*
- *We have laboriously presented tests and matrix element used*
- *We have fi nally presented phase space parametrization as well*
- *Our presentation of tangent space details and how expansion work was still insuffi ciently detailed but I hope it was better than any time before.*
- *Thank you for motivation !!*
- *As PHOTOS work for arbitrary number of charges in fi nal state, it is potentially suitable for emissions of gluons.*
- *There due to increase of number of sources tangent space become non summable.*
- *this diffi culty seem to be possible to overcome*
- *lack of orchester testbed with many spin amplitudes (also with approximations) seem to be a more serious obstacle*
- *also redefi nition of sense attributed to PDF will be serious issue.*

Future

- Breaktrough in precison of PHOTOS achieved
- Potentially interesting for QCD, but huge training field missing
- Long and uncertain project but offers mathematical fun
- Potentially offers understanding on how to get rid of ordering approximations; and ME for radiation kernels, not just matching.
- Certain discomfort may come because we talk about a prototype only.
- Further acivity require study of different processes at semi first order.
- Some paralelism with recent developments in AcerMC seem to be refreshing.
- Chances are slim, but potential of solution large.
- Performance of QED and scalar QED with PHOTOS is beyond my wildest dreams!