Model of PDF with kT ordering

"Hadronic Final States and Parton Density Functions"

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Motivation

- Parton shower generators are essentially leading order tools.
- At this time many NLO calculations are available in the market which should be implemented in MC in order to improve its quality. It would be also desirable to establish connections between parton showers and resummation techniques.
- There many **(infinite number)** possibilities to define PDF and we can choose a definition which will be best suited for our particular problem.
- Our main goal is to construct model of ISR which can be used in studies of transverse momentum distribution in DY processes.
- We have also a chance to get connection with Collins-Soper-Sterman resummation in transverse momentum.

Basic assumptions in the model

In the soft region covered by the parton shower models we can introduce in principle any kind of ordering.
In this talk I would like to investigate closer this possibility.
Having transverse momenta as independent variables simplify studies of the observables directly related to transverse momentum.
Our model is maximally simplified ISR model. We consider only two partons in their center of mass frame with $s=Q^2$.
They can emit other partons while they approach interaction point. We neglect transitions between different kind of partons.

Kinematics and basic notation

We parametrize the momenta of emitted particles using light-cone variables. Define the rapidity variables:

$$\xi_i = \frac{k_i^-}{k_i^+} = \frac{k_{Ti}^2}{k_i^{+2}}$$

In the parton shower models it customary to use variables z to parametrize momenta of emitted particles during consecutive branchings:

$$k_i^+ = Q(1 - z_i)x_{i-1},$$

$$x_i = z_1 z_2 \ldots z_i.$$

The conservation of "+" momenta is introduced as a delta function:

$$\delta(x^+ - 1 + \sum_i^n \frac{k_i}{Q\sqrt{\xi_i}})$$

PDF in one hemisphere

Our one hemisphere PDF is defined as follows:

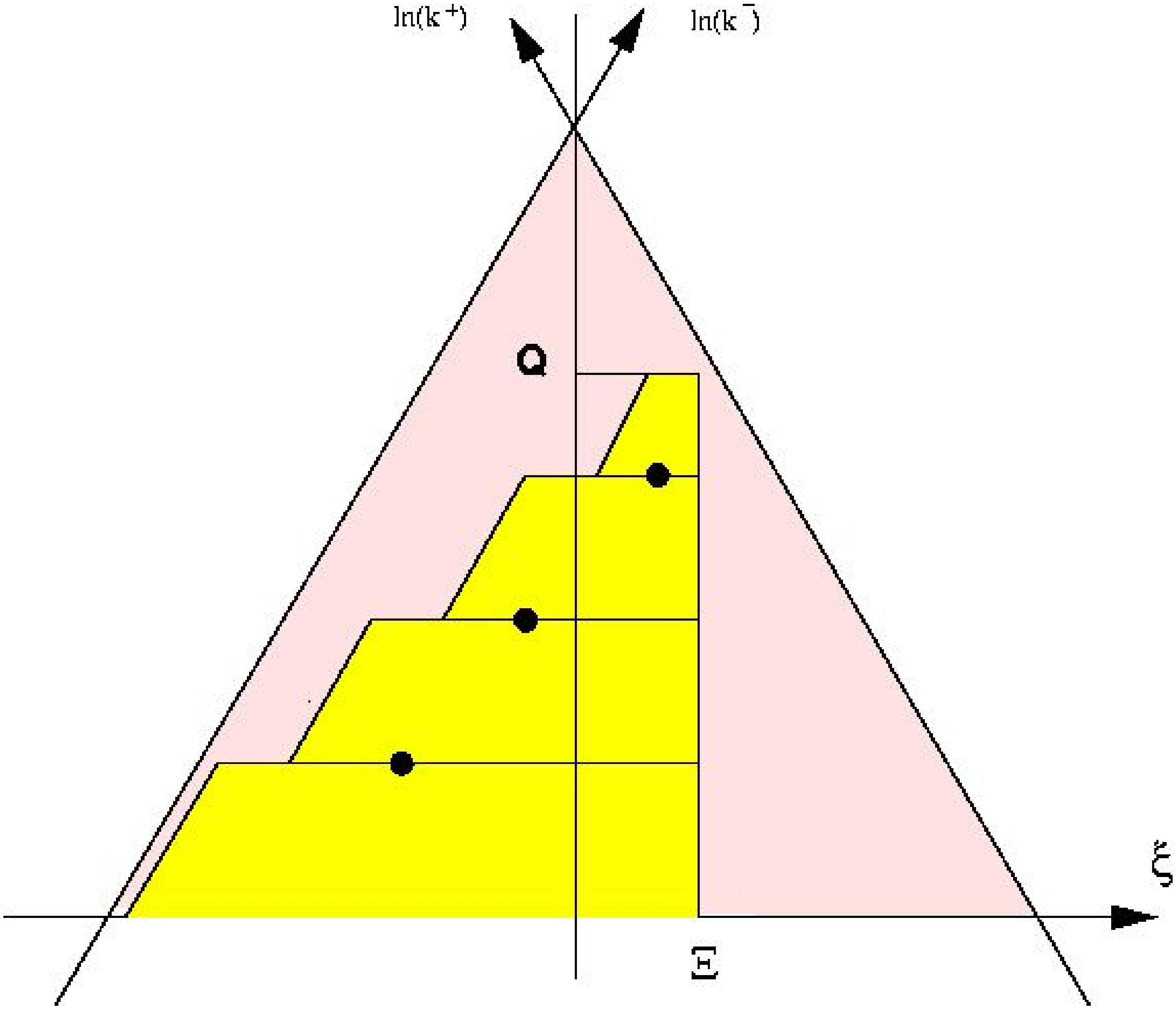
$$\bar{D}_q(Q,\Xi,x^+) = e^{-\phi(Q,\lambda|1)}\delta(1-x^+) + \sum_{n=1}^{\infty}\left(\prod_{i=1}^{n}\int_{\lambda}^{Q}\frac{dk_i}{k_i}\theta(k_{i+1}-k_i)e^{-\phi(k_{i+1},k_i|x_i)}\int_{\frac{k_i^2}{Q^2x_{i-1}^2}}^{\Xi}\frac{d\xi_i}{\xi_i}\mathcal{P}(k_i,z_i(\xi_i,x_{i-1}))\int\frac{d\varphi_i}{2\pi}\right) e^{-\phi(k_1,\lambda|1)}\times\delta(x-\prod_j z_j) \quad (9)$$

The appropriate Sudakov formfactor is defined as:

$$\phi(k_{i+1},k_i|x_i) = \int_{k_i}^{k_{i+1}}\frac{dk}{k}\int_{k^2/x_i^2Q^2}^{\Xi}\frac{d\xi'}{\xi'}\mathcal{P}(k_i,z(\xi',x_i))$$

In the following we will use simplified version of the kernel P=1 and neglect running of the coupling constant.

Phase space in one hemisphere PDF



Evolution equations

For our model we can obtain evolution equations:

For large Ξ

$$\frac{\partial}{\partial \log \Xi} \bar{D}_q(Q, \Xi, x^+) = 0 + O(1/\sqrt{\Xi})$$

Evolution equation in log(Q) reads

$$\frac{\partial}{\partial \log Q} \bar{D}_q(Q, \Xi, x^+) = 2 \int_x^1 \frac{dy}{(1-y)_+} \frac{1}{y} \bar{D}_q(Q, \Xi y^2, \frac{x^+}{y}).$$

Now we can prove the unitarity condition

$$\frac{\partial}{\partial \log Q} \int dx^+ \bar{D}_q(Q, \left(\frac{1}{x^+}\right)^2, x^+) = 0$$

PDF with kT ordering (2 hemispheres)

The parton distribution function can be defined as:

$$\begin{aligned}
\bar{D}(Q, x^+, x^-) &= e^{-\phi(Q,\lambda|1,1)}\delta(1-x^+)\delta(1-x^-) + \\
&\sum_{n=1}^{\infty}\int_{\lambda}^{Q} e^{-\phi(Q,k_n|x^+,x^-)}\left[\prod_{i=1}^{n}\frac{dk_i}{k_i}\theta(k_i-k_{i-1})e^{-\phi(k_i,k_{i-1}|x^+_{i-1},x^-_{i-1})}\int_{\frac{k_i^2}{Q^2 x^{+2}_{i-1}}}^{\frac{Q^2 x^{-2}_{i-1}}{k_i^2}}\frac{d\xi_i}{\xi_i}\int\frac{d\varphi_i}{2\pi}\right] \\
&\times\delta(x^+ - 1 + \sum_{\xi_i<1}\frac{k_i}{Q\sqrt{\xi_i}})\delta(x^- - 1 + \sum_{\xi_i>1}\frac{k_i}{Q}\sqrt{\xi_i})
\end{aligned}$$

This function however has certain pathologies because maximal kT can not in fact reach Q (negative probabilities) . In fact there are two large scales: maximal transverse momentum and Q, which sets overall normalization.

Modified PDF in 2 hemispheres

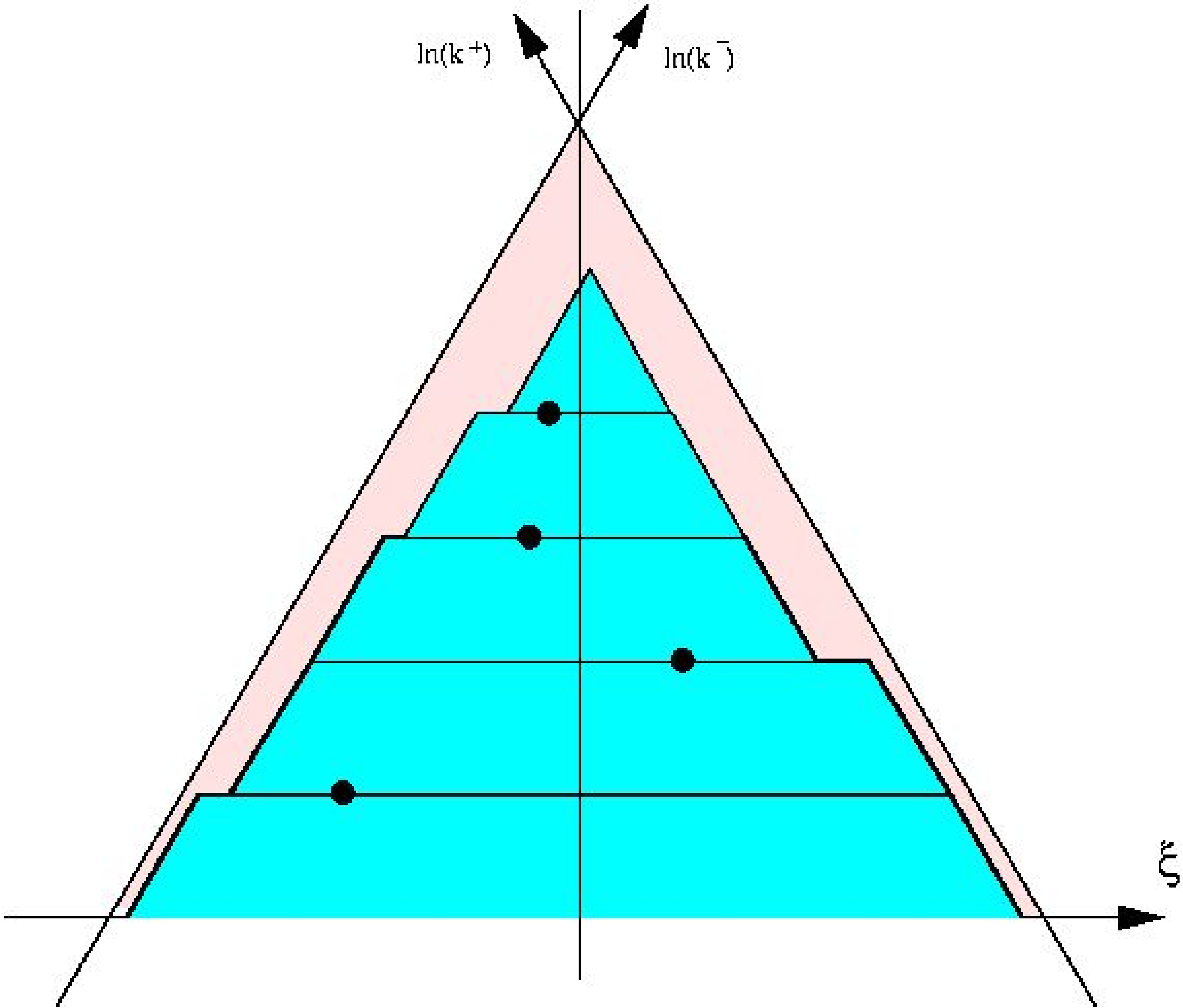
We propose modification in PDF. The unintegrated PDF reads:

$$\begin{aligned}\bar{D}(\bar{P}, x^+, x^-) \;=\;& e^{-\phi(Q,\lambda|1,1)}\delta(1-x^+)\delta(1-x^-)+\\ & \sum_{n=1}^{\infty}\int_{\lambda}^{\bar{P}} e^{-\phi(\bar{P},k_n|x^+,x^-)}\left[\prod_{i=1}^{n}\frac{dk_i}{k_i}\theta(k_i-k_{i-1})e^{-\phi(k_i,k_{i-1}|x^+_{i-1},x^-_{i-1})}\int_{\frac{k_i^2}{Q^2x^{+2}_{i-1}}}^{\frac{Q^2x^{-2}_{i-1}}{k_i^2}}\frac{d\xi_i}{\xi_i}\int\frac{d\varphi_i}{2\pi}\right]\\ & \times\delta(x^+-1+\sum_{\xi_i<1}\frac{k_i}{Q\sqrt{\xi_i}})\delta(x^--1+\sum_{\xi_i>1}\frac{k_i}{Q}\sqrt{\xi_i})\end{aligned}$$

We introduce another scale which sets the upper limit for evolution in kT.

A natural choice for $\bar{P}$ is: $\bar{P} = Q\sqrt{x^+x^-}$

Phase space in 2 hemisphere PDF



The formfactor

$$\phi(k_2,k_1|x^+,x^-)=\frac{1}{2}[log^2(\frac{x^+x^-Q^2}{k_1^2})-log^2(\frac{x^+x^-Q^2}{k_2^2})]$$

In DGLAP equation the final point of evolution is not limited and formfactor can rise to infinity. This is not true for evolution in kT. The formfactor increases with increasing k_2 to the maximal value, then decreases outside physical region.

Other important properties of the model

- A nice feature is that condition $s' = \left(p_Q + p_{\bar{Q}} - \sum_i k_i\right)^2 \approx Q^2 x^{plus} x^{minus} - \vec{k}_n^2 > 0$ is automatically satisfied.
- 2 hemisphere model does not obey such simple evolution equation as 1 hemisphere model.
- The problem in getting simple evolution equation is caused by the central region of rapidity on the boundaries of hemispheres.
- The normalization condition is not exactly satisfied, which indicates that we introduced NLO effects to our model simply by restrictions in phase space.

Approximate MC algorithm

Assume we already know k_i and ξ_i

- Generate random number R
- The probability for stopping the evolution is

$$p = e^{-\frac{1}{2}\log^2 \frac{x_i^+ x_i^- Q^2}{k_i^2}}$$

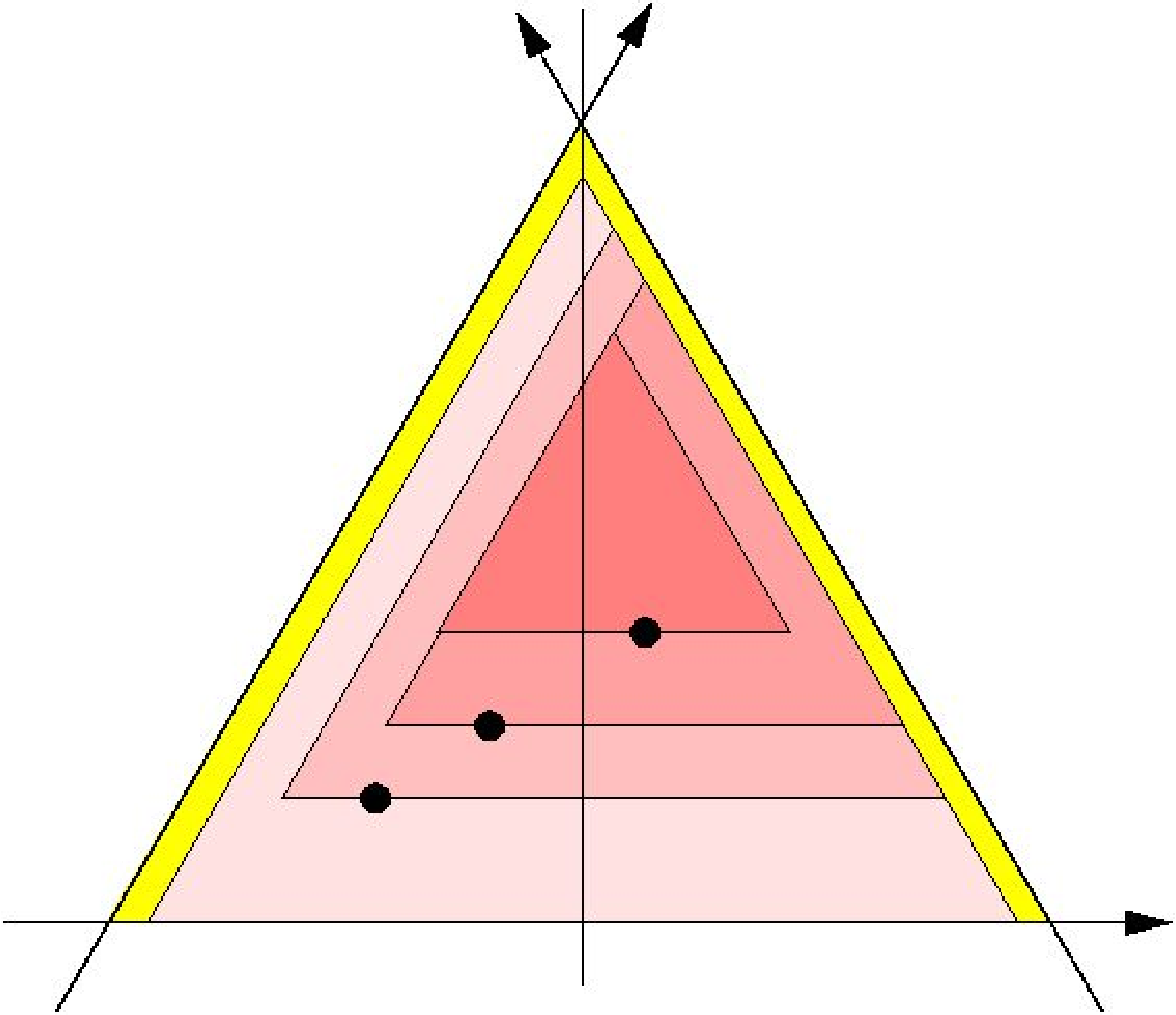
If R<p then we are done. At this point we approximate x+ and x-, by xi's at each step.

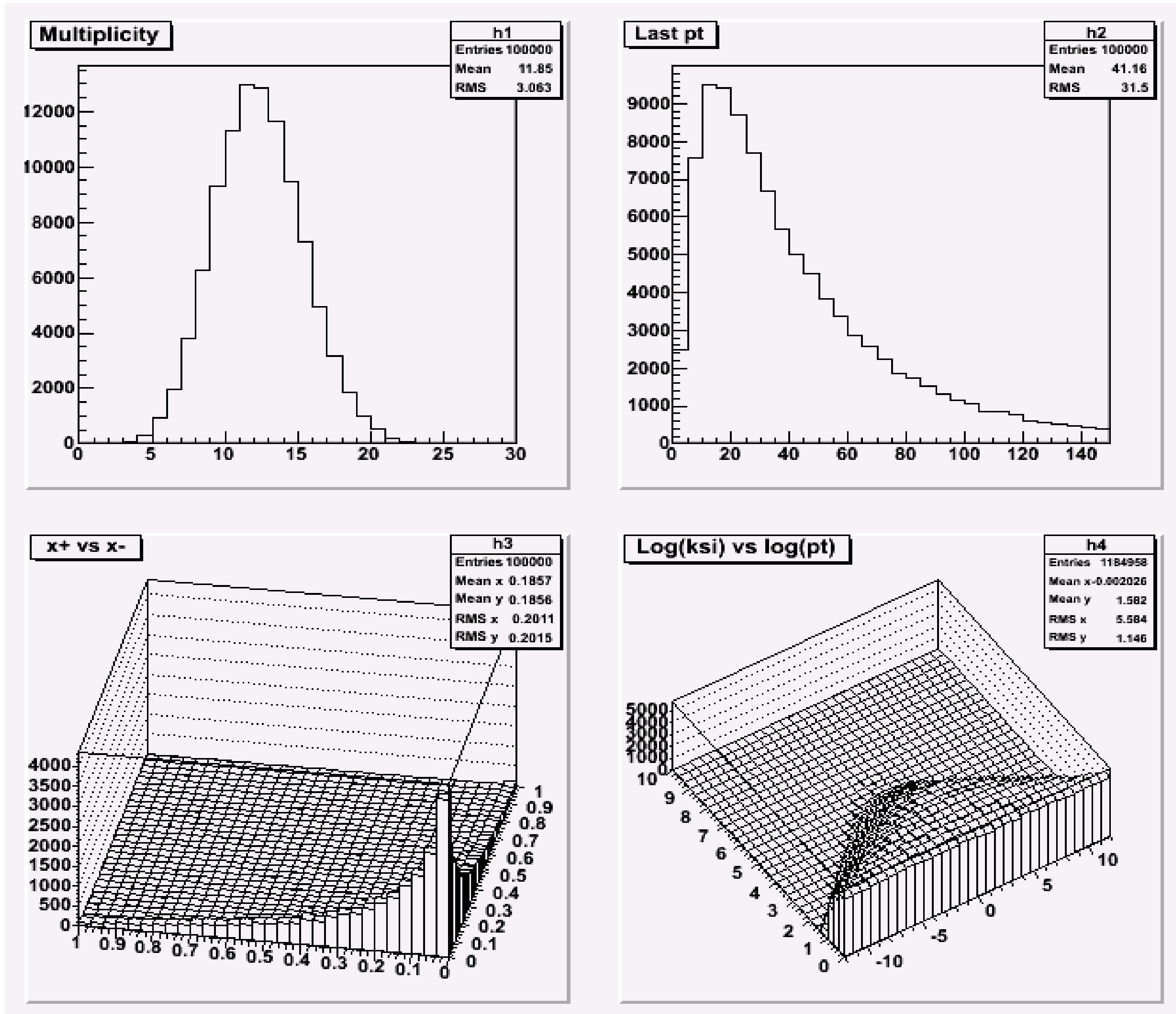
Otherwise solve the equation: $R = e^{-\phi(k_i, k_{i+1})}$ for k_{i+1}

- Generate next rapidity value (trivial)
- Go to first point.

This algorithm is not exact. Therefore some emissions occur above available kT limit. I suspect that if we simply discard such events we get exact algorithm.

Markovian algorithm in phase space





Multiplicity
h1
Entries 100000
Mean 11.85
RMS 3.063
Last pt
h2
Entries 100000
Mean 41.16
RMS 31.5
x+ vs x-
h3
Entries 100000
Mean x 0.1857
Mean y 0.1856
RMS x 0.2011
RMS y 0.2015
Log(ksi) vs log(pt)
h4
Entries 1184958
Mean x -0.002026
Mean y 1.582
RMS x 5.584
RMS y 1.146

CMC algorithm for PDF

- In CMC algorithm we do not to worry about moving boundary for kT emissions.
- The initial proposition is to use two independent CMC generators for each hemisphere.
- A first apparent difficulty is hidden in the formfactor which depends on the detailed history of emissions and introduces complicated interactions between hemispheres.
- However we can try to attach this dependency to additional MC weight. If this weight would behave sufficiently well then we can easily combine two events from distinct hemispheres into one complete event.

Conclusions

- We have constructed PDF with kT ordering, which can be used as a model of ISR for Drell-Yan proceses.
- We saw that by introducing simple restriction in phase space we generate NLO effects. Evolution equations are much more complicated.
- MC algorithms of both Markovian and non-Markovian type for this PDF can be relatively easily constructed.
- More work is needed to introduce other effects: flavour transition, running coupling constant etc.
- At the end we have to link the model with the NLO calculations (this is probably most complicated part)

Acknowledgments

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