Small-x Dipole Evolution Beyond the Large-N_c Limit

Emil Avsar

In Collaboration with Gösta Gustafson and Leif Lönnblad

Department of Theoretical Physics,

Lund University, Sweden

Outline

- Parton and Dipole Cascades in QCD.
- Small-x Evolution With Dipoles:Mueller's Approach
- Improving Mueller's Dipole Model:
  - Energy-Momentum Conservation.
  - Colour Suppressed Effects.
- DIS at HERA.
- pp at the Tevatron.
- Summary and Conclusions.

Parton Cascades in e^+e^- and DIS

- In e^+e^- : $\sigma_{tot} \approx \sigma_0(1 + \frac{\alpha_s}{\pi})$. Fixed by low order matrix element.
- Parton cascade $\rightarrow$ final state properties.
- In DIS *both* σ_{tot} and final state properties depend on parton cascade.
- High Q^2 and large $x \rightarrow$ DGLAP successful.
- Small-x evolution determined by BFKL eq. Gives σ_{tot} to LL and NLL accuracy.

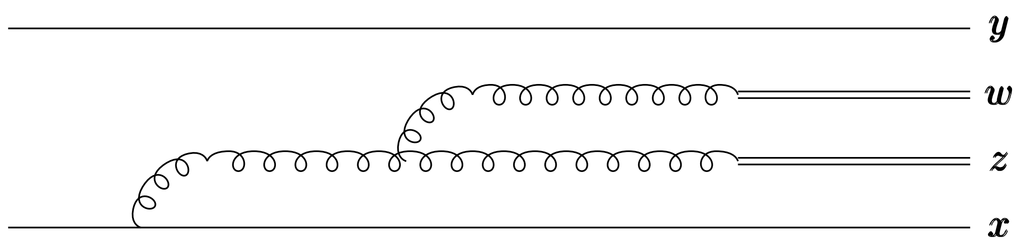
Cascades in DIS

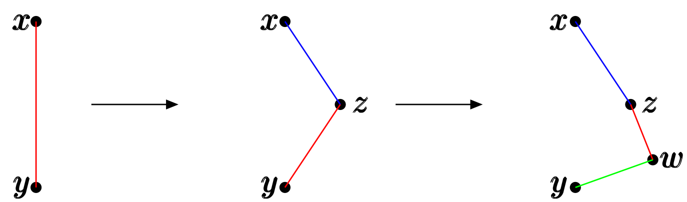
- Problems with BFKL: To LL, too steep rise of $\sigma \sim 1/x^{\lambda}$, $\lambda \sim 0.5$, also NLL very large.
- CCFM and LDC interpolate between BFKL and DGLAP. σ_{tot} ok but final state properties not fully successful.
- Higher energies at fixed resolution $\rightarrow$ higher gluon density. Expect nonlinear effects due to gluon overlap.
- Nonlinear evolution: GLR, BK, JIMWLK.

From Gluons to Dipoles

- Alternative picture of cascade: Colour neutral dipoles.
- In transverse momentum space:
  - Soft Radiation Model implemented in ARIADNE gives best description of final states at HERA. Does not predict σ_{tot}.
- In transverse coordinate space:
  - Saturation easier to take into account.
  - GBW model. Includes saturation, σ_{tot} and diffraction ok.
  - Reformulation of BFKL to LL accuracy: The Mueller Dipole Model.

Mueller's Dipole Model





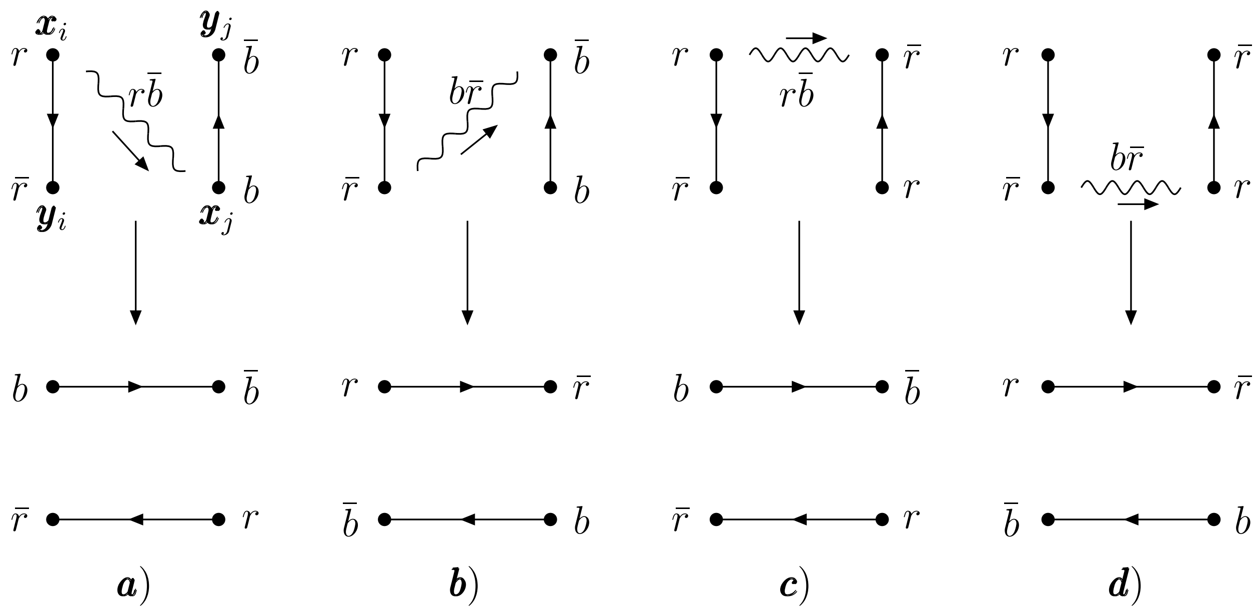
- Decay probability given by

$$
\frac{d\mathcal{P}}{dY} = \frac{\bar{\alpha_s}}{2\pi}\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}d^2\boldsymbol{z}
$$

$$
\bar{\alpha_s} = \frac{\alpha_s N_c}{\pi}
$$

Dipole Scattering

- Dipoles scatter through one gluon exchange in inelastic amplitude.



$$f_{ij} = \frac{\alpha_s^2}{8}\left[\log\left(\frac{(\boldsymbol{x}_i - \boldsymbol{y}_j)^2(\boldsymbol{y}_i - \boldsymbol{x}_j)^2}{(\boldsymbol{x}_i - \boldsymbol{x}_j)^2(\boldsymbol{y}_i - \boldsymbol{y}_j)^2}\right)\right]^2$$

Multiple Scatterings

- Single $\mathbb{P}$ exchange: $\sum_{ij} f_{ij}$, gives BFKL.
- Multiple scatterings. Dipole interactions uncorrelated $\rightarrow$ Amplitude exponentiate: $T = 1 - S = 1 -$exp$(- \sum_{ij} f_{ij})$.

$$\sigma_{tot} = 2 \int d^2\boldsymbol{b} \langle 1 - \exp(-\sum_{ij} f_{ij}) \rangle.$$

- Unitarity restored. Can be studied numerically in a MC, early simulations by Salam.

Dipoles and Pomerons:Evolution of S

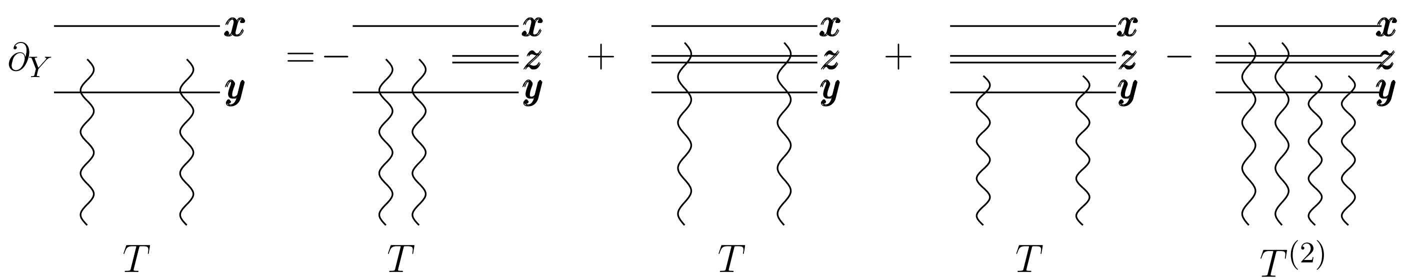
- Large-N_c B-JIMWLK hierarchy can be understood in terms of dipole evolution.
- If $Y \to Y + \Delta Y$ dipole can survive with prob: $1 - \int \frac{d\mathcal{P}}{dY}$.

$$\partial_Y S_{\boldsymbol{xy}} = \int_{\boldsymbol{z}} \frac{d\mathcal{P}}{dY} \{-S_{\boldsymbol{xy}} + S^{(2)}_{\boldsymbol{xz};\boldsymbol{zy}}\}$$

- Simple interpretation: Either survive, or evolve with prob. density $\frac{d\mathcal{P}}{dY d^2\boldsymbol{z}}$.
- Evolution of pomerons: replace S by $T = 1 - S$, scattering amplitude.

Dipoles and Pomerons: Evolution of T

$$\partial_Y T_{\boldsymbol{xy}} = \int_{\boldsymbol{z}} \frac{d\mathcal{P}}{dY} \{-T_{\boldsymbol{xy}} + T_{\boldsymbol{xz}} + T_{\boldsymbol{zy}} - T^{(2)}_{\boldsymbol{xz};\boldsymbol{zy}}\}$$



- If $T^{(2)}_{\boldsymbol{xz};\boldsymbol{zy}} = T_{\boldsymbol{xz}} T_{\boldsymbol{zy}} \Rightarrow$ BK equation.
- Nonlinear term gives saturation as $T \to 1$.

Problems With the Dipole Model

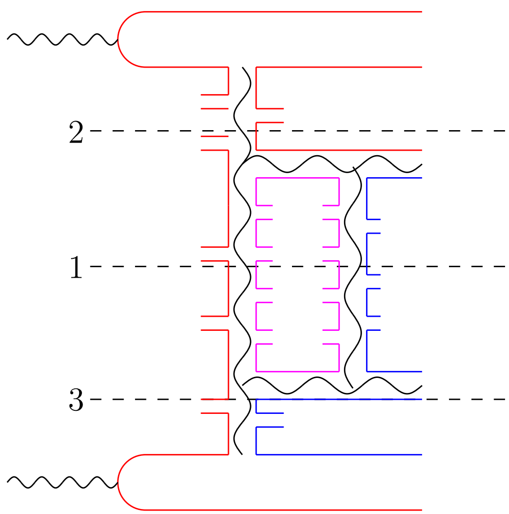
- $\int \frac{d\mathcal{P}}{dY} = \infty$. Cut off,$\rho$, needed such that $(\boldsymbol{x}-\boldsymbol{z})^2, (\boldsymbol{z}-\boldsymbol{y})^2 \geq \rho^2$.
- Cancels against virtual emissions $\rightarrow \sigma_{tot}$ finite as $\rho \rightarrow 0$. Colour transparency.
- In numerical calculations $\rho \neq 0$. Small ρ give very many dipoles $\rightarrow$ Inefficient simulation.
- Small dipole $\Rightarrow$ well localized gluons: High $p_\perp \sim 1/r$.
- Identification $p_\perp \sim 1/r \Rightarrow$ Analogy with LDC and CCFM.

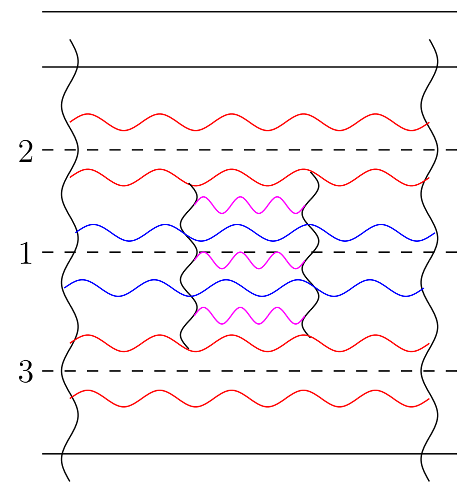
Energy-Momentum Conservation

- Divergent number of small dipoles $\&$ high $p_\perp \Rightarrow$ Violation of energy-momentum conservation.
- Cascade contains many virtual dipoles.
- EM Cons: Multiplicity reduced $\Rightarrow$ exponential increase $\sigma \sim 1/x^\lambda$ much reduced.
- Saturation delayed and small in DIS within HERA energy regime, for $Q^2 \gtrsim 1$GeV^2.
  JHEP 0507:062, hep-ph/0503181.

Violation of Frame Independence

- $f_{ij} \sim \alpha_s$ but $\frac{d\mathcal{P}}{dY} \sim \bar{\alpha_s} = \frac{N_c \alpha_s}{\pi} \Rightarrow f_{ij}$ formally colour suppressed.
- Multiple collisions give pomeron loops. No loops in evolution. Only loops cut in specific frame included.





- Formalism not frame independent.

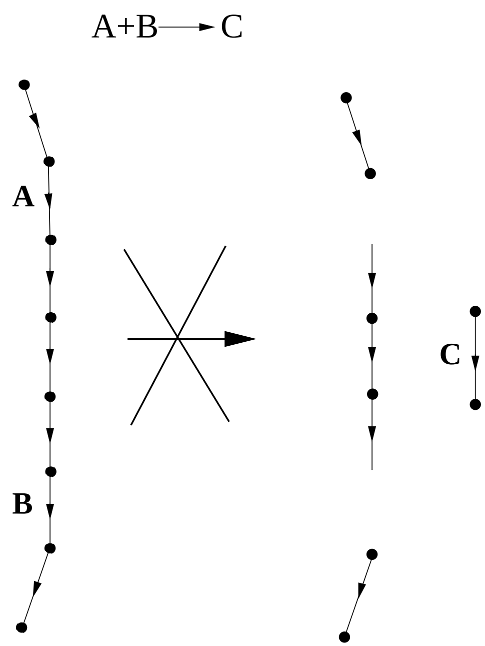
Higher Order Multipoles

- Beyond large N_c dipole basis overcomplete. Two dipoles can have same colour state: Quadrupole.
- Also seen from B-JIMWLK hierarchy. More complicated colour structures at each step of evolution.
- Dipole approximation very successful in e^+e^-. Try to find a working approximation using independent dipoles.
- Quadrupole field: Try approximate by two dipoles.

Additional Dipole Vertices

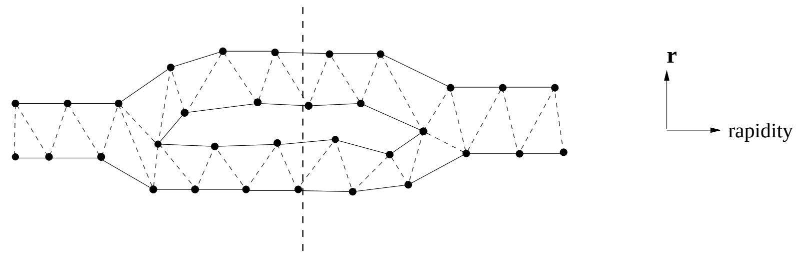
- Symmetric evolution requires also pomeron mergings. $\partial_Y T^k \sim T^{k-1}$.
- Many attempts to include additional dipole vertices: $2 \to 1$ etc.
- In our formalism dipoles are *directed* and *connected* in chains. Not just a collection uncorrelated dipoles.

Dipole Merging?

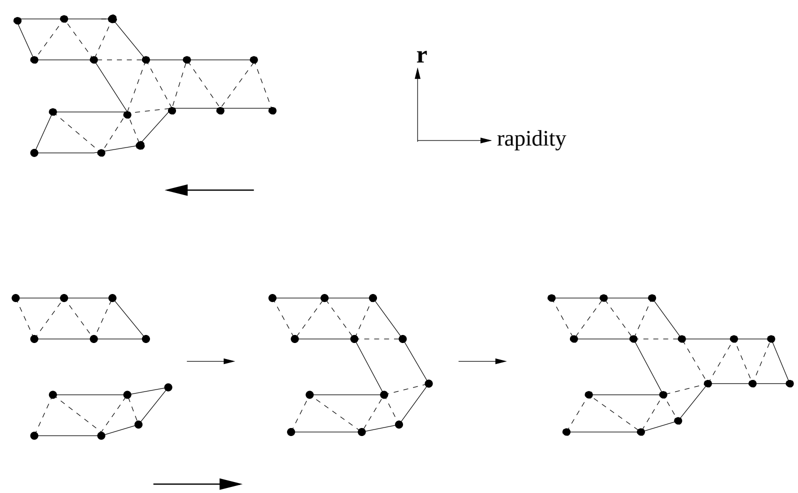


- A dipole cannot vanish in the cascade leaving loose ends.
- Note that $2\mathbb{P} \to 1\mathbb{P}$ does not require $2 \to 1$ dipole vertex.

Generating the Loops

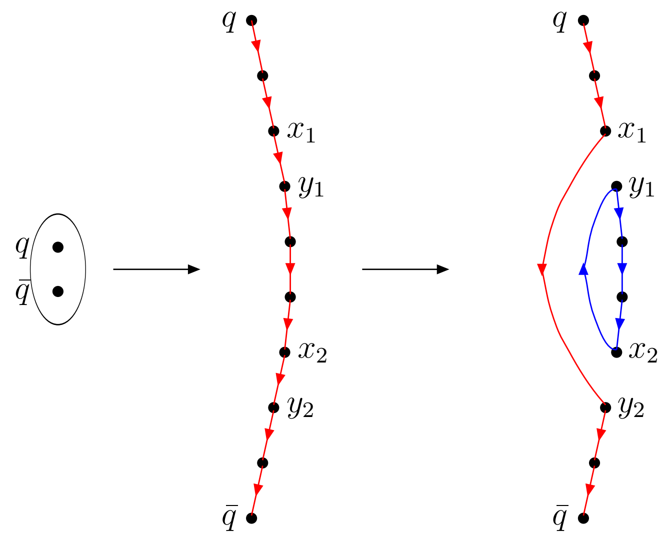


- Loops can be generated by $1 \to 2$ splitting $+2 \to 2$ “swing”.



Dipole Swing

- $(x_1, y_1) + (x_2, y_2) \to (x_1, y_2) + (x_2, y_1)$. Dipole swing or colour recoupling.



$$\Delta P = \lambda \frac{(\boldsymbol{x}_1 - \boldsymbol{y}_1)^2 (\boldsymbol{x}_2 - \boldsymbol{y}_2)^2}{(\boldsymbol{x}_1 - \boldsymbol{y}_2)^2 (\boldsymbol{x}_2 - \boldsymbol{y}_1)^2} \Delta y$$

More on the Swing

- λ phenomenological parameter. Determines how fast swing happens in y.
- In MC: Randomly assign each dipole a colour index $1, 2, \ldots, N_c^2 - 1$.
- Swing allowed if two dipoles have same colour index.
- Adjust so that result almost frame independent. We choose $\lambda = 1$.

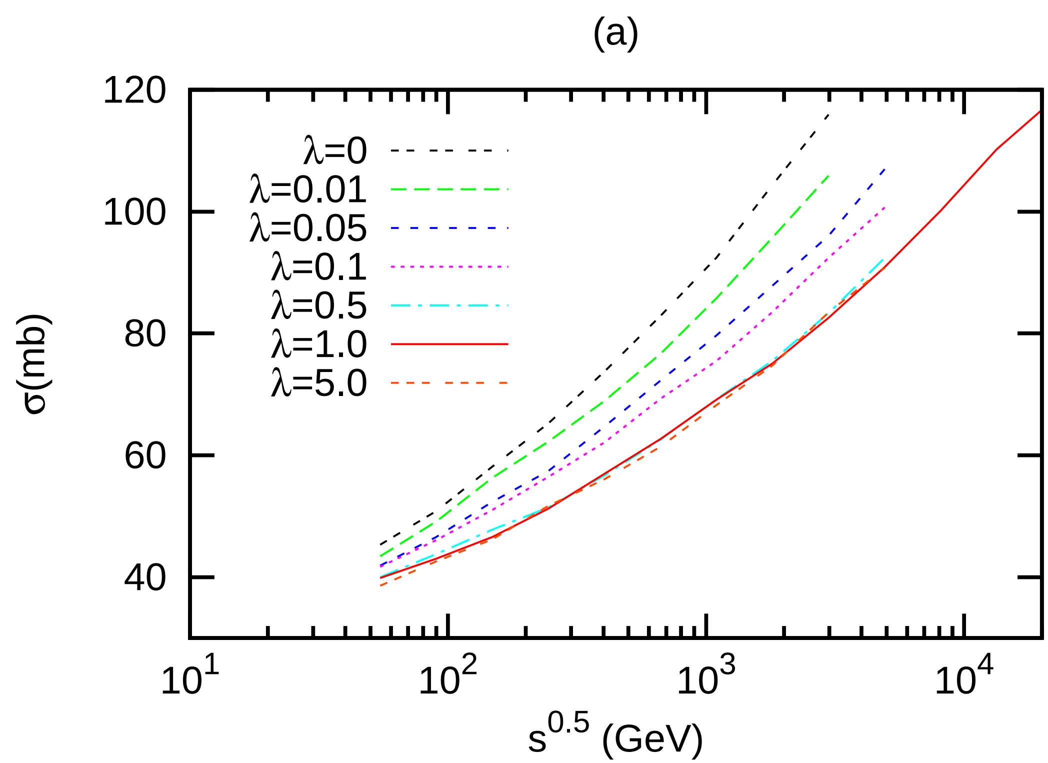
Dipole Picture for $\gamma^* p$ and pp

- Early suggested that semi-hard parton subcollisions $\&$ minijets important in pp collisions.
- Responsible for the rising cross section. Tevatron data successfully described by the PYTHIA MC.
- $\gamma^* p$ and pp collisions in the dipole picture. Need initial states for γ^* and p.
- $\gamma^* \rightarrow q\bar{q}$ well known. Described by ψ_L and ψ_T.

The Initial Proton

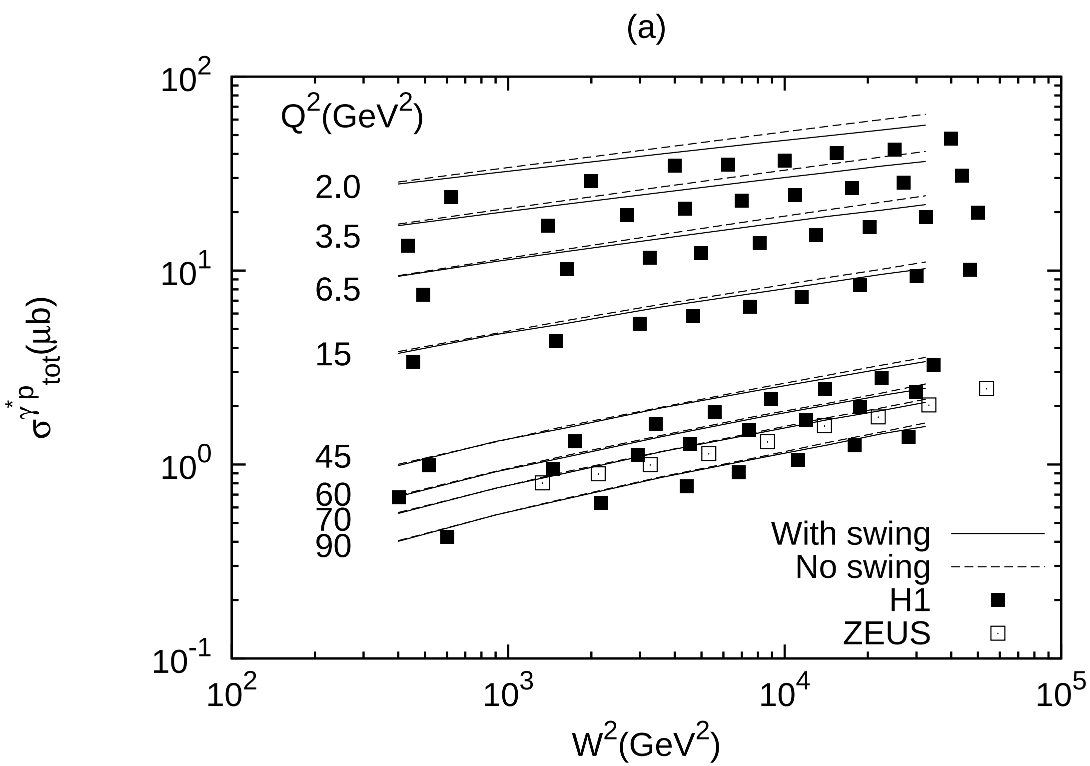
- Cannot fully be described by PQCD but needs some model assumption.
- 3 valence quarks in antisymmetric colour netural state emit single gluon as 3 dipoles.
- Model: “Triangle” configuration of 3 dipoles radiating independently.
- Size of dipoles: Gaussians with average size~ 3.1GeV$^{-1} = r_p$, determined by fit to data.
- Confinement: Dipole sizes limited. Gaussian suppression of new dipoles with average ~ 3.1GeV$^{-1} = r_{max} = r_p$.

λ Dependence



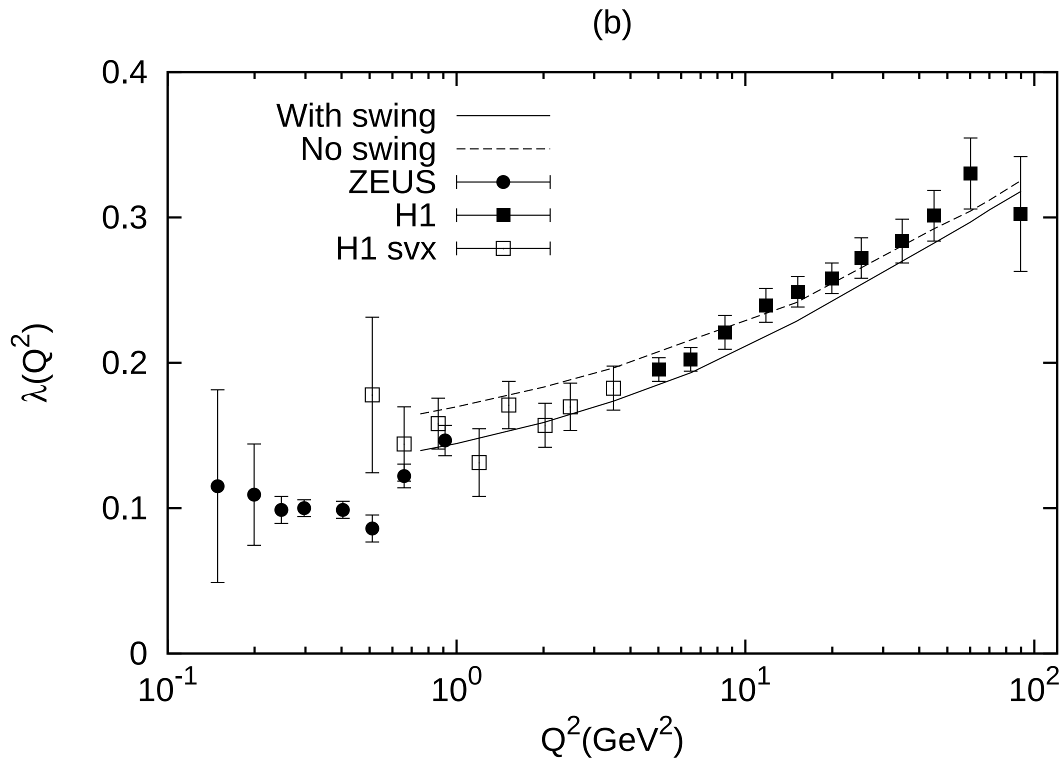
- Swing process saturate for $\lambda \gtrsim 0.5$. Swing effectively instantaneous. hep-ph/0610157

Results for DIS



$$\sigma^{tot}_{\gamma^* p} = \int d^2\boldsymbol{r} \int_0^1 dz\{|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2\}\sigma(z,\boldsymbol{r})$$

F_2 slope at HERA

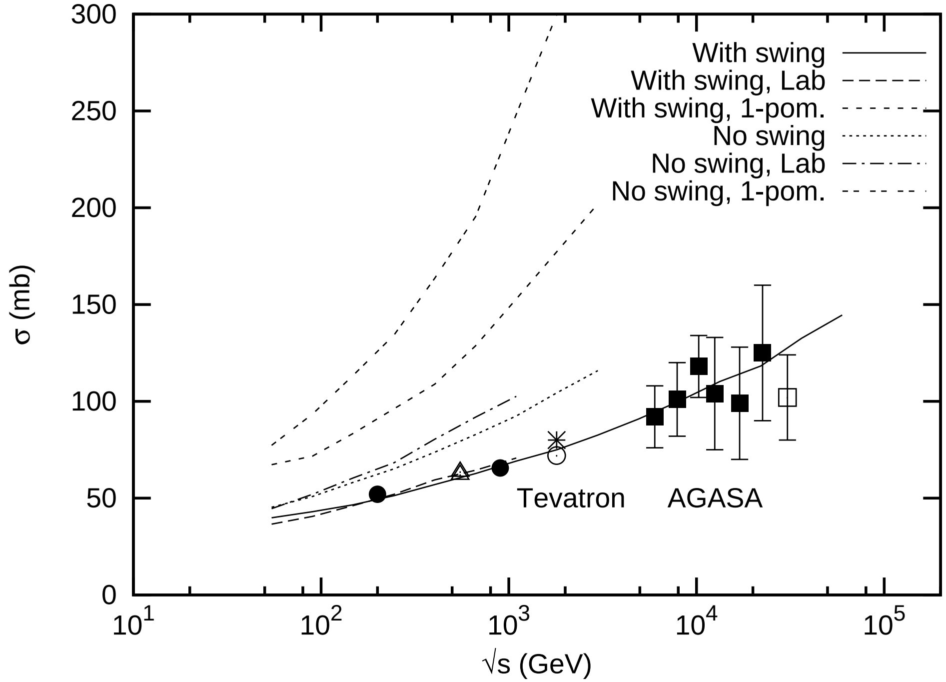


- $F_2 \sim x^{-\lambda_{eff}}$. hep-ph/0610157

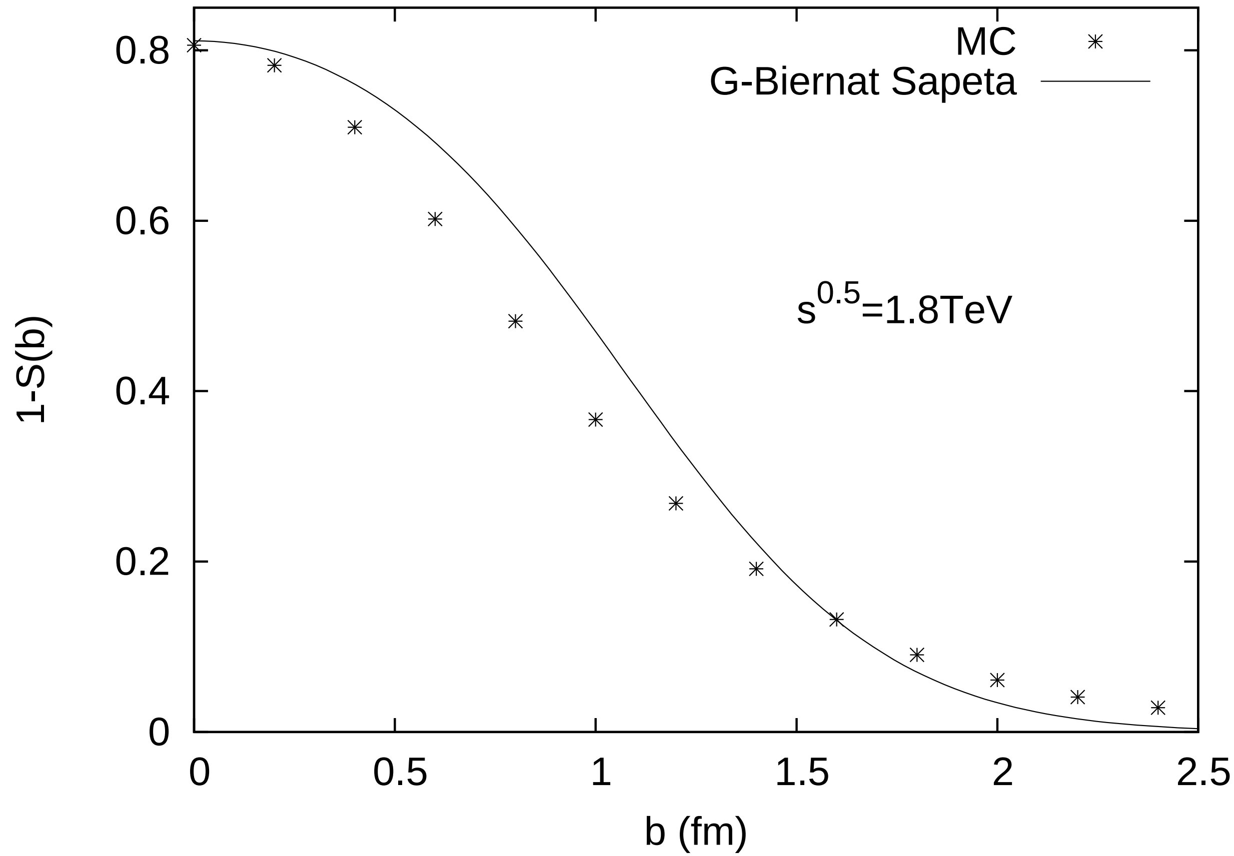
pp Total Cross Section

Saturation very important (small effect at HERA).

Dipole swing $\Longrightarrow$ Near frame independence.



pp Impact Parameter Profile



$S = \langle \text{exp}(-\sum_{ij} f_{ij}) \rangle$. GB-S = two parameter Gaussian fit to data.

Summary and Conclusions

- We have presented a dipole model based on a set of fairly simple ingredients.
- Using these in a MC we reproduce σ_{tot} both for $\gamma^* p$ at HERA and for pp from ISR energies to the Tevatron and beyond.
- Our results depend effectively on two parameters: Λ_{QCD} and $r_p = r_{max}$ proton size and cutoff for large dipoles.
- Swing contains λ. If λ large enough, results independent of λ.

Summary and Conclusions

- Results almost frame independent $\Longrightarrow$ Consistent treatment of saturation effects.
- Intention to find explicit frame independent formalism.
- Next step diffraction, AGK rules: how long are the chains?
- Extend model to simulate exclusive final states.
- Would give further insight into high energy evolution in QCD.