PDF-fitting procedure for the Monte Carlo solutions of evolution equations in QCD

Przemysław Stokłosa

IFJ-PAN and AGH-UST

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New parton shower MC project is currently constructed by IFJ-PAN group from Krakow, based on evolution in rapidity space

- Such parton shower with non-standard evolution (CCFM-like,$\alpha(Q(1-z))$,$\alpha(Q(x\frac{1-z}{z}))$ etc) requires dedicated PDF fits as a part of the project
- MC is slow by its nature (permille accuracy $\approx$ 3 days of running). Therefore MC cannot be used directly for fitting - fast procedure must be designed

Evolution equation

$$\partial_t D_A(x,t) = P_{AB}(x,t) \otimes^x D_B(x,t), \tag{1}$$

$$D_A(x,t=0) = D^0_A(x,\alpha^A_1,\ldots,\alpha^A_k). \tag{2}$$

$D_A(x,t)$ denotes PDF of the type A with $A = q_i, \bar{q}_i, g;\ i = 1.\ldots,n$. and t is the evolution time. The splitting kernels $P_{AB}(x,t)$ include also coupling constants and the convolution symbol $\otimes^x$ stands for

$$\big(f(x,\alpha)\otimes^x g(x,\beta)\big)(x) = \int_x^1 dydz\delta(x-yz)f(y,\alpha)g(z,\beta). \tag{3}$$

Decomposition

The decomposition of the solution $D^A(x,t)$ can be written as follows

$$D_A(x,t) = D^\delta_{AB}(x,t) \otimes^x D^0_B(x,\alpha^B_1,\ldots,\alpha^B_k) \tag{4}$$

$$\partial_t D^\delta_{AB}(x,t) = P_{AC}(x,t) \otimes^x D^\delta_{CB}(x,t), \tag{5}$$

$$D^\delta_{AB}(x,0) = \delta_{AB}\delta(1-x). \tag{6}$$

- Whole dependence of fitted patemeters (α^B_j) is limited to D^0_B
- Only single MC run needed for D^δ_{AB} (independent of α^B_j)

Integration procedure

- Convolution $\otimes^x$ is a 1-dim integral, can be done numericaly in a fast and accurate way.
- MC histogram of D^δ is parametrized by polynominal second order functions.
- Because we use log-scale for x, value at x close to 1 must be extrapolated from neighbouring bins
- This extrapolation could be avoided if linear scale were used, but it is not necessary for 10^{-3} precision

Test of integration procedure

We use 2-dim system of gluon (G) and singlet (Σ) PDF-s

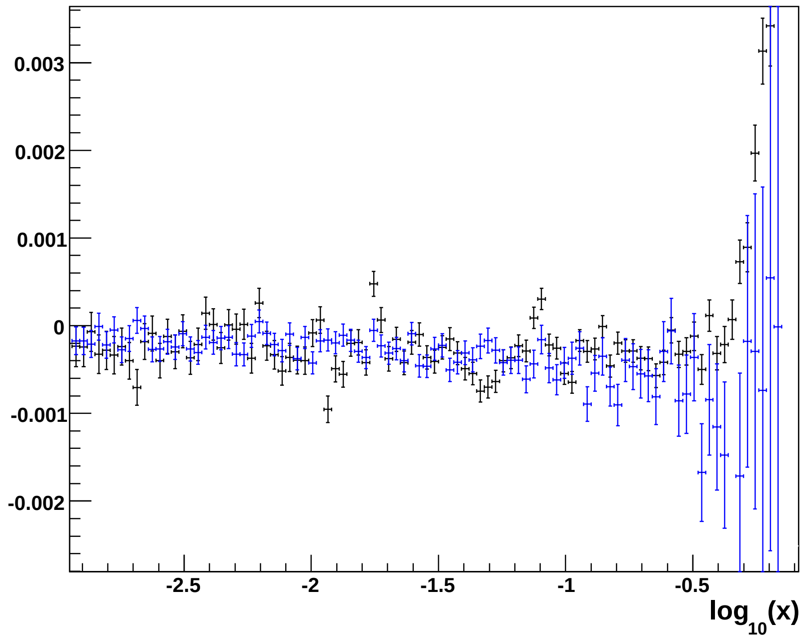
$$\begin{aligned} D_G(x,t) &= D^{\delta}_{GG}(x,t) \otimes D^0_G(x) + D^{\delta}_{G\Sigma}(x,t) \otimes D^0_{\Sigma}(x), \\ D_{\Sigma}(x,t) &= D^{\delta}_{\Sigma G}(x,t) \otimes D^0_G(x) + D^{\delta}_{\Sigma\Sigma}(x,t) \otimes D^0_{\Sigma}(x). \end{aligned} \tag{7}$$

Initial conditions

$$\begin{aligned} D^0_G(x) &= 1.908 \cdot x^{-1.2}(1-x)^{5.0}, \\ D^0_{\Sigma}(x) &= D^0_{sea}(x) + D^0_u(x) + D^0_d(x), \\ D^0_{sea}(x) &= 0.6733 \cdot x^{-1.2}(1-x)^{7.0}, \\ D^0_u(x) &= 2.187 \cdot x^{-0.5}(1-x)^{3.0}, \\ D^0_d(x) &= 1.230 \cdot x^{-0.5}(1-x)^{4.0}. \end{aligned} \tag{8}$$

Test of convolution Gluon and Singlet (LO)

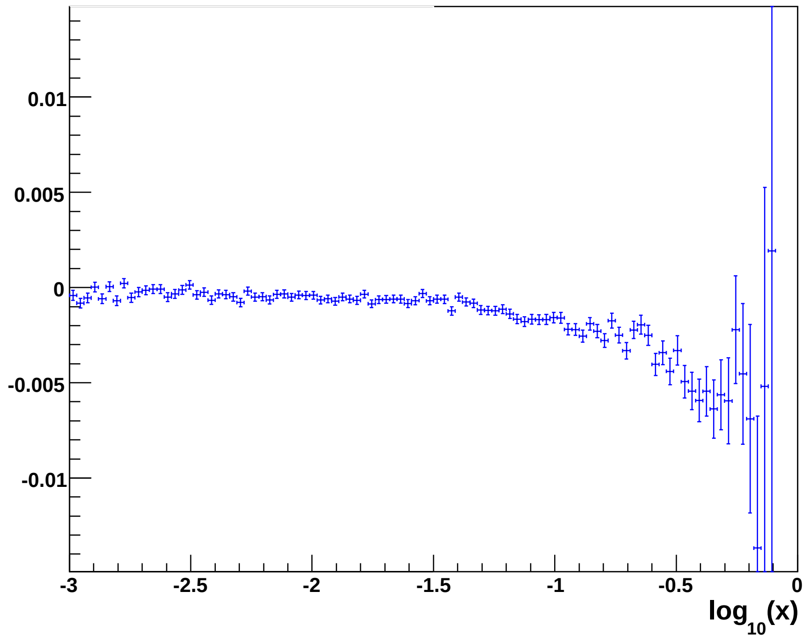
Ratio of gluon (blue) and singlet (black) generated directly from initial distribution and obtained from convolution of D_δ. Evolution from $Q = 1$ to $Q = 100GeV$, LO type



Agreement $5 \cdot 10^{-4}$ except of $x \sim 1$, where D is very small any way

Test of convolution Gluon (NLO)

Ratio of gluon (blue) and singlet (black) generated directly from initial distribution and obtained from convolution of D_δ. Evolution from $Q = 1$ to $Q = 100 GeV$, NLO type



Agreement 1×10^{-3} except of $x \sim 1$, where D is very small anyway

Fitting (PDFs)

- To check correction of the procedure we fitted PDFs to PDFs
- We used the same $G - \Sigma$ system
- We used MINUIT and minimized χ^2

$$\chi^2(\alpha_1^G, \ldots, \alpha_k^G; \alpha_1^\Sigma, \ldots, \alpha_k^\Sigma) = \sum_{A=G,\Sigma} \sum_i \frac{(D_A^X(x_i, t, \alpha_1^A, \ldots, \alpha_k^A) - D_A^Y(x_i, t))^2}{e_A^Y(x_i, t)^2} \tag{9}$$

X and Y denote evolution type: if $X = Y$ - technical test, if $X \neq Y$ - physical difference

Fitting PDF to PDF-test (LO)

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	5.000	-1.200	2.187	3.000	-0.500	1.230	4.000	-0.500
F	1.907	4.994	-1.199	2.186	3.008	-0.501	1.229	3.991	-0.505

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.67	7.00	-1.200
F	0.68	7.06	-1.199

Comparison of fi tted values of coeffi cients with the original ones used for generation for the LO evolution at $Q = 100$ GeV. I – input values, F – fi tted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$. Accuracy mostly within 2σ.

Fitting PDF to PDF-test (NLO)

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	5.000	-1.200	2.187	3.000	-0.500	1.230	4.000	-0.500
F	1.905	5.002	-1.200	2.187	3.013	-0.503	1.230	3.997	-0.513

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.673	7.000	-1.200
F	0.678	7.094	-1.198

Comparison of fi tted values of coeffi cients with the original ones used for generation for the NLO evolution at $Q = 100$ GeV. I – input values, F – fi tted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$. Accuracy mostly within 2σ.

Fitting PDF to PDF

Fitting different evolutions: Generated NLO DGLAP, fitted LO DGLAP with $\alpha(Q(1-z))$

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	5.000	-1.200	2.187	3.000	-0.500	1.230	4.000	-0.500
F	1.617	4.764	-1.202	1.125	2.495	-0.339	0.503	3.735	-0.539

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.673	7.000	-1.200
F	0.515	8.825	-1.465

$Q = 100$ GeV. I – input values, F – fitted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$ The change in parameters is big (up to 50%)

Fitting F2

In LO we have

$$F_2(x,t) = \sum_{A=q,\bar{q}} \int_0^1 d\xi D_A(\xi,t) x e_A^2 \delta(x-\xi) = \sum_{A=q,\bar{q}} D_A(x,t) x e_A^2. \tag{10}$$

$$F_2(x,t,\vec{\alpha}_1,\dots,\vec{\alpha}_k) = x \sum_{A=q,\bar{q}} e_A^2 D_{AB}^{\delta}(x,t) \otimes^x D_B^0(x,\alpha_1^B,\dots,\alpha_k^B). \tag{11}$$

and we fi t with MINUIT with χ^2

$$\chi^2(a,b,\cdots) = \sum_n \sum_i \frac{\big(F_2(x_i,t_n,\vec{\alpha}_1,\dots,\vec{\alpha}_k) - F_{2exp}(x_i,t_n)\big)^2}{e_{F_{2exp}}(x_i,t_n)^2}. \tag{12}$$

Fitting F2

In NLO nontrivial coeffi cent functions appear

$$
\begin{aligned}
F_2(x,t) = x \sum_{A=q,\bar{q}} e_A^2 D_A(x,t) \otimes^x \left(\delta\,(1-x) + \frac{\alpha_s}{2\pi} C_A^{\overline{MS}}\,(x)\right) \\
+\, x \sum_{A=q,\bar{q}} e_A^2 D_g(x,t) \otimes^x \frac{\alpha_S}{2\pi} C_g^{\overline{MS}}\,(x)
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
C_q^{\overline{MS}} &= C_F \left(2 \left(\frac{\ln(1-z)}{1-z} \right)_+ - \frac{3}{2} \left(\frac{1}{1-z} \right)_+ - (1+z)\ln(l-z) - \frac{1+z^2}{1-z}\ln(z) \right. \\
&\quad \left. + 3 + 2z - \left(\frac{\pi^2}{3} + \frac{9}{2} \right) \delta(1-z) \right),
\end{aligned}
\tag{14}
$$

$$
C_g^{\overline{MS}} = T_R \left(\left((1-z)^2 + z^2\right) \ln\left(\frac{1-z}{z}\right) - 8z^2 + 8z - 1 \right).
$$

Fitting F2 to F2 obtained from QCDNum16 - test (LO)

Fitting two structure functions: F2 (LO) obtained from Monte Carlo to F2 from QCDNum16

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	5.000	-1.200	2.187	3.000	-0.500	1.230	4.000	-0.500
F	1.908	4.988	-1.199	2.187	3.07	-0.502	1.241	4.08	-0.507

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.673	7.000	-1.200
F	0.674	6.944	-1.200

$Q = 100$ GeV. I – input values, F – fitted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$

Fitting F2 to F2 obtained from QCDNum16 - test (NLO)

Fitting two structure functions: F2 (NLO) obtained from Monte Carlo to F2 from QCDNum16

	α_1^G	α_2^G	α_3^G	α_1^u	α_2^u	α_3^u	α_1^d	α_2^d	α_3^d
I	1.908	5.000	-1.200	2.187	3.000	-0.500	1.230	4.000	-0.500
F	1.908	4.979	-1.199	2.196	3.06	-0.503	1.25	4.04	-0.51

	α_1^{sea}	α_2^{sea}	α_3^{sea}
I	0.673	7.000	-1.200
F	0.675	6.86	-1.200

$Q = 100$ GeV. I – input values, F – fitted values. The initial distributions are $\alpha_1^A x^{-\alpha_2^A}(1-x)^{\alpha_3^A}$

Summary

- We have developed framework for fitting PDFs from Monte Carlo solutions of evolution equations.
- It is based on one-dimensional numerical convolution.
- It is fast.
- Some numerical tests have been succefully performed at the level of PDFs and at the level of F2.
- More test needed...