Photon-Photon Collisions: Ambiguity and Duality in QCD factorization theorem

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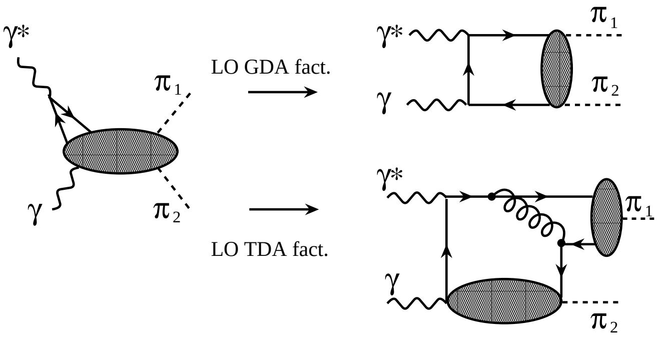
Factorization in "s"− and "t"− channels: GDAs and TDAs



Factorization in overlap region: GDAs vs. TDAs

When both "s" and "t" are small compared to Q^2, both GDA- and TDA- schemes of factorization can be performed for $\gamma^*\gamma \to \pi\pi$.

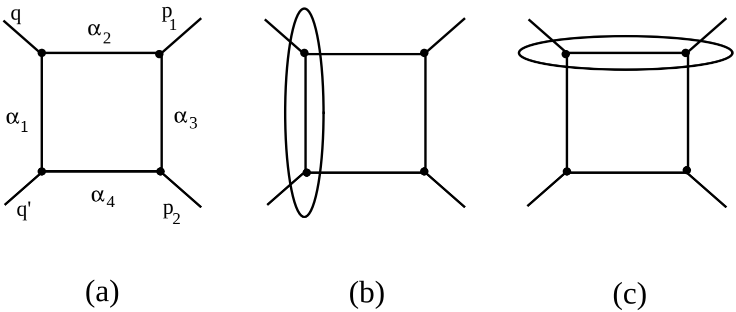
Therefore, the question now is:

ADDITIVITY: GDA $+$ TDA

or

DUALITY: GDA $=$ TDA ?

Duality in φ_E^3 model



Asymptotic Formula (Laplace int.) $\Longleftrightarrow$ Factorization Formula

The contribution of the leading "box" diagram can be written as

$$\mathcal{A}(s,t,m^2) = -\frac{g^4}{16\pi^2\Lambda^4}\,\tilde{\mathcal{A}}\,,$$

$$\tilde{\mathcal{A}} = \int_0^1 \frac{\prod_{i=1}^{4} d\beta_i}{\tilde{D}^2} \exp\Bigg[-\frac{Q^2}{\Lambda^2}\frac{\beta_1\beta_2\bar{\beta}_3\bar{\beta}_4}{\tilde{D}} - \frac{s}{\Lambda^2}\frac{\beta_2\beta_4\bar{\beta}_1\bar{\beta}_3}{\tilde{D}}$$

$$-\frac{t}{\Lambda^2}\frac{\beta_1\beta_3\bar{\beta}_2\bar{\beta}_4}{\tilde{D}} - \frac{m^2}{\Lambda^2}\frac{\tilde{D}}{\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3\bar{\beta}_4}\Bigg],$$

where $\tilde{D} = \beta_1\bar{\beta}_2\bar{\beta}_3\bar{\beta}_4 + \ldots + \bar{\beta}_1\bar{\beta}_2\bar{\beta}_3\beta_4$ and $\bar{\beta}_i = 1 - \beta_i$.

Three Regimes

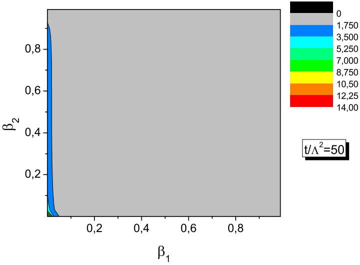
(a) $s \ll Q^2$ while t is of order Q^2;

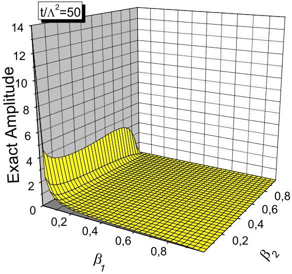
(b) $t \ll Q^2$ while s is of order Q^2;

(c) $s, \ t \ll Q^2$;

Regime (a): $s \ll Q^2$ while t is of order Q^2 (GDA)

$s/\Lambda^2 = 1$, $t/\Lambda^2 = 50.0$, $Q^2/\Lambda^2 = 100.0$.





- the β_2-wing $\Longrightarrow$ 85% to the whole amplitude
- the region : $\beta_1, \beta_2 \in [0, 0.1] \Longrightarrow 5$ %

By analytic integration over the region where $\beta_1 \sim 0$ (corresponding to the "GDA factorization"):

$$\tilde{\mathcal{A}}^{\text{as}}_{\text{GDA}}(s,t,m^2) = \int\limits_0^1 \frac{d\beta_2\, d\beta_3\, d\beta_4}{\tilde{D}_0^2} \exp\left(-\frac{s}{\Lambda^2}\frac{\beta_2\beta_4\bar{\beta}_3}{\tilde{D}_0} - \frac{m^2}{\Lambda^2}\frac{\tilde{D}_0}{\bar{\beta}_2\bar{\beta}_3\bar{\beta}_4}\right) \left[\frac{Q^2}{\Lambda^2}\frac{\beta_2\bar{\beta}_3\bar{\beta}_4}{\tilde{D}_0} + \frac{t}{\Lambda^2}\frac{\beta_3\bar{\beta}_2\bar{\beta}_4}{\tilde{D}_0} + \frac{m^2}{\Lambda^2}\right]^{-1},$$

where $\tilde{D}_0 = \beta_2\bar{\beta}_3\bar{\beta}_4 + \bar{\beta}_2\beta_3\bar{\beta}_4 + \bar{\beta}_2\bar{\beta}_3\beta_4$.

One should stress that when $s/\Lambda^2 = 1.0$, $t/\Lambda^2 = 50.0$, the ratio between the asymptotic and the exact amplitude is

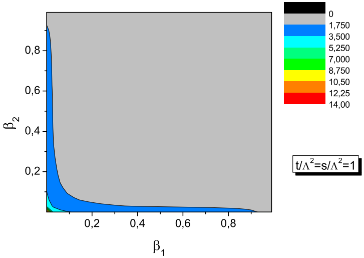
$$R = \frac{\tilde{\mathcal{A}}^{\text{as}}_{\text{GDA}}}{\tilde{\mathcal{A}}} = 1.01\,.$$

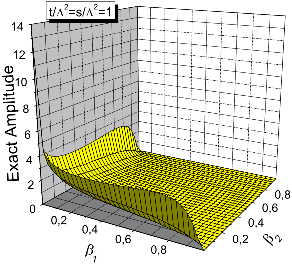
Regime (b): $t \ll Q^2$ while s is of order Q^2 (TDA)

For this regime, the asymptotic formula is given by the following replacement:

$$\mathcal{A}^{\text{as}}_{\text{TDA}}(s, t, m^2) \stackrel{s\leftrightarrow t}{\Longrightarrow} \mathcal{A}^{\text{as}}_{\text{GDA}}(t, s, m^2)\,.$$

Regime (c): $t, s \ll Q^2$ (overlap)

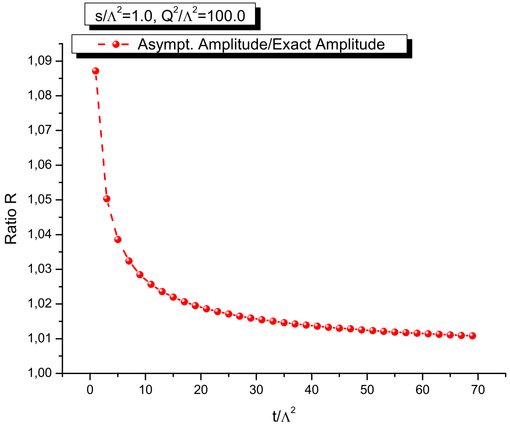




Option 1: $\tilde{A} \approx \left\{ \int_{\text{Reg.-1}} + \int_{\text{Reg.-3}} + \int_{\text{Reg.-4}} \right\} \Rightarrow$ 80% of the ex.ampl.

Option 2: $\tilde{A} \approx \tilde{A}^{\text{as}}_{\text{GDA}}(s, t, m^2)\Big|_{t \to s} \quad \Rightarrow \quad R = \tilde{A}^{\text{as}}_{\text{GDA}} / \tilde{A}$

Analyzing the ratio $R = \tilde{A}^{as}_{GDA}/\tilde{A}$ in terms of t/Λ^2, we obtain that



Conclusions for φ^3_E-model

Regime (a) and (b): NO Ambiguity in Factorization.

Regime (c)

Option 1: NO Factorization $\Rightarrow$

$\tilde{\mathcal{A}} \approx (\beta_1 - \mathrm{wing}) \oplus (\beta_2 - \mathrm{wing}) \equiv$ "ADDITIVITY"

Option 2: Factorization $\Rightarrow$

$\tilde{\mathcal{A}} \approx$ either $\tilde{\mathcal{A}}^{as}_{GDA}$ or $\tilde{\mathcal{A}}^{as}_{TDA} \equiv$ "DUALITY"

Duality in QCD

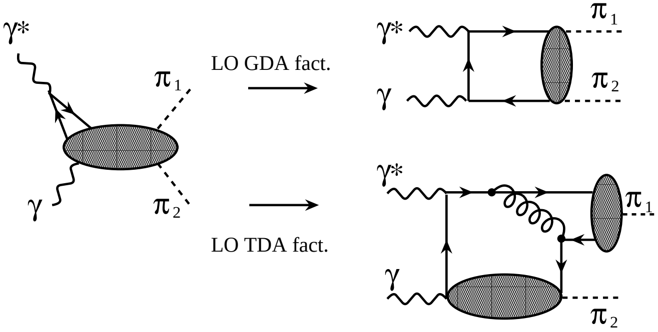
We consider the process:

$$\gamma_L^*(q) + \gamma_T(q') \to \pi(p_1)\pi(p_2).$$

The conditions to observe duality are

- the helicity amplitude: $\mathcal{A}_{(0,+)} = \varepsilon_\mu^{(0)} T^{\mu\nu}_{\gamma\gamma^*} \varepsilon_\nu'^{(+)}$
- $\xi \approx 1$ (small s) for TDA Factorization
- $\zeta \approx 0$ (small t) for GDA Factorization

Diagrams



Parametrization of photon-to-pion matrix elements

The $\gamma \to \pi^-$ matrix elements, entering the TDA-factorized amplitude, can be parameterized in the form

$$\langle \pi^-(p_2)|\bar{\psi}(-z/2)\gamma_\alpha\gamma_5[-z/2; z/2]\psi(z/2)|\gamma(q', \varepsilon')\rangle \stackrel{\mathcal{F}}{=} \frac{e}{f_\pi}\varepsilon'_T \cdot \Delta_T P_\alpha A_1(x, \xi, t)\,,$$

where $P = (p_2 + q')/2$, and $\Delta = p_2 - q'$.

To normalize the axial-vector TDA, A_1, we express A_1 in terms of the axial-vector form factor measured in the weak decay $\pi \to l\nu_l\gamma$, i.e.,

$$\int_{-1}^{1} dx\, A_1(x,\, \xi,\, t) = 2\, f_\pi\, F_A(t)/m_\pi\,,$$

where $f_\pi = 0.131\,\text{GeV}$, $m_\pi = 0.140\,\text{GeV}$, and $F_A(0) \approx 0.012$.

Thus, the helicity amplitude associated with the TDA mechanism reads

$$\mathcal{A}^{\text{TDA}}_{(0,j)} = \frac{\varepsilon'^{(j)} \cdot \Delta^T}{Q} \mathcal{F}^{\text{TDA}},$$

$$\mathcal{F}^{\text{TDA}} = [4\,\pi\,\alpha_s(Q^2)] \frac{C_F}{2\,N_c} \Big(\mathrm{tw}{-}2\;\;\mathrm{DA}\Big) \Big(\mathrm{tw}{-}2\;\;\mathrm{TDA}\Big),$$

where

$$\Big(\mathrm{tw}{-}2\;\;\mathrm{DA}\Big) = \int\limits_0^1 dy\,\phi_\pi(y) \left(\frac{1}{y} + \frac{1}{\bar{y}}\right)$$

$$\Big(\mathrm{tw}{-}2\;\;\mathrm{TDA}\Big) = \int\limits_{-1}^1 dx\,A_1(x,\xi,t)\left(\frac{e_u}{\xi - x} - \frac{e_d}{\xi + x}\right),$$

where $\xi^{-1} = 1 + 2W^2/Q^2$; the 1-loop $\alpha_s(Q^2)$ in the $\overline{\mathrm{MS}}$-scheme with $\Lambda_{\mathrm{QCD}} = 0.312$ GeV for $N_f = 3$.

The GDA helicity amplitude is (Wandzura-Wilczek approx.)

$$\mathcal{A}^{\text{GDA}}_{(0,j)} = \frac{\varepsilon'^{(j)} \cdot \Delta^T}{Q} \mathcal{F}^{\text{GDA}},$$
$$\mathcal{F}^{\text{GDA}} = 2\frac{W^2+Q^2}{Q^2}(e_u^2+e_d^2)\Big(\mathrm{tw}{-}3\ \mathrm{GDA\,WW}\Big),$$

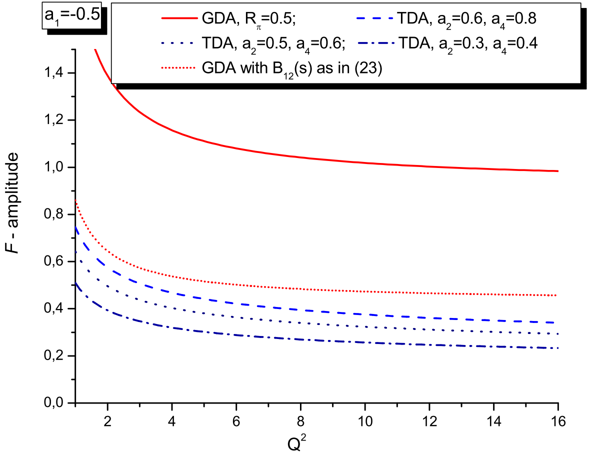
where

$$\Big(\mathrm{tw}{-}3\ \mathrm{GDA\,WW}\Big) = \int\limits_0^1 dy\, \partial_\zeta \Phi_1(y,\zeta,W^2)\left(\frac{\ln \bar{y}}{y} - \frac{\ln y}{\bar{y}}\right),$$

where $2\zeta - 1 = \beta \cos\theta^\pi_{cm}$ and $\cos\theta^\pi_{cm} = 2t/(W^2+Q^2) - 1$ and the partial derivative is defined by $\partial_\zeta = \partial/\partial(2\zeta-1)$.

Modeling of non-perturbative objects

- Twist 2 TDA $A_1(x,\xi) \stackrel{\xi\approx 1}{\Longrightarrow}$
  $\varphi_\gamma(\frac{1+x}{2}) \sim \left\{ a_1^D = -0.5,\; a_2 \in [0.3,\, 0.6],\; a_4 \in [0.4,\, 0.8] \right\}$
- Twist 3 GDA $\Phi_3^{WW}(z,\zeta)$, $\Phi_A^{WW}(z,\zeta) \stackrel{\zeta\approx 0}{\Longrightarrow}$
  $\Phi_1(z) \sim \left\{ \tilde{B}_{10}(W^2),\; \tilde{B}_{12}(W^2) \right\}$
  $\tilde{B}_{10}(W^2)$ can be discarded, while $\tilde{B}_{12}(W^2)$ can be modeled either by simplest ansatz $\beta^2 10 R_\pi/(9N_f)$ or by the resonance formula .
  (The relevant phase shift of the $\pi\pi$ scattering is defined by $\delta_2(W_0 = 0.8) \approx 0.03\pi$)



a1=-0.5
GDA, Rπ=0.5;
TDA, a2=0.6, a4=0.8
TDA, a2=0.5, a4=0.6;
TDA, a2=0.3, a4=0.4
GDA with B12(s) as in (23)
F - amplitude
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
2
4
6
8
10
12
14
16
Q2

Duality in QCD

Thus, duality in QCD we observed means that

$$\mathcal{A}^{\text{TDA}}_{(0,+)} = \frac{\varepsilon'^{(+)} \cdot \Delta^T}{Q} [4\,\pi\,\alpha_s(Q^2)] \Big(\text{tw}-2\ \text{DA}\Big) \Big(\text{tw}-2\ \text{TDA}\Big),$$

and

$$\mathcal{A}^{\text{GDA}}_{(0,+)} = \frac{\varepsilon'^{(+)} \cdot \Delta^T}{Q} \Big(\text{tw}-3\ \text{GDA}\Big),$$

are equivalent each other.

SSA and Duality

$$\mathcal{A}^{\mathrm{SSA}} = \frac{d\sigma_{\rightarrow} - d\sigma_{\leftarrow}}{d\sigma_{\rightarrow} + d\sigma_{\leftarrow}} = \frac{\mathrm{Im}[\,\rho_k^{(+,0)}\,]\,\mathrm{Im}\Big[\overset{*}{\mathcal{A}}_{(+,+)}\ \mathcal{A}_{(0,+)}\Big]}{4\,\rho_k^{(+,+)}\,\big|\mathcal{A}_{(+,+)}\big|^2}\ .$$

where

$$\rho_k^{(i,i')} \overset{\mathrm{def}}{=} [\,\overset{*}{\varepsilon}{}^{(i)} \cdot \mathcal{L}(k_1,k_2)\,]\,[\,\varepsilon^{(i')} \cdot \mathcal{L}^{+}(k_1,k_2)\,]\ ,$$

$$\mathcal{L}_\alpha(k_1,k_2) \overset{\mathrm{def}}{=} \bar{u}(k_2)\gamma_\alpha u(k_1)$$

Conclusions

- ★ We have shown that when it happens that both Mandelstam variables s and t are much less than the large momentum scale Q^2, with the variables s/Q^2 and t/Q^2 varying in the interval (0.001, 0.7), then the TDA and the GDA factorization mechanisms are equivalent to each other and operate in parallel.
- ★ We demonstrated that duality may serve as a tool for selecting suitable models for the non-perturbative ingredients of QCD factorization of various exclusive amplitudes.
- ★ We also observed that twist-3 GDAs appear to be dual to the convolutions of leading-twist TDAs and DAs, multiplied by a QCD effective coupling.

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