Heavy quark and target mass effects on the virtual photon in QCD

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Photon 2009 at DESY, Hamburg

Contents

1. Introduction

- Future linear collider experiment (ILC)

Two-photon process $e^+e^- \to (e^+e^-\gamma\gamma) \to e^+e^-X$

Viewed as a deep-inelastic electron-photon scattering when $P^2 \ll Q^2$

We can study the structures of photon

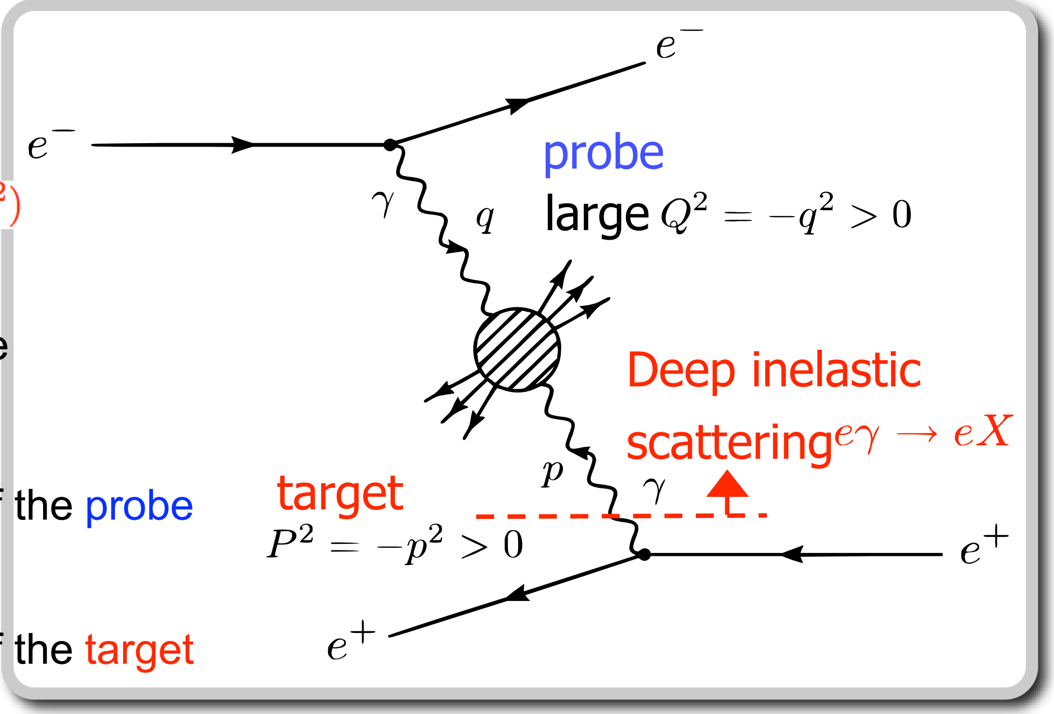
$F_2^\gamma(x, Q^2, P^2)$ and $F_L^\gamma(x, Q^2, P^2)$

$x = \dfrac{Q^2}{2p \cdot q}$:Bjorken variable

$0 \le x \le 1$

$-Q^2 = q^2 \le 0$:mass squared of the probe photon

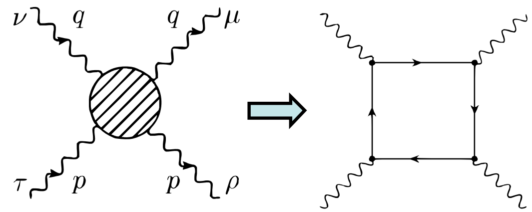
$-P^2 = p^2 \le 0$:mass squared of the target photon



F_2^γ in Perturbative QCD

- For real photon target $(P^2 \approx 0)$ $\quad P^2 \ll Q^2$

Simple parton model



$$F_2^\gamma(x,Q^2) = \alpha \left[\boxed{\frac{1}{\alpha_s(Q^2)}} A + B + B' + \mathcal{O}(\alpha_s) \right]$$

(LO) $\quad$ (NLO) $\quad$ Hadronic piece

$\sim \ln \frac{Q^2}{\Lambda^2}$

OPE+RGE
lowest order in α

Witten (1977)
Bardeen-Buras (1979)

Point-like contribution dominates $\sim \ln Q^2$

Walsh-Zerwas (1973)

⇨ NNLO extension $\quad$ Moch-Vermaseren-Vogt (2002, 2006)

- For highly virtual photon target $(\Lambda^2 \ll P^2 \ll Q^2)$

$$F_2^\gamma(x,Q^2,P^2) = \alpha \left[\frac{1}{\alpha_s(Q^2)} \tilde{A} + \tilde{B} + \mathcal{O}(\alpha_s) \right]$$

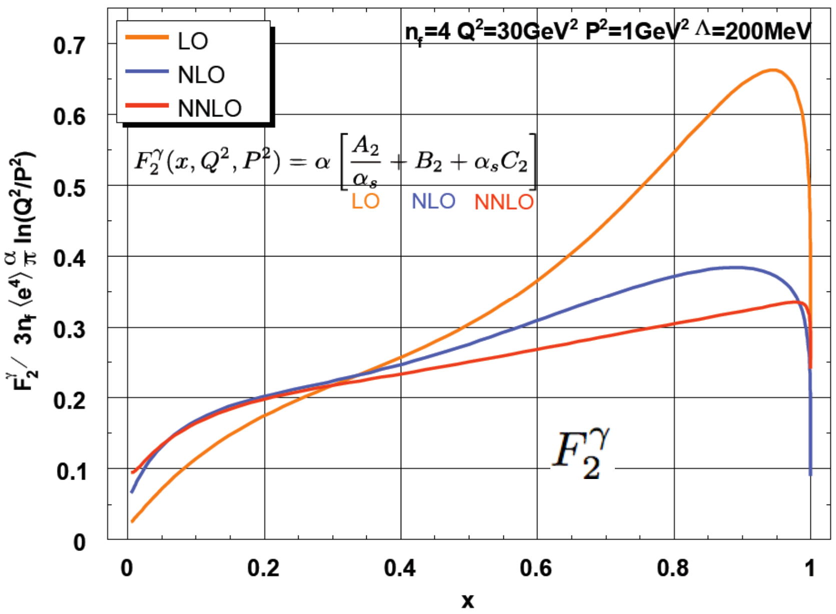
Λ : QCD scale parameter

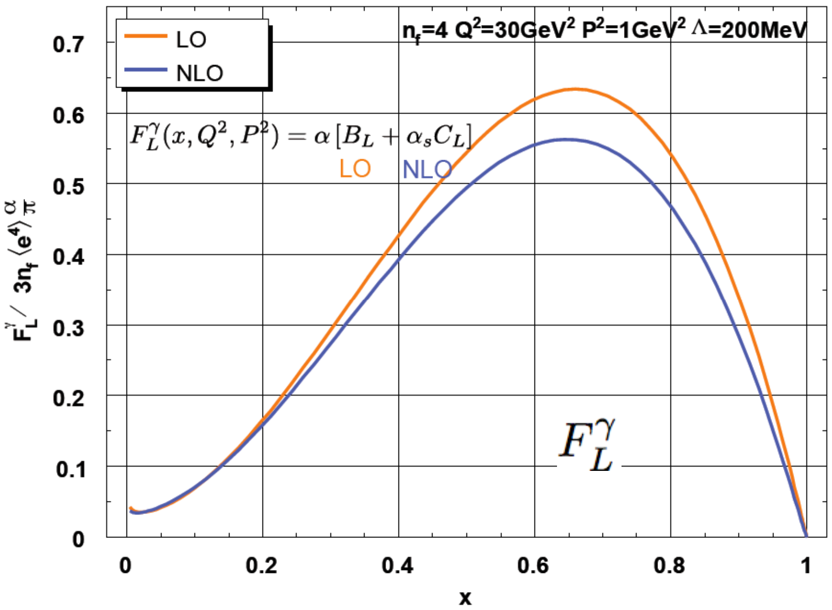
(LO) $\quad$ (NLO) $\quad$ Uematsu-Walsh (1981,1982)

Hadronic piece can also be dealt with perturbatively
Definite prediction of F_2^γ, its shape and magnitude, is possible

- NNLO ($\alpha\alpha_s$) extension $\quad$ Ueda-Uematsu-KS (2007)

Motivated by the calculation of 3-loop anomalous dimensions
Vogt-Moch-Vermaseren (2004,2006)



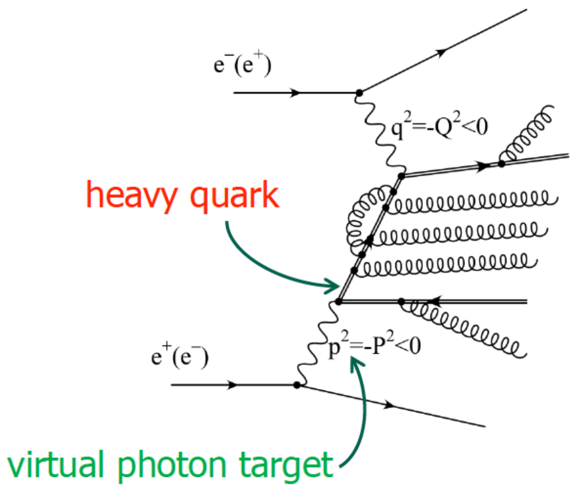


- NNLO QCD analysis performed with 3-loop splitting fns. and 2-loop coefficient fns. for massless quarks

- Here we investigate heavy quark mass and target mass effects on virtual photon structure fns.

The heavy quark effects in DIS processes

- In the case of nucleon target, the heavy quarks are treated as being absent in the intrinsic parton components but radiatively generated from the gluon and light quarks
- For virtual photon target, the heavy quarks are treated in the same way as light quarks and generated from the photon target (and also from gluon and light quarks)
- Heavy quark mass effects for the real photon case are studied in the literature
  M. Gluck, E. Reya & A. Vogt,
  F. Cornet, P. Jankowski & M. Krawczyk, A. Lorca, ...

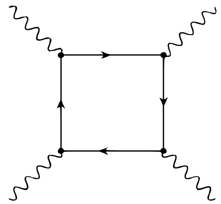


- We study heavy quark mass effects in the framework of mass-independent renormalization group method

Naïve QPM vs. PLUTO & L3 data

- Effective photon structure function

$$F^{\gamma}_{\rm eff}(x,Q^2,P^2) = F^{\gamma}_2(x,Q^2,P^2) + \frac{3}{2}F^{\gamma}_L(x,Q^2,P^2)$$



- Naïve QPM

8 structure functions: $W_{TT}, W_{ST}, W_{TS}, W_{SS}, \cdots$ Budnev, Chernyak, Ginzburg

$$F^{\gamma}_{\rm eff}(x,Q^2,P^2) = \left(\frac{5}{\tilde{\beta}^2}-3\right)x\left[W_{TT}-\frac{1}{2}W_{TS}\right]+\frac{5}{\tilde{\beta}^2}x\left[W_{ST}-\frac{1}{2}W_{SS}\right]$$

$$\tilde{\beta} = \sqrt{1-\frac{P^2Q^2}{(p\cdot q)^2}} \qquad \beta = \sqrt{1-\frac{4m^2}{(p+q)^2}} \qquad L = \ln\frac{1+\beta\tilde{\beta}}{1-\beta\tilde{\beta}}$$

- Heavy quark mass threshold effects: $x^H_{\rm max} = \dfrac{1}{1+\frac{4m_H^2}{Q^2}+\frac{P2}{Q^2}} \longleftarrow (p+q)^2 \geq 4m_H^2$

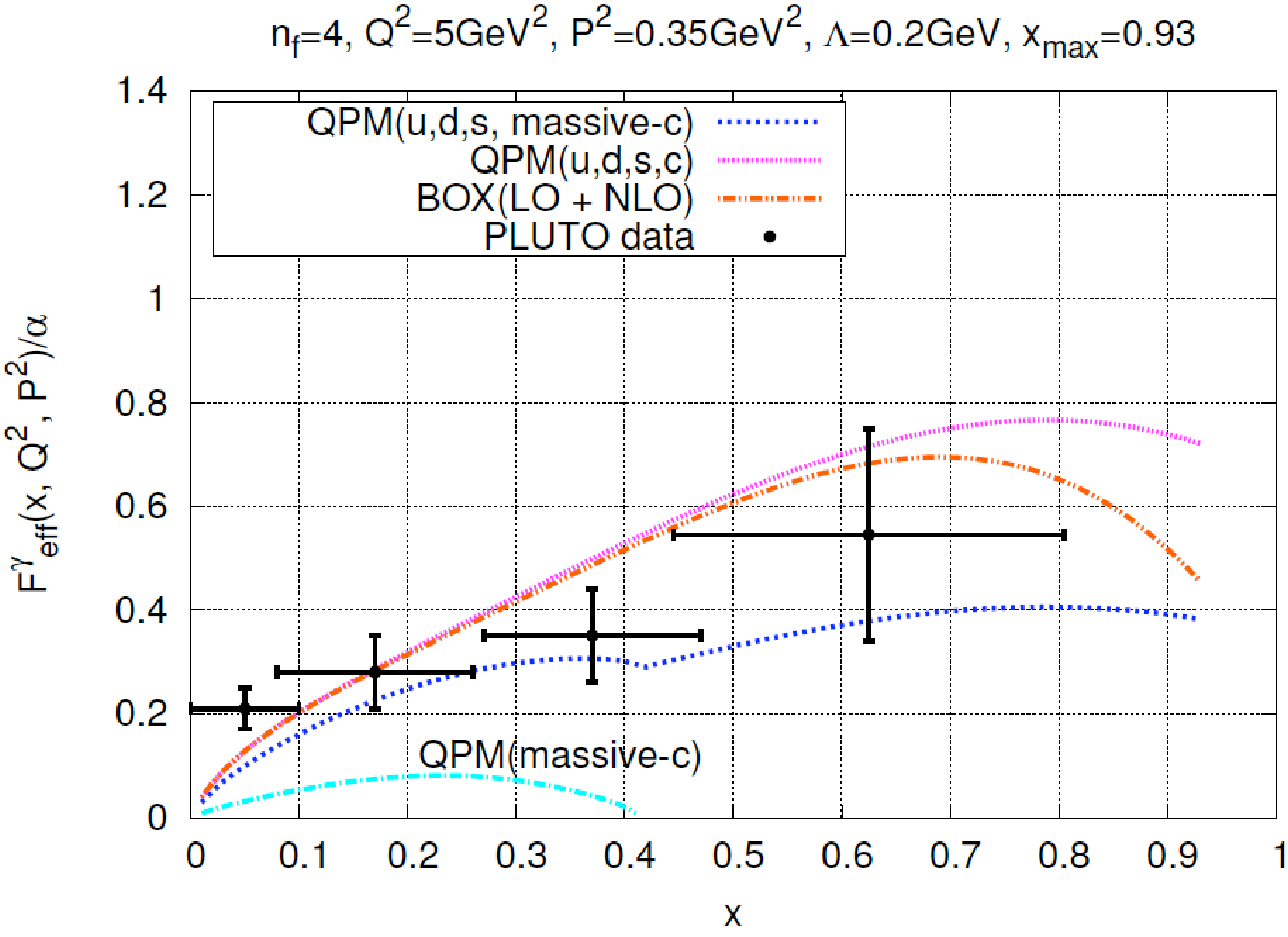
- Heavy quark mass inputs:

$$m_c = 1.3\ \mathrm{GeV} \qquad m_b = 4.2\ \mathrm{GeV}$$

- Box(LO+NLO) graph: all quarks massless, $\left(\frac{Q^2}{P^2}\right)^k$ powers are neglected

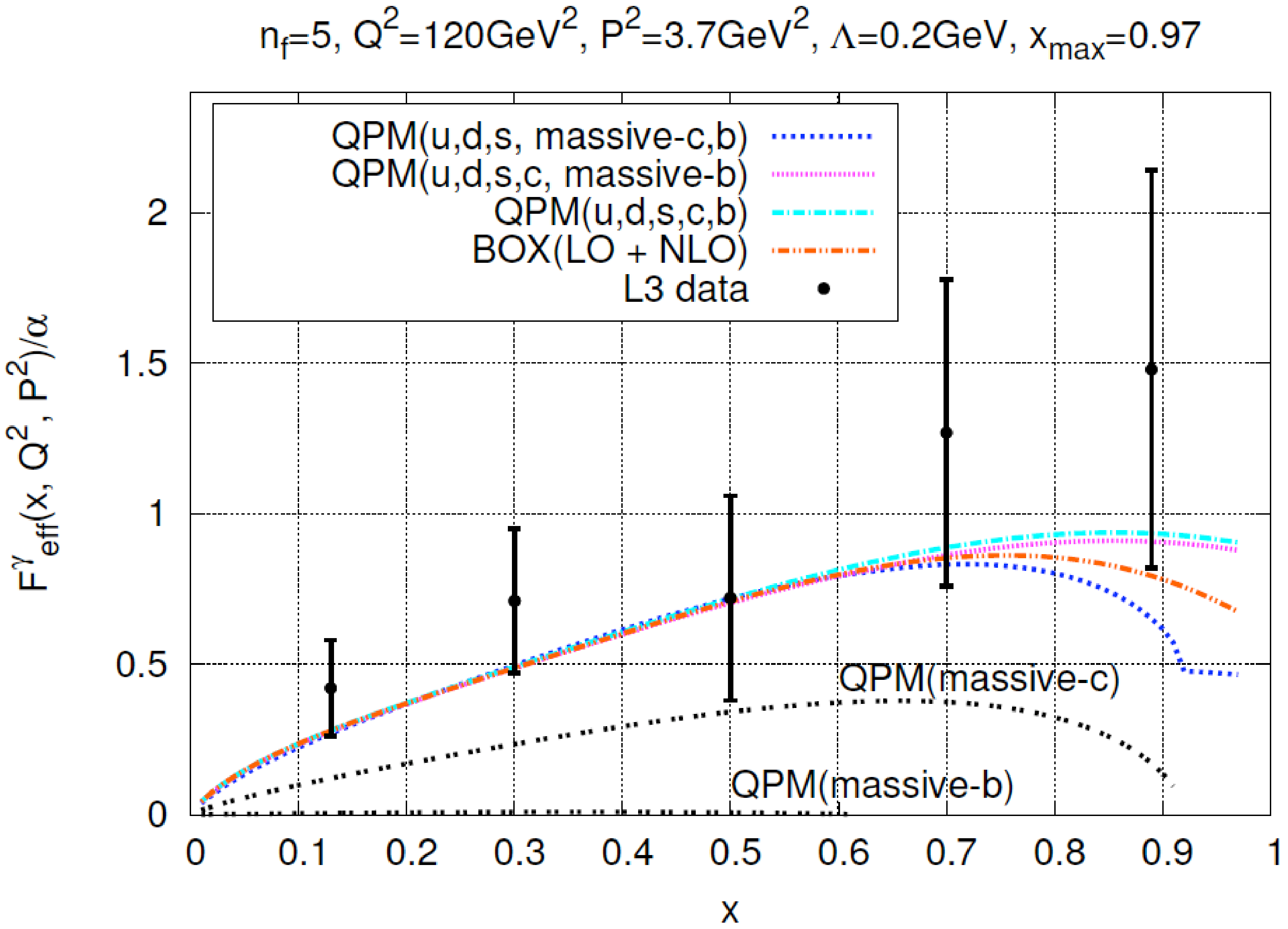
$$\beta = 1 \qquad L \Rightarrow \left(\ln\frac{Q^2}{P^2} - 2\ln x\right)$$

Naïve QPM vs. PLUTO data for F^{γ}_{eff}



$$e_c^4 = \frac{16}{81}$$

Naïve QPM vs. L3 data for $F_{\rm eff}^{\gamma}$



$$e_b^4 = \frac{1}{81}$$

$$e_c^4 = \frac{16}{81}$$

2. Evolution equations with heavy quark

n_f :number of flavours

Let us consider the case: $n_f - 1$ massless quarks and 1 heavy quark

The evolution equation:

$$\frac{d}{d\ln Q^2}\vec{q}^{\gamma}(x,Q^2,P^2) = \int_x^1 \frac{dy}{y}\left[\vec{q}^{\gamma}(y,Q^2,P^2)\hat{P}\left(\frac{x}{y},Q^2\right)\right] + \vec{k}(x,Q^2,P^2)$$

$$\vec{q}^{\gamma}(x,Q^2,P^2) = (q_L^{\gamma}, q^H, G^{\gamma}, q_{NS}^{\gamma})$$

$$q_L^{\gamma} \equiv \sum_{i=1}^{n_f-1} q^i$$ Singlet quark

$$q_{NS}^{\gamma} \equiv \sum_{i=1}^{n_f-1} e_i^2 q_{NS}^i$$ Non-singlet quark

$$q_{NS}^i \equiv q^i - \frac{1}{n_f-1} q_L^{\gamma}$$

Splitting functions

$$\hat{P} \equiv \begin{pmatrix} P_{qq}^S & P_{LH} & P_{LG} & 0 \\ P_{HL} & P_{HH} & P_{HG} & 0 \\ P_{GL} & P_{GH} & P_{GG} & 0 \\ 0 & 0 & 0 & P_{qq}^{NS} \end{pmatrix}^T$$

Photon-parton splitting function

$$\vec{k} = (k_L, k_H, k_G, k_{NS})$$

Splitting functions

$\tilde{P}^{S}_{qq}$ starts from $\mathcal{O}(\alpha_s^2)$

$$
\begin{aligned}
P^S_{qq} &= \tilde{P}_{qq} + \frac{n_f - 1}{n_f}\tilde{P}^S_{qq}, \quad P_{LH} = \frac{n_f - 1}{n_f}\tilde{P}^S_{qq}, \quad P_{LG} = (n_f - 1)\tilde{P}_{qG}, \\
P^S_{HL} &= \frac{1}{n_f}\tilde{P}^S_{qq}, \quad P_{HH} = \tilde{P}_{qq} + \frac{1}{n_f}\tilde{P}^S_{qq}, \quad P_{HG} = \tilde{P}_{qG}, \\
P^S_{GL} &= \tilde{P}_{Gq}, \quad P_{GH} = \tilde{P}_{Gq}, \quad P_{GG} = \tilde{P}_{GG}, \quad P^{NS}_{qq} = \tilde{P}_{qq} .
\end{aligned}
$$

Photon-parton splitting functions

$$
k_L = \sum_{i=1}^{n_f-1} \tilde{P}_{i\gamma}, \quad k_H = \tilde{P}_{H\gamma}, \quad k_G = \tilde{P}_{G\gamma}, \quad k_{NS} = \sum_{i=1}^{n_f-1} e_i^2 \left(\tilde{P}_{i\gamma} - \frac{1}{n_f - 1} \sum_{j=1}^{n_f-1} \tilde{P}_{j\gamma} \right)
$$

Mass-independent renormalization group eq.

$$
\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_m(g)m\frac{\partial}{\partial m} - \gamma_n(g,\alpha)\right]_{ij} C^j_n\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \bar{g}(\mu^2), \alpha\right) = 0
$$

Coefficient functions

The heavy quark mass effects in parton picture and OPE

$$F_2^\gamma(x,Q^2,P^2)=\vec{q}^{\,\gamma}(y,Q^2,P^2,m^2)\otimes\vec{C}\left(\frac{x}{y},\frac{\overline{m}^2}{Q^2},\bar{g}(Q^2)\right)$$

Photon structure function PDF Coefficient function

mass dependence

Parton interpretation of twist-2 operators $\vec{O}_n$

No mass dependence

$$\int_0^1 dx\,x^{n-1}\vec{q}^{\,\gamma}(x,Q^2,P^2,m^2)$$

$$=\vec{A}_n\left(1,\frac{\overline{m}^2(P^2)}{P^2},\bar{g}(P^2)\right)T\exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg\frac{\gamma_n(g,\alpha)}{\beta(g)}\right]$$

where

$$\langle\gamma(P^2)|\vec{O}_n(\mu^2)|\gamma(P^2)\rangle=\vec{A}_n\left(\frac{P^2}{\mu^2},\frac{\overline{m}^2(\mu^2)}{\mu^2},\bar{g}(\mu^2)\right)$$

Perturbatively calculable !

Mass dependence

3. Heavy quark mass effects

- We compute the deviation arising from heavy quark mass effects on the photon matrix elements of twist-2 quark & gluon operators and on the coefficient fns. up to NLO in QCD

$$M_2^\gamma(n,m^2) \;=\; M_2^\gamma(n,m^2=0)+\Delta M_2^\gamma(n,m^2)$$

$$\Delta M_2^\gamma(n,Q^2,P^2,m^2)=\int_0^1 dx x^{n-2}\Delta F_2^\gamma(x,Q^2,P^2,m^2)$$

$$=\frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left[\sum_{i=\pm,NS}\Delta\mathcal{A}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right]\right.$$
$$\left.+\sum_{i=\pm,NS}\Delta\mathcal{B}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right]+\Delta\mathcal{C}^n\right]+\mathcal{O}(\alpha_s)$$

1-loop anomalous dim.

$$d_i^n=\lambda_i^n/2\beta_0$$
$$(i=\pm,NS)$$

In the massive quark limit

$$\Lambda^2_{\rm QCD} \ll P^2 \ll m^2 \ll Q^2$$

e_H : Heavy quark charge

$$\langle e^2 \rangle_{n_f} = \sum_{i=1}^{n_f} e_i^2/n_f$$

We obtain

$$\Delta \mathcal{A}^n_{NS} = -12\beta_0 e_H^2 (e_H^2 - \langle e^2 \rangle_{n_f})(\Delta \tilde{A}^\psi_{nG}/n_f)$$

$$\Delta \mathcal{A}^n_{\pm} = -12\beta_0 e_H^2 \langle e^2 \rangle_{n_f} (\Delta \tilde{A}^\psi_{nG}/n_f) \frac{\gamma^{0,n}_{\psi\psi} - \lambda^n_{\mp}}{\lambda^n_{\pm} - \lambda^n_{\mp}}$$

$$\Delta \mathcal{B}^n_{NS} = 0\ , \quad \Delta \mathcal{B}^n_{\pm} = 0\ , \quad \Delta \mathcal{C}^n = 12\beta_0 e_H^2 (\Delta \tilde{A}^\psi_{nG}/n_f)$$

where

$$\tilde{A}^\psi_{nG}/n_f = 2\left[-\frac{n^2+n+2}{n(n+1)(n+2)} \ln \frac{m^2}{P^2} + \frac{1}{n} - \frac{1}{n^2} + \frac{4}{(n+1)^2} - \frac{4}{(n+2)^2} - \frac{n^2+n+2}{n(n+1)(n+2)} \sum_{j=1}^{n} \frac{1}{j} \right]$$

4. Numerical analysis

Y. Kitadono, T. Ueda, T. Uematsu and KS

Prog. Theor. Phys. 121(2009) 054019

We evaluate

Effective photon structure function

$$F_{\mathrm{eff}}^{\gamma}(x,Q^2,P^2)=F_2^{\gamma}(x,Q^2,P^2)+\frac{3}{2}F_L^{\gamma}(x,Q^2,P^2)$$

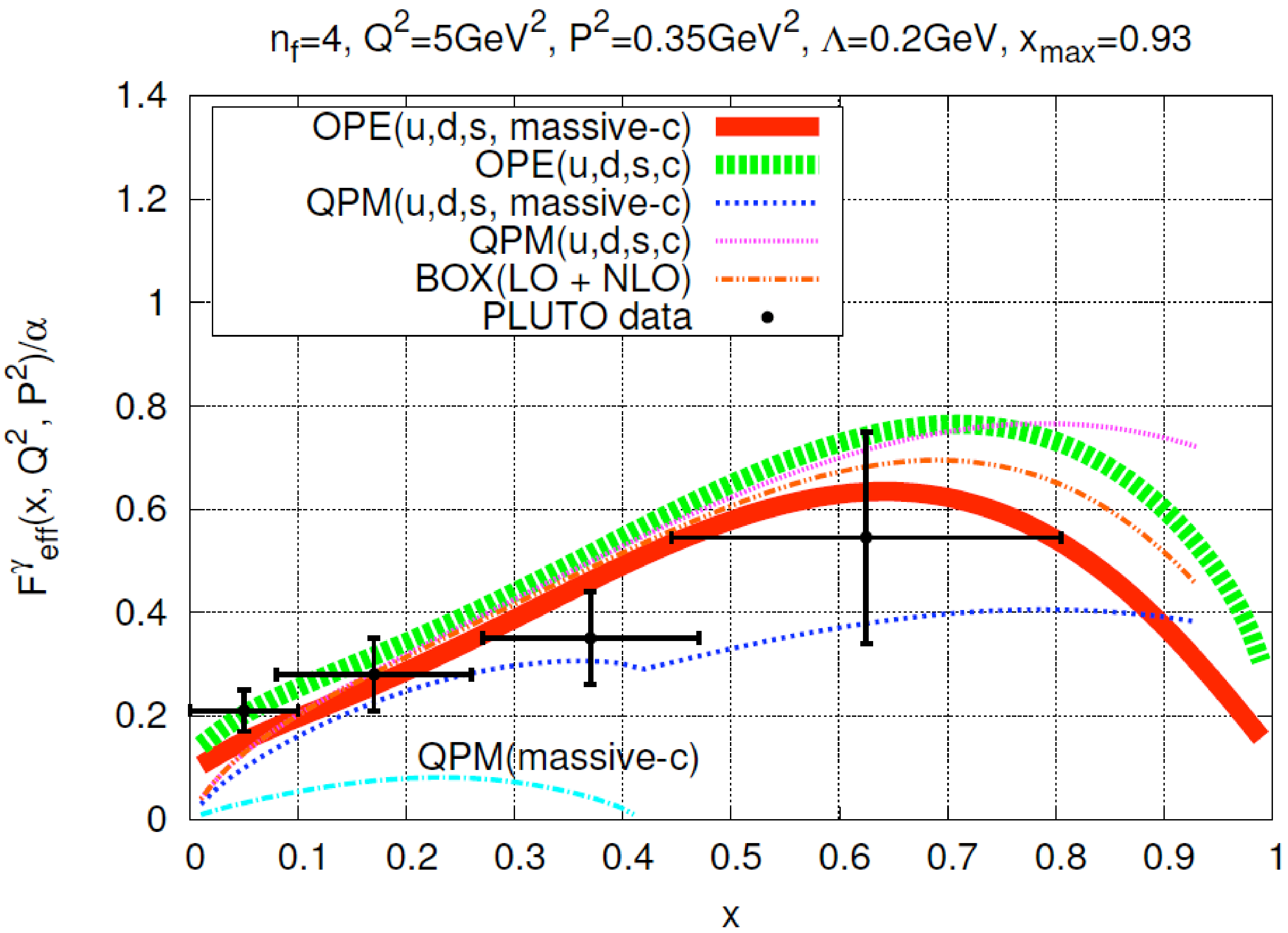
up to NLO in QCD and compare with the exisiting exp. data from PLUTO & L3

Heavy quark mass inputs:

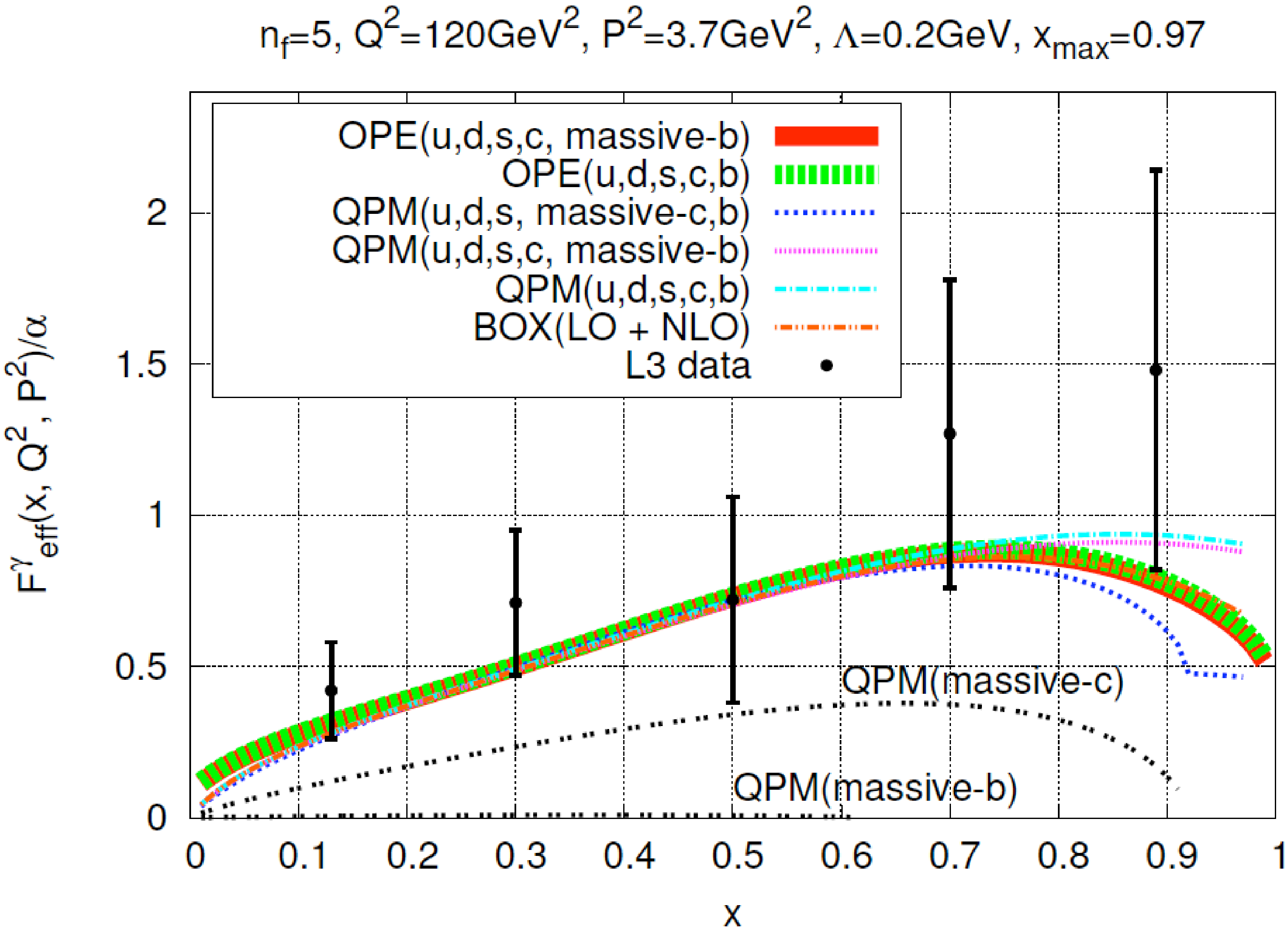
$m_c = 1.3\ \mathrm{GeV}$ (for PLUTO)

$m_b = 4.2\ \mathrm{GeV}$ (for L3)

NLO QCD prediction vs. PLUTO data



NLO QCD prediction vs. L3 data



5. Target mass effects

Y. Kitadono, T. Ueda, T. Uematsu and KS
Phys. Rev. D77, 054019 (2008)

Our previous analysis on $F_2^\gamma(x, Q^2, P^2)$ up to NNLO

Ueda-Uematsu-KS(07)

- The power corrections of the form $(P^2/Q^2)^k \ (k = 1, 2, \cdots)$ coming from target mass effects or higher-twist effects are ignored
- For a real photon $(P^2 = 0)$ - - - no target mass corrections
- For a virtual photon $(P^2 \neq 0)$ - - - target mass effects (TME)
- The maximal value of x

$$x_{\max} = \frac{1}{1 + \frac{P^2}{Q^2}} \quad \Leftarrow \quad (p+q)^2 \geq 0$$

- The structure functions should vanish at $x_{\max}$
- TME have been studied for nucleon structure functions

O. Nachtmann, H. Georgi and H. Politzer, S. Wandzura, H. Kawamura and T. Uematsu, …..

TME for the polarized virtual photon structure functions H. Baba, T. Uematsu and KS

Nachtmann Moments

- OPE at short distance

$$
\begin{aligned}
i\int d^4x e^{iq\cdot x} T(J_\mu(x)J_\nu(0)) \;=\; & \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \sum_{\substack{n=0\\ n=\text{even}}} \left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_n} \sum_i C^i_{L,n} O_i^{\mu_1\cdots\mu_n} \\
& + \left(-g_{\mu\lambda}g_{\nu\sigma}q^2 + g_{\mu\lambda}q_\nu q_\sigma + g_{\nu\sigma}q_\mu q_\lambda - g_{\mu\nu}q_\lambda q_\sigma\right) \sum_{\substack{n=2\\ n=\text{even}}} \left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_{n-2}} \sum_i C^i_{2,n} O_i^{\lambda\sigma\mu_1\cdots\mu_{n-2}}
\end{aligned}
$$

- The photon matrix elements

$$
\begin{aligned}
\langle\gamma(p)|O_i^{\mu_1\cdots\mu_n}|\gamma(p)\rangle_{\text{spin av.}} &= A_n^i(\mu^2, P^2)\{p^{\mu_1}\cdots p^{\mu_n} - \text{trace terms}\} \\
&\equiv A_n^i(\mu^2, P^2)\{p^{\mu_1}\cdots p^{\mu_n}\}_n \,.
\end{aligned}
$$

$\{p^{\mu_1}\cdots p^{\mu_n}\}_n$: totally symmetric traceless tensor

- Contraction

$$
q_{\mu_1}\cdots q_{\mu_n}\{p^{\mu_1}\cdots p^{\mu_n}\}_n = a^n C_n^{(1)}(\eta)
$$

$$
\begin{aligned}
q_{\mu_1}\cdots q_{\mu_{n-2}}\{p^\lambda p^\sigma p^{\mu_1}\cdots p^{\mu_{n-2}}\}_n = \frac{1}{n(n-1)} \Bigg[& \frac{g^{\lambda\sigma}}{Q^2} a^n 2C^{(2)}_{n-2}(\eta) + \frac{q^\lambda q^\sigma}{Q^4} a^n 8C^{(3)}_{n-4}(\eta) \\
& + p^\lambda p^\sigma a^{n-2} 2C^{(3)}_{n-2}(\eta) + \frac{p^\lambda q^\sigma + q^\lambda p^\sigma}{Q^2} a^{n-1} 4C^{(3)}_{n-3}(\eta) \Bigg]
\end{aligned}
$$

where $C_n^{(\nu)}(\eta)$ Gegenbauer polynomials

$$
(1-2\eta t + t^2)^{-\nu} = \sum_{n=0}^{\infty} C_n^{(\nu)}(\eta) t^n
$$

$$
a = -\frac{1}{2}PQ \qquad \eta = -\frac{p\cdot q}{PQ}
$$

Nachtmann Moments

- $$\begin{aligned} M^{\gamma}_{2,n} &= \int_0^{x_{max}} dx \frac{1}{x^3}\xi^{n+1}\left[\frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)}\right] F_2^{\gamma}(x,Q^2,P^2) \\ &= \sum_i C^i_{2,n}(Q^2,P^2,g)A^i_n(P^2) \end{aligned}$$

  where $r \equiv \sqrt{1-\frac{4P^2x^2}{Q^2}}$, $\xi \equiv \frac{2x}{1+r}$

  Without TME

  $$M^{\gamma}_{2,n} = \int_0^1 dx x^{n-2} F_2^{\gamma}(x,Q^2,P^2)$$

- Inversion of Nachtmann moments

$$F_2^{\gamma}(x,Q^2,P^2) = \frac{x^2}{r^3}F(\xi) - 6\kappa\frac{x^3}{r^4}H(\xi) + 12\kappa^2\frac{x^4}{r^5}G(\xi)$$

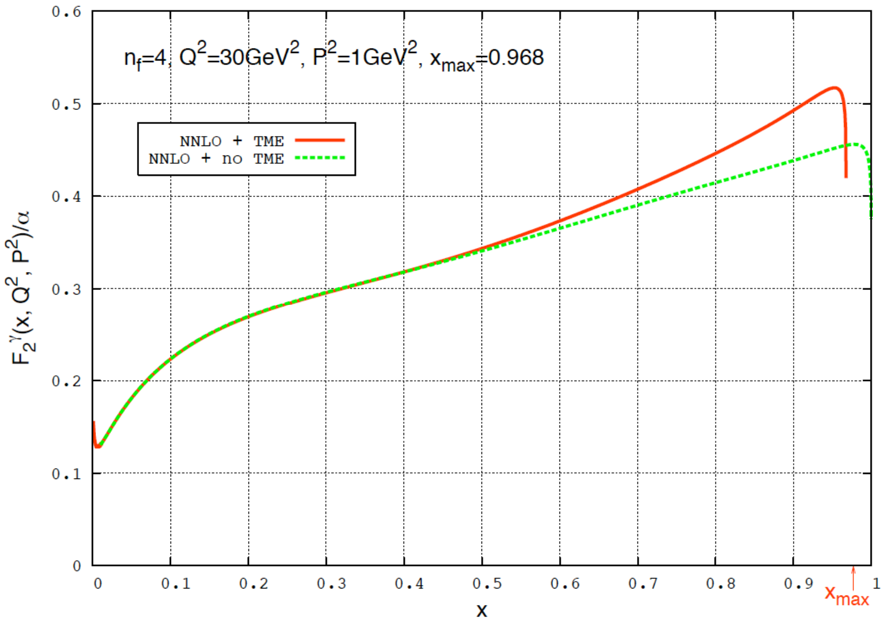
$$\kappa = \frac{P^2}{Q^2}$$

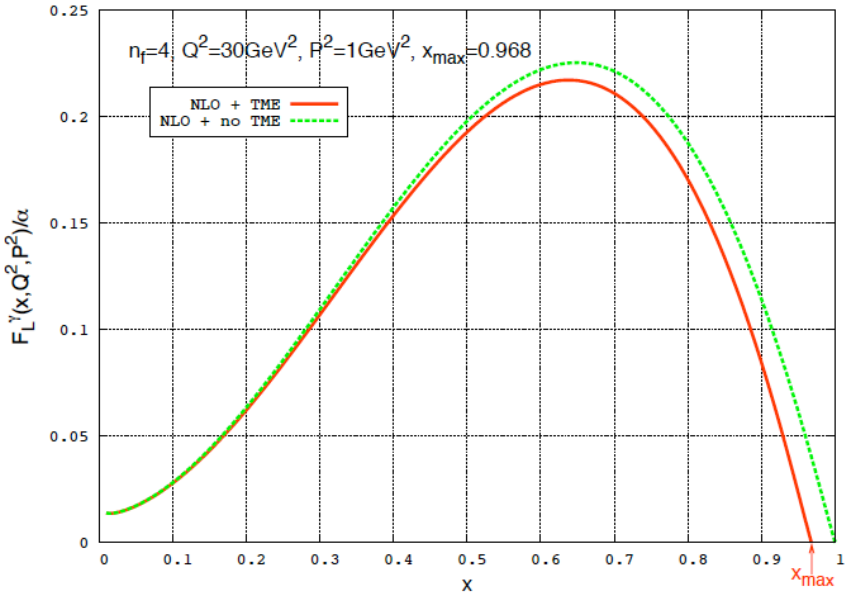
$$G(\xi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, \xi^{-n+1}\left\{\frac{M^{\gamma}_{2,n}}{n(n-1)}\right\}$$

$$H(\xi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, \xi^{-n}\frac{M^{\gamma}_{2,n}}{n}$$

$$F(\xi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, \xi^{-n-1}M^{\gamma}_{2,n}$$

Numerical Results (TME)

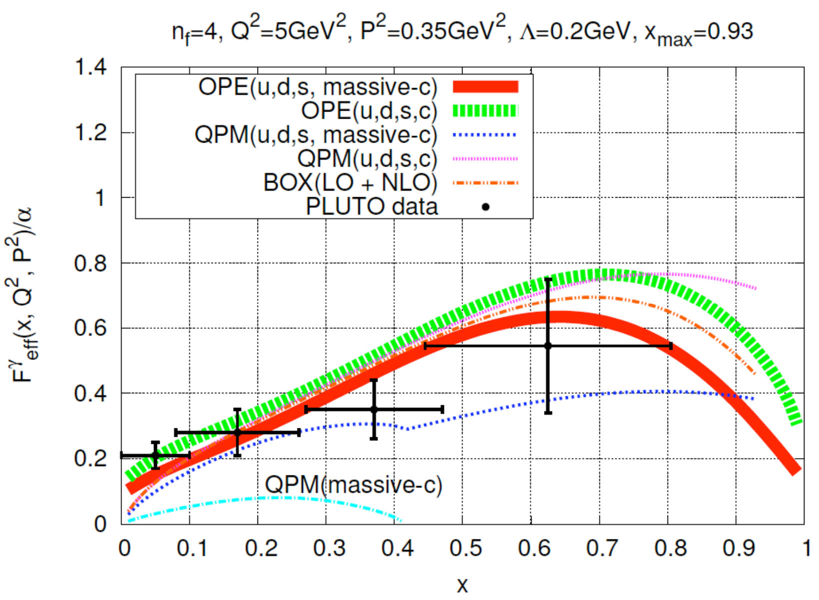


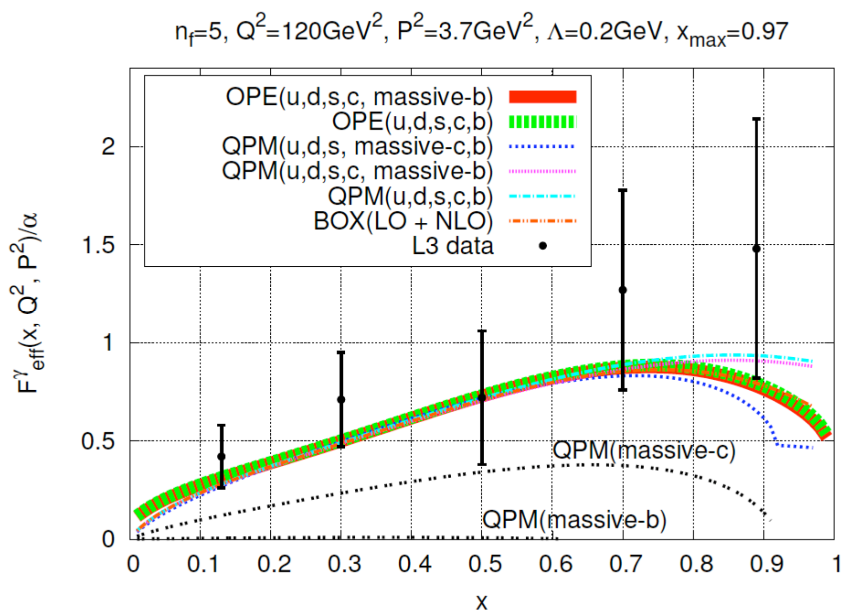


up to NNLO ($\alpha\alpha_s$) in QCD for massless quarks

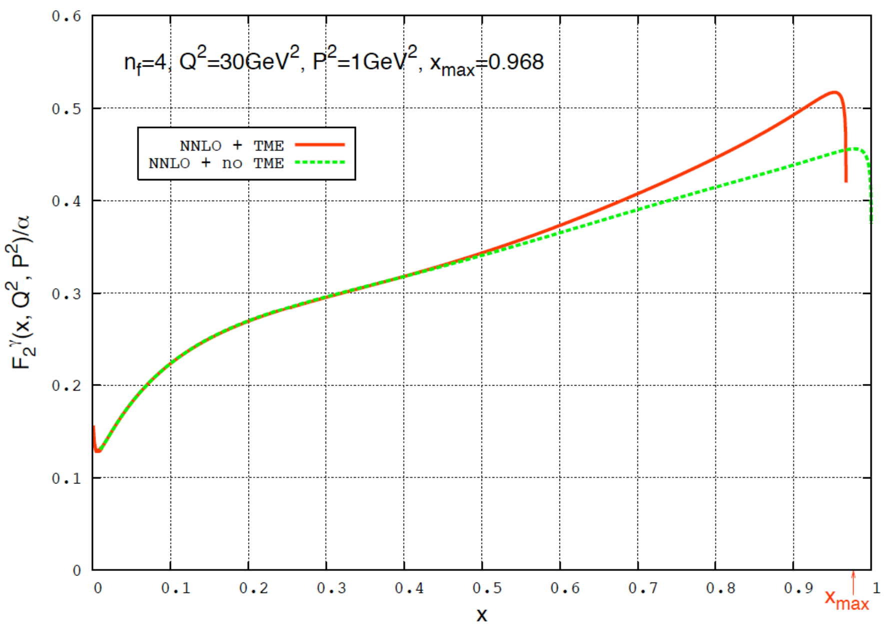
6. Summary

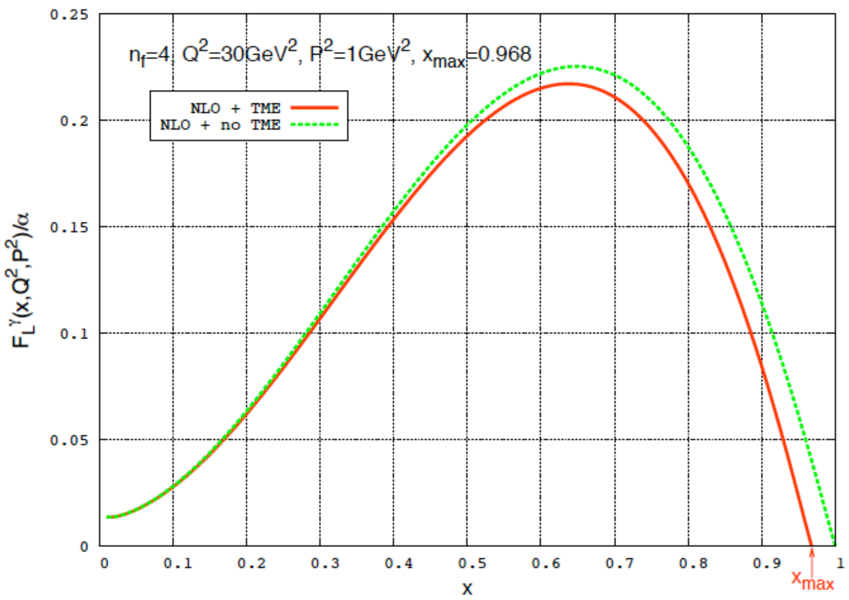
- Heavy quark mass effects on the virtual photon structure functions were investigated in the framework of OPE and the mass-independent renormalization group method up to NLO (α)
- Heavy quark mass effects appear in the photon matrix elements of the twist-2 quark & gluon operators and coefficient functions
- NLO QCD predictions for $F_{\rm eff}^{\gamma}(x, Q^2, P^2)$ are compared with the exp. data of PLUTO & L3





- We also analyzed the target mass effects on the virtual photon structure functions up to NNLO ($\alpha\alpha_s$) for massless quarks





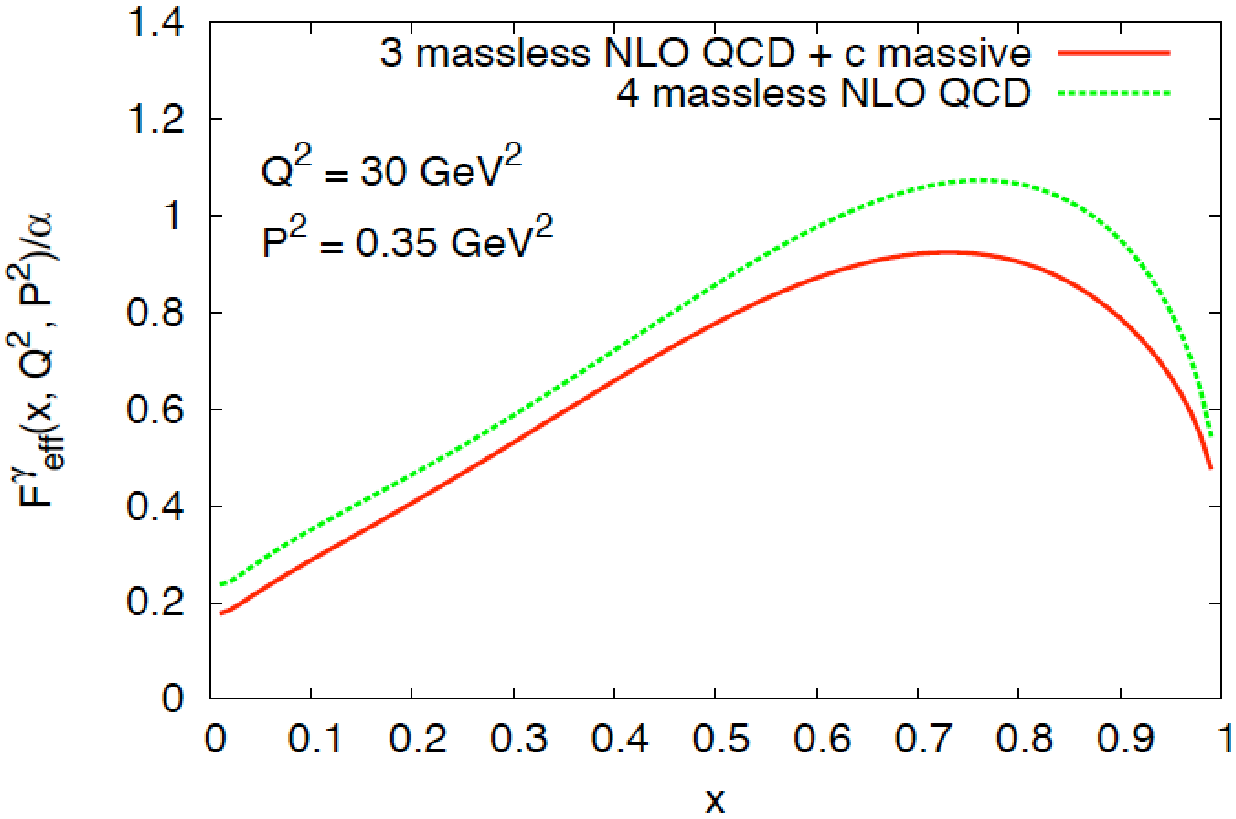
Future works

- Heavy quark effects and target mass effects should be considered together

  Y. Kitazono, T. Ueda, T. Uematsu, K.S. in preparation

- We have only considered the twist-2 operators and not the higher-twist effects. Hence we have not included the kinematical threshold effects. So the next subject is how to take account of these kinematical threshold effects into the QCD analysis

Prediction for $F_{\rm eff}^{\gamma}$ with flavor effects



when massive quark limit is satisfied

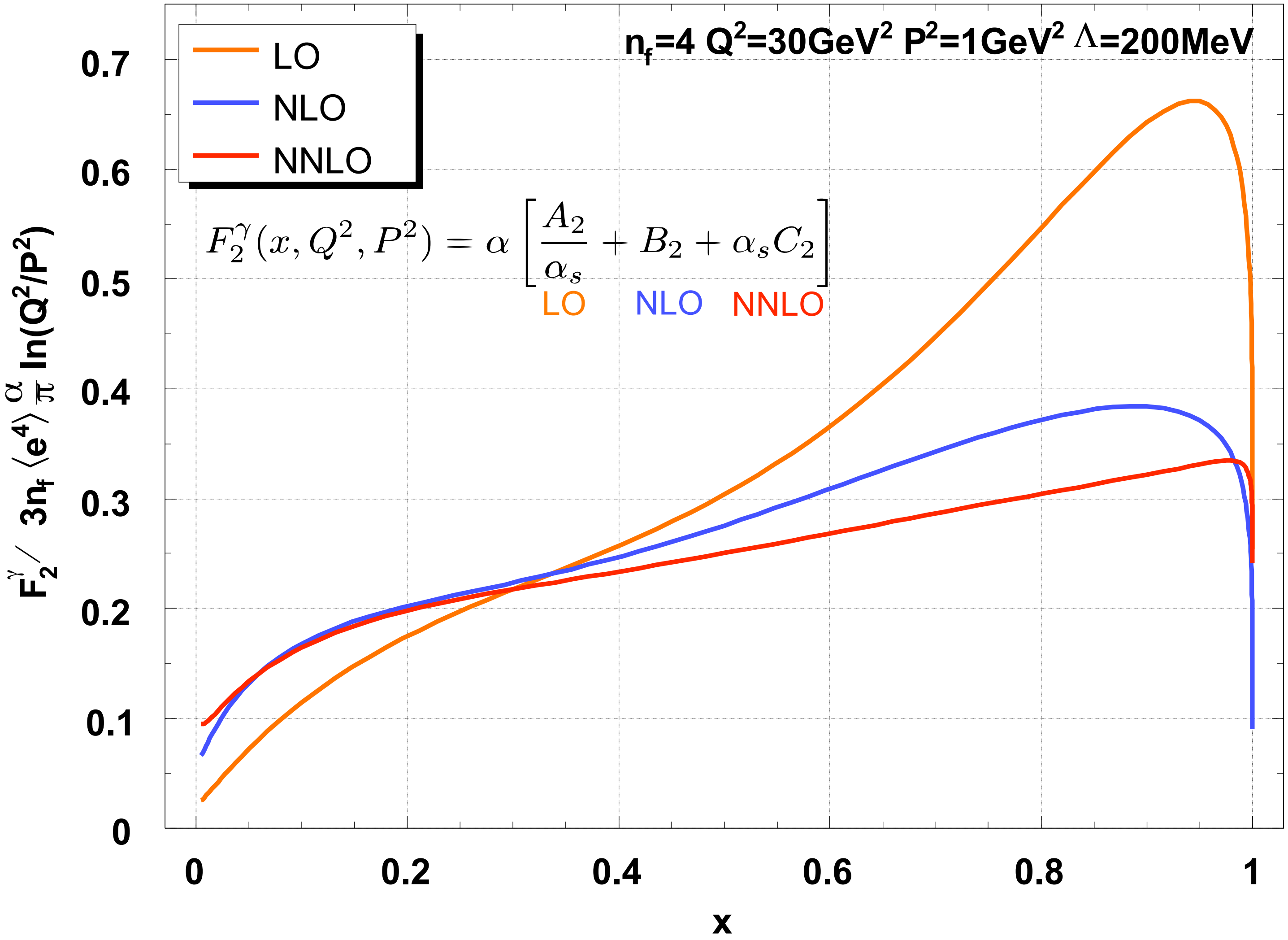
$$\Lambda^2_{\rm QCD} \ll P^2 \ll m^2 \ll Q^2$$

Extra Slides

QPM calculation of W_{TT}

$$
\begin{aligned}
\widetilde{W}_{TT}|_{PM} &= W_{TT}|_{PM}\frac{1}{\frac{\alpha}{2\pi}\delta_\gamma} \\
&= L\Bigg\{-\frac{8x^2}{\widetilde{\beta}}\frac{m^4}{Q^4}+\frac{\widetilde{\beta}^2-(1-2x)^2}{\widetilde{\beta}^3}\ \frac{m^2}{Q^2} \\
&\qquad +\frac{1}{8\widetilde{\beta}^5}\left[(1-\widetilde{\beta}^2)(\widetilde{\beta}^4+3)-8x\left\{\widetilde{\beta}^4-2\widetilde{\beta}^2+3\right\}\right]\frac{P^2}{Q^2} \\
&\qquad +\frac{1}{4\widetilde{\beta}^5}\left[-\widetilde{\beta}^6+(2x^2-4x+7)\widetilde{\beta}^4+(8x-11)\widetilde{\beta}^2+3(2x^2-4x+3)\right]\Bigg\} \\
&+ \frac{\beta}{1-\beta^2\widetilde{\beta}^2}\Bigg\{-16x^2\frac{m^4}{Q^4}+\frac{2\left[(1-2x)^2+(4x-1)\widetilde{\beta}^2\right]}{\widetilde{\beta}^2}\frac{m^2}{Q^2} \\
&\qquad +\frac{1}{4\widetilde{\beta}^4}\left[(1-\widetilde{\beta}^2)(\widetilde{\beta}^4+2\beta^2\widetilde{\beta}^2-3)+8x\left\{\beta^2\widetilde{\beta}^4-2(\beta^2+1)\widetilde{\beta}^2+3\right\}\right]\frac{P^2}{Q^2} \\
&\qquad +\frac{1}{2\widetilde{\beta}^4}\Big[(2\beta^2+1)\widetilde{\beta}^6+\left\{2x^2+4\beta^2x-6\beta^2-5\right\}\widetilde{\beta}^4 \\
&\qquad\quad +\left\{4\beta^2x^2-8(\beta^2+1)x+6\beta^2+11\right\}\widetilde{\beta}^2-3(2x^2-4x+3)\Big]\Bigg\}
\end{aligned}
$$

Numerical Plot $F_2^\gamma(x, Q^2, P^2)$



Result