# Theory of <br> generalized parton distributions and deeply virtual Compton scattering 

 Dieter Müller
## Ruhr-Universität Bochum

K. Kumerički, DM, K. Passek-Kumerički (KMP-K), hep-ph/0703179 GPD fits at NLO and NNLO of H1/ZEUS data KMP-K, 0805.0152 [hep-ph]
constructive critics on ad hoc GPD model approach [lot of good news] first applications of dispersion integral approach
KMP-K, 0807.0159 [hep-ph]; KM 0904.0458 [hep-ph] flexible GPD model for small $x$ and fits of H1/ZEUS data dispersion integral fits of HERMES and JLAB data

## outline:

- Photon leptoproduction (DVCS)
- GPD properties \& representations
-Strategies to analyze DVCS data
* ad hoc GPD models to provide estimates
- flexible GPD models: Are we ready? (H1/ZEUS fits)
- dispersion relation approach
(global fit example)
- Summary


## Photon leptoproduction $e^{ \pm} N \rightarrow e^{ \pm} N \gamma$

 measured by H1, ZEUS, HERMES, CLAS, HALL A collaborations planed at COMPASS, JLAB@12GeV, perhaps at ?? EIC,$$
\frac{d \sigma}{d x_{\mathrm{Bj}} d y d\left|\Delta^{2}\right| d \phi d \varphi}=\frac{\alpha^{3} x_{\mathrm{Bj}} y}{16 \pi^{2} \mathcal{Q}^{2}}\left(1+\frac{4 M_{B j}^{2}{ }^{2}}{\mathcal{Q}^{2}}\right)^{-1 / 2}\left|\frac{\mathcal{T}}{e^{3}}\right|^{2},
$$



## interference of DVCS and Bethe-Heitler processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}} \ldots$ elastic form factors $F_{1}, F_{2}$ (helicity amplitudes)

$$
\begin{array}{ll}
\left|\mathcal{T}_{\mathrm{BH}}\right|^{2}=\frac{e^{6}\left(1+\epsilon^{2}\right)^{-2}}{x_{\mathrm{Bj}}^{2} y^{2} \Delta^{2} \mathcal{P}_{1}(\phi) \mathcal{P}_{2}(\phi)}\left\{c_{0}^{\mathrm{BH}}+\sum_{n=1}^{2} c_{n}^{\mathrm{BH}} \cos (n \phi)\right\}, & \begin{array}{l}
\text { exactly known } \\
(\mathrm{LO}, \mathrm{QED})
\end{array} \\
\left|\mathcal{T}_{\mathrm{DVCS}}\right|^{2}=\frac{e^{6}}{y^{2} \mathcal{Q}^{2}}\left\{c_{0}^{\mathrm{DVCS}}+\sum_{n=1}^{2}\left[c_{n}^{\mathrm{DVCS}} \cos (n \phi)+s_{n}^{\mathrm{DVCS}} \sin (n \phi)\right]\right\}, \begin{array}{l}
\text { harmonics } \\
\frac{\square}{\text { helicity ampl. }} \frac{1: 1}{}
\end{array} \\
\mathcal{I}=\frac{ \pm e^{6}}{x_{\mathrm{Bj}} y^{3} \Delta^{2} \mathcal{P}_{1}(\phi) \mathcal{P}_{2}(\phi)}\left\{c_{0}^{\mathcal{I}}+\sum_{n=1}^{3}\left[c_{n}^{\mathcal{I}} \cos (n \phi)+s_{n}^{\mathcal{I}} \sin (n \phi)\right]\right\} . & \begin{array}{l}
\frac{\square}{\text { harmonics }} 1: 1 \\
\text { helicity ampl }
\end{array}
\end{array}
$$

## GPDs embed non-perturbative physics

GPDs appear in various hard exclusive processes,
[DM et. al (90/94)
Radyushkin (96)
Ji (96)]
e.g., hard electroproduction of photons (DVCS)

$\xrightarrow{t=\Delta^{2}}-\mathrm{fix}$

$$
\mathcal{F}\left(\xi, \mathcal{Q}^{2}, t\right)=\int_{-1}^{1} d x C\left(x, \xi, \alpha_{s}(\mu), \mathcal{Q} / \mu\right) F(x, \xi, t, \mu)+O\left(\frac{1}{\mathcal{Q}^{2}}\right)
$$

## CFF

Compton form factor
observable

## hard scattering part

perturbation theory (our conventions/microscope)

GPD
universal (conventional)
higher twist
depends on approximation
relations among harmonics and GPDs are based on $1 / \mathcal{Q}$ expansion: (all harmonics are expressed by twist-2 and -3 GPDs) [Diehl et. al (97)

Belitsky, DM, Kirchner (01)]

$$
\begin{aligned}
& \left\{\begin{array}{l}
c_{1} \\
s_{1}
\end{array}\right\}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}} \mathrm{tw}-2(\mathrm{GPDs})+O\left(1 / \mathcal{Q}^{3}\right), \quad c_{0}^{\mathcal{I}} \propto \frac{\Delta^{2}}{\mathcal{Q}^{2}} \mathrm{tw}-2(\mathrm{GPDs})+O\left(1 / \mathcal{Q}^{4}\right), \\
& \left\{\begin{array}{l}
c_{2} \\
s_{2}
\end{array}\right\}^{\mathcal{I}} \propto \frac{\Delta^{2}}{\mathcal{Q}^{2}} \mathrm{tw}-3(\mathrm{GPDs})+O\left(1 / \mathcal{Q}^{4}\right), \quad\left\{\begin{array}{l}
c_{3} \\
s_{3}
\end{array}\right\}^{\mathcal{I}} \propto \frac{\Delta \alpha_{s}}{\mathcal{Q}}(\mathrm{tw}-2)^{\mathrm{T}}+O\left(1 / \mathcal{Q}^{3}\right), \\
& c_{0}^{\mathrm{CS}} \propto(\mathrm{tw}-2)^{2}, \quad\left\{\begin{array}{l}
c_{1} \\
s_{1}
\end{array}\right\}^{\mathrm{CS}} \propto \frac{\Delta}{Q}(\mathrm{tw}-2)(\mathrm{tw}-3), \quad\left\{\begin{array}{l}
c_{2} \\
s_{2}
\end{array}\right\} \propto \alpha_{s}(\mathrm{tw}-2)(\mathrm{tw}-2)^{\mathrm{GT}}
\end{aligned}
$$

setting up the perturbative framework:
[Belitsky, DM (97);
Mankiewicz et. al (97);
$\checkmark$ twist-two coefficient functions at next-to-leading order Ji,Osborne (98)]
$\checkmark$ evolution kernels at next-to-leading order [Belitsky, DM, Freund (01)]
[KMP-K \&
$\checkmark$ next-to-next-to-leading order in a specific conformal subtraction scheme Schaefer 06]
$\checkmark$ twist-three including quark-gluon-quark correlation at LO $\begin{gathered}\text { [Anikin,Teryaev, Pire (00); } \\ \text { Belitsky DM (00); Kivel et. al] }\end{gathered}$
$\checkmark$ partial twist-three sector at next-to-leading order [Kivel, Mankiewicz (03)]
$\checkmark$ 'target mass corrections' (not well understood) [Belitsky DM (01)]

## GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

$$
\begin{aligned}
& e p \rightarrow e^{\prime} p^{\prime} \gamma \\
& e p \rightarrow e^{\prime} p^{\prime} \mu^{+} \mu^{-} \\
& \gamma p \rightarrow p^{\prime} e^{+} e^{-}
\end{aligned}
$$



- Hard exclusive meson production (flavor filter)
scanned area of the surface as a functions of lepton energy


$$
e p \rightarrow e^{\prime} p^{\prime} \mu^{+} \mu^{-}
$$

$e p \rightarrow e^{\prime} p^{\prime} \pi$ $e p \rightarrow e^{\prime} p^{\prime} \rho$ $e p \rightarrow e^{\prime} n \pi^{+}$ $e p \rightarrow e^{\prime} n \rho^{+}$

twist-two observables:
cross sections
transverse target spin asymmetries

- etc.


## Can one ‘measure’ GPDs?

- CFF given as GPD convolution:

$$
\begin{aligned}
\mathcal{H}\left(\xi, t, \mathcal{Q}^{2}\right) & \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} d x\left(\frac{1}{\xi-x-i \epsilon}-\frac{1}{\xi+x-i \epsilon}\right) H\left(x, \eta=\xi, t, \mathcal{Q}^{2}\right) \\
& \stackrel{\mathrm{LO}}{=} i \pi H^{-}\left(x=\xi, \eta=\xi, t, \mathcal{Q}^{2}\right)+\mathrm{PV} \int_{0}^{1} d x \frac{2 x}{\xi^{2}-x^{2}} H^{-}\left(x, \eta=\xi, t, \mathcal{Q}^{2}\right)
\end{aligned}
$$

- $H\left(x, x, t, Q^{2}\right)$ viewed as spectral function ( $s$-channel cut):

$$
H^{-}\left(x, x, t, Q^{2}\right) \equiv H\left(x, x, t, Q^{2}\right)-H\left(-x, x, t, Q^{2}\right) \stackrel{\text { LO }}{=} \frac{1}{\pi} \Im \mathrm{~m} \mathcal{F}\left(\xi=x, t, Q^{2}\right)
$$

- CFFS satisfy dispersion relations (not the physical ones, threshold $\xi_{0}$ set to 0 )

$$
\Re \mathrm{e} \mathcal{F}\left(\xi, t, Q^{2}\right)=\frac{1}{\pi} \mathrm{PV} \int_{0}^{1} d \xi^{\prime}\left(\frac{1}{\xi-\xi^{\prime}} \mp \frac{1}{\xi+\xi^{\prime}}\right) \Im \mathrm{m} \mathcal{F}\left(\xi^{\prime}, t, Q^{2}\right)+\mathcal{C}\left(t, Q^{2}\right)
$$

access to the GPD on the cross-over line $\eta=x$ (at LO )

## GPD Properties

GPDs are intricate functions: $H\left(x, \eta=\xi, t, \mu^{2}=\mathcal{Q}^{2}\right)$

## a non-trivial interplay of variable dependence

- $t$-dependence dies out at large $x$ (spectator models, indicated by lattice \& XQS-model)
- effective Regge behavior (from phenomenology) at small $x$; unknown $\eta$-dependence
- evolution depends on the GPD shape
at least four phenomenological important GPDs for each parton
GPD-constraints:
- reduction to PDFs:

$$
q\left(x, \mu^{2}\right)=\lim _{\Delta \rightarrow 0} H\left(x, \eta, t, \mu^{2}\right)
$$

- generalized form factor sum rules, e.g.: (polynomiality, GPD support property)
- Ji's sum rule

$$
\begin{aligned}
& F_{1}(t)=\int_{-1}^{1} d x H\left(x, \eta, t, \mu^{2}\right) \\
& \frac{1}{2}=\int_{-1}^{1} d x x(H+E)\left(x, \eta, t=0, \mu^{2}\right)
\end{aligned}
$$

- positivity constraints (valid at LO) [P. Pobylitsa 02]
(strongly constraining variable interplay in the outer region)


## A partonic duality interpretation

## quark GPD (anti-quark $x \rightarrow-x$ ):

$F=\theta(-\eta \leq x \leq 1) \omega\left(x, \eta, \Delta^{2}\right)+\theta(\eta \leq x \leq 1) \omega\left(x,-\eta, \Delta^{2}\right)$
$\omega\left(x, \eta, \Delta^{2}\right)=\frac{1}{\eta} \int_{0}^{\frac{x+\eta}{1+\eta}} d y x^{p} f\left(y,(x-y) / \eta, \Delta^{2}\right)$
dual interpretation on partonic level:


central region $-\eta<x<\eta$ mesonic exchange in $t$-channel

## support extension <br> is unique [DM et al. 92]


ambiguous ( $D$-term) [DM, A. Schäfer (05) KMP-K (07)]

outer region $\eta<x$ partonic exchange in $s$-channel

## Modeling \& Evolution

outer region governs the evolution at the cross-over trajectory

$$
\mu^{2} \frac{d}{d \mu^{2}} H\left(x, x, t, \mu^{2}\right)=\int_{x}^{1} \frac{d y}{x} V\left(1, x / y, \alpha_{s}(\mu)\right) H\left(y, x, \mu^{2}\right)
$$

GPD at $\eta=x$ is ‘measurable' (LO)


## Overview: GPD representations



SL(2,R) (conformal) expansion (series of local operators)
one version is called Shuvaev transformation, used in ‘dual' (t-channel) GPD parameterization
light cone wave function overlap
(Hamiltonian approach in light-cone quantization)

Radyushkin (97);
Belitsky, Geyer, DM, Schäfer (97);
DM, Schäfer (05); ....
Shuvaev (99,02); Noritzsch (00)
Polyakov $(02,07)$
Diehl, Feldmann,
Jakob, Kroll $(98,00)$
Diehl, Brodsky,
Hwang (00)
each representation has its own advantages, however, they are equivalent (clearly spelled out in [Hwang, DM 07])


## Strategies to analyze DVCS data <br> GPD model approach:

ad hoc modeling: VGG code [Goeke et. al (01) based on Radyuskin'sDDA]
(first decade) BKM model [Belitsky, Kirchner, DM (01) based on RDDA] `aligned jet' model [Freund, McDermott, Strikman (02)] Kroll/Goloskokov (05) based on RDDA [not utilized for DVCS] `dual' model [Polyakov,Shuvaev 02;Guzey,Teckentrup 06;Polyakov 07]
" -- " [KMP-K (07) in MBs-representation]
Bernstein polynomials [Liuti et. al (07)]
physical models: not applied [Radyuskin et.al (02); Tiburzi et.al (04); Hwang DM (07)]...
flexible models: any representation by including unconstrained degrees of freedom (for fits) KMP-K (07/08) for H1/ZEUS in MBs-representation
What is the physical content of 'invisible’ (unconstrained) degrees of freedom?

## Extracting CFFs from data: real and imaginary part

i. (almost) without modeling [Guidal, Moutarde (08/09)]
ii. dispersion integral fits [KMP-K (08),KM (08/09)]
iii. flexible GPD modeling [KM (08/09)]

## Ready for flexible GPD model fits?



$\square$
GeParD a $N(N) L O$ routine for the evaluation of gen. FF
 (in terms of gen. FF)
the answer is YES for small $x$ and NO for JLAB@6GeV kinematics:

- reasonable well motivated hypotheses of GPDs (moment) must be implemented
- many parameters - Is a least square fit an appropriate strategy?
- some technical, however, straightforward work is left (reevaluation of observables)


## DVCS fit's for H1 and ZEUS data

DVCS cross section measured at small $x_{\mathrm{Bj}} \approx 2 \xi=\frac{2 \mathcal{Q}^{2}}{2 W^{2}+\mathcal{Q}^{2}}$

$$
40 \mathrm{GeV} \lesssim W \lesssim 150 \mathrm{GeV}, \quad 2 \mathrm{GeV}^{2} \lesssim \mathcal{Q}^{2} \lesssim 80 \mathrm{GeV}^{2}, \quad|t| \lesssim 0.8 \mathrm{GeV}^{2}
$$

predicted by

$$
\begin{aligned}
\frac{d \sigma}{d t}\left(W, t, \mathcal{Q}^{2}\right) & \left.\approx \frac{4 \pi \alpha^{2}}{\mathcal{Q}^{4}} \frac{W^{2} \xi^{2}}{W^{2}+\mathcal{Q}^{2}}\left[|\mathcal{H}|^{2}-\frac{\Delta^{2}}{4 M_{\mathrm{p}}^{2}}|\mathcal{E}|^{2}+|\widetilde{\mathcal{H}}|^{2}\right]\left(\xi, t, \mathcal{Q}^{2}\right)\right|_{\xi=\frac{\mathcal{Q}^{2}}{2 W^{2}+\mathcal{Q}^{2}}} \\
& \text { suppressed contributions <<0.05>> } \overbrace{0} \text { relative } O(\xi)
\end{aligned}
$$

- LO data are described with - huge (wrong) $t$-slope [Belitsky, DM, Kirchner (01)] - inconsistent GPDs [Freund, McDermott, Strikman (03)]
- missing factor of $1 / 4$ [Guzey, Teckentrup $(06,08)$ ]
- NLO works with ad hoc GPD models [Freund, McDermott (02)]
results strongly depend on employed PDF parameterization

$\Rightarrow$
do a simultaneous fit to DIS and DVCS [KMP-K (07)]
use flexible GPD models in a two-step fitt [KMP-K (08)]
good DVCS fits at LO, NLO, and NNLO with flexible GPD ansatz





## quark skewness ratio from DVCS fits @ LO

$$
R=\frac{\Im \mathrm{m} A_{\mathrm{DVCS}}}{\Im \mathrm{~m} A_{\mathrm{DIS}}} \stackrel{L O}{=} \frac{H(\xi, \xi)}{H(2 \xi, 0)} \approx 2^{\alpha} r
$$

$$
r=\frac{H(\xi, \xi)}{H(\xi, 0)}
$$




- @LO the conformal ratio is ruled out for sea quark GPD
- generically, one finds a zero-skewness effect over a large $Q^{2}$ lever arm
- this zero-skewness effect is non-trivial to realize in conformal space ( $\mathrm{SO}(3)$ sibling poles are required)
- CFF H posses "pomeron behavior" $\xi^{-\alpha(Q)-\alpha^{\prime}(Q) t}$
$\checkmark \alpha$ increases with growing $Q^{2}$
$\checkmark \alpha^{\prime}$ decreases growing $Q^{2}$
$-t$-dependence: exponential shrinkage is disfavored $\quad\left(\alpha^{\prime} \approx 0\right)$ dipole shrinkage is visible $\left(\alpha^{\prime} \approx 0.15\right.$ at $\left.Q^{2}=4 \mathrm{GeV}^{2}\right)$
- (normalized) profile functions

$$
\rho \propto \int d^{2} \vec{\Delta}_{\perp} e^{i \vec{b} \cdot \vec{\Delta}_{\perp}} H\left(x, 0, t=-\vec{\Delta}_{\perp}^{2}\right)
$$



## Ready for dispersion relation fits?


the answer is YES, however, more data are needed:

- to pin down the GPD models (on the cross over line $\eta=x$ )
- to overcome the hypotheses of $H$ (and twist-two) dominance
efficient code is needed


## Global GPD fit example: HERMES \& JLAB






- quality of global fit is good, e.g.,
$\chi^{2} /$ d.o.f. $\approx 28 / 34$
- model depended extraction of GPD $H(x, x, t)$ and subtraction constant

$$
H(x, x, t)=n r 2^{\alpha}\left(\frac{2 x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b} \frac{1}{\left(1-\frac{1-x}{1+x} \frac{t}{M^{2}}\right)^{p}}
$$

within some assumptions

- prediction for COMPASS asymmetry $\quad A_{\mathrm{BCSA}}=\frac{d \sigma^{\uparrow+}-d \sigma^{\downarrow-}}{d \sigma^{\uparrow+}+d \sigma^{\downarrow-}}$




## Summary

## GPDs are intricate and (thus) a promising tool

$>$ to reveal the transverse distribution of partons
$>$ to address the spin content of the nucleon
$>$ providing a bridge to non-perturbative methods (e.g., lattice)

## hard photon leptoproduction (DVCS)

- possesses a rich structure, allowing to access various CFFs/GPDs
- it is elaborated at twist-three (partly NLO) and NNLO
- it is widely considered as a theoretical clean process


## compatible strategies to analyze DVCS data

* analytic formulae, fitter code to extract CFFs
* flexible GPD models + fitting (minimizing $X^{2}$ )
* dispersion integral technique for fixed target experiments


## Back up slides are coming

## (partonic) 'quantum’ numbers in GPD representations



## SL(2,R) representations for GPDs

- support is a consequence of Poincaré invariance (polynomiality)

$$
H_{j}\left(\eta, t, \mu^{2}\right)=\int_{-1}^{1} d x c_{j}(x, \eta) H\left(x, \eta, t, \mu^{2}\right), \quad c_{j}(x, \eta)=\eta^{j} C_{j}^{3 / 2}(x / \eta)
$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

$$
\mu \frac{d}{d \mu} H_{j}\left(\eta, t, \mu^{2}\right)=-\frac{\alpha_{s}(\mu)}{2 \pi} \gamma_{j}^{(0)} H_{j}\left(\eta, t, \mu^{2}\right)
$$

- inverse relation is given as series of mathematical distributions:

$$
H(x, \eta, t)=\sum_{j=0}^{\infty}(-1)^{j} p_{j}(x, \eta) H_{j}(\eta, t), p_{j}(x, \eta) \propto \theta(|x| \leq \eta) \frac{\eta^{2}-x^{2}}{\eta^{j+3}} C_{j}^{3 / 2}(-x / \eta)
$$

- various ways of resummation were proposed:
- smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
- mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
- dual parameterization (a mixture of both) [M. Polyakov, A. Shuvaev (02)]
- based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
- Mellin-Barnes integral [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]


## GPD ansatz at small x from t-channel view

* at short distance a quark/anti-quark state is produced, labeled by conformal spin $j+2$
* they form an intermediate mesonic state with total angular momentum $J$ strength of coupling is $f_{j}^{J}, J \leq j+1$
* mesons propagate with $\frac{1}{m^{2}(J)-t} \propto \frac{1}{J-\alpha(t)}$
* decaying into a nucleon anti-nucleon pair with given angular momentum J , described by an impact form factor


$$
F_{j}^{J}(t)=\frac{f_{j}^{J}}{J-\alpha(t)} \frac{\bigvee_{1}}{\left(1-\frac{t}{M^{2}(J)}\right)^{p}}
$$

! GPD $E$ is zero if chiral symmetry holds (partial waves are Gegenbauer polynomials with index 3/2)
$D$-term arises from the $\mathrm{SO}(3)$ partial wave $J=j+1$ ( $j \rightarrow-1$ )

## Can the skewness function be constrained from lattice?

- relation among measurable and GPD Mellin moments at $\eta=0$ :

$$
\int_{0}^{1} d \xi \xi^{j} \Im \mathrm{~m} \mathcal{F}\left(\xi, t, \mathcal{Q}^{2}\right) \stackrel{\mathrm{LO}}{=} \pi f_{j}\left(t, \mathcal{Q}^{2}\right)\left[1+\delta_{j}\left(t, \mathcal{Q}^{2}\right)\right]
$$

- deviation factors: $\delta_{j}\left(t, \mu^{2}\right)=\frac{\int_{0}^{1} d x x^{j} S\left(x, t, \mu^{2}\right) F\left(x, \eta=0, t, \mu^{2}\right)}{\int_{0}^{1} d x x^{j} F\left(x, \eta=0, t, \mu^{2}\right)}$
are given by a series of operator expectation values with increasing spin $j+n+1$

$$
\delta_{j}\left(t, \mu^{2}\right)=\sum_{\substack{n=2 \\ \text { even }}}^{\infty} \frac{f_{j+n}^{(n)}\left(t, \mu^{2}\right)}{f_{j}\left(t, \mu^{2}\right)}, \quad f_{j}^{(n)}\left(t, \mu^{2}\right)=\left.\frac{1}{n!} \frac{d^{n}}{d \eta^{n}} f_{j}\left(\eta, t, \mu^{2}\right)\right|_{\eta=0}
$$

- lattice can evaluate $j=0,1,2,(3)$, i.e., $n=2: \delta_{0}\left(t, \mu^{2}=4 \mathrm{GeV}^{2}\right) \approx 0.2+$ ? thanks to
- ? wrong expectation from evolution: $\quad \delta_{j} \sim \frac{2^{j+1} \Gamma(5 / 2+j)}{\Gamma(3 / 2) \Gamma(3+j)}-1$
the analog small $\times$ prediction is ruled out $\quad \delta_{0} \sim 0.5 \quad \delta_{1} \sim 1.5$
[Shuvaev et al. (99)]

