Leading theoretical uncertainties in the muon g-2

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OUTLOOK

- SM prediction
- LO hadronic (e^+e^-)
- LO hadronic (τ)
- Final comments

Collaboration with: M. Davier, A. Höcker, B. Malaescu, X. Mo, G. Toledo, P. Wang, C. Yuan, Z. Zhang [DHLMMTWYZ (09)]

Accuracy: experiment

Muon magnetic moment: $\vec{\mu} = g \frac{e}{2m} \vec{s}$
(g=2 Dirac theory)

Anomalous moment: $a = \frac{g-2}{2}$

Onderwater's talk

Final E821 assuming CPT

$10^{11} \times a_\mu^{exp} = 116592080 \pm 63$

- Sensitivity to New Physics
- Sensitivity to all SM interactions
- Insensitive to m_H, m_t
- Forced theory to similar accuracy
- Discrepancy with SM at 3.5σ

E969 goal:

± 15

- Force to improve theory in hadronic sector
- Could establish NP at $> 6\sigma$ level

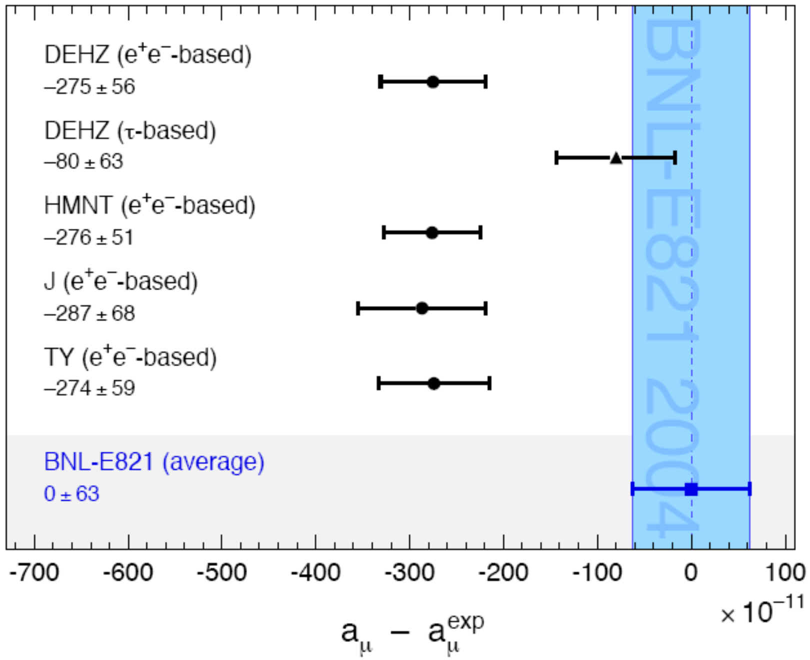
Miller, de Rafael & Lee Roberts, Rep.Part.Phys.70, (2007)

Accuracy: theory

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$$

- Currently better than experimental accuracy
- Limited by photon-hadron interactions (HVP, L×L)
- Discrepancy in HVP e^+e^- vs. τ based data

Circa 2008



Recent reviews: Miller, de Rafael & Lee Roberts, Rep.Part.Phys.70, (2007); Jegerlehner & Nyffeler, arXiv:0902.3360; Davier & Marciano, Ann. Rev. Nuc. P. Sci. (2004)

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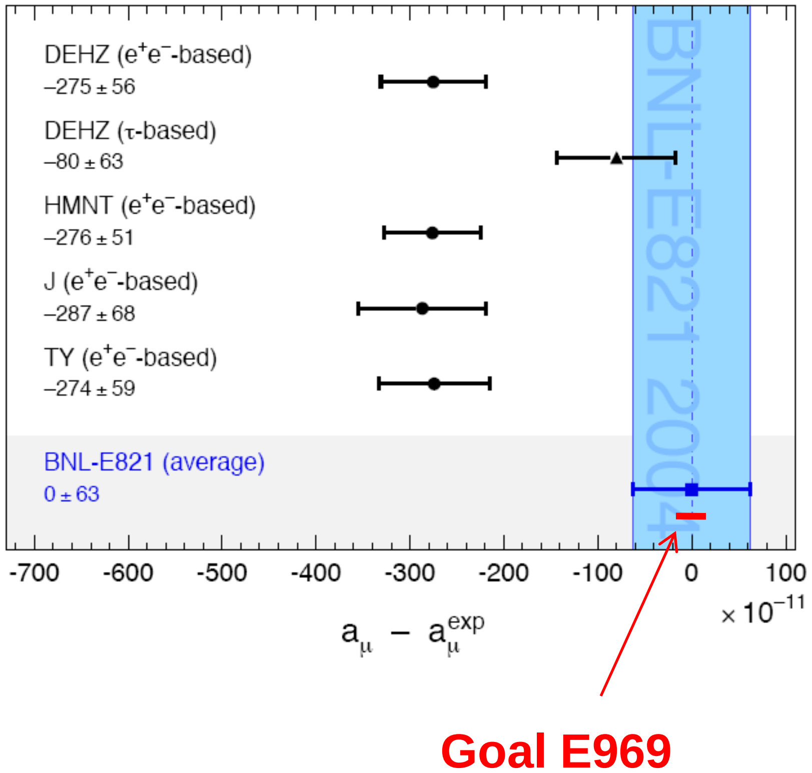

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QED contribution

$$a_\mu^{QED} = 0.5\left(\frac{\alpha}{\pi}\right)$$

$$+0.765857410(27)\left(\frac{\alpha}{\pi}\right)^2$$

$$+24.05050964(87)\left(\frac{\alpha}{\pi}\right)^3$$

$$+130.9916(80)\left(\frac{\alpha}{\pi}\right)^4$$

$$+663(20)\left(\frac{\alpha}{\pi}\right)^5$$

γ & lepton loops only

Schwinger (1948)

•

•

•

Numerical Kinoshita,Nio (2006)

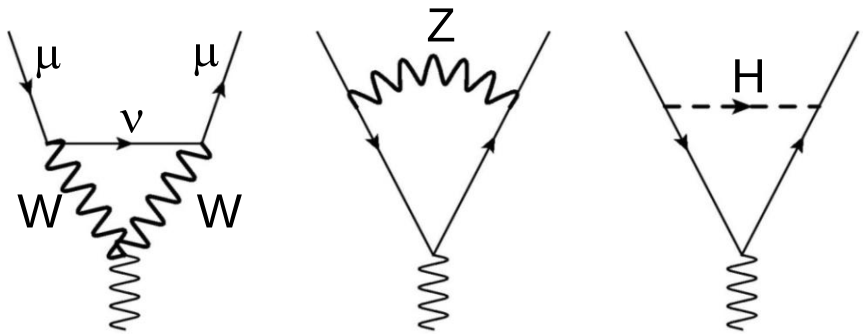
Leading Log: Kinoshita-Nio (2006); Kataev (2006); Baikov et al (2008); Aoyama et al (2008)

Using α^{-1}=137.035999710(96) from measured a_e [Gabrielse et al (2006)]

$$10^{11} \times a_\mu^{QED} = 116584718.09 \pm 0.14_{O(\alpha^5)} \pm 0.04_{\alpha}$$

Electroweak contribution

Heavy weak bosons in loops



$$a_\mu^{EW} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3} \left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{m_{Z,H}^2} \log\left(\frac{m_{Z,H}^2}{m_\mu^2} \right) \right) \right]$$

Fujiwaka et al(72), Bardeen et al (72)..

$$- \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{\alpha}{\pi} \right) \left[155.5(4.0)_{m_H} (1.8)_{had.loops} \right]$$

Czarnecki et al (95-02)

$$+ O(10^{-12})$$

Czarnecki et al (03)
Degrasi et al(03)

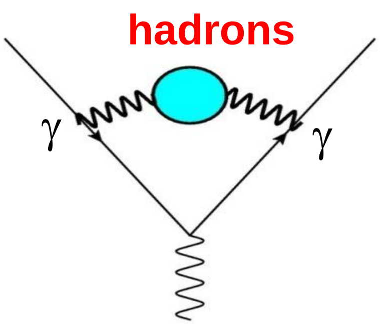
$$10^{11} \times a_\mu^{EW} = 154 \pm 2_{m_H} \pm 1_{had.loops}$$

G. López Castro

Hadronic contributions

$$a_\mu^{had} = a_\mu^{had,\ LO} + a_\mu^{had,\ HO} + a_\mu^{had,\ l\times l}$$

$$\alpha^2 \qquad \alpha^3 \qquad \alpha^3$$

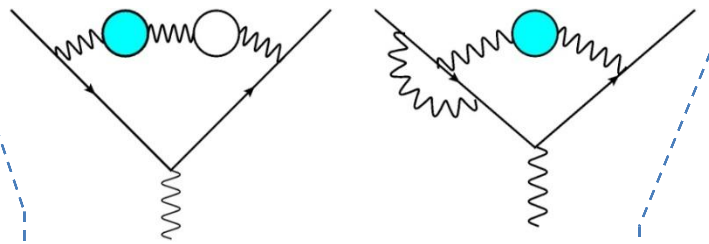


1 HVP insertion;

~7000

accuracy :< 1%

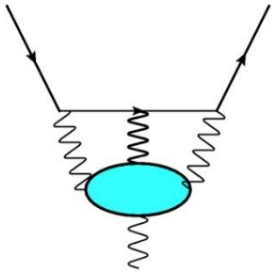
<0.3%



1 HVP + γ, leptonic or another HVP loop

$$10^{11} \times a_\mu^{had,\ HO} = -98 \pm 1$$

Hagiwara et al (2003)



More details in

Prades' talk

$$10^{11} \times a_\mu^{had,\ l\times l} = 105 \pm 26$$

Prades et al (2009)

Hadronic Leading order

Dispersion integral

$$a_\mu^{had,LO} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} \, \frac{\sigma^0(e^+e^- \to hads)}{\sigma^0(e^+e^- \to \mu^+\mu^-)}$$

[Bouchiat-Michel (61); Gourdin-de Rafael (69)]

QED kernel *K(s)* decreases with *s* :
- Low energies → data
- High energies → QCD

Importance of low energy data:

- 92% of $a_\mu^{had,\,LO}$ comes from $\sqrt{s} < 1.8$ GeV
- 73% of $a_\mu^{had,\,LO}$ comes from 2π (large ρ contribution)
- 63% of theoretical uncertainty comes 2π

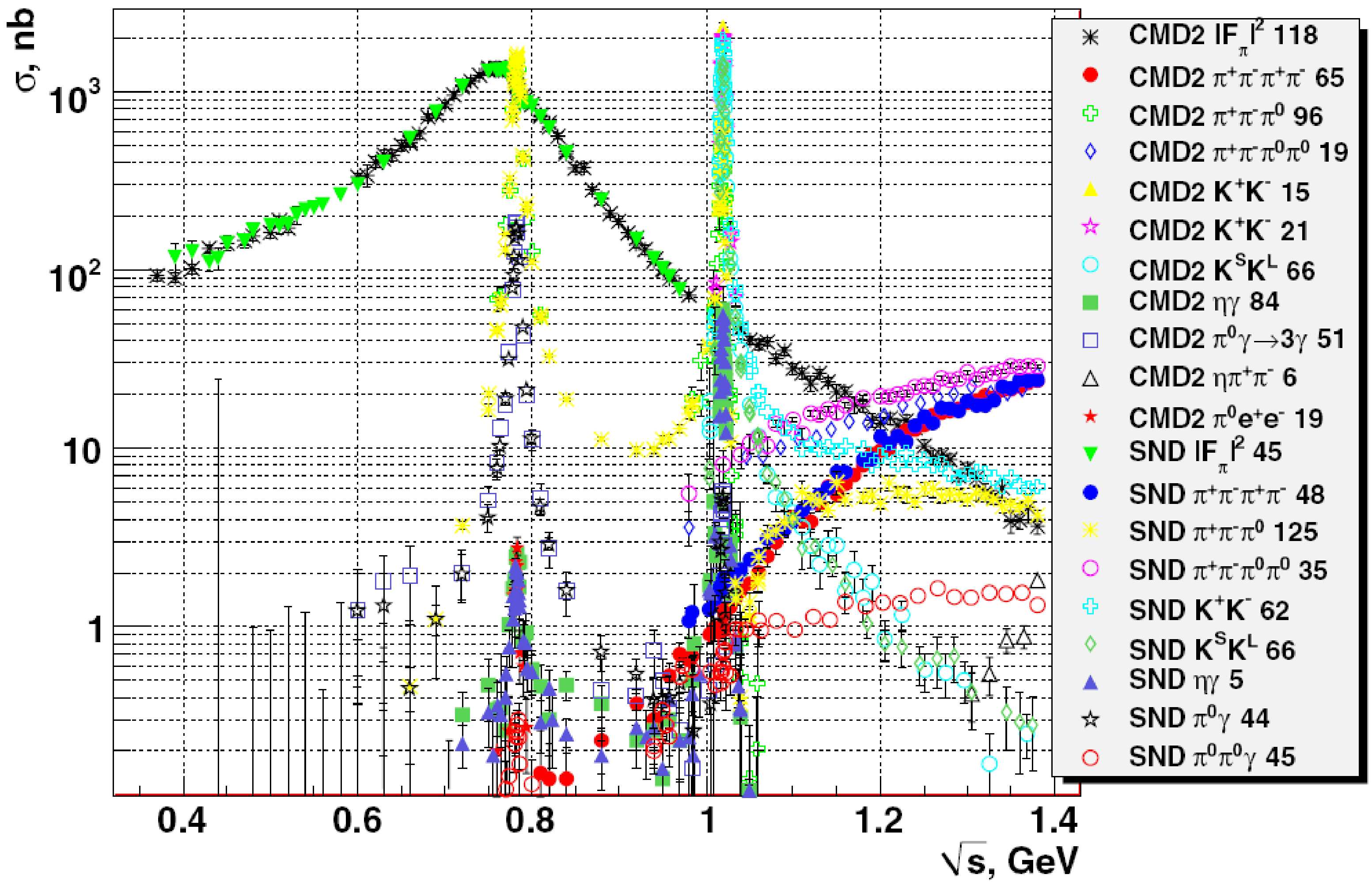
Recent measurements of 2π channel

Exp.	Method	E_{cm} (GeV)	σ (%)	year
$CMD-2$	$scan$	$0.37-0.52$	0.7	$2005-07$
$CDM-2$	$scan$	$0.60-0.97$	$0.6-0.8$	"
$CMD-2$	$scan$	$1.04-1.38$	$1.3-4.2$	"
SND	$scan$	$0.39-0.42$	3.2	$2005-06$
SND	$scan$	$0.43-0.97$	1.3	"
$KLOE$	ISR	$0.59-0.97$	0.9	2005
$KLOE$	ISR	$0.59-0.97$	0.6	2008
$BABAR$	ISR	$0.50-3.0$	0.56_{ρ}	$TAU\,08$

Difficult to reach 0.3% required to match future E969

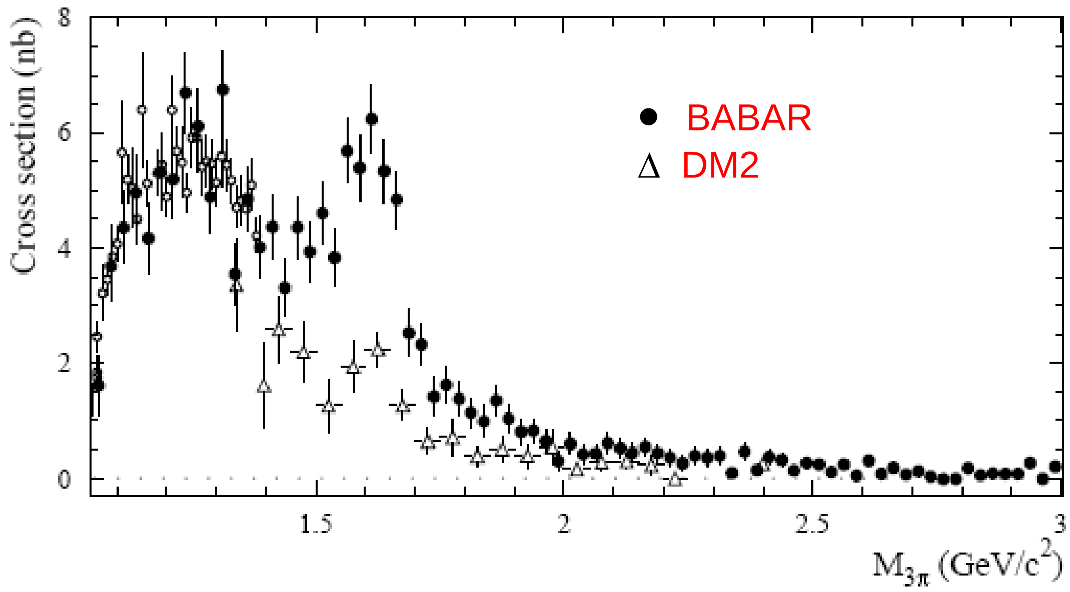
Novosibirsk CMD-2 & SND results

Logashenko, TAU08



BABAR 3π channel (I=0), ISR

Aubert et al (2004)



$10^{11}\times\delta a_\mu = 24.5 \pm 2.6 \pm 0.3$

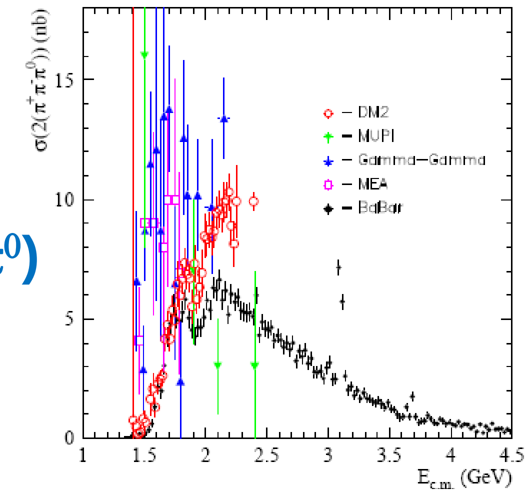
$32.5 \pm 0.9 \pm 0.1$

BABAR data higher than DM2 above 1.4 GeV

BABAR 4π, 6π channels (I=1)

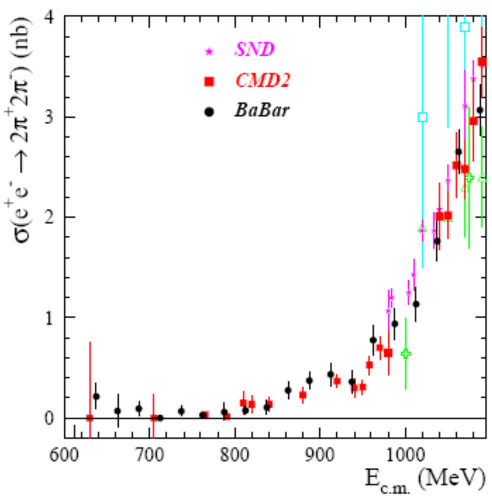
Aubert et al (2005-2007)

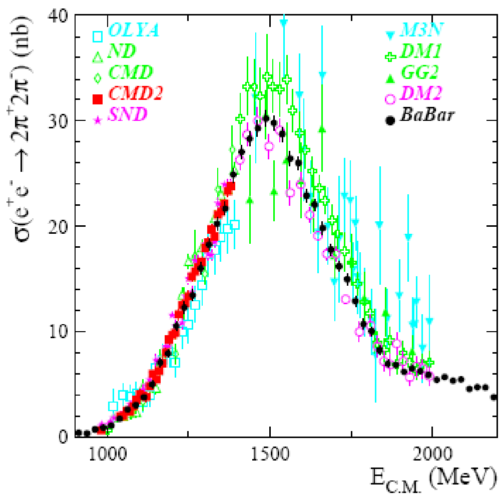
$2(\pi^+\pi^-\pi^0)$



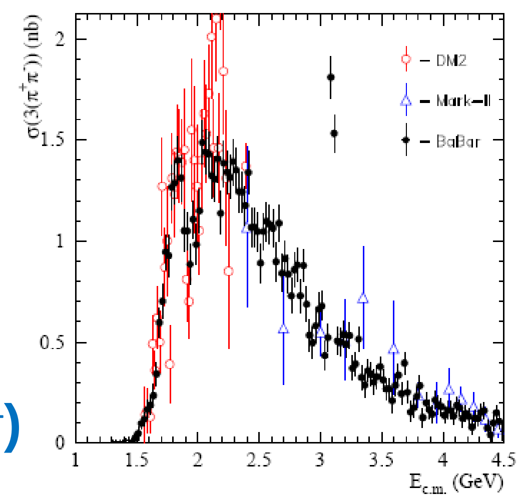
δa_μ: $(14 \pm 3) \longrightarrow (9 \pm 1)$

$2(\pi^+\pi^-)$





$3(\pi^+\pi^-)$



Good agreement with CMD-2 & SND; better accuracy

δa_μ: $(1 \pm 1) \longrightarrow (1.1 \pm 0.2)$

New evaluation: (1) including/excluding KLOE data
(2) HVPtools dressed $\rightarrow$ bare σ
(3) Taylor expansion below 0.36 GeV

Channel	E (GeV)	$10^{11}\times a_\mu^{had,LO}$ (ee)	
2π, *low* exp	$2m_\pi - 0.36$	94.7 ± 1.0	
2π	$0.36 - 1.8$	4926.3 ± 35.0	
$\pi^+\pi^-2\pi^0$	≤ 1.8	$168.0 \pm 13.0 \pm 2.0_{rad}$	
$2\pi^+2\pi^-$	≤ 1.8	$131 \pm 4 \pm 0_{rad}$	
$Other Exc. + QCD$	$2m_\pi - \infty$	$1564 \pm 19 \pm 4_{rad} \pm 7_{QCD}$	
$Total$	$2m_\pi - \infty$	$6884(41)(19)_{rad}(7)_{QCD}$ $6897(48)(19)_{rad}(7)_{QCD}$	$w/KLOE$ $wo/KLOE$

DHLMMTWYZ (09)

Davier et al ('06~'07)

$$F_\pi(s) = 1 + \frac{1}{6}\langle r^2 \rangle_\pi s + c_1 s^2 + c_2 s^3$$

Summary of SM contributions and experiment

Contrib.	$10^{11}\times a_\mu$
QED	116584718.09 ± 0.15
EW	$154 \pm 2 \pm 1$
had, HO	-98 ± 1
$had, l \times l$	105 ± 26
$E821$	116592080 ± 63

Recent evaluations of had. (LO & l×l).

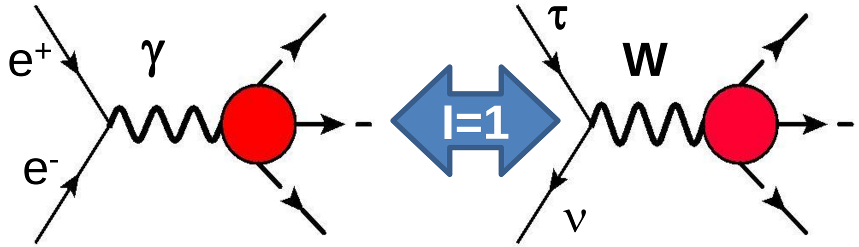
- **Discrepancy > 3σ confirmed,**
- **Theory uncertainty dominated by LO hadronic contribs.**

Calcul.	$a_\mu^{had,LO}$	a_μ^{SM}	$a_\mu^{exp} - a_\mu^{SM}$
$DEHZ-03$	6963 ± 72	116591842 ± 77	$238 \pm 99\,[3.4\sigma]$
$HMNT-03$	6961 ± 62	116591840 ± 67	$240 \pm 92\,[2.6\sigma]$
$DEHZ-06$	6909 ± 44	116591788 ± 51	$292 \pm 81\,[3.6\sigma]$
$HMNT-06$	6894 ± 46	116591773 ± 53	$307 \pm 82\,[3.7\sigma]$
$J-08$	6923 ± 60	116591802 ± 65	$278 \pm 90\,[3.1\sigma]$
$DH....YZ09$	6884 ± 45 6897 ± 52	116591766 ± 53 116591779 ± 58	$317 \pm 82\,[3.9\sigma]$ $301 \pm 86\,[3.5\sigma]$

Lattice 6670 ± 200 (**T. Blum, TAU08**):

CVC and τ data

Use τ lepton spectral functions:
Alemany, Davier & Höcker (1998).



Advantages:

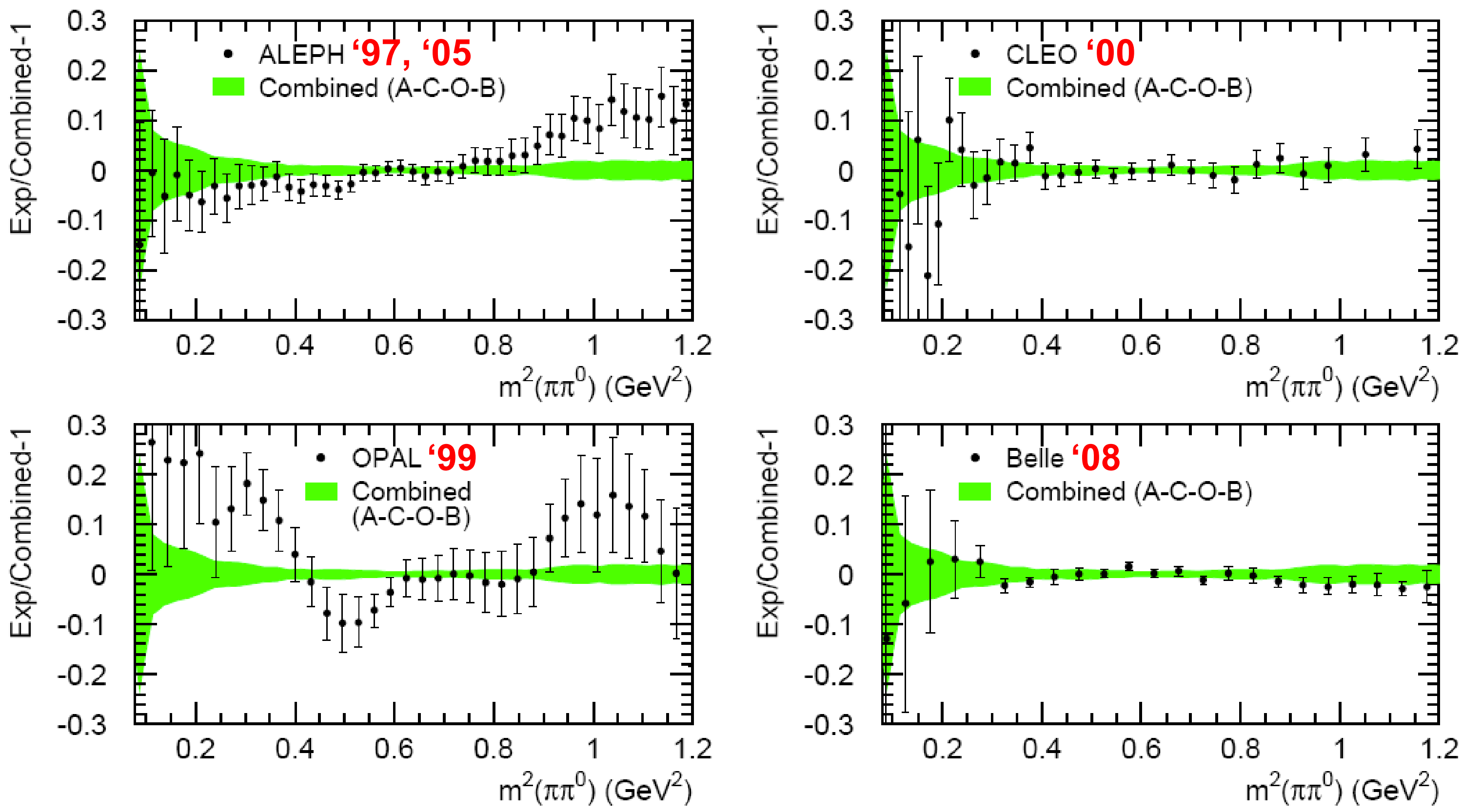
- Can improve precision of had, LO (factor $\sqrt{2}$ combined w/e^+e^-);
- Check consistency of e+e- data (test of CVC)

Disadvantages:

- e+e- data more directly related to had, LO
- Isospin breaking corrections not well understood at present

[CVC tested at the level of $10^{-3} \sim 10^{-4}$ in superallowed beta transitions
arXiv: 0812.1202 ($\rightarrow$ $|V_{ud}|=0.95425 \pm 0.00022$)]

Comparison of ALEPH, BELLE, CLEO & OPAL data with combined mass spectrum ($\tau^{\pm}\to\pi^{\pm}\pi^{0}\nu$)



DHLMMTWYZ (2009)

Using tau data: CVC and IB corrections

CVC hypothesis (1958)

ee→2nπ τ→(2nπ)ν

$$\sigma_{X^0}^{I=1}\left(e^+e^-\right)=\left(\frac{4\pi\alpha^2}{s}\right)\cdot\frac{m_\tau^2}{6|V_{ud}|^2}\frac{B_{X^-}}{B_e}\left(\frac{1}{N_X}\frac{dN_X}{ds}\right)\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{R_{IB}(s)}{S_{EW}}$$

Case of 2π

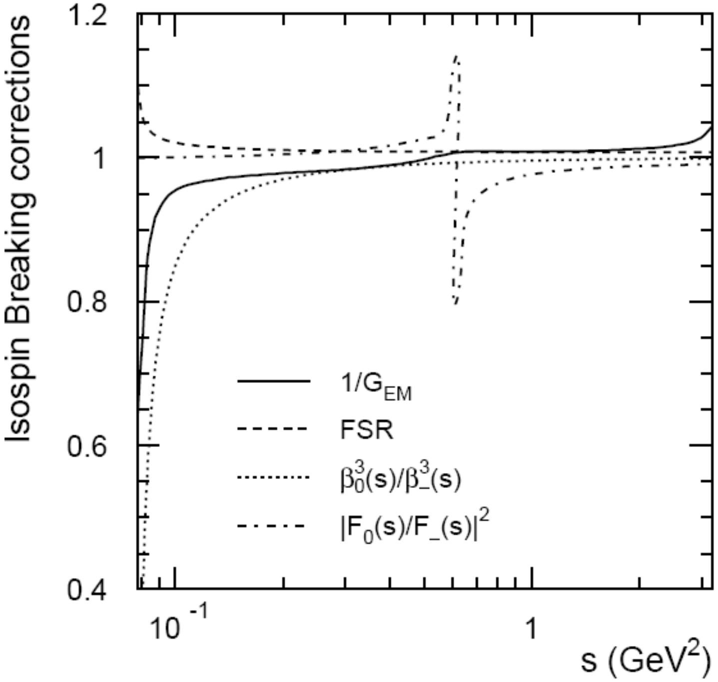
$$R_{IB}(s)=\frac{FSR(s)}{G_{EM}(s)}\frac{\beta_0^3(s)}{\beta_-^3(s)}\cdot\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

- Remove LD photonic corrections
- Add FSR and I=0(ρ−ω)
- Place correct phase space
- Introduce π and ρ mass difference
- Introduce ΔΓρ

Cirigliano, Ecker, Neufeld (01-02)
Flores, GLC, Toledo (2005-2007)

New analysis 2π: CVC and IB corrections

- Belle data, high statistics (2008)
- New IB corrections in m_ρ & Γ_ρ. $\Delta\Gamma_\rho$ includes soft & hard photon corrections to $\rho\to\pi\pi$ (•)
- New LD photonic corrections to $\tau\to\pi^+\pi^0\nu$ decays (•)
- Removed $\omega\to\pi^0\gamma$ events from radiative decays (•)



$\Delta a_\mu^{\text{had, LO}}(\tau)=(-166.0 \pm 12.4_{IB})\cdot 10^{-11}$

- **Flores, GLC, Toledo (2007)**
- **Flores, GLC, Toledo (2006)**
- **DHLMMTWYZ (2009)**

Compared to previous value (**DEHZ03**)

$\Delta a_\mu^{\text{had, LO}}(\tau)=(=(-92 \pm 24_{IB})\cdot 10^{-11}$

Impact of IB corrections on τ-based HVP contribution

Channel	E (GeV)	$10^{11}\times a_\mu^{had,LO}(ee)$	$10^{11}\times a_\mu^{had,LO}(\tau)$
2π, low exp	$2m_\pi - 0.36$	94.7 ± 1.0	$96.8 \pm 1.5 \pm 0.4_{BR} \pm 1.7_{IB}$
2π	$0.36 - 1.8$	4926.3 ± 35.0	$5049.0 \pm 14.3 \pm 21.8_{BR} \pm 11.8_{IB}$
$\pi^+\pi^-2\pi^0$	≤ 1.8	$168.0 \pm 13.0 \pm 2.0_{rad}$	$214.0 \pm 13.0 \pm 6.0_{IB}$
$2\pi^+2\pi^-$	≤ 1.8	$131 \pm 4 \pm 0_{rad}$	$123 \pm 1 \pm 4_{IB}$
$Other Exc. + QCD$	$2m_\pi - \infty$	$1564 \pm 19 \pm 4_{rad} \pm 7_{QCD}$	$--$
$Total$	$2m_\pi - \infty$	$6884(41)(19)_{rad}(7)_{QCD}$ $6897(48)(19)_{rad}(7)_{QCD}$	$7047(37)(7)_{rad}(7)_{QCD}(14)_{IB}$

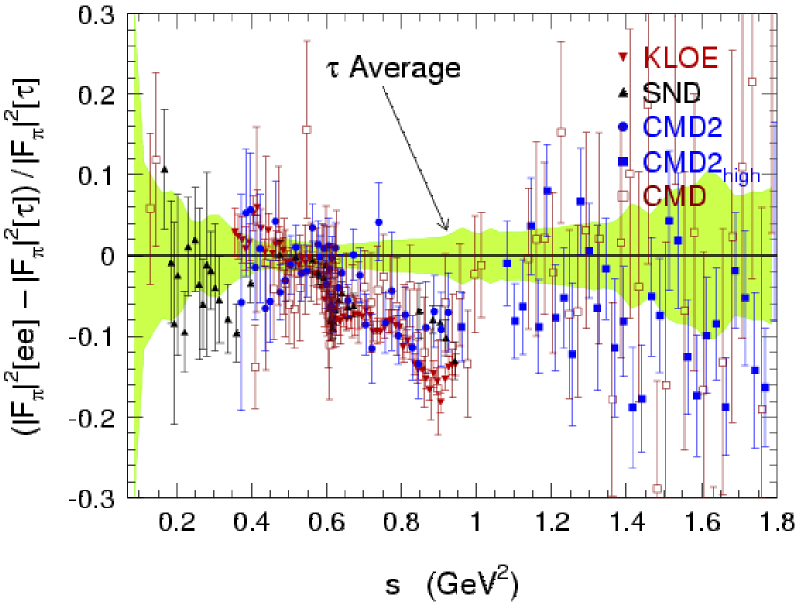
$$a_\mu^{\pi\pi,LO}(ee) - a_\mu^{\pi\pi,LO}(\tau) = \begin{cases} -125 \pm 35_{ee} \pm 29_{\tau+IB}, & w/\,KLOE \\ -112 \pm 43_{ee} \pm 29_{\tau+IB}, & wo/\,KLOE \end{cases}$$

$$a_\mu^{\exp} - a_\mu^{SM} = \begin{cases} 151 \pm 79, & \tau\ data \\ 314 \pm 82, & ee\ w/\,KLOE \\ 301 \pm 86, & ee\ wo/\,KLOE \end{cases}$$

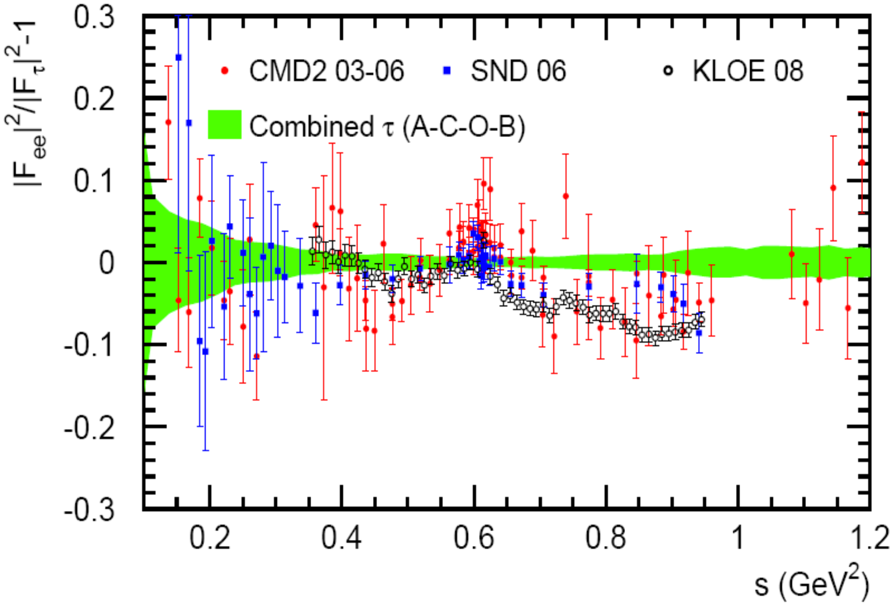
Impact of IB corrections on $|F_\pi(s)|^2$

DHLMMTWYZ (09)

Before 2008



After Belle, KLOE &new IB corrections



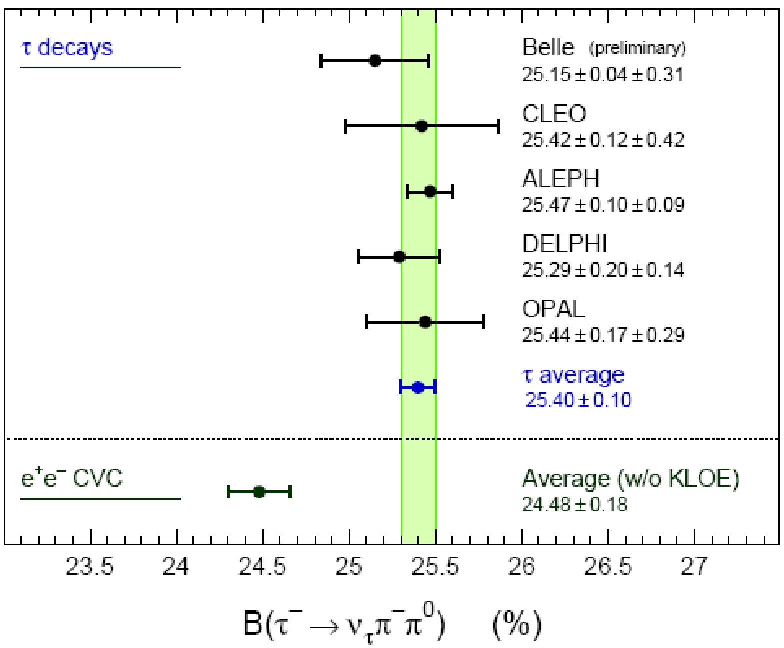
Deviations up to 15%

Deviations less than 8%

Impact of new IB on the tau BR

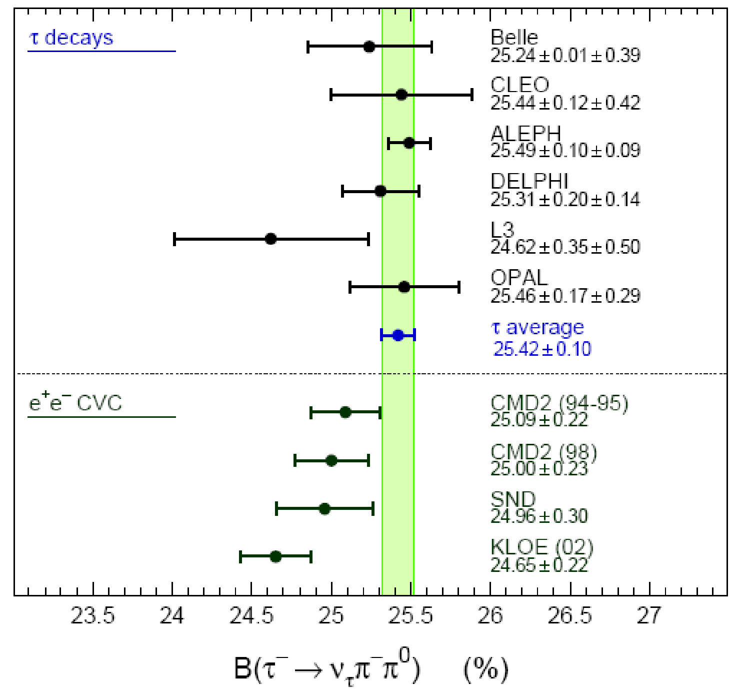
$$B(\tau \to \pi\pi\nu) = B_{\pi\pi}(CVC, e^+e^-) + \underbrace{\Delta B_{\pi\pi}(IB)}_{(0.73\pm0.10)\%}$$

M. Davier, TAU06



Discrepancy at 4.5σ

With KLOE & new IB corrections

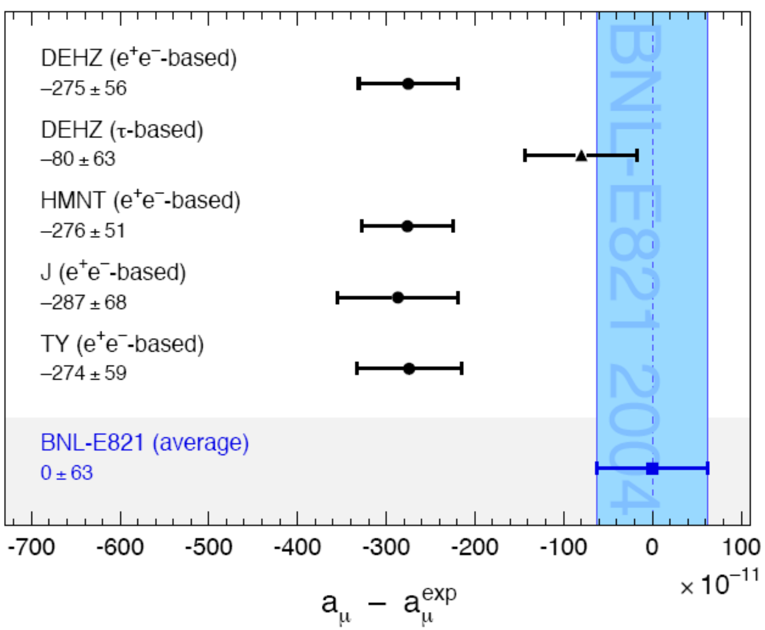


1.9σ only

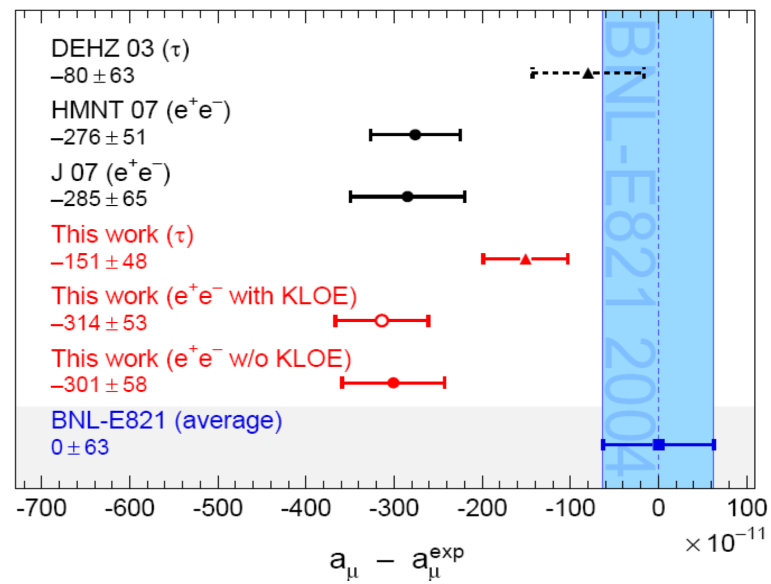
Comparison of SM predictions based on e^+e^- and τ lepton data

DHLMMTWYZ (09)

Previous to 2008



With Belle, KLOE & new IB corrections



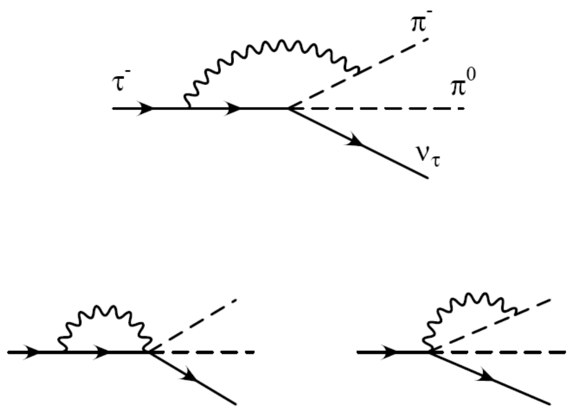
- **Discrepancy e+e- & tau reduced, but persists…**
- **For first time τ-based prediction does not overlap with E821**

Final comments

- Persistent discrepancy theory-experiment at >3σ level
- Theory errors smaller than experimental ones, dominated by hadronic HVP ($\pm$44) and L$\times$L ($\pm$26~35). New data (Babar, Belle, KLOE; CLEO-c) may reduce HVP error ($\pm 44 \rightarrow \pm 22$?),
- τ decay data can improve accuracy (w.a. w/e^+e^-, $\rightarrow \pm$ 25).
  * New IB corrections improve agreement with e^+e^- (CVC),
- If HVP contribution increases to close the gap, Higgs mass upper bound reduces $m_H^{95} \leq 133$ GeV and cause "tension" with direct bounds (**Marciano, Passera, Sirlin 2008**)
- A new measurement of the a_μ could establish New Physics

Backup slides

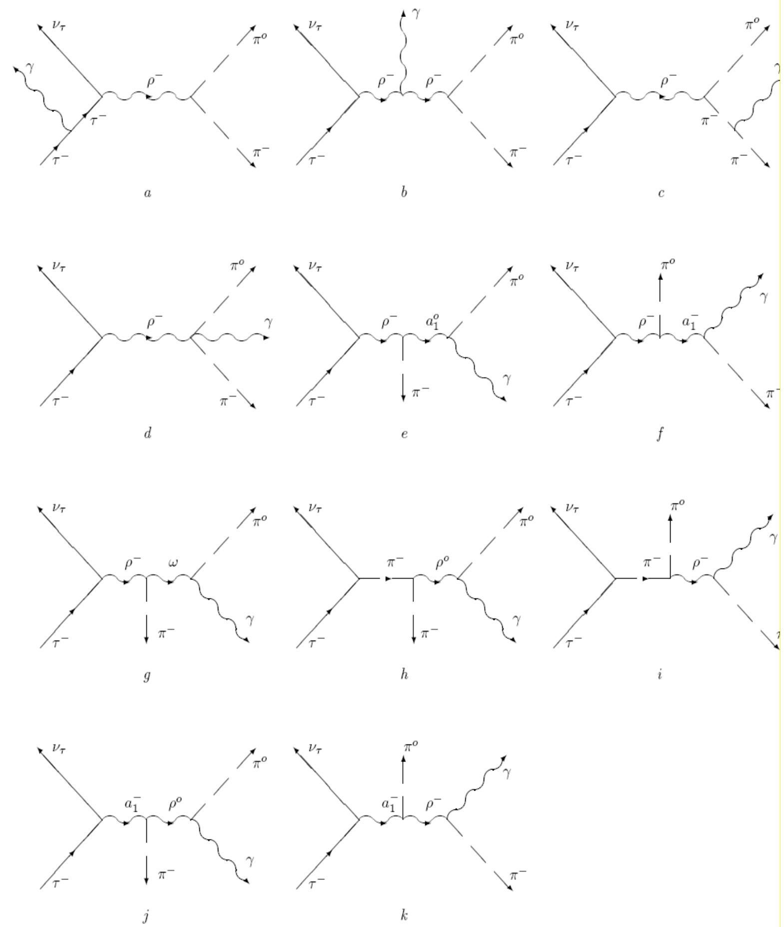
L-D radiative corrections to $\tau \to \pi\pi\nu$



$$\frac{d\Gamma^0}{dt} \Rightarrow \frac{d\Gamma^0}{dt} \times G_{EM}(t)$$

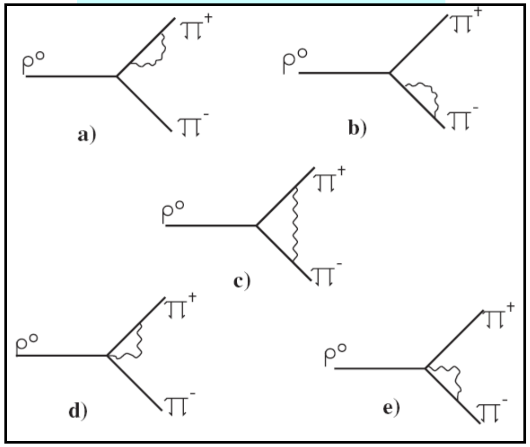
$$G_{EM}(t) = G^0_{EM}(t) + G^{MD}_{EM}(t)$$

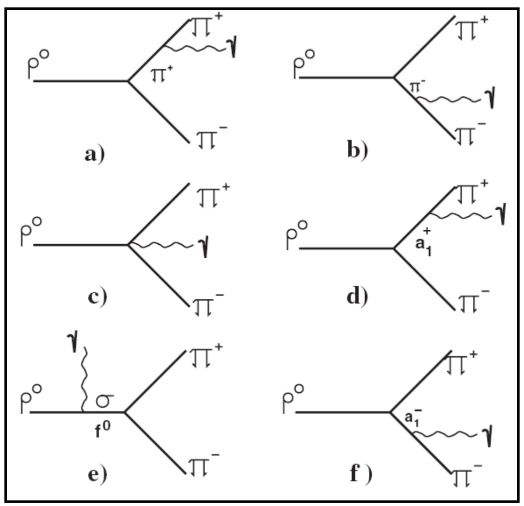
$$t = (p_{\pi^+} + p_{\pi^0})^2$$



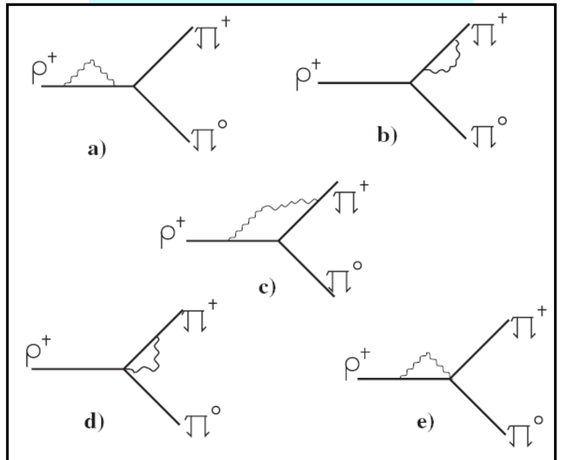
Cirigliano, Ecker & Neufeld , JHEP, (2002);
Flores, Flores, GLC, Toledo , PRD, (2006)

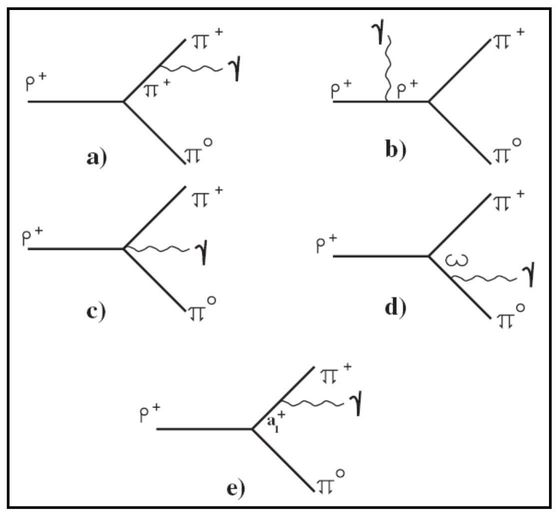
$\rho^0 \to \pi^+\pi^-(\gamma)$





$\rho^\pm \to \pi^\pm\pi^0(\gamma)$





Up to 0.1% accuracy

$$\Delta\Gamma_\rho = \Gamma(\rho^0 \to \pi^+\pi^-(\gamma)) - \Gamma(\rho^+ \to \pi^+\pi^0(\gamma)) = +0.76\ MeV$$

Previous values used:

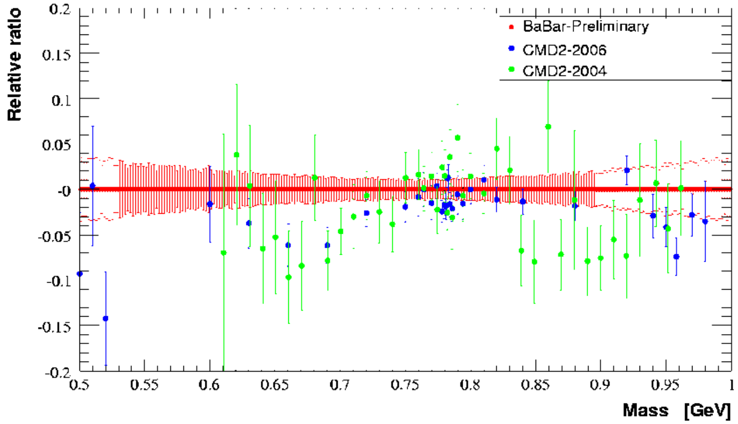
(-0.42 ± 0.45) DEHZ03

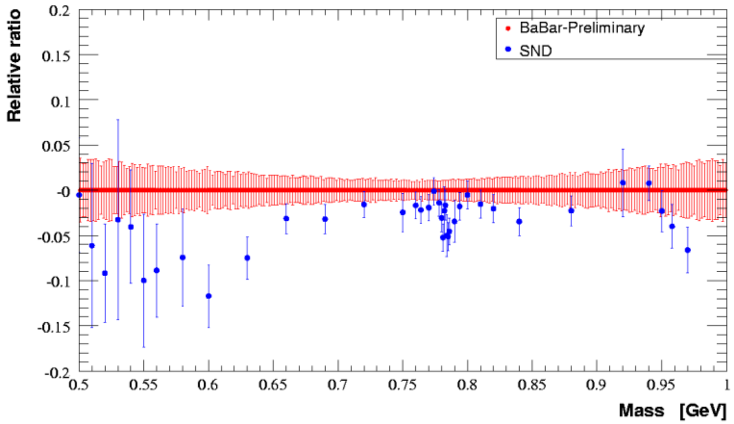
(-0.61 ± 0.45) ChPT01

Flores, GLC, Toledo PRD 76,(2007)

BABAR reported preliminary data on $\pi^+\pi^-(\gamma)/\mu+\mu-(\gamma)$ using ISR method (M. Davier, TAU08)

Compared to other e+e- data in 0.5 -> 1 GeV





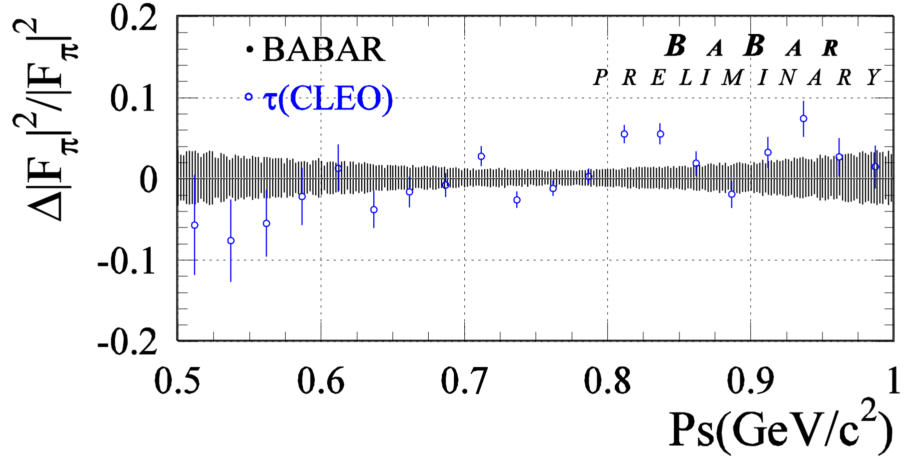
Contribution of $\pi+\pi-$ data in 0.5 to 1 GeV regionn:

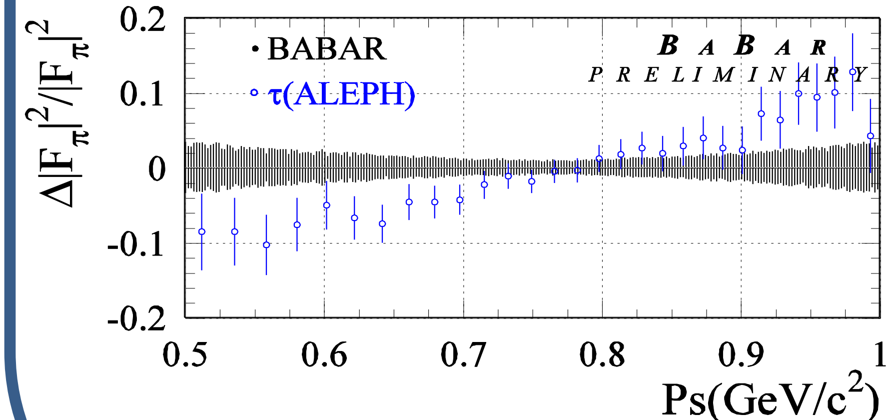
$(462.5\pm0.9\pm3.1)\times10^{-10}$ (BABAR)

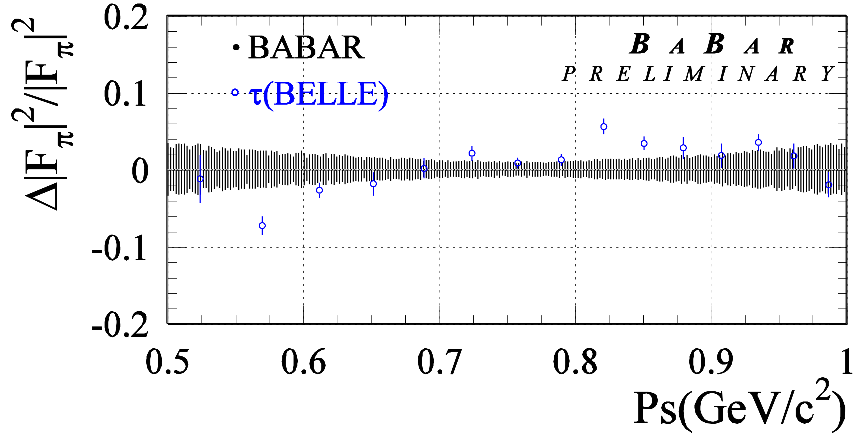
$(462.5\pm0.9\pm3.1)\times10^{-10}$ (previous)

Reduces discrepancy from 3.3σ to 1.7 σ

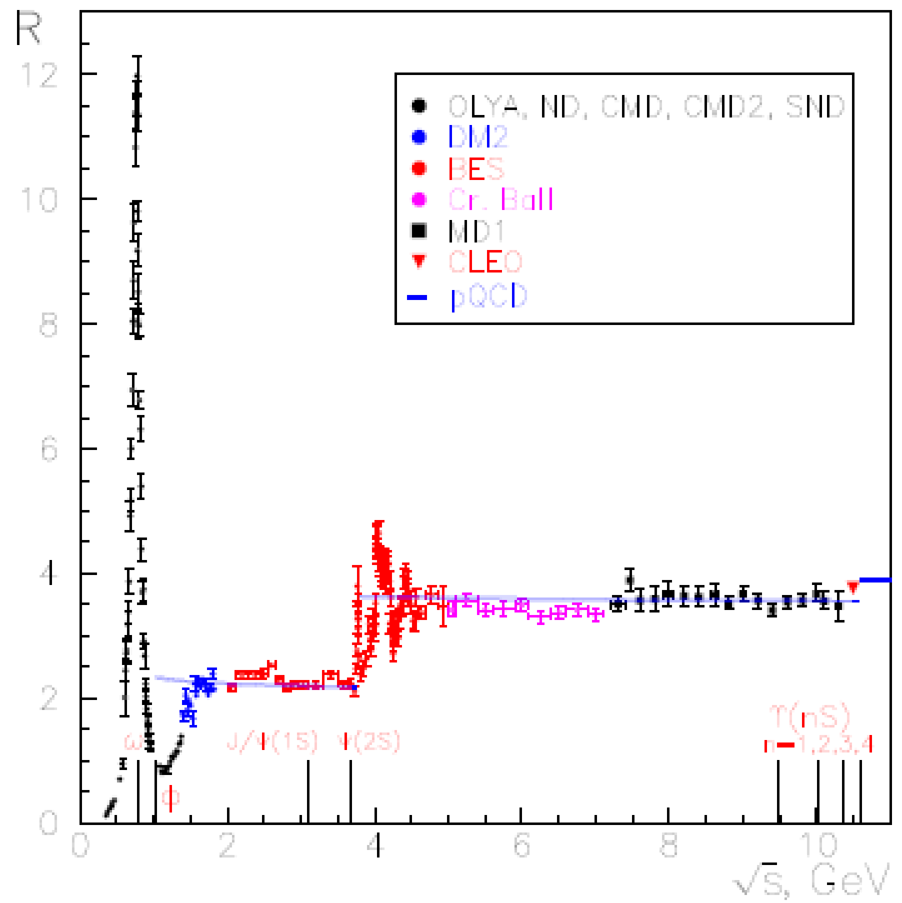
Babar data (without ρ–ω) compared to IB corrected τ data in the 0.5 to 1 GeV region



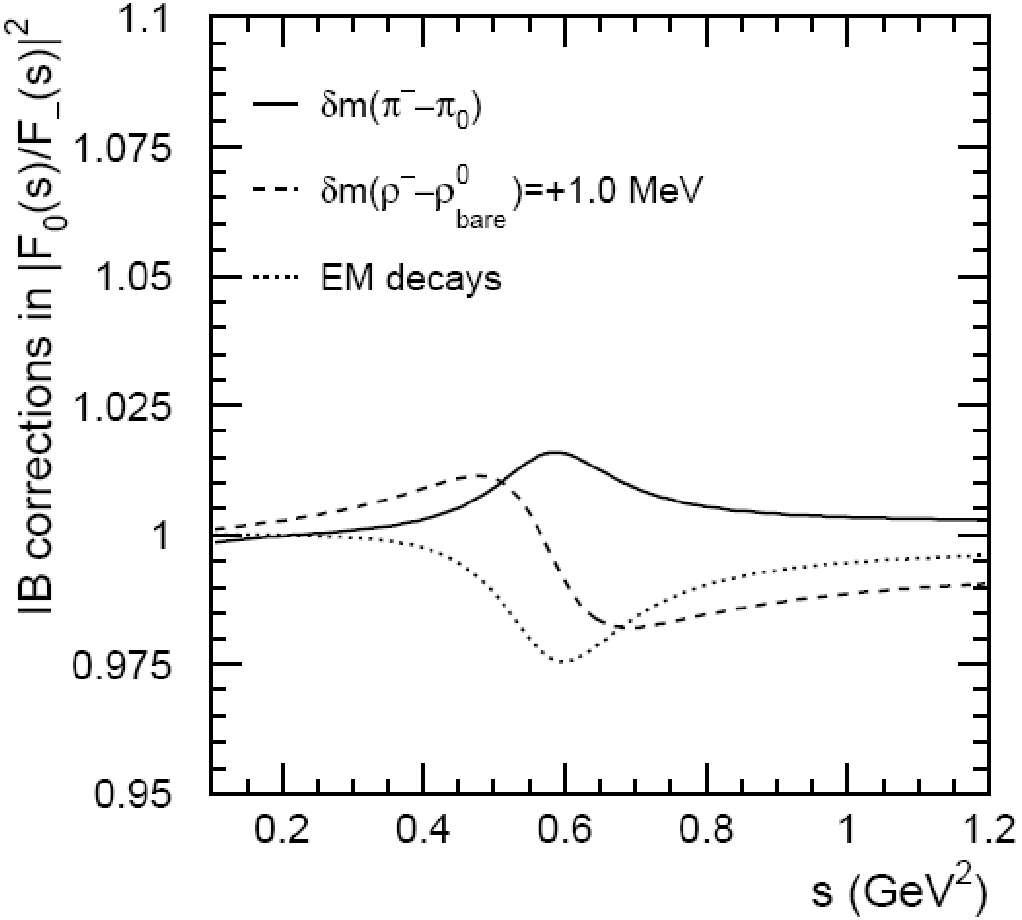




Measurements of R below 10 GeV and QCD seems in good agreement



IB corrections in pion form factors



Impact of IB corrections in the test of CVC

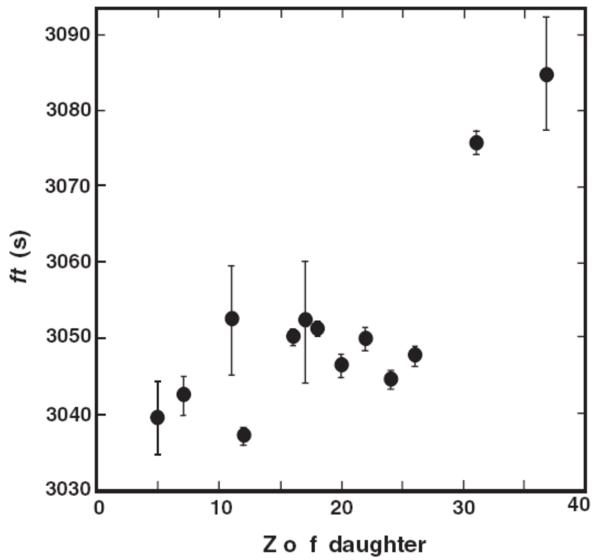
Source	$\Delta a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model
S_{EW}	-12.21 ± 0.15	
G_{EM}	-1.87 ± 0.56	
FSR	$+4.65 \pm 0.23$	
ρ–ω interference	$+2.43$	$+2.07$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.76	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.12 \pm 0.03$	$+3.71 \pm 0.33$
$m_{\rho^\pm} - m_{\rho^0_{\mathrm{bare}}}$	$-0.07^{+0.06}_{-0.03}$	$-0.34^{+0.30}_{-0.22}$
$\pi\pi\gamma$, electrom. decays	-5.96 ± 0.30	
Total	-16.60 ± 1.24	

Source	$\Delta\mathcal{B}^{\mathrm{CVC}}_{\pi^-\pi^0}$ (10^{-2}) GS model	KS model
S_{EW}	$+0.57 \pm 0.01$	
G_{EM}	-0.07 ± 0.01	
FSR	-0.19 ± 0.01	
ρ–ω interference	0.01	0
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	$+0.19$	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	-0.22	-0.20
$m_{\rho^\pm} - m_{\rho^0_{\mathrm{bare}}}$	$+0.10 \pm 0.09$	$+0.11 \pm 0.10$
$\pi\pi\gamma$, electrom. decays	$+0.34 \pm 0.02$	
Total	$+0.73 \pm 0.10$	

DHLMMTWYZ (2009)

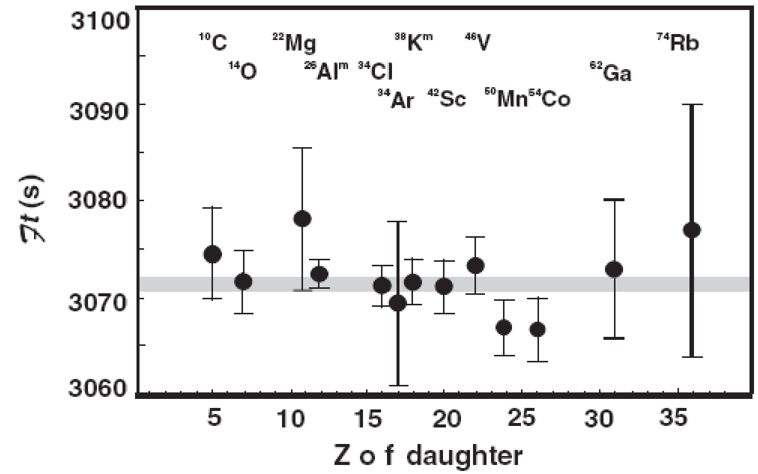
CVC tested at the level of 10^{-4} in $0^+ \to 0^+$ $e\nu$ Fermi transitions!

$\Rightarrow |V_{ud}| = 0.97418(26)$ [Towner & Hardy, PRC77, (2008)]



t=0

Isospin Corrections



$$ft = \frac{K}{G_F^2 |V_{ud}|^2 |M_F|^2};$$

$$M_F = \langle f | Q_- | i \rangle = \sqrt{2}$$

$$\mathcal{F}t = ft(1+\delta_R)(1-\delta_C)$$

$$M_F = \langle f | Q_- | i \rangle = \sqrt{2}(1-\delta_C/2)$$