# Photon and $\rho$ from chiral quarks

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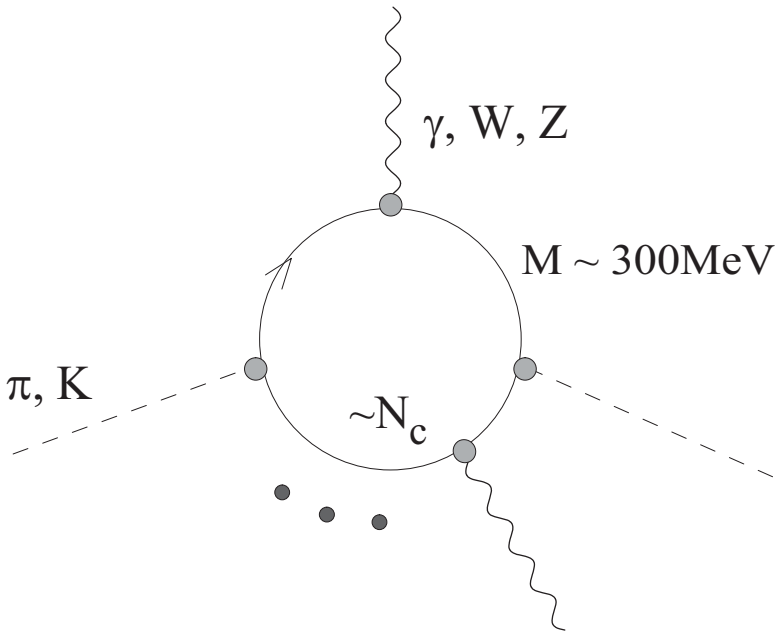
# More information:

- *Photon distribution amplitudes and light-cone wave functions in chiral quark models*, **Alexander E. Dorokhov**, WB, ERA, Phys. Rev. D74 (2006) 054023
- *Pion-photon Transition Distribution Amplitudes in the Spectral Quark Model*, WB, ERA, Phys. Lett. B649 (2007) 49
- *Generalized parton distributions of the pion in chiral quark models and their QCD evolution*, WB , ERA, **Krzysztof Golec-Biernat**, Phys. Rev. D77 (2008) 034023
- *Gravitational and higher-order form factors of the pion in chiral quark models*, WB, **Enrique Ruiz Arriola**, Phys. Rev. D78 (2008) 094011

Chiral quark models used in similar studies by the Bochum, Valencia, and Jagellonian groups

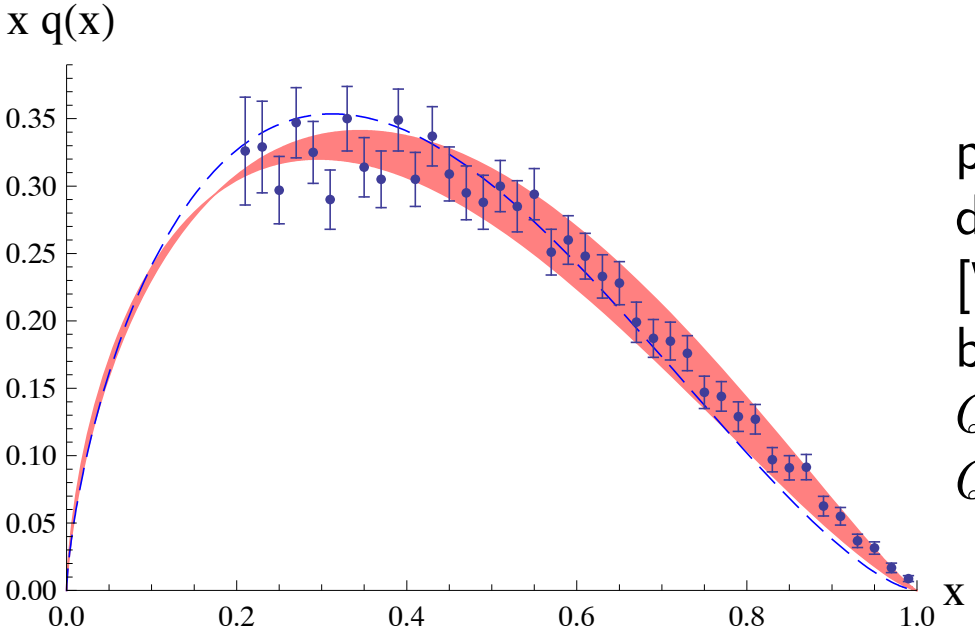
# Chiral quark models




- soft regime $\rightarrow$ chiral sym. breaking
- NJL (Nobel 2008), instanton liquid, DSE
- relatively few parameters (traded for $f_\pi$, $m_\pi$, ...)
- very many processes can be computed!
- no confinement - careful not to open the $q\overline{q}$ threshold
- low quark-model scale $\sim 320 - 500$ MeV – need for evolution
- local and non-local (instanton-based) models

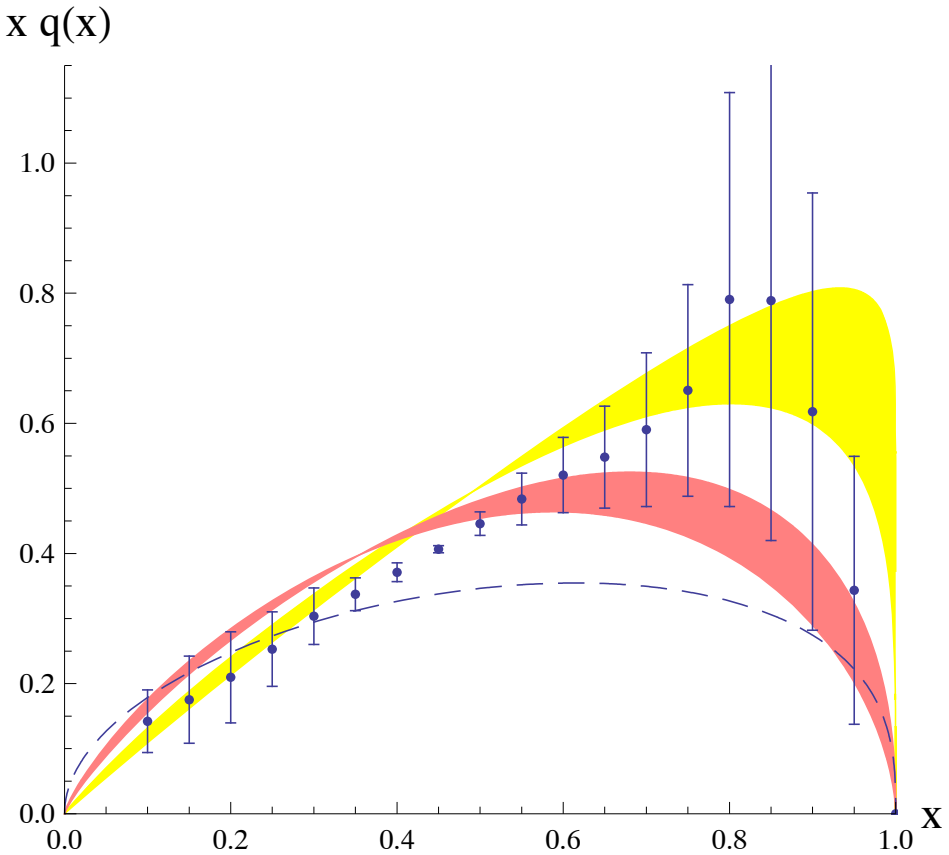
# PDF of the pion, QM vs. E615

LO DGLAP QCD evolution of the non-singlet part to the scale $Q^2 = (4\ \mathrm{GeV})^2$ of the E615 Fermilab experiment:



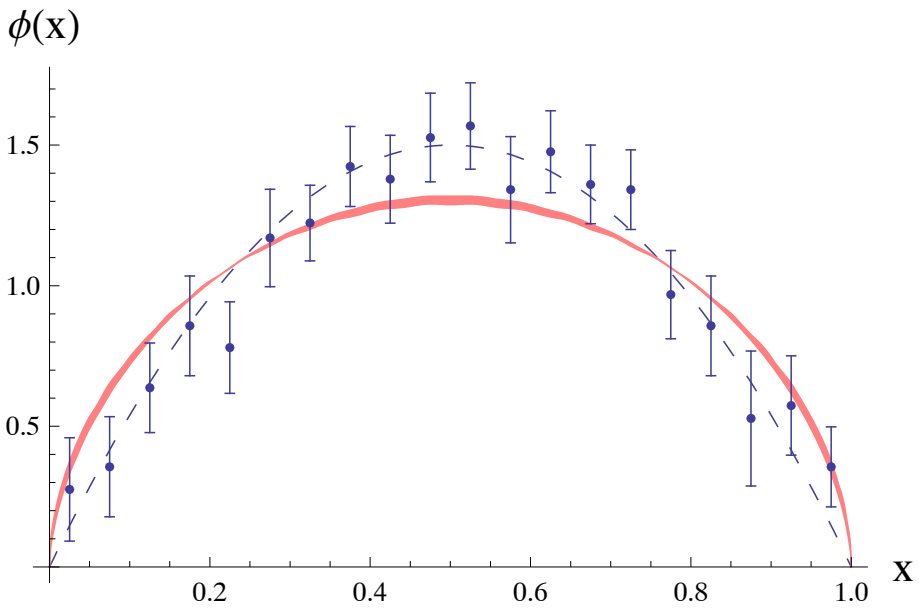

points: Drell-Yan from E615
dashed: reanalysis of data [Wijesooriya et al., 2005]
band: valence QM PDF evolved to $Q = 4$ GeV from the QM scale $Q_0 = 313^{+20}_{-10}$ MeV

# PDF of the pion, QM vs. lattice

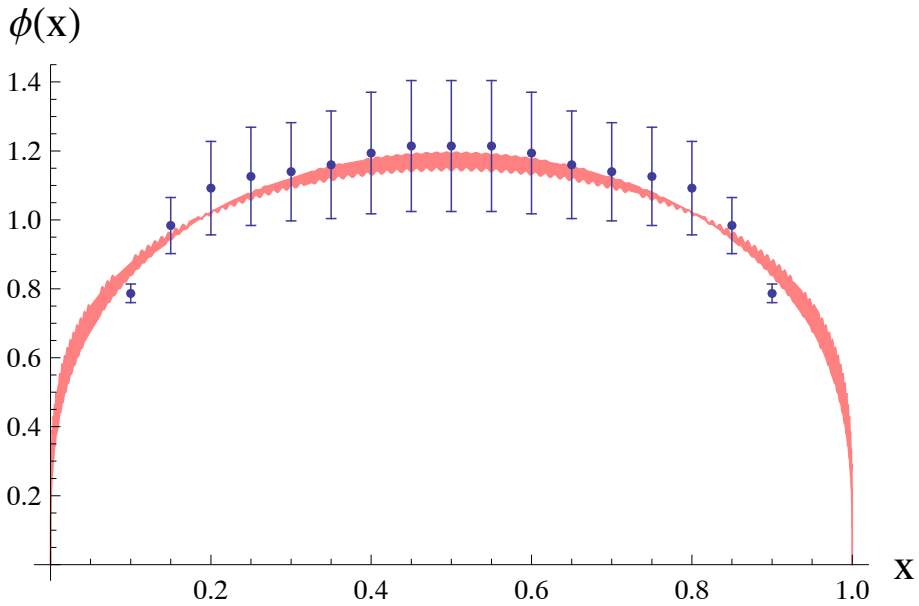



points: transverse lattice [Dalley, van de Sande, 2003]
yellow: QM evolved to 0.35 GeV
pink: QM evolved to 0.5 GeV
dashed: GRS parameterization at 0.5 GeV

# PDA of the pion, QM vs. E791 and lattice data

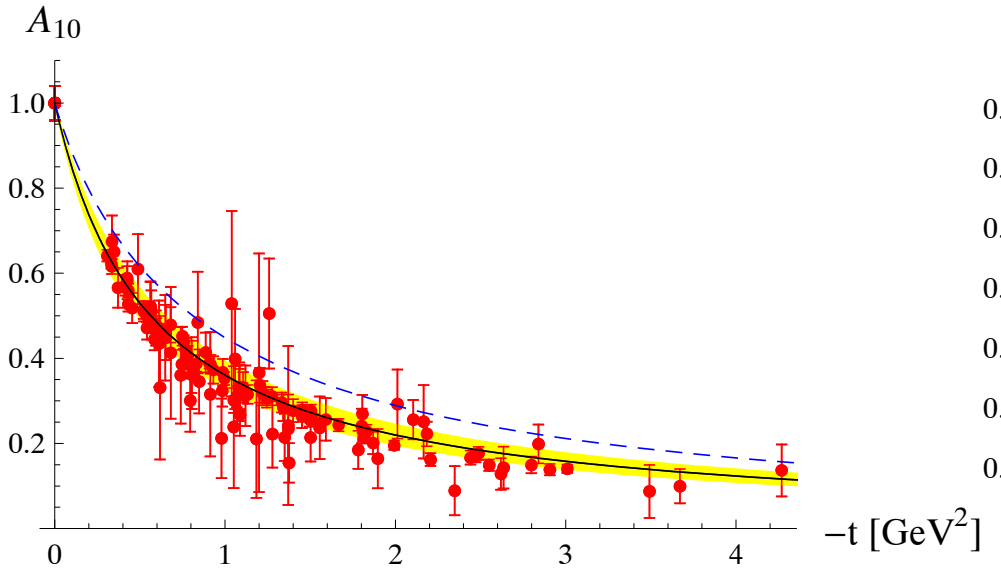


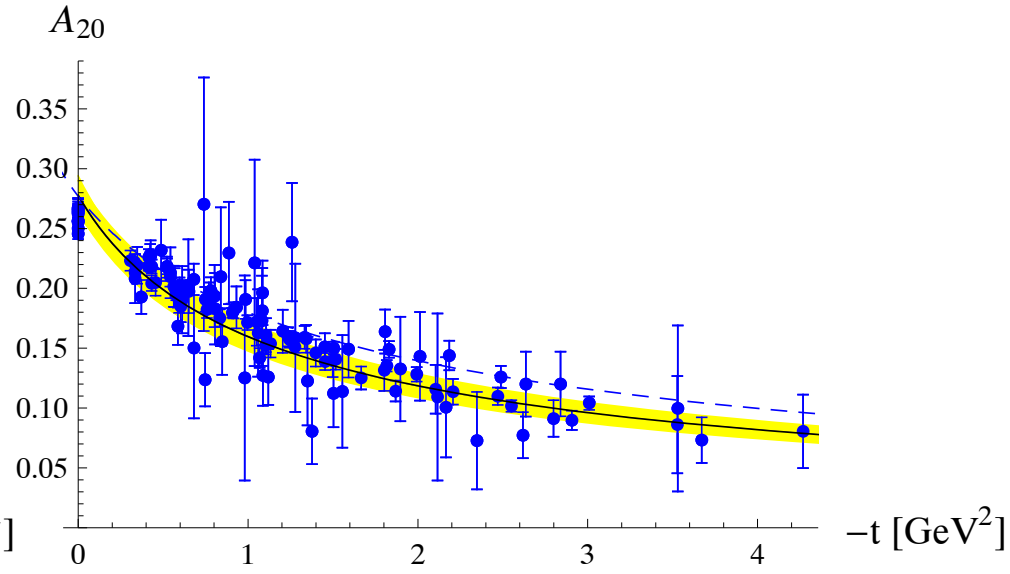

points: E791 data from di-jet production in $\pi + A$
band: QM at $Q = 2$ GeV
dashed line: asymptotic form ($Q \to \infty$)




points: transverse lattice data [Dalley, van de Sande, 2003]
band: QM at $Q = 0.5$ GeV

# Pion form factors vs. lattice




The EM FF (left) and the quark part of the gravitational form factor $\Theta_1$ (right) in SQM (solid line) and NJL (dashed line), compared to data from [Brömmel et al., 2005-7]

Quark-model relation: $\langle r^2 \rangle_\Theta = \frac{1}{2} \langle r^2 \rangle_V$

Matter more concentrated than charge!

# Photon DA's: definitions

Twist-2 components of the photon Distribution Amplitude
[Ali+Braun 95, Ball+Braun 96– ]

$$\langle 0|\overline{q}(z)\sigma_{\mu\nu}[z,-z]q(-z)|\gamma^{\lambda}(q)\rangle =$$

$$ie_q\langle\bar{q}q\rangle\chi_{\rm m}f^{t}_{\perp\gamma}\left(q^2\right)\left(\epsilon^{(\lambda)}_{\perp\mu}p_\nu-\epsilon^{(\lambda)}_{\perp\nu}p_\mu\right)\int_0^1 dxe^{i(2x-1)q\cdot z}\phi_{\perp\gamma}(x,q^2)+h.t.$$

$$\langle 0|\overline{q}(z)\gamma_{\mu}[z,-z]q(-z)|\gamma^{\lambda}(q)\rangle =$$

$$e_q f_{3\gamma}f^{v}_{\|\gamma}\left(q^2\right)p_\mu\left(\epsilon^{(\lambda)}\cdot n\right)\int_0^1 dxe^{i(2x-1)q\cdot z}\phi_{\|\gamma}(x,q^2)+h.t.$$

$$\langle 0|\overline{q}(z)\gamma_{\mu}\gamma_5[z,-z]q(-z)|\gamma^{\lambda}(q)\rangle = h.t.$$

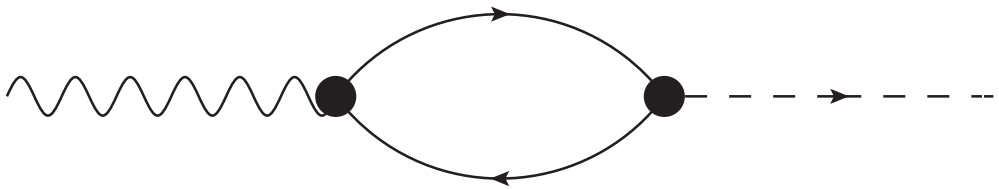
where

$$\epsilon^{(\lambda)}\cdot q=0,\ \epsilon^{(\lambda)}\cdot n=0 \text{ for real photons}$$

$$p_\mu=q_\mu-\frac{q^2}{2}n_\mu,\ n_\mu=\frac{z_\mu}{p\cdot z},\ e^{(\lambda)}_\mu=\left(e^{(\lambda)}\cdot n\right)p_\mu+\left(e^{(\lambda)}\cdot p\right)n_\mu+e^{(\lambda)}_{\perp\mu}$$

# Quark-model evaluation

Photon/$\rho$-to-current transition:



We have used

- Non-local quark model (instanton-based), with Gaussian regulator
- NJL model with PV subtraction
- Spectral Quark model (SQM), implementing the VMD

# Constants

QCD predicts the scale dependence for the quark condensate $\langle 0 \left| \overline{q} q \right| 0 \rangle$, its magnetic susceptibility $\chi_{\mathrm{m}}$, and for $f_{3\gamma}$. At LO

$$\langle 0 \left| \overline{q} q \right| 0 \rangle \big|_{\mu} = L^{-\gamma_{\overline{q}q}/b} \, \langle 0 \left| \overline{q} q \right| 0 \rangle \big|_{\mu_0}$$

$$\chi_m \big|_{\mu} = L^{-(\gamma_0 - \gamma_{\overline{q}q})/b} \, \chi_m \big|_{\mu_0} \, , \quad f_{3\gamma} \big|_{\mu} = L^{-\gamma_f/b} \, f_{3\gamma} \big|_{\mu_0}$$

where $r = \alpha_s \left(\mu^2\right) / \alpha_s \left(\mu_0^2\right)$, $b = \left(11 N_c - 2 n_f\right)/3$, $\gamma_{\overline{q}q} = -3 C_F$, $\gamma_0 = C_F$, $\gamma_f = 3 C_A - C_F/3$, with $C_F = 4/3$ and $C_A = 3$ for $N_c = 3$.

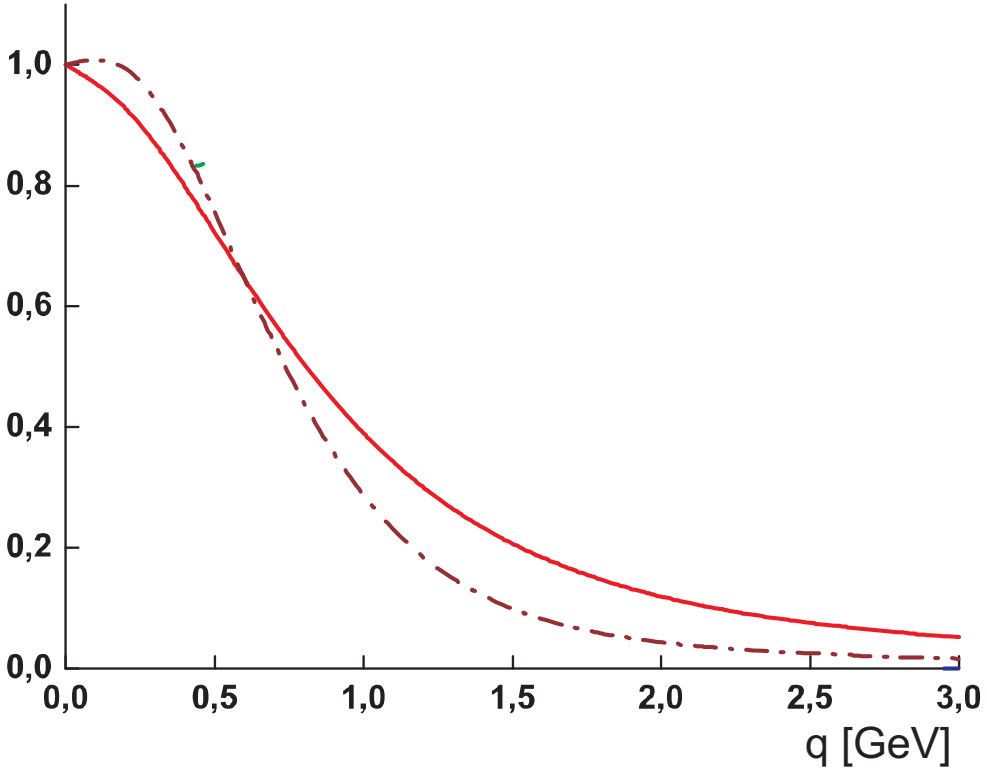
| quantity at 1 GeV | non-local | SQM | QCD s.r. | VMD |
|---|---|---|---|---|
| $\left(-\langle 0 \left| \overline{q} q \right| 0 \rangle\right)^{1/3}$ [GeV] | 0.24 | 0.24 | $0.24 \pm 0.02$ | - |
| $\chi_m$ [GeV$^2$] | 2.73 | 1.37 | $3.15 \pm 0.3$ | 3.37 |
| $f_{3\gamma}$ [GeV$^{-2}$] | -0.0035 | -0.0018 | $-0.0039 \pm 0.0020$ | -0.0046 |

SQM similar to NJL, local models have larger $r$ than the non-local model

QCD s.r. and VDM from [Braun+Ball+Kivel 03]

# Form factors

$f^t_{\perp\gamma}(q^2)$, $f^v_{\|\gamma}(q^2)$, non-local model



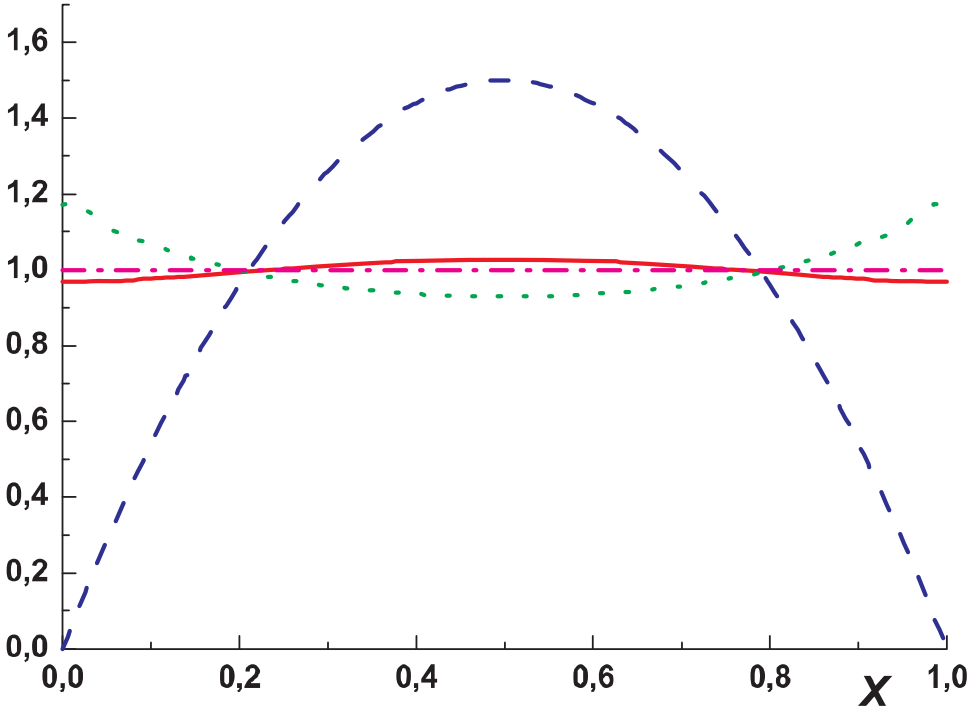

- Local models very similar
- Typical fall-off scale of $\sim m_\rho$
- In SQM

$$f^{t,\mathrm{SQM}}_{\perp\gamma}(q^2) = \frac{m_\rho^2}{m_\rho^2 + q^2}$$

# Photon DA's

evaluation at the QM scale

$\phi_{\perp\gamma}(x, q^2 = 0)$ – chirally-odd transversity DA




non-local local approx. to n.l. – Bochum asymptotic, $6x(1-x)$

- $\phi_{\perp\gamma}(x, q^2 = 0) = 1$ in local models
- $\sim 1$ in non-local models
- For virtual photon SQM gives simple formulas:

$$\phi_{\|\gamma^*}(x, q^2) = \frac{1 + \frac{q^2}{m_\rho^2}}{\left(1 + \frac{4q^2}{m_\rho^2} x(1-x)\right)^{3/2}}$$

- In the limit $q^2 \to -m_\rho^2$ it becomes $\delta\left(x - \frac{1}{2}\right)$
- $\phi_{\perp\|}(x, q^2 = 0) = 1$ in local models

# Photon's LCWF

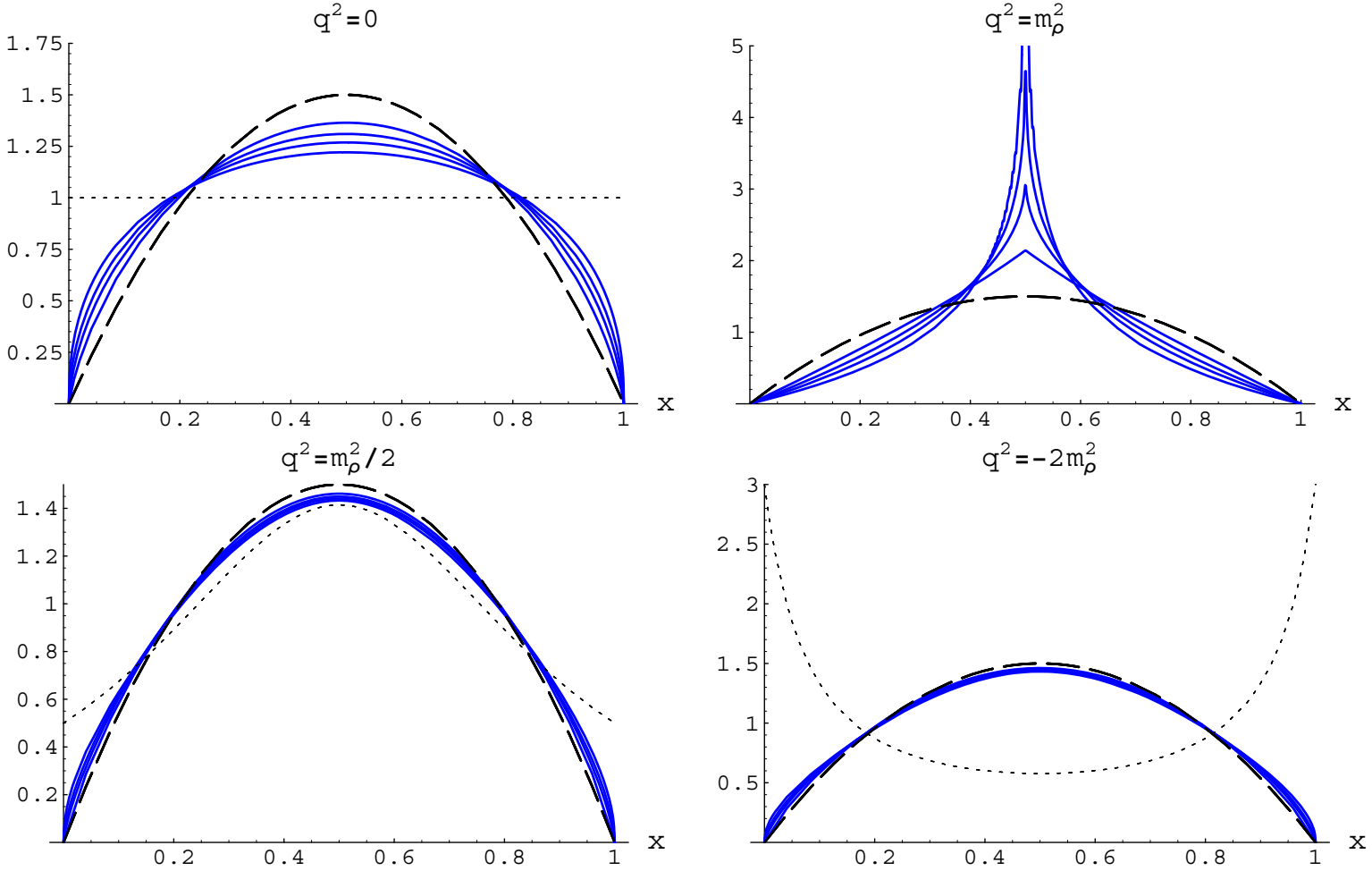
The light-cone wave function ($k_\perp$-unintegrated) has a simple form in SQM (at the QM scale):

$$\Phi_{\perp\gamma}(x, \boldsymbol{k}_\perp) = \frac{6}{m_\rho^2(1 + 4\boldsymbol{k}_\perp^2/m_\rho^2)^{5/2}}$$

Note the power-law fall-off at large transverse momenta, $\Phi_{\perp\gamma}(x, \boldsymbol{k}_\perp) \sim 1/k_\perp^5$. In cross section this leads to tails $\sim 1/k_\perp^{10}$. For the virtual photon

$$\Phi_{\perp\gamma^*}(x, \boldsymbol{k}_\perp) = \frac{6\left(1 + \frac{q^2}{m_\rho^2}\right)}{m_\rho^2\left(1 + 4\frac{\boldsymbol{k}_\perp^2 + q^2 x(1-x)}{m_\rho^2}\right)^{5/2}}$$

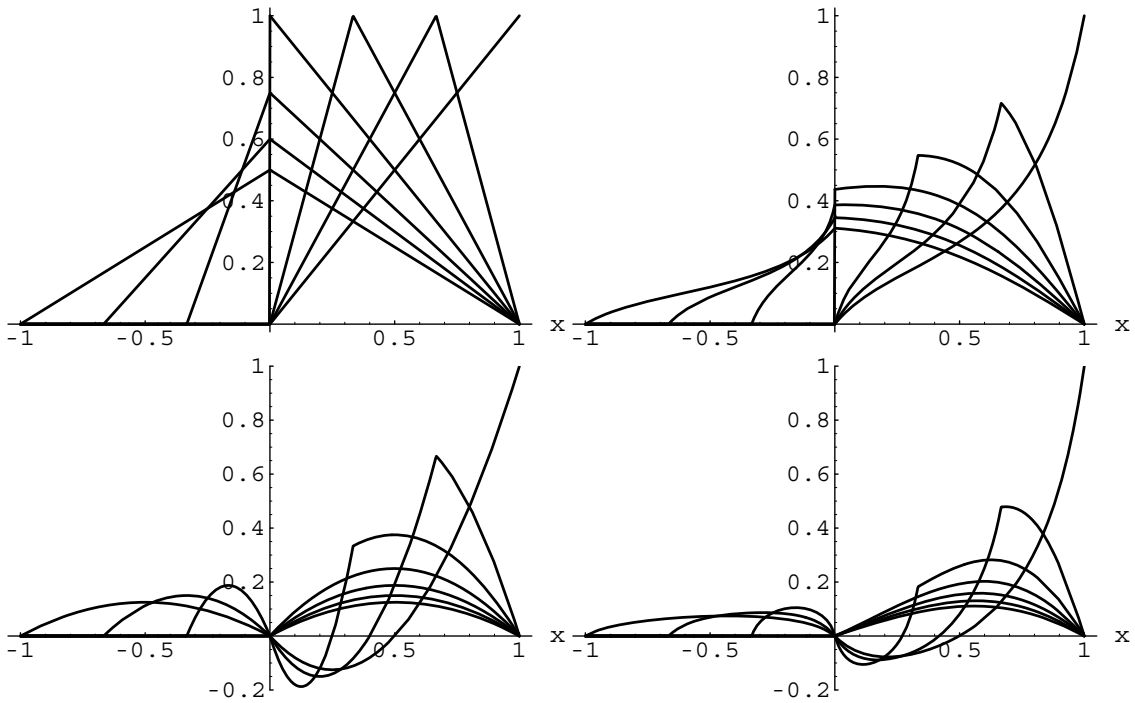
# LO ERBL of the tensor DA of the virtual photon




Initial conditions (dots) from SQM. Solid lines: evolution to scales $1$, $2.4$, $10$, and $1000$ GeV, asymptotic form $6x(1-x)$: dashed line

# Pion-photon Transition Distribution Amplitudes

[Pire+Szymanowski 05] – as GPD, but between the $\pi$ and $\gamma$ states




Top: vector TDA at $q^2 = 0$ (left) and $q^2 = 0.4$ GeV$^2$ (right) several values of $\zeta$: $-1$, $-2/3$, $-1/3$, $0$, $1/3$, $2/3$, and $1$. Bottom: same for the axial TDA (SQM at the QM scale)

1. Chiral quark models provide a link between high- and low-energy analyses. They yield in a fully dynamical way the initial conditions for the QCD evolution, which is essential to bring the predictions to experimental/lattice scales
2. Numerous predictions for soft matrix element involving the Goldstone bosons and photons
3. Scale in chiral quark models is very low, 320-500 MeV, QCD evolution "fast"
4. Simple analytic formulas – useful to understand general properties, (*e.g.*, no factorization of the $t$-dependence
5. For the pion, with the LO QCD evolution the overall agreement with the available data and lattice simulations is very reasonable (PDF, DA, generalized form factors, GPD, TDA, ...)
6. [ Pire+Szymanowski 09 ] $l\bar{l}$ photoproduction $\rightarrow \phi_{\perp\gamma}\left(\frac{\alpha Q^2}{q^2+Q_\perp^2}\right) h_1^q()$
7. Predictions can be further tested with future lattice simulations also for the photon/$\rho$